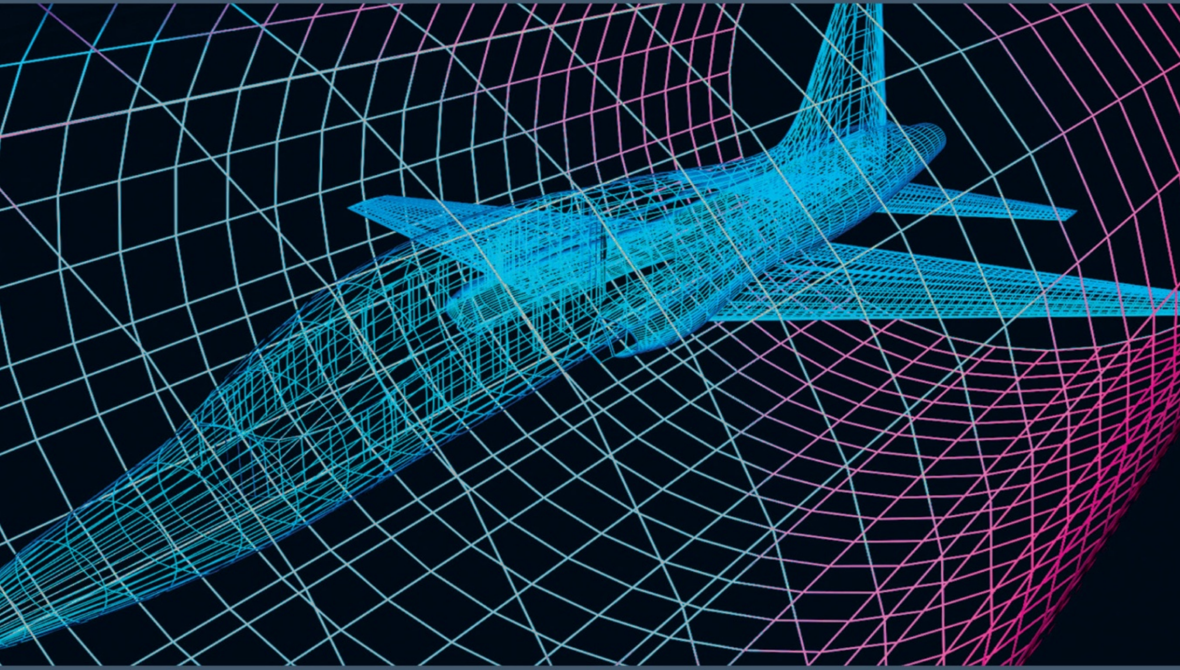
MECHANICAL ENGINEERING AND SOLID MECHANICS SERIES

Mechanics of Aeronautical Solids, Materials and Structures

Christophe Bouvet



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Contents

Foreword

This book follows a long-standing tradition of mechanical engineering tuition, which is already a century old and comes from the Toulouse mechanical engineering scientific community, now merged into the Institut Clément Ader (www.institut-clement-ader.org). Just as all of its illustrious predecessors, this book is very timely and illustrates the specificity and originality of the approaches that we have developed, which have both a high scientific standing and a quasi-permanent connection with the aeronautical industry. This publication provides the reader with the necessary knowledge and techniques to calculate structures and decompartmentalize disciplines and fields. The aeronautical engineer will find all of the helpful information he or she needs within these pages: the basis of continuum mechanics, the finite element method, and knowledge of materials, metals and composites, both within linear and non-linear fields. The information is presented in an extremely clear and educational manner. The reader may draw on an impressive series of exercises with detailed corrections, something which is not so commonly found.

Bruno CASTANIÉ
INSA Toulouse
Institut Clément Adler

Preface

This volume, on the mechanics of solids and materials, as well as aeronautical structures, aims to give an overview of the necessary notions for structure sizing within the aeronautics field. It begins by establishing all of the classic notions of mechanics: stress, strain, behavior law and sizing criteria. Also covered are notions that are specific to aeronautics, with a particular emphasis on the notion of limit loads and ultimate loads.

Different problem-solving methods, particularly the finite element method, are then introduced. The methods are not classically presented and instead energy minimization is drawn on in order to minimize the number of equations, all while remaining within a framework that we may comprehend “with their hands”.

The book then addresses the subject of plasticity, showcasing its influence on structure sizing, and especially the advantages it has for sizing criteria.

