

Research in Mathematics Education

Series Editors: Jinfa Cai · James A. Middleton

Chiara Andrà

Domenico Brunetto

Esther Levenson

Peter Liljedahl *Editors*

# Teaching and Learning in Maths Classrooms

Emerging Themes in Affect-related  
Research: Teachers' Beliefs, Students'  
Engagement and Social Interaction

 Springer

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Series editors

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Chiara Andrà • Domenico Brunetto •  
Esther Levenson • Peter Liljedahl  
Editors

# Teaching and Learning in Maths Classrooms

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Teachers' Beliefs, Students' Engagement  
and Social Interaction

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# Foreword

The acronym MAVI stands for **MA**thematical **VI**ews and speaks to the focus of the conference in a broad and inclusive sense, that is: affective issues in Mathematics Education.

The conference is an unavoidable appointment for any researcher interested in the role of beliefs, motivation, attitudes, emotions, will and values in mathematics teaching and learning processes. Theoretical and methodological issues are brought forth and/or refined by a group of researchers who have the sole intent of enjoying the discussion of (new) ideas, welcoming anybody who has a different perspective and getting the best to improve his/her own research.

In 2015, the 21st edition of this annual international conference took place in Milan and attracted new researchers, “besides the ones belonging to the group since many years.” Germany and Finland are the birthplaces for the conference, in that Guenter Toerner and Erkki Pehkonen from respective countries have launched the first edition of it. Since then, researchers from both Germany and Finland have attended the various editions of MAVI, together with colleagues from Austria, Italy, Sweden, Israel, Spain, Estonia, Denmark, Australia and Canada. In 2015, the MAVI conference was enriched by the presence of researchers from Japan and Nigeria.

We all have different backgrounds, different research interests and different academic statuses. Special attention is paid to young researchers, who represent the majority of the contributors. The spirit of the conference is, in fact, not only inclusive: it is dedicated to Ph.D. students and young researchers, who are welcome to come and present the status of their research in order to get insightful feedback from their colleagues. Extended time is dedicated to the discussion of each presentation, so that the balance between the time for frontal presentation and discussion is in favour of the latter. No keynote speakers, no plenaries, no parallel sessions: the entire group participates in the whole conference, and no distinction is made among participants on the basis of their experience, academic status or age.

Those who intend to participate have to submit a contribution, which goes through a peer-review process of different phases: in phase 1, before the conference starts, two reviewers read the paper and submit their advice; in phase 2, each author reviews his/her paper, prepares for the conference presentation and during the

conference receives questions, feedback, suggestions and comments during a long discussion dedicated to his/her work; in phase 3, after the conference, the paper is revised again, on the basis of what the author has learned from the discussion.

The result is a high-quality collection of cutting-edge research reports. Year after year, new research themes emerge, others are extended and deepened, and foundational constructs are debated and enriched with new perspectives. This is what the reader will find in the next pages.

MAVI21 Conference Organizers

Chiara Andrà  
Domenico Brunetto

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# Contributors

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**Dr. Ruthi Barkai** is a researcher and teacher educator at the School of Education, Tel Aviv University, and a lecturer at the College. In her Ph.D. study she investigated content knowledge and pedagogical content knowledge of practicing high school teachers regarding proofs and proving in high school. Her research interests include the following: developing mathematical thinking among preschool students and their teachers; teachers' training; connections between proving and reasoning; professional development of pre-service and practicing mathematics teachers at elementary and high school levels.

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**Domenico Brunetto** is a young researcher in Mathematics Department at the Polytechnic of Milan. His research interests are: the usage of multimedia (specially the MOOCs) during the classroom practices, the interaction between students in small and large groups, and the network analysis. My research project aims to analyse and model the student interactions both on line and in classroom, using opinion dynamics models properly adapted for the teaching-learning context.

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# Chapter 1

## Introduction

**Esther Levenson**

This book is essentially made up of the 25 papers presented at the 21st MAVI conference in Milan. On the one hand, it may appear to the reader as a mere collection of papers. On the other hand, several of the papers have a common research theme, although the focus may be on different elements. Some of the studies are directly related to previous studies presented at MAVI, written by long-time members of the MAVI community. Other studies, although not directly related to previously presented MAVI papers, are indirectly related and when taking a look at the bigger picture, add to our understanding of the research presented. This introduction is written and organized in order to help the reader get the most out of this book by describing the common threads that run along the papers while placing them in the larger picture of MAVI conferences.

The first section is dedicated to classroom practices and beliefs regarding those practices. Three papers take a look at prospective or practicing teachers' views of different practices such as decision-making (Gonzales), the roles of explanations in the classroom (Levenson and Barkai), and the use of play in mathematics classrooms (Lake). A fourth paper, Tirosh et al., investigates preschool teachers' self-efficacy beliefs for solving patterning tasks. This paper may be seen as a direct continuation of previous studies reported in MAVI (e.g., Tirosh et al. 2011, 2014) regarding teachers' self-efficacy beliefs for various mathematical tasks carried out in preschool, showing the relationships between teachers' knowledge and self-efficacy beliefs. One paper (Ahtee, Näveri and Pehkonen) reports solely on students' views and focuses on the way they perceive their teacher's activities during a mathematics lesson. The methodology used in this paper, having students draw a picture of a mathematics lesson, was also used by Pehkonen et al. (2011), and presented in the 17th MAVI conference. Taking into consideration that classroom practices

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are experienced by both teachers and students, Palmer and Karlsson look at both teachers' and students' perspectives, in this case, focusing on problem-solving. Previously, problem solving beliefs were discussed only from a teachers' point of view (e.g., Näveri et al. 2011; Pehkonen 1999). In this book, however, Palmer and Karlsson report that students and teachers have different images of problem solving that may influence the way teachers teach problem solving and the way learners learn problem solving.

