

NEW DIRECTIONS IN THE
PHILOSOPHY OF SCIENCE

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A PHILOSOPHICAL
ANALYSIS OF
CHAOS THEORY

Lena C. Zuchowski



New Directions in the Philosophy
of Science

Series Editor
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A Philosophical Analysis of Chaos Theory

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Introduction

Abstract The book's strategy and content will be introduced. Over the last 40 years, chaos theory has had a huge impact on science and philosophy. This is evidenced by the astonishing volume of chaos-related publications; even a cursory survey shows that chaos has been detected virtually everywhere, from cardiac rhythms to Joyce's *Ulysses* (e.g. Kellert, 2008). Given the high export appeal of chaos theory, it is surprising that there are fundamental aspects of the field that still remain poorly understood and, in some cases, permanently debated.

Keywords Chaos · Introduction

In particular: (i) it is still not clear how chaos should be defined and how the large number of coexisting chaos definitions relate to each other (e.g. Smith, 1998; Werndl, 2009c); (ii) there are still (largely unarticulated) questions about the faithfulness and predictiveness of the numerical and theoretical models on which chaos theory is based; and, finally, (iii) it has not been unequivocally resolved whether there is chaos in nature (e.g. Kellert, 1993) and how it should be diagnosed (e.g. Pool, 1989; Hastings et al., 1993). The three aspects are not independent of each other and it is evident that difficulties (i) and (ii) contribute to difficulty (iii). Together, they have made it very difficult to judge the diverse contributions to chaos theory comparatively and to enforce universal standards of quality and rigour.

This book aims to clarify aspects (i)–(iii) by providing a structured survey of the construction, diagnosis and evaluation of chaotic models. Although the book follows a survey approach in that it aims to achieve a certain degree of comprehensiveness and to comparatively cover different aspects of chaos theory, it is not a mere review. I will pursue a modelling-centred strategy and thereby aim to provide the first in-depth analysis of all three stages, i.e. construction, diagnosis and evaluation, of modelling in chaos theory. This allows me to draw on a large amount of recently developed work on the use of models in science, which has so far not been applied to the field of chaos theory. In particular, the book uses, and develops further, several results of both the fictionalist approach to modelling (e.g. Frigg, 2010; Toon, 2012; Suarez, 2013) as well as the work on horizontal modelling by Bokulich (2003), which were not available to authors of earlier philosophical analyses of chaos theory (e.g. Kellert, 1993; Smith, 1998).

Throughout the book, I will follow a strategy of rational reconstruction, that is I will aim to develop analytic frameworks that allow the exposition of the relevant concepts with the greatest possible clarity. These frameworks are intended to be tools in the conceptual analyses presented here. I thereby neither aim to be descriptively accurate on an individual level, that is to paint a detailed picture of the individual practices of chaos scientists, nor normatively prescribing, that is to offer advice to scientists on how the reconstructed concepts should be used. Instead, I aim to develop frameworks in which the content and the use of a given concept can be displayed clearly. The merits of the analytic frameworks developed in this book should therefore be evidenced in the quality of the analyses provided.

The book is subdivided into three main content chapters. These chapters deal with the construction, diagnosis and evaluation of chaotic models, respectively.

In [Chapter 2](#), I will analyse the construction of vertical and horizontal models in chaos theory. I will review some selected material on scientific modelling, which will be crucial to the discussion in the remainder of the book ([Section 2.2](#)). In particular, I will discuss the construction and evaluation of two different classes of models: vertical and horizontal models. I will introduce the inferential account of model evaluation developed by Suarez (2013). This account conceptualizes the evaluation of vertical models as the evaluation of a conditional $C \rightarrow B$ to be transferred from the model to the target system. Underlying this framework is the assumption that scientists are not only interested in the occurrence of a certain behaviour B but also in the sufficient conditions C for this

behaviour. Within this framework for the analysis of model evaluation, the concept of model faithfulness will be defined.

