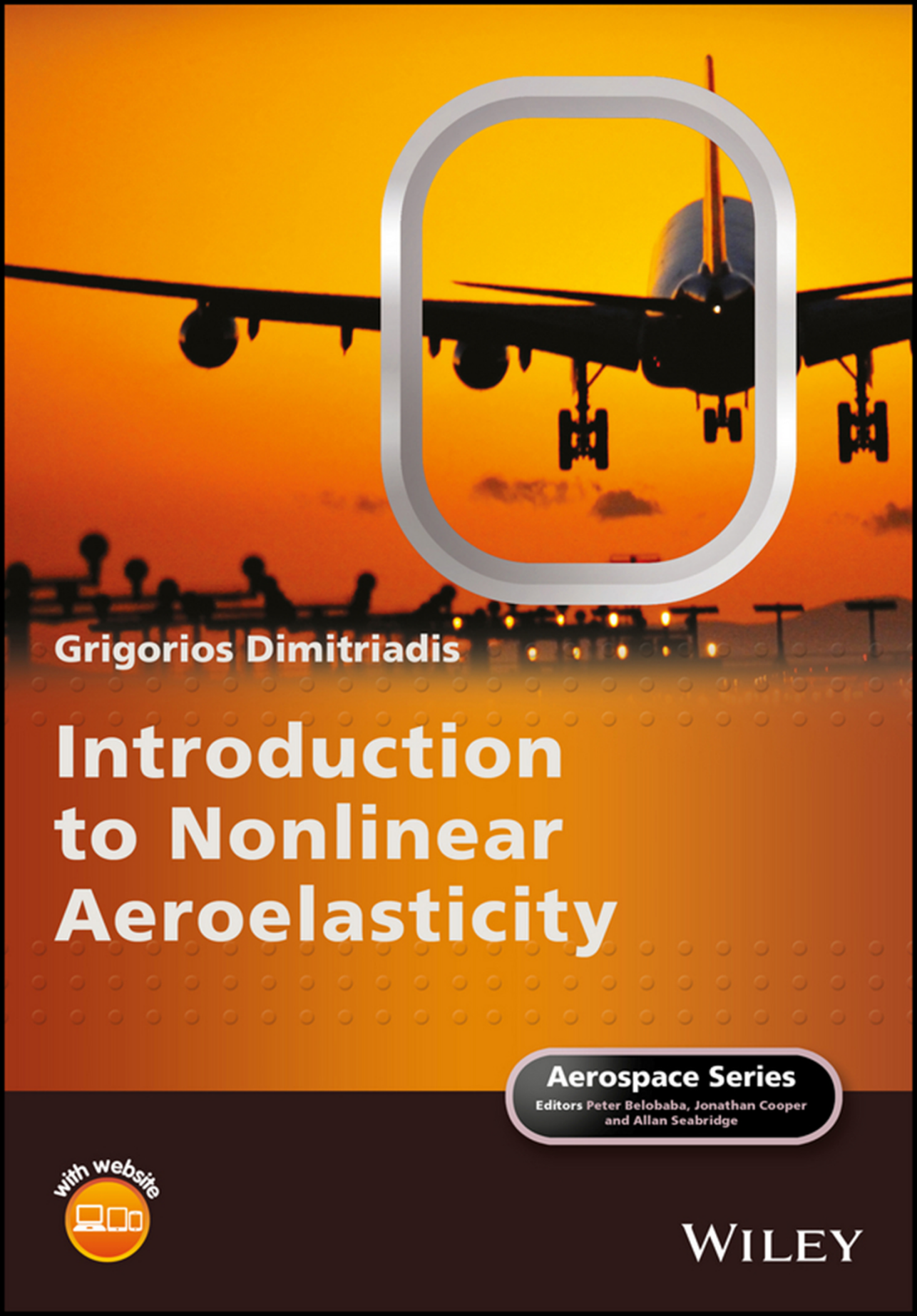




Grigorios Dimitriadis

Introduction to Nonlinear Aeroelasticity

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INTRODUCTION TO NONLINEAR AEROELASTICITY

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Preface

Aeroelasticity is the study of the static and dynamic interaction between airflow and flexible structures. Classical aeroelasticity deals with linearised problems; all displacements are small, all springs are perfect, all contacts are smooth and all flows are attached. Nonlinear aeroelasticity does exactly the opposite; it studies the static and dynamic interaction between airflow and flexible structures in the presence of large deformations, friction, freeplay in actuators, backlash in gears, nonlinear control laws, flow separation, oscillating shock waves and other nonlinear phenomena. The combined output of researchers in the field has now reached the level of maturity necessary to make nonlinear aeroelasticity an important and useful branch of engineering.

This book is an introduction to nonlinear aeroelasticity, which means that it aims to present the phenomena of interest and the most common analysis methodologies. The emphasis of the discussion is on application, so that all theories are accompanied by practical examples solved by means of Matlab codes. The latter are available to the reader on the Wiley website; they have been tested on Matlab versions 2013 and 2014 but could also be compatible with earlier versions. The reader should note that the purpose of the codes is to illustrate the examples and the underlying theories. They solve the particular problems for which they were written but they should not be seen as general nonlinear dynamic analysis codes that can be directly applied to different problems.

