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ACS Without an Attitude

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This book is dedicated to the many students/colleagues that attended the lectures, whose active participation greatly improved the lectures and this book.

Preface

If you do a casual search for books that contain the word attitude in their title, you'll find yourself drowning in a sea of over 500 volumes. Of course, most of those books relate more to personal self-improvement than to spacecraft dynamics, but even when the subject is limited to the areas of aerospace and astrodynamics you'll still find a fair number of items from which to choose. So, what distinguishes this text from those many other candidates? This book attempts to de-emphasize the formal mathematical description of spacecraft onboard attitude and orbit applications in favor of a more qualitative, concept-oriented presentation of these topics (whether or not we ultimately achieved that goal is something we'll have to leave to you, dear reader, to decide). As such, it would most likely be described by an attitude control analyst as a (hopefully) amusing light read rather than an essential reference bible. And it certainly would not be the first text an Attitude Control Subsystem (ACS) flight software (FSW) designer would grab if he needed a specification of a Kalman filter algorithm. ACS Without an Attitude is instead intended for programmers and testers new to the field who are seeking a commonsense understanding of the subject matter they're coding and testing in the hope that they'll reduce their risk of introducing or missing the key software bug that causes an abrupt termination in their spacecraft's mission and their careers.

ACS Without an Attitude is organized in four major sections. Section One (Chaps. 1–3) contains the attitude, orbit, and dynamics background material required to understand the downstream spacecraft applications. Section Two (Chaps. 4 and 5) is a survey of the spacecraft sensors and actuators used to measure and control the spacecraft attitude and orbit. Section Three (Chaps. 6–11) examines how sensor data is combined with reference data to measure attitude and orbit, and how desired or commanded attitude parameters are compared with measured attitude parameters to determine what should be done to maintain the current pointing, or modify it to satisfy future needs. Finally, Section Four (Chap. 12) is a survey of mission characteristics and how attitude and orbit geometries are selected to accomplish mission objectives.

The information presented in these sections was originally collected to support an informal set of lectures in 1999 and 2000 instigated by my Branch Chief, Elaine

Shell (Flight Software Branch, NASA Goddard Space Flight Center), who also realized that bullet charts are an ineffective means to document information, hence this book. The following is a list of textbooks and documents I drew on (hopefully not too blatantly) while preparing for my talks, as well as additional references used when writing this book:

1. Spacecraft Attitude Determination and Control, edited by James R. Wertz, Kluwer Academic Publishers (1978).
2. Space Mission Analysis and Design, edited by Wiley J. Larson and James R. Wertz, Microcosm, Inc. and Kluwer Academic Publishers (1992).
3. Reducing Space Mission Cost, edited by James R. Wertz and Wiley J. Larson, Microcosm Press and Kluwer Academic Publishers (1996).
4. Satellite Technology and Its Applications, by P.R.K. Chetty, TAB Professional and Reference Books (1991).
5. An Introduction to the Mathematics and Methods of Astrodynamics, by Richard H. Battin, AIAA Education Series (1987).
6. Modern Inertial Technology Navigation, Guidance, and Control, by Anthony Lawrence, Springer (1998).
7. Modern Control Systems, by Richard C. Dorf and Robert H. Bishop, Addison-Wesley Publishing Company (1995).
8. Feedback Control of Dynamic Systems, by Gene F. Franklin, J. David Powell, and Abbas Emami-Naeini, Addison-Wesley Publishing Company (1991).
9. Modern Control Engineering, by Katsuhiko Ogato, Prentice-Hall (1997).
10. Goddard Trajectory Determination System (GTDS) Mathematical Theory, Revision 1, edited by A.C. Long, J.O. Cappellari, Jr., C.E. Velez, and A.J. Fuchs, NASA/GSFC Flight Dynamics Division Code 550 (1989).
11. Fundamentals of Astrodynamics, by Roger R. Bate, Donald D. Mueller, and Jerry E. White, Dover Publications, Inc. (1971).
12. Classical Mechanics, by Herbert Goldstein, Addison-Wesley Publishing Company (1950).
13. Fundamentals of Spacecraft Attitude Determination and Control, by Landis F. Markley and John L. Crassidis, Springer Science and Business Media (2014). (An excellent book for both subject history and in-depth mathematical analysis.)

In addition, here are some references targeted to specific topics discussed in the later chapters:

1. Hermite Polynomials, Legendre polynomials, and spherical harmonics: Mathematical Methods for Physicists (seventh edition), by G. Arfkin, H. Weber and F. Harris, Academic Press, Inc. (2013), Sect. 18.1.
2. Runge-Kutta integrator: Numerical Methods for Scientists and Engineers by R.W. Hamming, McGraw-Hill (1962).

One of the strengths of my original set of lectures was the participation of several of my NASA/GSFC Guidance, Navigation, and Control (GN&C) colleagues who supplemented (and often, and graciously, corrected) my presentations with material drawn from their extensive experience here at GSFC. And when my pitches started to drag a bit, they also helped liven things up by recounting some of their many war stories accumulated during their years of applying clean-cut mathematical algorithms to uncooperative real-life spacecraft. My crew of semi-regular experts included

1. Gary Welter
2. Landis Markley
3. Dave Quinn
4. Dave McGlew
5. Bruce Bromberg

As the years have rolled by since the first version of the manuscript, the material in the book has been updated many times, stimulated by new missions and new ACS technologies, as well as new teaching experiences gained repeating the course. Finally, as I reached the point I could no longer bear to edit the material again, Gary Welter, Dave Simpson, and Chris Rouff have ridden to the rescue to co-author with me this final version.

