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Ahmer Mehmood

Viscous Flows

Stretching and Shrinking of Surfaces

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Mathematical Engineering

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Stretching and Shrinking of Surfaces

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To my respected Parents and beloved Family

Preface

The boundary-layer flow past bodies of finite lengths has a long history as old as the concept of boundary-layer itself. Such kind of flows had completely been explored till the completion of first fifty years of the boundary-layer theory. In contrast, the boundary-layer flow due to moving continuous surfaces was first introduced in 1961, almost six decades later to the idea of boundary-layer. Besides the interesting nature of this flow, it has so far not been explored in complete. Even the two-dimensional self-similar case of this flow cannot be claimed to be fully explored and understood, despite the presence of hundreds of published research papers on this flow. The biggest misfortune with this flow is that it had never been studied completely; rather, the developments on this flow had been contributed in bits. The situation is far worse in the cases of axisymmetric and three-dimensional flows of this class. Only the self-similar laminar flows of this type have so far been investigated in literature, and no attention has been given to the non-similar and turbulent flows at all.

A critical review of the published literature on this topic reveals the presence of huge number of those published research papers which do involve incorrect and misleading analyses. Unfortunately, after getting published, such researches become an authentic reference regarding the further propagation and justification of such misleading erroneous analyses. In this way, the research on this topic has, by a lot, went rotten because of the publication of huge number of erroneous research papers. Unfortunately, the published wrong results are immediately adopted by the others instead of correcting them. In such a messy situation, it is really quite hard to correct all such erroneous literature by making all the audience aware of such mistakes.

A thorough review of the available literature on this topic concludes that the majority of the errors have arose due to the incorrect self-similar formulation of the governing systems; examples can be given of the problems concerning shrinking surfaces or those involving local parameters in the governing equations. Therefore, it seems that if the concept of self-similarity could be explained in detail and the construction of self-similar variables of these flows could be made available, then the errors are expected to be minimized to an appreciable extent. Such an elaboration can further be expected to be helpful to the researchers in the exploration of further self-similar flows of this class.

After having a realization of the above facts, the author had continuously been worried, since last few years, regarding the correction of aforementioned incorrect analyses. Writing a correction or comment to every such paper was, however, quite inconvenient in this regard. Finally, it was decided to identify the root causes of such incorrect analyses and the way out toward their correction and to report this all in the form of a book at once. In this regard, the incomplete understanding of the self-similarity was identified to be the major root cause behind all such incorrect analyses, at the most, as pointed out in the above paragraph. In view of these facts, the primary objective of this book is threefold: first, to elaborate the general criterion of self-similarity by reporting the general self-similarity criterion for the planar and the axisymmetric cases; second, the presentation of correct shrinking surface flow analysis which could negate most of the "mysterious" facts associated with this flow; and third, to introduce the non-similar flows of this class in the said two cases, namely the planar and the axisymmetric ones. In this regard, the self-similarity criterion for this class of flows has completely been determined and the associated self-similar governing systems have been developed. Correct self-similar formulation of the shrinking sheet flow has been reported, and the self-similar shrinking sheet flow has been discussed in detail. A comparison between the current and the already existing formulations has been made in order to clarify the situation. The non-similar flows of this class have been formulated in general; some particularly chosen non-similar flows in the planar and axisymmetric cases have also been discussed. The identification of temporal self-similarity and the criterion of semi-similarity have been included. Finally, the turbulent flow due to stretching surfaces has also been considered.

Fundamental knowledge of fluid mechanics and the boundary-layer theory is essential for the understanding of the presented material. This book particularly focuses the students and the initial researchers in this field. Therefore, the presentation of the material is quite straightforward with a bit detail and sufficient explanation. However, the presented material is also expected to be of equal importance for the specialized and established researchers in this field.

This book has mainly been distributed into four major parts. The first part includes some fundamental essential knowledge and the explanation of the concept of self-similarity. Part II contains the self-similar flows due to moving continuous surfaces including the planar and axisymmetric flows. Spatial and temporal non-similarity has been modeled in Part III, whereas the turbulent flows due to moving continuous surfaces have been considered in Part IV.

First four chapters constitute the Part I of this book. Boundary-layer character of the flows due to moving continuous surfaces has been explained in Chap. 1. The governing boundary-layer equations and the momentum integral equations corresponding to the planar and axisymmetric flows have been developed in Chap. 2. The concept and restrictions of self-similarity have been explained in detail in Chap. 3, whereas an introduction to the suitable solution techniques has been given in Chap. 4. The criterion of self-similarity for the wall velocities has been determined in detail for both the planar and axisymmetric flows in Chap. 5. Flows due to stretching and shrinking surfaces have been discussed in Chaps. 6 and 7, respectively. The restriction on the wall suction/injection velocities and on the variable thickness of the continuous surfaces, in view of self-similarity criterion determined in Chap. 5, has also been determined in these chapters. Similarity criterion of the unsteady flows due to moving continuous surfaces has been derived in Chap. 8. The aforementioned Chaps. 5–8 have been included in Part II. Non-similar flows due to moving continuous surfaces have been introduced in Part III consisting of Chaps. 9–11. The planar and axisymmetric non-similar flows have been considered in Chaps. 9 and 10, respectively, whereas the temporal non-similarity has been considered in Chap. 11. The Part IV includes only one chapter (Chap. 12) concerning the turbulent flow due to moving and stretching continuous surfaces.