Finally, the physics of the two main materials in aeronautics, namely aluminum and composite materials, is discussed, so as to shed light on the sizing criteria outlined in the previous chapters.

The corrected exercises help the student to test their understanding of the different topics.

What is so original about this book is that from the outset, it places itself within the field of aeronautics. Sizing criteria are indeed rather specific to this field. Nevertheless, the notions discussed remain valid for the majority of industrial fields: in Mechanical Engineering and Finite Elements these notions in fact remain the same.

Another original aspect of this work is that it consolidates basic continuum mechanics with a very succinct description of finite elements, and a description of the material aspect of the main materials used in aeronautical structures, that being aluminum and composites. This publication is therefore a summary of the basic knowledge deemed necessary for the ("Airbus") engineer working within research departments. The book is simultaneously aimed at both students who are beginning their training and also engineers already working in the field who desire a summary of the basic theories.

Lastly, the publication aims to limit the amount of formulas provided as much as possible, in order to highlight the significance of the physical. Any readers who may be interested in demonstrations are advised to refer to more specific and theoretical works, such as [COI 01, DUV 98, GER 73, HEA 77, KHA 95, LEM 96, MIR 03, SAL 01, UGU 03] and [THU 97], etc.

Christophe BOUVET
January 2017

Introduction

I.1. Outlining the problem

Let us consider a solid S that is subjected to imposed displacements and external forces.

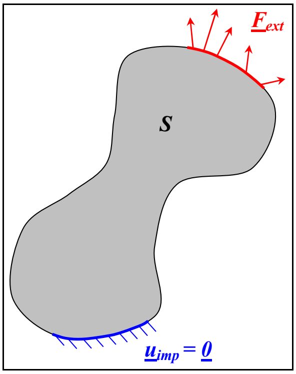

Figure I.1. *Outlining the problem. For a color version of this figure, see www.iste.co.uk/bouvet/aeronautical.zip*

The aim of the mechanics of deformable solids is to study the internal state of the material (notion of stress) and the way in which it becomes deformed (notion of strain) [FRA 12, SAL 01, LEM 96].

In mechanics, a mechanical piece or system may be designed:

– to prevent it from breaking;

– to prevent it from becoming permanently deformed;

– to prevent it from becoming too deformed, or;

– for any another purposes.

A solid shall be deemed a continuous medium, meaning that it shall be regarded as a continuous set of material points with a mass, representing the state of matter that is surrounded by an infinitesimal volume.

Mechanics of deformable solids enables the study of cohesive forces (notion of stress) at a point M, like the forces exerted on the small volume surrounding it, called a Representative Elementary Volume (REV). For metals, the REV is typically within the range of a tenth of a millimeter.

The matter in this REV must be seen as continuous and homogeneous:

– if it is too small, the matter cannot be as seen homogeneous: atomic piling, inclusion within matter, grains, etc. (for example: for concrete, the REV is within the range of 10 cm);

– if it is too big, the state of the cohesive forces in its center will no longer represent the REV state.

1

Stress

1.1. Notion of stress

1.1.1. *External forces*

There are three types of external forces:

– concentrated forces: this is a force exerted on a point (in Newton units, noted as N). In practice, this force does not actually exist. It is just a model. If we were to apply a force to a point that has zero surface, the contact pressure would be infinite and the deformation of the solid would therefore induce a non-zero contact surface. Nevertheless, it can still be imagined for studying problems with a very concentrated contact type load between balls. The results will thus yield an infinite stress and will need to be interpreted accordingly;

– surface forces, which will be noted as $\underline{F}_{ext}$ for the rest of this volume (in Pascal units, it is noted Pa). This type of force includes contact forces between two solids as well as the pressure of a fluid. Practically, any concentrated force can be seen as a surface force distributed onto a small contact surface;

– volume forces, which will be noted as $\underline{f}_v$ for the rest of this book (in N/m^3). Examples of volume forces are forces of gravity, electromagnetic forces, etc.

Incidentally, in this book you will notice that vectors are underlined once and matrices (or tensors of rank 2), which you will come across further on, are underlined twice.

1.1.2. *Internal cohesive forces*

We wish to study the cohesive forces of the solid S, at point M and which is in equilibrium under the action of external forces. The solid is cut into two parts E_1 and E_2 by a plane with a normal vector $\underline{n}$ passing through M. The part E_1 is in equilibrium under the action of the external forces on E_1 and the cohesive force of E_2 on E_1.