Lou Hallock—2010

Well … life goes on, including delays from other obligations. Lou passed the torch to us, his three amigos, when he retired in 2011, along with encouragement to put our own stamp on the book (sometimes with a "you broke it, you bought it" response to editorial suggestions). We've also corralled a couple of colleagues (Scott Starin and Tim McGee) to provide some review and feedback on the near-final text. (Our thanks to you both.) So, here is the multi-chef result, we think well-flavored—though some of you may find certain sections more "in your face" than "without an attitude". Time to let it go. Enjoy.

Gary Welter, Dave Simpson and Chris Rouff—2016

Contents

Acronyms

ACE	Attitude Control Electronics
ACS	Attitude Control Subsystem
AOS	Acquisition of Signal
AST	Advanced Star Tracker
AU	Astronomical Unit
CCD	Charge-Coupled Device
CDR	Critical Design Review
CGRO	Compton Gamma Ray Observatory
CPU	Central Processing Unit
CSA	Celestial Sensor Assembly
CSS	Coarse Sun Sensor
CXO	Chandra X-Ray Observatory
DCM	Direction Cosin Matrix
DSCOVR	Deep Space Climate Observatory
DoD	Department of Defense
DOY	Day of Year
DSS	Digital Sun Sensor
ECI	Earth-centered Inertial
emf	Electromotive force
EOS	Earth Observing System
ESA	European Space Agency
ESA	Earth Scanner Assembly
FDF	Flight Dynamics Facility
FEEP	Field Emission Electric Propulsion
FES	Fine Error Sensor
FGS	Fine Guidance Sensor
FHST	Fixed-Head Star Tracker
FOG	Fiber Optic Gyro
FOV	Field of View
FSS	Fine Sun Sensor

FSW	Flight Software
GCI	Geocentric Inertial
GEO	Geosynchronous Earth Orbit
GMT	Greenwich Mean Time
GOES	Geostationary Operational Environmental Satellite
GPS	Global Positioning System
GSFC	Goddard Space Flight Center
GTDS	Goddard Trajectory Determination System
HCI	Heliocentric Inertial
HEAO-2	High Energy Astronomy Observatory-2
HGA	High-Gain Antenna
HRG	Hemispheric Resonating Gyro
HST	Hubble Space Telescope
ICOB	Inclined-Center-of-Box
IFOG	Interferometric Fiber Optic Gyro
IFOV	Instantaneous Field of View
IGRF	International Geomagnetic Reference Field
IR	Infrared
IRU	Inertial Reference Unit
ISEE-1	International Sun–Earth Explorer-1
IUE	International Ultraviolet Explorer
JD	Julian Day
JPL	Jet Propulsion Laboratory
JWST	James Webb Space Telescope
KSC	Kennedy Space Center
LISA	Laser Interferometer Space Antenna
LEO	Low Earth Orbit
LMSC	Lockheed Missiles and Space Company
LOS	Loss of Signal
L-R-C	Inductance Resistance Capacitance
LVA	Local Vertical Acquisition
MJD	Modified Julian Day
MT	Magnetic Torquer
MTB	Magnetic Torquer Bar
NASA	National Aeronautics and Space Administration
NCC	Network Control Center
NMR	Nuclear Magnetic Resonance
OBC	Onboard Computer
PD	Proportional–Derivative
PI	Proportional–Integral
PID	Proportional–Integral–Derivative
RFOV	Reduced Field of View
RG	Rate Gyro
RGA	Rate Gyro Assembly
RIG	Rate Integrating Gyro

RLG	Ring Laser Gyro
RPO	Revolution Per Orbit
RWA	Reaction Wheel Assembly
RXTE	Rossi X-ray Timing Explorer
SAMPEX	Solar Anomalous and Magnetospheric Particle Explorer
SDOF	Single-Degree-of-Freedom
SI	Science Instrument
SLP	Solar-Lunar-Planetary
SMM	Solar Maximum Mission
SOHO	Solar and Heliospheric Observatory
ST 5	Space Technology 5
TAI	International Atomic Time
TAM	Three-Axis Magnetometer
TDB	Barycentric Dynamical Time
TDT	Terrestrial Dynamical Time
TDOF	Two-Degree-of-Freedom
TDRS	Tracking Data and Relay Satellite
TDRSS	Tracking Data and Relay Satellite System
TFOV	Total Field of View
TONS	TDRSS Onboard Navigation System
TRIAD	Tri-Axial Attitude Determination
TRMM	Tropical Rainfall Measuring Mission
TT	Terrestrial Time
UT	Universal Time
UTC	Universal Time Coordinate
WMAP	Wilkinson Microwave Anisotropy Probe
YGC	Yaw Gyro Control

Chapter 1
Attitude Conventions and Definitions

Most people who have had even the most casual contact with spacecraft (no, not a Close Encounter of the First Kind) have heard the term "orbit" used and have at least a rough idea of what the word means. By contrast, the first time you hear the phrase "attitude determination" or "attitude control" a whole host of rather exotic images may cross your mind, for example Michael Caine's portrayal of Harry Palmer in the movie *The Ipcress File*. In some respects, these impressions are not totally far-fetched, and there are some similarities between psychiatry and spacecraft attitude control. Although the field of psychoanalysis is possessed of a rich literature documenting its knowledge and a host of psychoanalytical techniques for analyzing a subject have been established, it is difficult even for a talented psychiatrist to determine in the short-term a subject's state of mind. There are many reasons for this problem, but four that strike a resonant chord relative to the aerospace profession are that:

1. many forces can influence a subject's behavior;
2. the relative importance and impact of those forces can be difficult to judge;
3. events from the past must be blended with current events to predict current behavior;
4. there is no direct means to measure the subject's state of mind.