Of major interest to MAVI participants, and a long-debated issue, is the relationship between teachers' professed beliefs and classroom practice (e.g., Ribeiro and Carrillo 2011), and teacher change (Philippou and Christou 1996). This is the focus of the second section in this book. Investigating teachers' professed beliefs is a challenge. Andrà employs the use of teachers' auto-biographical narratives, claiming that metaphors may open a window on the structure of teachers' beliefs, while Beswick asked teachers to respond to an open written questionnaire describing their best and worst mathematics students. Semi-structured interviews were used to explore teachers' conceptions of arithmetic as a specific mathematical discipline (Eichler, Bräunling and Männer) and to investigate the impact of the physical environment on students' learning (Fahlstrom).

Three papers in the second section deal directly with teacher change—Brunetto and Kontorovich, Heyd-Metsuyanım, and Liljedahl. Teacher change is notoriously difficult, even when the teachers themselves are interested in changing their practice. At times, this difficulty is caused by teachers' emotions and their identification with students' emotions. For example, even when teachers agree that classroom norms should be developed such that students feel comfortable making mistakes, teachers tend to emotionally identify with their students and to avoid cognitively demanding and discussion-based instruction (Heyd-Metsuyanım). Emotions and change were also linked in the previous MAVI conference where Liljedahl (2014) related how prospective teachers' emotions are linked to the hierarchy of their motives. In this volume, Liljedahl discusses how teachers' active participation in task design and task piloting can promote changes in their mathematics practice.

The third section of this book centers on the undercurrents of teaching and learning mathematics, what goes on just beneath the surface, but rises in various situations, causing tensions and inconsistencies. Two papers take into consideration parents, one paper focusing on teachers' conflicting views of parent involvement (Rouleau and Peter Liljedahl) and one focusing on parents' own conflicting views of their involvement (Albersmann and Bosse). Conflicting views and tensions are not necessarily detrimental. Kontorovich and Zazkis show how presenting learners with tasks that give rise to conflicting views, may stimulate learning. Inconsistencies are sometimes caused by the tensions felt between affective and social concerns. These tensions may influence patterns of participation (Tuohilampi), attitudes towards the place of mathematics in science education (Aderonke, Oyebola, and Akinloye), as well as how one identifies themselves (Branchetti and Morselli) and others (Hess-Green and Heyd-Metzuyanım) as mathematics learners. While in this section,

Branchetti and Morselli, and Hess-Green and Heyd-Metzuyanım, discuss learners' identities, in past MAVI conferences, several studies investigated teachers' identities of themselves as mathematics teachers (e.g., Lutovac and Kaasila 2012; Palmer 2013).

The last section of this book takes a look at emerging themes in affect-related research. Some of the papers relate to the development of new research tools (Goldin, Gırat) while others describe extending research to new directions by re-analyzing existing data (Pieronkiewicz, Sumpster). At the 20th MAVI conference, Törner noted that as early as the 1940s, researchers investigated the influence of attitudes on assessment. In this section, instead of investigating affective elements which influence assessment, two papers discuss attitudes towards assessment. Cusi, Morselli, and Sabena investigate the role of technologically enhanced formative assessment methods, while Signorini investigates teachers' emotions and beliefs towards standardized mathematics assessment, comparing differences between school levels and discussing their educational relevance.

As can be seen from this introduction, many of the papers presented in this book continue traditional MAVI themes while others build on those themes towards new directions. Although the book was divided into sections according to themes, we invite the reader to search for commonalities between papers in different sections, and to explore additional themes and avenues of affect research in mathematics education.

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**Part I**  
**Classroom Practices: Explanation,  
Problem-Solving, Patterning,  
Decision-Making, Drawings and Games**

# Chapter 2

## Prospective Primary Teachers' Beliefs Regarding the Roles of Explanations in the Classroom

Esther Levenson and Ruthi Barkai

**Abstract** This study classifies and discusses the views of 23 prospective primary teachers in Israel regarding the roles of explanations in the mathematics classroom, explanations given by teachers and those given by students. Results indicated that prospective teachers perceive explanations as playing various roles although greater emphasis is placed on building content knowledge than on developing a mathematical disposition. Results also hinted that perspectives of explanations may reflect on teachers' beliefs regarding mathematics and their beliefs regarding the teaching and learning of mathematics.

### 2.1 Introduction

Explanations are central to mathematics education. They are given during various instructional activities such as concept handling, carrying out procedures, and conjecturing. Mathematics educators promote the giving of explanations in the classroom as a means for encouraging communication and enhancing mathematical reasoning (NCTM 2000). How prospective teachers (PTs) view the roles of explanations, both explanations given by teachers as well as explanations given by students, may eventually affect how they use explanations in the classroom. For teacher educators, who are interested in developing not only PTs' mathematics knowledge, but also their pedagogical content knowledge, it is important to recognize that knowledge is often intertwined with mathematical and pedagogical beliefs (Kinach 2002). Thus, the first aim of this study is to investigate PTs' views regarding the roles of teachers' explanations and the roles of students' explanations in the mathematics classroom. We differentiate between teachers' and students' explanations because the teacher and students sometimes play different roles in the classroom, which in turn may affect the roles of explanations given by each. Taking into consideration that different beliefs are often inter-related (Beswick 2005), we

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