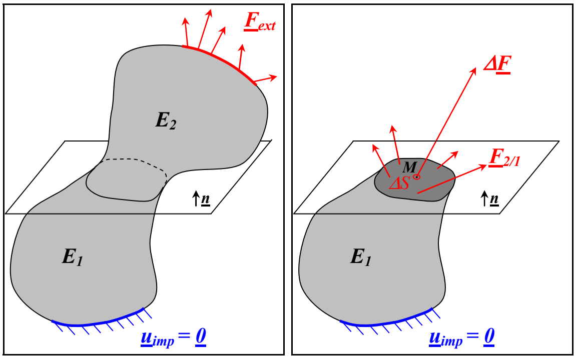

Figure 1.1. *Principle of internal cohesive forces. For a color version of this figure, see www.iste.co.uk/bouvet/aeronautical.zip*

Let ΔS be the surface around M and $\Delta \underline{F}$ be the cohesive force of 2 on 1 exerting on ΔS, then the stress vector at the point M associated with the facet with a normal vector $\underline{n}$ is called:

$$\underline{\sigma}(M,\underline{n}) = \lim_{\Delta S \to 0} \frac{\Delta \underline{F}}{\Delta S} = \frac{d\underline{F}}{dS} \qquad [1.1]$$

The unit is N/m^2 or Pa and we generally use MPa or N/mm^2.

Physically, the stress notion is fairly close to the notion of pressure that can be found in everyday life (the unit is even the same!), but as we will see further on, pressure is but only one particular example of stress.

1.1.3. *Normal stress, shear stress*

We define the different stresses as:

– normal stress, the projection of $\underline{\sigma}(M, \underline{n})$ onto $\underline{n}$, noted as σ;

– shear stress, the projection of $\underline{\sigma}(M, \underline{n})$ onto the plane with normal n, noted as τ.

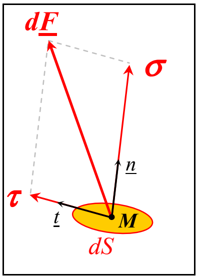

Figure 1.2. *Decomposition of a stress vector. For a color version of this figure, see www.iste.co.uk/bouvet/aeronautical.zip*

Thus, σ represents the cohesive forces perpendicular to the facet, meaning the traction/compression, and τ the forces tangential to the facet, meaning the shear. In a physical sense, the pressure found in our everyday lives is simply a normal compression stress.

We then definitely have:

$$\underline{\sigma}(M,\underline{n}) = \sigma\underline{n} + \tau\underline{t} \tag{1.2}$$

NOTE.– *$\underline{n}$ and $\underline{t}$ must be unit vectors*.

And conversely:

$$\begin{cases} \sigma = \underline{\sigma}(M,\underline{n}) \cdot \underline{n} \\ \tau = \underline{\sigma}(M,\underline{n}) \cdot \underline{t} \end{cases} \tag{1.3}$$

1.2. Properties of the stress vector

1.2.1. *Boundary conditions*

If $\underline{n}$ is an external normal, then:

$$\underline{\sigma}(M,\underline{n}_{ext}) = \underline{F}_{ext} \tag{1.4}$$

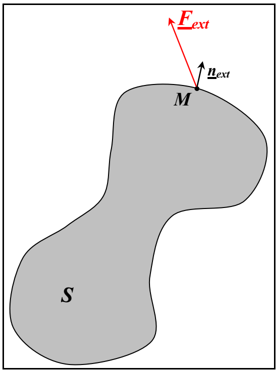

Figure 1.3. *External force and associated normal vector. For a color version of this figure, see www.iste.co.uk/bouvet/aeronautical.zip*

NOTE.– *$\underline{F}_{ext}$ is in MPa, and a normal external vector is always moving from the matter towards the exterior.*

So, $\underline{F}_{ext}$ can be seen as a stress vector exerted on *S*, particularly if the surface is a free surface:

$$\underline{\sigma}(M, \underline{n}_{ext}) = \underline{0} \tag{1.5}$$

These relations are important as they translate the stress boundary conditions on the structure. In order for this to be the solution to the problem (see Chapter 3), these relations are part of a group of conditions that are needed to verify a stress field.