Now replace this image of a patient lying on the couch pouring out his heart to a well-paid psychiatrist with that of a spacecraft in orbit telemetering data to a not-so-well paid Attitude Control Subsystem (ACS) analyst, and let's run through the checklist of difficulties. A spacecraft is subject to a wide variety of perturbations on its pointing (item 1), the effects of which can only be approximated even with the help of complex mathematical models (item 2). One of the standard tools for accurate measurement of attitude errors (i.e., deviations of the measured orientation of the spacecraft in 3-dimensional space from its desired orientation), the Kalman filter, combines a (limited) historical baseline with the most recent measurements, weighting fresher data higher than staler data (item 3). And, unlike the highly controlled conditions of a laboratory or a vacuum chamber, no direct measurements can be

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made of a spacecraft's orientation in orbit (item 4). One must deduce its orientation from attitude sensor data, i.e., one must see through the "eyes" of the spacecraft rather than seeing the spacecraft state directly. So, as with psychoanalysis, attitude determination and control requires a great deal of analytical rigor in its methodologies along with a somewhat artistic flair in selecting which combination of factors should be evaluated and how those factors should be weighted relative to each other.

Some of you may find Sects. 1.2–1.6 a bit tedious, with its many explanations of different ways to express attitude. One of our reviewers suggested an early plug for Sect. 1.7, which explains why they are all needed. So take heart as you proceed; justification will be forth-coming - we promise. (Thanks Scott.)

1.1 Definition of the Inertial Reference Frame

As is the case for any serious discipline, attitude determination and control possesses its own special vocabulary and conventions that must be absorbed prior to learning the key material that can make its practitioner truly dangerous to his spacecraft. To this end, in this chapter, we'll present the fundamental concepts used to define attitude, along with (to a minimal degree) the various mathematical expressions of attitude and how they relate to each other. To describe the orientation of an object in space, it is first necessary to specify a frame of reference relative to which directions in space and the object's orientation is defined. For spacecraft in orbit about the Earth, it is usually most convenient to define a reference frame whose origin is located at the Earth's center.

The reference frame z-axis is defined to be a unit vector parallel to the Earth's spin axis. By convention, a spin axis direction is defined such that if the fingers of a right hand are curled along the spinning motion, the "thumbs up" thumb direction will be parallel to the positive spin axis direction; this is called a right-handed rotation. By conservation of angular momentum (more on that in Chap. 3), and the fact that the Earth's mass distribution is very nearly spherically symmetric, the direction of the Earth's spin axis is very nearly constant in time …at least over typical human time scales. The reference frame x-axis is associated with another quasi-invariant, the unit vector pointing from the Earth to the Sun at the instant of the vernal equinox. The vernal equinox is the time during the year when the Sun moves along its "orbital" path (as seen from the Earth's point of view) across the Earth's equator from the Southern hemisphere to the Northern hemisphere. This equatorial transit is also called the First Point of Aries, a reference back thousands of years ago to the time when the constellations of the zodiac were originally named and the Sun was in the constellation of Aries at the moment of the vernal equinox. The plane of the Sun's orbit about the Earth (or less parochially, the Earth's orbit about the Sun) is called the *ecliptic*, and is tilted (on average) about 23.44° relative to the Earth's equatorial plane. Finally, taking the cross product of z-axis and x-axis yields the third member of the orthonormal triplet, the y-axis. The reference frame whose axes are defined by this (x, y, z) triplet is called the *Geocentric Inertial (GCI) frame*, where the term

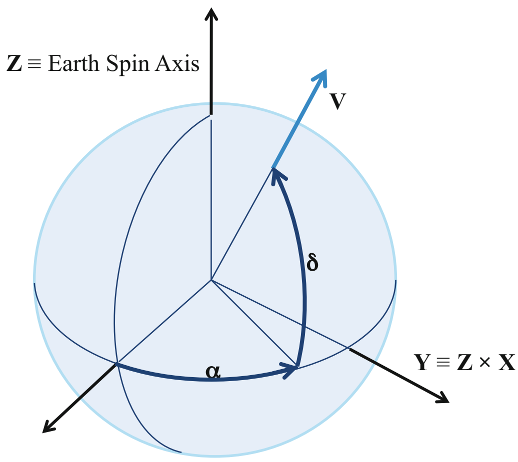


Fig. 1.1 A direction vector **V** relative to the GCI reference frame, rotated from the x-axis based on angles α (along the equator) and δ *above* or *below* the equator

geocentric means "Earth-centered" while the term *inertial* means that the reference frame is fixed relative to the "stationary" stars in the celestial sphere. For folks not into Greek and the Earth goddess Gaia, the reference frame is also called the Earth-centered Inertial (ECI) frame. Figure 1.1 illustrates how a direction vector (**V**) can be defined relative to the GCI frame.

If the Earth's mass distribution were spherically symmetric, the GCI frame described in the preceding paragraph would be truly fixed in space. However, because the Earth is in fact shaped like an oblate spheroid (i.e., it has an equatorial bulge), the gravitational intereaction of the Sun and Moon with the Earth's equatorial bulge causes the Earth's spin axis to precess (i.e., cone) and nutate (i.e., wobble) like a top spinning on the floor. The precession rate (generated by the Sun's torque on the bulge) is 50 arcsec/year, so it takes about 26,000 years for the Earth's spin axis (i.e., the North celestial pole) to complete a single coning period about the North ecliptic pole. (The ecliptic poles define a line perpendicular to the ecliptic plane.[1]) This rotation of the North celestial pole about the North ecliptic pole is also called the *precession of the equinoxes*. The half cone angle is the average angle between the ecliptic pole and the Earth's spin axis, i.e., 23.44°. The amplitude of the wobble (as the spin axis precesses) is about 9.2 arcseconds and arises from the Moon's torque on the Earth's equatorial bulge, with the period of the wobble equal to 18.6 years, which also is the precession period of the Moon's orbital plane due to Solar gravitational influences. Because of these motions, the GCI frame isn't truly "inertial", which is perhaps another reason (in addition to the insistence on viewing the Sun as orbiting

[1] Wertz, Spacecraft Attitude Determination and Control, p. 27.

the Earth rather than the generally accepted Copernican model) for renaming the Geocentric Inertial frame as the Egocentric Inertial frame. In any event, one can't really talk about GCI coordinates without associating those coordinates with a time.