I will then present two case studies to illustrate the construction of vertical and horizontal models in chaos theory: the construction of three models based on the logistic equation ([Section 2.3](#)) and of two models based on the Lorenz equations ([Section 2.4](#)).

In [Chapter 3](#), I will analyse the criteria for and definitions of chaos. The chapter has two main theses: (i) I will maintain that there are five main criteria to diagnose chaos and (ii) that various combinations and embodiments of these criteria are used to build different chaos definitions.

I will begin this chapter by introducing an analytic framework that views chaos definitions as twofold decomposable: into five main criteria and into different technical embodiments of these criteria. The use of this framework will then be illustrated in a case study of the diagnoses of chaos of the logistic models ([Section 3.2](#)). This case study will introduce the five criteria that I consider to be constitutive of virtually all existing chaos definitions: determinism, transitivity, periodicity, aperiodicity and sensitive dependence on initial conditions (SDIC).

I will then discuss these five criteria in detail ([Section 3.3](#)). I will show that the criteria are similarity categories and can assume many different formal or semi-formal embodiments. It will become apparent that the different embodiments of a criterion can be used to make this criterion applicable to a specific class of models. The fact that many embodiments are applicable only to a specific class of models mitigates any conceptual conflicts between the different criteria.

I will then demonstrate how the most prevalent chaos definitions are composed of different combinations of embodiments of the five core criteria ([Section 3.4](#)). Five definitions will be analysed in detail: Devaney chaos; the definition of chaos as mixing; the definition of chaos in terms of positive Lyapunov exponents; stochastic chaos; and the definition of chaos in terms of strange attractors. I will maintain that many of these definitions are targeted towards specific classes of models and that the use of different combinations of criteria in different definitions can be viewed as a means of highlighting those properties of these models that will be the most important for their future investigative use. The coexistence of many different chaos definitions can therefore be viewed as a consequence of the variety of models used in chaos theory.

In [Chapter 4](#), I will discuss the evaluation of models in chaos theory. Building on the framework for the transference of conditionals from

models to their target systems developed by Suarez (2013), I will show that, for vertical chaotic models, the evaluation process can be decomposed into three crucial steps: (i) one needs to determine which type of chaos and which proposed sufficient conditions for its occurrence are to be evaluated, that is one needs to determine a conditional $C \rightarrow B$ to be transferred from the model to the target system; (ii) the existence of chaos in the model's target system needs to be ascertained and (iii) the faithfulness of the model should be evaluated. While it is possible to clearly separate these steps conceptually, actual evaluations of vertical chaotic models do not necessarily employ these steps in order or give equal weight to all steps. The use of this analytic framework will be demonstrated in a case study of the evaluation of the logistic models (Section 4.2).

I will then discuss each of the three steps in detail (Section 4.3). It will become apparent that there are two types of chaotic conditionals to be transferred from vertical models: conditionals that posit forms non-linearity and iteration as sufficient conditions for the occurrence of chaos (type 1); and conditionals that posit non-linearity and discreteness as sufficient conditions for chaos (type 2). It will be shown that the determination of which conditionals holds true in a model is usually technically difficult and often involves investigative work with related horizontal models. I will therefore be able to specify the investigative function of horizontal models more precisely as aiding investigations of the properties of related vertical models during the first step of these latter models' evaluation. The use of horizontal models in chaos theory will also be discussed in a separate section of the chapter (Section 4.4).

I will maintain that the second step of the model evaluation process, that is determining the existence of chaos in the target system, is often difficult to complete for chaotic models. This difficulty will be traced back to a difficulty in determining the fulfilment of the criterion of determinism.

Finally, I will discuss the model faithfulness of chaotic models. A particular conceptual result of this analysis will be the realization that numerical models can only model chaos faithfully, if the chaotic behaviour is seen as part of a type 1 conditional. Accordingly, a significant part of the modelling activity of scientists in chaos theory can be interpreted as attempts to establish these conditionals in numerical models. This last point, and the general interplay of different models in chaos theory, will then be illustrated in a case study of the evaluation of the Lorenz models (Section 4.5).