EXAMPLE: TANK UNDER PRESSURE.–

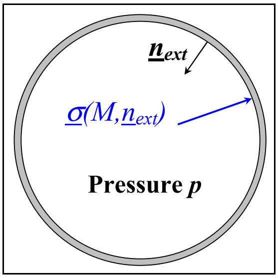

Figure 1.4. *Tank under pressure*

For every point on the internal wall of the tank, we find:

$$\underline{\sigma}(M, \underline{n}_{ext}) = -p \cdot \underline{n}_{ext} \qquad [1.6]$$

With the external normal vector moving towards the center of the circle, from where the normal and shear stresses are:

$$\begin{cases} \sigma = \underline{\sigma}(M, \underline{n}_{ext}) \cdot \underline{n}_{ext} = -p \\ \tau = \underline{\sigma}(M, \underline{n}_{ext}) \cdot \underline{t} = 0 \end{cases} \qquad [1.7]$$

Given that the normal stress is negative and the shear stress is zero, the material is subjected to pure compression. The first relation shows that the physical notion of pressure is simply a normal stress of compression: hence the minus sign before the pressure!

1.2.2. *Torsor of internal forces*

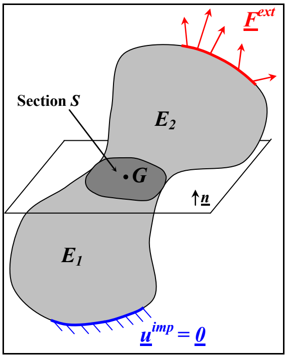

Figure 1.5. *Set of internal forces. For a color version of this figure, see www.iste.co.uk/bouvet/aeronautical.zip*

The torsor of internal forces of 2 on 1 at G, the center of gravity of S, is:

$$\left\{ T_{2/1}^{coh} \right\}_G = \left\{ \begin{matrix} \underline{R}_{2/1} \\ \underline{M}_{2/1}(G) \end{matrix} \right\} \qquad [1.8]$$

At first sight, the torsor notion may seem primitive but it enables us to simply consolidate the force with the moment. Should the notion of torsor bother you, you may settle for referring to it in plainer language as force and moment. However, you should not forget that when speaking about internal forces between 2 parts of a solid, it needs to be remembered that there is a force (in N) and a moment (in N.mm). The ambiguity comes from the term "force", which is used for a force (in the common everyday sense of the word), and as a whole, force + moment!

Let us now seek to link this set of internal forces to the previously discussed stress vector. We then have:

$$d\underline{F}_{2/1}(M) = \underline{\sigma}(M,\underline{n}).dS \tag{1.9}$$

therefore:

$$\begin{cases} \underline{R}_{2/1} = \sum\limits_{M \in S} d\underline{F}_{2/1}(M) = \iint_S \underline{\sigma}(M,\underline{n}).dS \\ \underline{M}_{2/1}(G) = \sum\limits_{M \in S} d\underline{M}_{2/1}^{(M)}(G) = \sum\limits_{M \in S} \underline{GM} \wedge d\underline{F}_{2/1}(M) \\ \qquad\qquad\qquad\qquad = \iint_S \underline{GM} \wedge \underline{\sigma}(M,\underline{n}).dS \end{cases} \tag{1.10}$$

These relations are somewhat (or very) complex, but physically, they simply translate the fact that if we add up all of the stress vectors on section S, then we will obtain the force of part E_2 on part E_1. Lastly, we should not forget that when we add up the stress vectors, we will obtain not only a force, but also a moment (which obviously depends on the point at which it is calculated).

These relations can also be written on an external surface as:

$$\begin{cases} \underline{R}_{ext/1} = \iint_{S_{ext}} \underline{\sigma}(M,\underline{n}_{ext}).dS = \iint_{S_{ext}} \underline{F}_{ext}.dS \\ \underline{M}_{ext/1}(G) = \iint_{S_{ext}} \underline{GM} \wedge \underline{\sigma}(M,\underline{n}_{ext}).dS = \iint_{S_{ext}} \underline{GM} \wedge \underline{F}_{ext}.dS \end{cases} \tag{1.11}$$

These relations are important because in practice, although we know the resultant $\underline{R}_{ext/1}$ or $\underline{M}_{ext/1}$, we do not generally know $\underline{F}_{ext}$. In fact, an external force is practically applied via the intermediary of a beam, a screed, a jack, etc., and the applied resulting force (or the moment) is known, but the way in which it is divided is unknown.