The time identifies the direction of the Earth's spin axis relative to the stars in the Sun's neighborhood, and thereby the orientation of the GCI axes (at that time) relative to the fixed stars. Further, the time identifier comes in different "flavors". If the time is *mean of epoch*, the nutation effects are ignored (i.e., the cone angle is set to 23.44°), but the precession effects are included for the specified epoch time. Nowadays, a standard epoch time is January 1, 2000. So in a Goddard Space Flight Center (GSFC) Flight Dynamics Facility (FDF) ephemeris file, consisting of a series of GCI position and velocity vector pairs spaced at equal time intervals (say, once per minute) over an extended time interval (say, one week), each vector's components are specified relative to the GCI frame defined at the precession angle corresponding to January 1, 2000. Mean of epoch is a popular time choice for star catalogs.

If the time is *true of epoch*, both nutation and precession effects are included, and the same rotation is applied to each vector in the GSFC FDF ephemeris file. If the time is *mean of date*, each vector in the file will include the precession effects associated with the attached time for the vector, so each position and velocity vector pair will be rotated by a slightly different amount. This means, in effect, that the components of each vector pair in the file are specified with respect to a slightly different reference frame. Similarly, with *true of date* times, the nutation and precession effects associated with the attached time for each individual vector pair are included. Unless otherwise requested, GSFC FDF-generated ephemeris files are typically true of date, potentially making them inconsistent with a mean of epoch star catalog. In other words, unless care is taken, the star catalog star vectors may be defined relative to a different GCI frame than the spacecraft position and velocity vectors. This problem, of course, can be avoided by requesting the ephemeris data in mean of epoch format, with the same precession time for both the ephemeris and star catalog data.

To avoid the complexity of having an inertial reference frame (GCI) that's only inertial if you compensate for the fact that it is really moving relative to the celestial objects you're most concerned with (stars in the Sun's neighborhood), one could select a Sun-centered (i.e., Heliocentric) Inertial (HCI) reference frame. However, that choice costs you the convenience gained by choosing as the origin the gravitational body (the Earth) about which most GSFC spacecraft orbit. So for most GSFC missions, it's less work and less confusing to select GCI as the reference relative to which you'll define your spacecraft's orientation. An exception is missions where the spacecraft orbits a Lagrange point (stable or pseudo-stable gravitational equilibrium points relative to two gravitational objects orbiting each other, as will be discussed in Chap. 12). For such cases, for example a Sun-Earth Lagrange point, the "natural" reference frame to pick for many calculations is a Sun-centered rotating (i.e., non-inertial) frame that "freezes" the gravitational objects and the Lagrange point in that frame.

The GCI and HCI reference frames described above are examples of a right-handed reference frames. Mathematically, their orthogonal unit vectors ($\mathbf{x}$, $\mathbf{y}$, $\mathbf{z}$) satisfy the relations: $\mathbf{x} \times \mathbf{y} = \mathbf{z}$, $\mathbf{y} \times \mathbf{z} = \mathbf{x}$, and $\mathbf{z} \times \mathbf{x} = \mathbf{y}$. The right-handedness arises from imagining a right-handed rotation from $\mathbf{x}$-to-$\mathbf{y}$ having a spin axis along $\mathbf{z}$, a right-handed rotation from $\mathbf{y}$-to-$\mathbf{z}$ having a spin axis along $\mathbf{x}$, and a right-handed rotation from $\mathbf{z}$-to-$\mathbf{x}$ having a spin axis along $\mathbf{y}$. One can also have left-handed frames; these satisfy $\mathbf{x} \times \mathbf{y} = -\mathbf{z}$, $\mathbf{y} \times \mathbf{z} = -\mathbf{x}$, and $\mathbf{z} \times \mathbf{x} = -\mathbf{y}$. Right-handed frames are the standard; we will only be using such frames in this book.

1.2 Defining Attitude via Euler Angles (Right Ascension, Declination, and Roll)

Getting back to our favorite (for now) reference frame, the GCI, one of the first things you want to be able to describe, even before talking about the orientation of the spacecraft, is where important celestial objects are located within the frame. Strictly speaking, since most celestial objects (like stars) are so far away, an attitude analyst is primarily interested in the direction of an object in GCI as opposed to its actual position vector (which would include both direction and distance). Just as only two angles, latitude and longitude, are required to specify the location of any point on the Earth's surface, only two angles are needed to specify the location of the center of any celestial object on the celestial sphere (see Fig. 1.1). The GCI counterpart of longitude is called *right ascension*. Right ascension is an azimuthal angle measured within the GCI's x-y plane and has a range of 0–360°, with a right-handed sense of rotation, and with 0° defined to be the First Point of Aries. Right ascension is computed by projecting onto the x-y plane the vector directed from the GCI origin to the celestial object center. The angle between the x-axis and the vector's projection is the object's right ascension. The GCI counterpart of latitude is called *declination*. Declination is an elevation angle that measures the angle between the x-y plane and the vector from GCI origin to the celestial object, and has a range of $-90°$ to $+90°$. For a given right ascension and declination, the equivalent three-dimensional vector is

$$(x, y, z) = [\cos(\alpha)\cos(\delta), \sin(\alpha)\cos(\delta), \sin(\delta)] \tag{1.1}$$

where

(x, y, z) = unitized 3-vector in GCI reference frame
α = right ascension (degrees)
δ = declination (degrees)

The reverse transformation is

$$\begin{aligned} \alpha &= \arctan(y/x) \\ \delta &= \arcsin(z) \end{aligned} \tag{1.2}$$

(The range of the arctan function in Eq. 1.2 is 0–360°, and must be assigned to the correct quadrant according to the signs of x and y. If both x and y are equal to zero (i.e. the declination is $\pm 90°$), then α is undefined.)