Islamabad, Pakistan Ahmer Mehmood

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January 2017 Ahmer Mehmood
Islamabad

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Introduction

The history of fluid mechanics is as old as the history of human beings. Every human in his life solves so many fluid mechanics problems whether consciously or unconsciously. But the manner he solves his daily life problems, such as dissolving sugar in the cup of tea by stirring a spoon in it or blowing the lump of hot food before taking it to mouth, is exactly in accordance with the scientific laws of fluid mechanics in convective phenomena. Similar examples can also be found in the Stone Age era when man had been using long, slim, and even fin-stabilized arrows for hunting the animals and birds. His understanding about the water flow from high level to low, in the process of irrigation, is also an example of utilizing the potential energy of water to make it to flow. Numerous similar examples can further be found from the practices of present and the history of ancient man where the above constitute only a few glimpses from it. Thus, the unconscious understanding of the human about the fluid flow and heat and mass transfer phenomena continuously turned into his conscious efforts toward the scientific exploration of the flow phenomena because of his day by day increasing problems of fluid mechanics.

The first, on the record, scientific theory in fluid mechanics is due to the Archimedes in which he presented his research as postulates of buoyancy. The viscous resistance in fluids was scientifically interpreted by Sir Isaac Newton in 1687 when he stated his famous law of viscosity. The law of fluid motion was first proposed by Daniel Bernoulli in 1730 and was further improved by Leonhard Euler in 1755. It is important to note that although the Newton's law of viscosity was discovered in 1687 and the Bernoulli's equation, after Euler's modification, in 1755, but they intentionally ignored the fluid friction. The fluid friction was taken into account by Navier and Stokes independently where they introduced the viscous terms to the equation of motion in 1827 and 1845, respectively. Consequently, the resulting equations were named as Navier–Stokes equations and are still recognized by this name. These equations are equally applicable to gasses and liquids following the Newton's law of viscosity. Later, the Osborn Reynolds distinguished the viscous flows into two categories on the basis of velocity magnitude. However, he also explored that this differentiation does not depend strictly upon the fluid velocity only but obviously upon the viscosity of fluid and the pipe radius also.

On the basis of this argument, he developed the famous *Reynolds number* which has great practical importance in laminar and turbulent flows having velocities less than the speed of sound.

In 1749, a French mathematician Jean le Rond D'Alembert, while working on the flow drag on a solid surface, concluded that "it seems to me that the theory (potential flow), developed in all possible rigor, gives, at least in several cases, a strictly vanishing resistance, a singular paradox which I leave to future Geometers (i.e., mathematicians—the two terms were used interchangeably at that time) to elucidate." On the other hand, the experimental results reflected significant viscous drag for the flows in water and air at that time. Based on the failure of the theory regarding the prediction of viscous drag, D'Alembert stated his results in the form of a famous paradox in 1752 which stayed unresolved till the year 1904. The D'Alembert's paradox states that "for incompressible and inviscid potential flow—the drag force is zero on a body moving with constant velocity relative to the fluid." At this stage, the reader is asked to realize that the D' Alembert's paradox stayed unresolved even after the development of the Navier–Stokes equations which completely incorporate the contribution of viscous forces to the momentum transport equation. The misfortune with the D'Alembert and the other scientists of that time was that they used to ignore the "little" air friction in the theoretical calculations causing them to reach the wrong conclusion of zero drag. On the other hand, such "little" air friction was impossible to be ignored in the experiments where it kept on resulting in significant viscous drag. This fact was actually realized by Ludwigs Prandtl, in his series of experiments, that in such flows, with less friction, the viscous effects are negligible in most of the flow domain but are in-ignorable in a very thin region near the solid surface. This observation made him able to split the entire flow domain into two major parts: a potential flow region where there is no flow resistance at all and the near wall region where the effects of viscous resistance are prominent. He called this thin region as the *thin shear layer* or the *boundary-layer*. This thin, near wall, region was actually being ignored by the previous scientists, thus preventing them to reach the correct results. The development of a concrete theory for the accommodation of this fact was another difficult step for which Prandtl himself introduced the order of magnitude analysis. He calculated the magnitude of every term in the Navier–Stokes equations and identified the contributing/surviving and vanishing/ignorable terms. The process is strictly based on his clear understanding of exact nature of the flow within the boundary-layer. This theory revealed that most of the terms in viscous part of the Navier–Stokes equation are ignorable, as did by the previous scientists, but not the all. Thus, the identification of surviving and non-surviving terms does mainly based on the order of magnitude analysis which is actually due to the Ludwigs Prandtl.

Before Prandtl, there were two divergent branches of fluid dynamics, namely the theoretical *hydrodynamics* and the *hydraulics*. In the era of D'Alembert, it was misbelieved that the theoretical hydrodynamics does not apply to many practical situations, such as air or water flows, where the viscous drag does play important role in actual. On this basis, the engineers of that time started developing their own