EXAMPLE: TRACTION.–

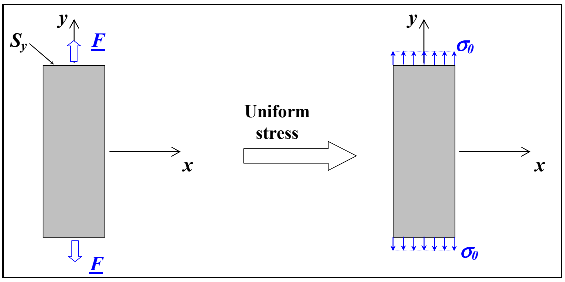

Figure 1.6. *Tensile test. For a color version of this figure, see www.iste.co.uk/bouvet/aeronautical.zip*

In a tensile test, we know that the resultant of the forces applied to S_y is worth $\underline{F}$:

$$\begin{cases} \iint_{S_y} \underline{\sigma}(M,\underline{y}).dS = \underline{F} \\ \iint_{S_y} \underline{GM} \wedge \underline{\sigma}(M,\underline{y}).dS = \underline{0} \end{cases} \quad [1.12]$$

However, in order to deduce that:

$$\underline{\sigma}(M,\underline{y}) = \frac{\underline{F}}{S_y} = \sigma_0 \cdot \underline{y} \quad [1.13]$$

we must add a homogeneity hypothesis of the force applied which remains to be verified. Incidentally, we can demonstrate that the two previous integrals are verified with this stress vector.

EXAMPLE: BENDING.–

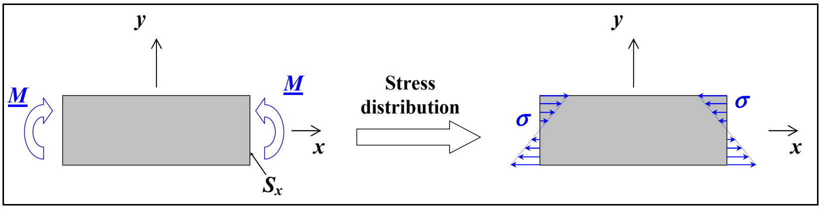

Figure 1.7. *Bending test. For a color version of this figure, see www.iste.co.uk/bouvet/aeronautical.zip*

In a pure bending test, we know that the resultant of the forces applied to S_x is worth $M.\underline{z}$:

$$\begin{cases} \iint_{S_X} \underline{\sigma}\left(M,\underline{x}\right).dS = \underline{0} \\ \iint_{S_X} \underline{GM} \wedge \underline{\sigma}\left(M,\underline{x}\right).dS = M.\underline{z} \end{cases} \qquad [1.14]$$

However, by deducing that on S_x:

$$\underline{\sigma}\left(M,\underline{x}\right) = \frac{-M}{I_z}.y.\underline{x} \qquad [1.15]$$

This formula is a classic example of the mechanics of material which we will discuss (and demonstrate) again when doing the exercises. Should you need to, you can read a more detailed publication, such as [AGA 08, BAM 08, CHE 08, DEL 08, DUP 09], etc.

Obviously, with the moment of inertia:

$$I_z = \iint_{S_x} y^2.dS \qquad [1.16]$$

we must add a linear distribution hypothesis of the stress applied which remains to be verified. Incidentally, we can demonstrate that the two previous integrals are verified with this stress vector.

1.2.3. *Reciprocal actions*

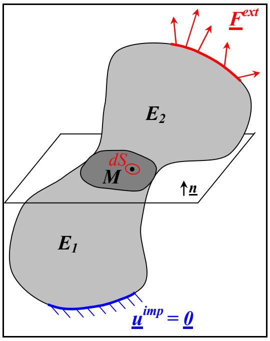

Figure 1.8. *Reciprocal actions. For a color version of this figure, see www.iste.co.uk/bouvet/aeronautical.zip*

According to the Law of Reciprocal Action, we have:

$$d\underline{F}_{2/1} = -d\underline{F}_{1/2} \qquad [1.17]$$

Yet:

$$\begin{cases} d\underline{F}_{2/1}(M) = \underline{\sigma}(M,\underline{n}).dS \\ d\underline{F}_{1/2}(M) = \underline{\sigma}(M,-\underline{n}).dS \end{cases} \qquad [1.18]$$

Hence:

$$\underline{\sigma}(M,\underline{n}) = -\underline{\sigma}(M,-\underline{n}) \qquad [1.19]$$

This can be translated by the fact that a fine slice of matter of surface dS, which has a normal vector $+\underline{n}$ on one side and $-\underline{n}$ on the other, is at equilibrium under the action of the two opposing forces $\underline{\sigma}(M, \underline{n}).dS$ and $\underline{\sigma}(M, -\underline{n}).dS$. Evidently, it is very much at equilibrium.