Vectors having the z-component zero (i.e., vectors in the *x-y* plane) have declination zero. The North Celestial Pole is defined to have declination 90°, while the South Celestial Pole has declination $-90°$. Note that if you try to project a vector directed to either pole onto the *x-y* plane you get a single point at the origin of the GCI frame, not a vector. This condition is referred to mathematically as a singularity, meaning the right ascension is undefined (i.e., cannot be calculated) for objects located at the poles. Alternately, if the declination is $\pm 90°$ the celestial object will be located at the North (South) Pole no matter what value of right ascension is specified.

As illustrated above, the definition of the direction of (ideally) infinitely distant point sources (which, practically speaking, includes the "fixed" stars) in GCI requires only two numbers, right ascension and declination. However, if you want to define the *orientation* of a three-dimensional object in GCI, more information is required.

To help visualize the situation, suppose you wanted to define the orientation of a spinning spacecraft in GCI at a specific instant in time. It is straightforward to specify the spacecraft's spin axis right ascension and declination. The only thing left that we care about is how components mounted on the spacecraft body are oriented with respect to the GCI as they rotate about the spacecraft spin axis at a constant rate (in the absence of perturbations). For example, suppose a camera is mounted on the spacecraft body looking out in a direction perpendicular to the spin axis. You would like to be able to say in what direction in GCI the camera was looking, so if it saw something interesting you could associate what it saw with known celestial objects, or alternately determine the location of a new celestial object so it could be studied in more detail in the future. A convenient way to do this would be to define the angle about the spacecraft spin axis at which the object was observed. (Note, this approach requires that the spin axis be fixed in direction, which is frequently close enough to true.) However, as we saw with right ascension and declination, to use angles to define the orientation of an object in space, we must also define a fiducial (i.e., a reference mark) from which the angle is to be measured.

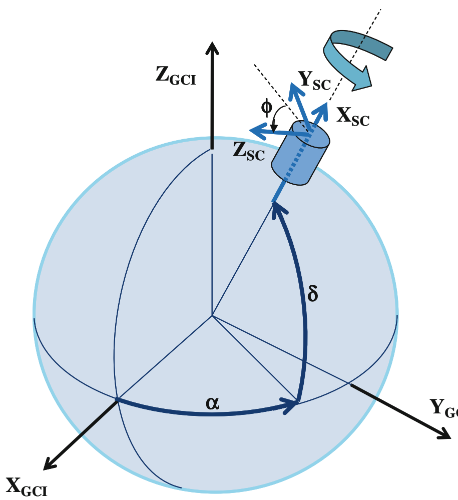
Continuing our imaginary spacecraft construction exercise, suppose there was a bright object sensor mounted on the spacecraft body looking out in a direction perpendicular to the spin axis. If the spacecraft were in sunlight, as the spacecraft rotated about its spin axis, the bright object sensor would spot the Sun (assuming the spin axis was not pointed directly at, or away from, the Sun) and you could note that time relative to an onboard clock. You could also note relative to an onboard clock when the camera saw its celestial object. Knowing the azimuthal angular separation between the camera and sensor boresights (assume it was measured when the spacecraft was built or was calibrated on-orbit), and knowing the rotation rate of the spacecraft (easily obtained by measuring the time differences between Sun sightings by the bright object sensor, given that the spin axis is fixed in space), you could compute the angle about the spin axis measured from the Sun direction to the direction of the celestial object. This third angle, along with the right ascension and declination of the spin axis, uniquely "pins" the spacecraft to the celestial sphere at the time of the camera observation.

The spinning spacecraft discussion is a nice practical exercise in visualizing how you can define a specific spacecraft's attitude. What we next want to do is define a more general approach that would be applicable to any spacecraft. Keeping the image of our spinning spacecraft, let's now visualize the spacecraft spin axis (defined to be the spacecraft x-axis) lined up with the GCI x-axis and the camera boresight (defined to be the spacecraft z-axis) lined up with the GCI z-axis. Note that since the spacecraft y-axis is the cross product of the spacecraft z- and x-axes, the spacecraft y-axis will line up with the GCI y-axis as well. Then we can define a unique orientation angle, called the *roll angle* (ϕ), that describes how the spacecraft's camera axis has been rotated from the GCI z-axis in the GCI y-z plane. The roll angle's range of rotation will be 0°–360° (or $-180°$ to $+180°$), with a right-handed sense of direction.

So now we can imagine putting the spacecraft through a set of three successive "right-handed" rotations about the spacecraft z-, y-, and x-axes with values $(\theta_Z, \theta_Y, \theta_X) = (\alpha, -\delta, \phi)$. The first two get the spacecraft x-axis (the spin axis for our previously imagined spinning spacecraft) pointed at the desired (α, δ) coordinates. (Note that by "right-hand" convention, a rotation that brings spacecraft pointing up into the $+Z$ hemisphere requires a negative rotation about the local Y spacecraft axis.) The final rotation adjusts the roll angle to the desired value (see Fig. 1.2).

Applying these three rotations in that order suffices to reorient the spacecraft from its original attitude (where its axes are aligned with the GCI reference axes) to any general attitude at which the spacecraft could be oriented. Rotating the spacecraft through successive rotations within a fixed reference frame is often called "performing active rotations". The same effect can also be achieved kinematically by conceptually holding the spacecraft fixed and rotating the reference frame (and implicitly the entire universe with it) by the negative of those rotations in the reverse order.