1.2.4. *Cauchy reciprocal theorem*

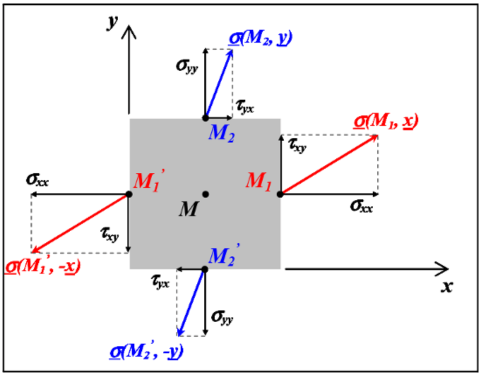

Figure 1.9. *Stress vectors on the faces of a square. For a color version of this figure, see www.iste.co.uk/bouvet/aeronautical.zip*

Let us put this in 2D, in order to make the demonstration easier.

A square is assumed to be infinitely small, therefore the stresses are assumed to be constant everywhere in the cube, hence we have:

$$\begin{cases} \underline{\sigma}\left(M_1,\underline{x}\right)=\underline{\sigma}\left(M,\underline{x}\right)=\sigma_{xx}\cdot\underline{x}+\tau_{xy}\cdot\underline{y} \\ \underline{\sigma}\left(M_2,\underline{y}\right)=\underline{\sigma}\left(M,\underline{y}\right)=\sigma_{yy}\cdot\underline{y}+\tau_{yx}\cdot\underline{x} \\ \underline{\sigma}\left(M_1',-\underline{x}\right)=-\underline{\sigma}\left(M,\underline{x}\right)=-\sigma_{xx}\cdot\underline{x}-\tau_{xy}\cdot\underline{y} \\ \underline{\sigma}\left(M_2',-\underline{y}\right)=-\underline{\sigma}\left(M,\underline{y}\right)=-\sigma_{yy}\cdot\underline{y}-\tau_{yx}\cdot\underline{x} \end{cases} \quad [1.20]$$

In the notation of τ_{xy}, the first "x" corresponds to the direction of the facet, meaning the normal vector on the cutting plane in question, and the second index "y" represents the direction of the stress.

The equilibrium equation on the square is written as:

$$\Sigma\left\{T_{ext/cube}^{coh}\right\}=\{0\} \quad [1.21]$$

which, for the force equation, induces the following:

$$\underline{\sigma}\left(M_1,\underline{x}\right).dy.dz+\underline{\sigma}\left(M_2,\underline{y}\right).dx.dz+\underline{\sigma}\left(M_1',-\underline{x}\right).dy.dz+\underline{\sigma}\left(M_1',-\underline{y}\right).dx.dz=\underline{0} \quad [1.22]$$

This is an automatically verified equation. For the moment equation in M at the center of the square, the below is induced:

$$\begin{aligned}&\underline{MM}_1\wedge\underline{\sigma}\left(M_1,\underline{x}\right).dy.dz+\underline{MM}_2\wedge\underline{\sigma}\left(M_2,\underline{y}\right).dx.dz \\ &\qquad+\underline{MM}_1'\wedge\underline{\sigma}\left(M_1',-\underline{x}\right).dy.dz+\underline{MM}_2'\wedge\underline{\sigma}\left(M_2',-\underline{y}\right).dx.dz=\underline{0}\end{aligned} \quad [1.23]$$

where the Cauchy reciprocity theorem is:

$$\tau_{xy}=\tau_{yx} \quad [1.24]$$

It can be shown in the same way in 3D:

$$\begin{cases}\tau_{xy}=\tau_{yx}\\ \tau_{xz}=\tau_{zx}\\ \tau_{yz}=\tau_{zy}\end{cases} \quad [1.25]$$

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