Fig. 1.2 A spacecraft's attitude expressed as a rotation sequence $(\alpha, -\delta, \phi)$



In other words, you get the same result vis-a-vis relative orientation of the spacecraft to the universe by flipping the order and signs of rotation from $(\alpha, -\delta, \phi)$ to $(-\phi, +\delta, -\alpha)$ and imagine them applied to the universe. This approach (rotating the reference frame rather than the object) is often called "performing passive rotations". It is the approach most common in classical dynamics textbooks, although for many the active approach seems easier to visualize.

Whether your rotation convention is active or passive, the set of angles utilized in performing the successive rotations is called an Euler angle sequence. The order of rotation is somewhat arbitrary, as is even the set of axes about which the rotations are performed. Specifically, for a given set of 3 mutually orthogonal rotation axes, there are 12 different rotation combinations that span the space, i.e., that can generate a set of successive rotations that can orient the spacecraft to any desired attitude. The values of the angles, of course, may change depending on the convention selected, so although the definitions of the angle-axis combination is arbitrary, once a convention has been selected all elements of the ground-flight system must follow that convention or pointing interpretation errors will result. The 12 allowed combinations of rotations are 3-1-3, 2-1-2, 1-2-1, 3-2-3, 2-3-2, 1-3-1, 3-1-2, 2-1-3, 1-2-3, 3-2-1, 2-3-1, 1-3-2. In this shorthand notation, "1" stands for the x-axis, "2" stands for the y-axis, and "3" stands for the z-axis.

1.3 Defining Attitude via Euler Angles (Roll, Pitch, and Yaw)

{Roll, Dec, R.A.} is just one of several popular applications of Euler angle formulations to ACS problems. So let's discuss some of the other Euler angle uses. The problem we've examined so far is how do you define the orientation of the spacecraft in the GCI frame, and we've defined a convenient formalism that satisfies that need. Another important problem is how to describe the difference between the way the spacecraft is pointed and the way you'd like to have it pointed. You could do a direct comparison between the measured and desired {Roll, Dec, R.A.}, but the simple differences between the measured and desired angles could give you a rather misleading impression of how large the error is and how much effort is required to correct it. The unreliability of the simple differences grows with the size of the angles (breakdown of first order approximations) and with how close you are to the celestial pole singularities. A better approach to the problem is to define two body reference frames, the desired or commanded body frame and the measured body frame. Of course, there really is only one body frame, and it usually is defined relative to "natural" spacecraft axes, such as the primary moments of inertia, science instrument boresights, etc. But if you substitute for the GCI frame the desired body frame, and substitute for the spacecraft frame the measured body frame, you can similarly define an Euler angle triplet that rotates you from the commanded frame to measured frame. The associated angles are a measure of the angular displacements between the two frames

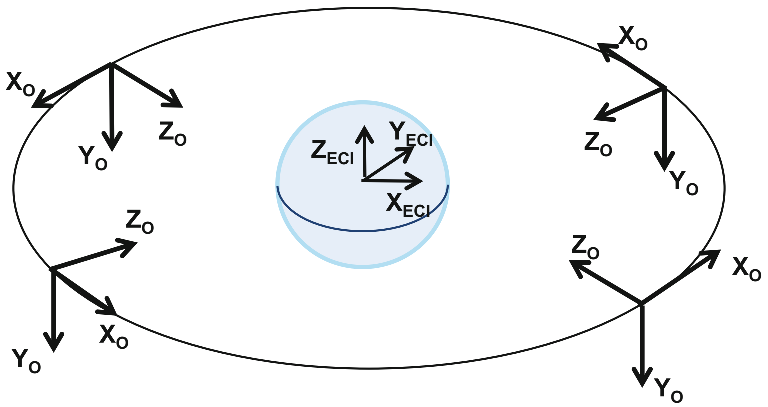


Fig. 1.3 Orbit reference frame; $\mathbf{Z}_0$ towards nadir, $\mathbf{X}_0$ parallel to velocity, $\mathbf{Y}_0 = \mathbf{Z}_0 \times \mathbf{X}_0$

for the selected axes of rotation. The names used for these displacement angles are {Roll, Pitch, and Yaw}, as compared to {Roll, Dec., and R.A.}, where unlike the declination angle's left-handed sense, the pitch angle is measured in a right-handed fashion. The ranges of the angles are the same. The roll and yaw angles have ranges of 0°–360° (or sometimes −180° to +180°), and the pitch angle has a range from −90° to +90°.

Note that the quantitative description of attitude error is just one use of the {Roll, Pitch, Yaw} formalism. If you substitute for the GCI frame the current commanded body frame and substitute for the spacecraft frame a future commanded body frame, you can obtain an Euler angle triplet that rotates you from where you currently want to be to where you want to go in the future. These Euler angles (and their associated axes) can then be used to control the spacecraft's motion when executing a slew (i.e., a large re-orientation of the spacecraft's pointing).

Another application of {Roll, Pitch, Yaw} is describing an Earth-pointing spacecraft's orientation relative to its orbit about the Earth. In the orbit frame, the z-axis (the yaw axis) is aligned with the direction of the nadir vector, the vector that points from the spacecraft center-of-mass to the Earth center-of-mass (similarly, the zenith vector points from Earth center to spacecraft center). The y-axis (the pitch axis) is aligned with the negative orbit-normal, with the x-axis (the roll axis) forming the last of the orthogonal triplet. Figure 1.3 illustrates how the orbit frame axes are dependent on the spacecraft position in the orbit, rotating once per orbit about the y-axis. For a perfect circular orbit, the x-axis will line up with the direction of the spacecraft orbital velocity vector.

1.4 Defining Attitude via the Direction Cosine Matrix

So an Euler angle description of the spacecraft attitude is an extremely flexible formulation that is easily adaptable to different applications (absolute pointing, relative pointing, and reorientation). It is also ideal for visualizing what's going on with your spacecraft. The problem is how do you convert your visual image to something useful to a flight (or ground) computer that is wonderfully adept at doing exactly what you tell it to do very quickly, but is (at least at this writing) utterly inept at imagining anything at all. That's where the *direction cosine matrix* comes in. The *direction cosine matrix* (DCM) recasts those three Euler angles into a 3×3 matrix that can be used with mathematical rigor to transform any 3-dimensional vector between two reference frames. Specifically, the DCM that transforms vectors from the GCI to the body frame is the attitude matrix. The individual Euler angle rotations themselves can be expressed as 3×3 rotation matrices and, as stated earlier, you can use these elementary single-axis rotations to build up a set of rotations that span the space, although (as discussed at the end of Sect. 1.2) the choice and order of the three rotations is somewhat arbitrary.

OK, so let's construct the attitude direction cosine matrix for a situation in which the spacecraft has been rotated from GCI via an Euler angle sequence (Φ_3, Φ_2, Φ_1), equal to $(\alpha, -\delta, \phi)$, as discussed in Sect. 1.2. First construct the 3×3 matrix that would transform a direction vector from the GCI frame (G) to intermediate frame $F1$ rotated from GCI by angle Φ_3 about axis-3 (i.e., the z-axis):

$$\mathbf{V}_{F1} = \boldsymbol{A}_{F1<G}\mathbf{V}_G = \begin{bmatrix} \cos(\Phi_3) & \sin(\Phi_3) & 0 \\ -\sin(\Phi_3) & \cos(\Phi_3) & 0 \\ 0 & 0 & 1 \end{bmatrix} \mathbf{V}_G$$

where the notation "$F1 < G$" indicates transformation into frame $F1$ from G. Next construct the matrix that would transform a direction vector from frame $F1$ to intermediate frame $F2$ rotated from $F1$ by angle Φ_2 about axis-2 (i.e., the y-axis):

$$\mathbf{V}_{F2} = \boldsymbol{A}_{F2<F1}\mathbf{V}_{F1} = \begin{bmatrix} \cos(\Phi_2) & 0 & -\sin(\Phi_3) \\ 0 & 1 & 0 \\ \sin(\Phi_3) & 0 & \cos(\Phi_2) \end{bmatrix} \mathbf{V}_{F1}$$

Finally, construct the matrix that would transform a direction vector from frame $F2$ to the final spacecraft frame (S) rotated from $F2$ by angle Φ_1 about axis-1 (i.e., the x-axis):

$$\mathbf{V}_S = \boldsymbol{A}_{S<F2}\mathbf{V}_{F2} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(\Phi_1) & \sin(\Phi_1) \\ 0 & -\sin(\Phi_1) & \cos(\Phi_1) \end{bmatrix} \mathbf{V}_{F2}$$

For each of the three matrices $\boldsymbol{A}_{F1<G}$, $\boldsymbol{A}_{F2<F1}$, and $\boldsymbol{A}_{S<F2}$, we encourage you to compute a few examples so that you're comfortable that the minus signs are in the

correct positions in the three matrices. Now that you have the individual pieces, put them together as:

$$\mathbf{V}_S = \boldsymbol{A}_{S<F2}\boldsymbol{A}_{F2<F1}\boldsymbol{A}_{F1<G}\mathbf{V}_G = \boldsymbol{A}_{S<G}\mathbf{V}_G \tag{1.3}$$

If you want to go in the opposite direction, i.e., transform a vector from the spacecraft frame to the GCI frame, the appropriate equation is:

$$\mathbf{V}_G = \boldsymbol{A}_{G<S}\mathbf{V}_S = [\boldsymbol{A}_{S<G}]^T\mathbf{V}_S = [\boldsymbol{A}_{F1<G}]^T[\boldsymbol{A}_{F2<F1}]^T[\boldsymbol{A}_{S<F2}]^T\mathbf{V}_S$$

where the superscript T indicates matrix transposition. Direction cosign matrices (a.k.a., rotation matrices) are a special subclass of 3×3 matrices for which the matrix inverse is equal to the matrix transpose.

Equation 1.3 specifies the particular Euler angle convention employed by the ground system for the Hubble Space Telescope (HST) mission, where the order for the roll, declination, and right ascension rotations were arrived at in what passes in aerospace work for a collegial atmosphere. Two analysts (including one of the authors), who were also best friends, argued with and yelled at each other for about an hour until they realized they were saying the same thing (the author was an "activist", the other analyst was a "passivist", but definitely not a "pacifist").[2]

Getting back to our original discussion of attitude formalisms, you might ask how is it that the 9 numbers of the DCM can say the same thing, nothing more and nothing less, as 3 Euler angles. That's 6 more numbers than we started with but no more content (please, no wisecracks about government projects). The reason this can be true is that unlike the independent nature of the 3 Euler angles, which can take on any values within their allowed ranges, the 9 direction cosine numbers are highly correlated with each other. The fact that the DCM is an orthonormal matrix requires that the columns of the matrix, when viewed as three 3-dimensional vectors, be mutually orthogonal (i.e., their dot products with each other yield value zero) and be normalized (i.e., their dot products with themselves yield value unity). That places 6 conditions (3 from orthogonality and 3 from normality) on the 9 numbers, each of which has a range from -1 to 1. Counting independent pieces of information, the 9 numbers of the DCM represent only 3 independent pieces of information, the

[2] While we're on the subject of convention confusions, there's a classic source of misunderstandings between spacecraft builders and spacecraft users that arises entirely from their respective roles. If you're building a spacecraft, you literally see the thing in front of you and have the perspective of a god on the outside looking down on (actually more likely up to, unless it's a very small spacecraft) your creation. So, for example, you see the star trackers attached to the body of the spacecraft and see the sensor's field of view (FOV) from the outside looking in. By contrast, if you're using a spacecraft, it's way above your head in orbit, so you imagine yourself to be a bug sitting inside the spacecraft (or, if you're a Trekkie, you imagine yourself to be Captain Kirk standing on the bridge of the Enterprise) looking out on the heavens through the windows provided by the sensors. This difference in perspective has led to numerous heated arguments between hardware providers and ACS analysts regarding reference frame polarities, and even worse, can produce an incorrect impression of agreement pre-launch, starting a ticking time bomb that will wait until a critical moment in the mission post-launch to rear its ugly head.

same as contained by the set of 3 Euler angles. You can play the same game with the DCM rows, but those 6 conditions can be shown to be redundant with the 6 constraints on the columns. But in return for expressing the same information in a less compact (and therefore less efficient) fashion and abandoning the visual ease of the Euler angle formulation, we've gained a straightforward means to transform vectors quantitatively between reference frames, an essential capability if we are to be able to determine and control the spacecraft pointing accurately in realtime. We also get as a side benefit a very convenient gimmick for computing the inverse pointing. Because a direction cosine matrix is orthonormal, its inverse is the same as its transpose (i.e., you get the inverse by exchanging rows and columns). So if you know the rotation matrix that takes vectors from the GCI frame to the body frame, all you have to do is flip the matrix indices to get the rotation matrix that turns body vectors into GCI vectors.

1.5 Defining Attitude via the Eigenvector and Rotation Angle

Still, utilizing 9 numbers to do the work of 3 seems pretty inefficient, although nowadays flight computer memory and computing power has improved to the point that this inefficiency no longer poses the serious problem it once did to the first generation of onboard computers (OBCs). A useful alternative to these two approaches (direction cosines vs. Euler angles) is obtained by breaking away from the key concept the first two approaches have in common, namely that a general rotation in 3-dimensional space should be built up from a series of 3 rotations about 3 axes. One could equally well view the process of rotating an object in 3-dimensional space as a single, right-handed rotation about a single generalized axis, the generalized axis being any unit vector in the starting reference frame. (That the rotation between any two reference frames can be expressed as a single rotation about a single axis was first demonstrated by Euler.) Because the axis of rotation is an invariant under its rotation, the components of the rotation vector in the starting reference frame will not only have the same numerical values in the ending reference frame, they will also have the same values at any intermediate "snapshot" along the way. For this reason, the rotation axis is also called the eigenvector, where "eigen" means "same" in German. Considering some of the applications we discussed earlier, the rotation angle could be the slew angle if you were maneuvering the spacecraft between attitudes, the angular error between the commanded and measured attitudes, or simply the angular displacement from coincidence with the GCI axes needed to orient the spacecraft at the specified pointing.

So any rotation also can be described by four numbers, one number specifying the angle of rotation and three numbers specifying the axis of rotation. Since the rotation axis is a unit vector, there is a normalization constraint on the rotation axis's three numbers, reducing that information content from three to two. Therefore the four numbers of the single rotation angle-plus-vector formulation represent just three independent pieces of information, the same as the Euler angle or DCM formulation. As for the Euler angle formulation, one can visualize what the rotation about that one axis will look like, but (as for the Euler angle formulation) you can't easily transform a vector between frames without (for example) first converting from the rotation angle-plus-vector formulation to a DCM. Equation 1.4[3] supplies a messy looking matrix that does that.

$$A = \begin{pmatrix} \cos\phi + e_1^2(1-\cos\phi) & e_1e_2(1-\cos\phi) + e_3\sin\phi & e_1e_3(1-\cos\phi) - e_2\sin\phi \\ e_1e_2(1-\cos\phi) - e_3\sin\phi & \cos\phi + e_2^2(1-\cos\phi) & e_2e_3(1-\cos\phi) + e_1\sin\phi \\ e_1e_3(1-\cos\phi) + e_2\sin\phi & e_2e_3(1-cos\phi) - e_1\sin\phi & \cos\phi + e_3^2(1-\cos\phi) \end{pmatrix} \tag{1.4}$$

where

$\boldsymbol{A}$ = the direction cosine attitude matrix
(e_1, e_2, e_3) = eigenvector
ϕ = slew angle

However, in the case of the single rotation angle-plus-vector formulation, there is a more direct analog to the Euler angle's DCM, the *attitude quaternion*, which is the subject of our next section.

1.6 Defining Attitude via Quarternions

Quaternions seem to generate a wide variety of emotional reactions among people in the aerospace field, such as fear, anger, disbelief, etc. Perhaps the parallels to psychoanalysis aren't so strained after all. Quaternion algebra was first introduced by Sir William Hamilton in the 1840s.[4] For a number of decades, it became the dominant method of expressing much of physics (including kinematics and electromagnetic theory), eventually being replaced by vector analysis starting in the mid 1880s. However, because of its compact form and associated numerical efficiency, the use of quaternion algebra has made a comeback in a number of fields, including attitude control for aerospace systems.

Quaternion algebra may be viewed as an extension of complex algebra, wherein the single imaginary element of the latter ($i = \sqrt{-1}$) is replaced with a triplet of imaginary elements (i, j, k) satisfying Eq. 1.5:

$$i^2 = j^2 = k^2 = -1 \tag{1.5a}$$

$$ij = k;\ jk = i;\ ki = j \tag{1.5b}$$

[3] Wertz, Spacecraft Attitude Determination and Control, p. 413.

[4] W. Hamilton, "On Quaternions, or On a New System of Imaginaries in Algebra", in 18 installments in volumes 25–36 of The London, Edinburgh, and Dublin Philosophical Magazine and Journal of Science, 1844–1850. See http://www.emis.de/classics/Hamilton/OnQuat.pdf.