Chapter 1 is a brief introduction to nonlinear aeroelasticity, summarising some of the main advances accomplished since the 1940s. Chapter 2 is an introduction to nonlinear dynamics, discussing issues such as fixed point stability and limit cycle oscillations on very simple aeroelastic systems. The solution methods used at this stage are still either qualitative or approximate; Chapter 3 presents in detail the numerical solution methods for calculating the time response of nonlinear systems that will be used throughout the rest of the book. Additional numerical methodologies for analysing nonlinear system responses are presented in Chapter 4.

The bulk of the dynamic phenomena typically encountered in nonlinear aeroelastic systems are demonstrated on a fundamental model in Chapter 5, while Chapter 6 presents additional phenomena caused by non-smooth nonlinearities. Numerical continuation approaches are introduced in Chapter 7 and are used throughout the rest of the book. The focus of Chapter 8 is low-speed aerodynamic nonlinearities caused by unsteady flow separation, while high-speed nonlinear aeroelastic phenomena are discussed in Chapter 9. Finally, Chapter 10 introduces nonlinear structural and linear aerodynamic modelling techniques for finite wings.

I have worked on nonlinear dynamic and aeroelastic research since 1995 and I would like to take this opportunity to thank all the people with whom I have collaborated on the subject over the years. In particular, I would like to thank Jonathan Cooper who introduced me to aeroelasticity and supervised my doctoral research. I would also like to thank Earl Dowell, Bob Kielb, Gareth Vio, Xavier Amandolese and Pascal Hemon who welcomed me in their departments during my sabbatical year.

Dimitriadis: Nonlinear Aeroelasticity – Series Preface Oct 2016

The field of aerospace is multi-disciplinary and wide ranging, covering a large variety of products, disciplines and domains, not merely in engineering but in many related supporting activities. These combine to enable the aerospace industry to produce innovative and technologically advanced vehicles. The wealth of knowledge and experience that has been gained by expert practitioners in the various aerospace fields needs to be passed onto others working in the industry and also researchers, teachers and the student body in universities.

The *Aerospace Series* aims to be a practical, topical and relevant series of books aimed at people working in the aerospace industry, including engineering professionals and operators, engineers in academia and allied professions, such as commercial and legal executives. The range of topics is intended to be wide ranging, covering design and development, manufacture, operation and support of aircraft, as well as topics such as infrastructure operations and current advances in research and technology.

Aeroelasticity is the scientific discipline that arises from the interaction of aerodynamic, elastic and inertial forces, and has a significant effect upon the design and performance of all aircraft. The influence of nonlinearities, appearing in structures, aerodynamics and control systems, can have a major influence upon aeroelastic behaviour; for instance, phenomena such as Limit Cycle Oscillations can only occur in nonlinear systems.

This book, *Introduction to Nonlinear Aeroelasticity*, provides an excellent introduction to the effects of structural and aerodynamic nonlinearities on aeroelastic behaviour and describes a number of methodologies to predict the resulting behaviour. The text is complemented with a comprehensive set of Matlab codes that will enable the reader to readily apply the methods themselves. This book makes a strong addition to the Wiley Series' existing content in aeroelasticity and related topics.

Peter Belobaba, Jonathan Cooper and Alan Seabridge

About the Companion Website

Don't forget to visit the companion website for this book:

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1

Introduction

Nonlinear aeroelasticity is the study of the interactions between inertial, elastic and aerodynamic forces on engineering structures that are exposed to an airflow and feature non-negligible nonlinearity. There exist several good textbooks on linear aeroelasticity for aircraft (Bisplinghoff et al. 1996; Fung 1993; Hodges and Alvin Pierce 2002; Wright and Cooper 2015). Dowell (2004) even includes chapters on nonlinear aeroelasticity and stall flutter, while Paidoussis et al. (2011) discusses a number of nonlinear aeroelastic phenomena occurring in civil engineering structures. However, there is no introductory text that presents the methodologies of nonlinear dynamics and applies them to a wide range of nonlinear aeroelastic systems. The present book aims to fill this gap to a certain degree. The subject area is vast and multidisciplinary and it would be impossible to fit every aspect of it in a textbook. The main omission is high fidelity numerical simulation using Computational Fluid Dynamics and Computational Structural Dynamics solvers; these methodologies are already the subject of a dedicated text (Bazilevs et al. 2013). The aerodynamic models used in this book are analytical, empirical or based on panel methods while the structural models are either analytical or make use of series solutions.

The book is introductory but it assumes knowledge of structural dynamics, aerodynamics and some linear aeroelasticity. The main linear aeroelastic phenomena of flutter and static divergence are discussed in detail because they can affect nonlinear behaviour, but the present work is by no means a complete text on linear aeroelasticity. Unsteady aerodynamic modelling is used throughout the book and discussed in Chapters 8, 10 and in the Appendix. However, again this book is not a complete reference on unsteady aerodynamics, linear or nonlinear. On the other hand, nonlinear dynamics and bifurcation analysis are presented in great detail as they do not normally feature in most undergraduate or even graduate Aerospace and Mechanical Engineering courses. The emphasis of all discussions is on the application rather than the rigorous derivation of the theorems; there already exist several classic textbooks for the latter (Kuznetsov 1998; Guckenheimer and Holmes 1983). More application-based works on nonlinear dynamics also exist (e.g. Strogatz 1994) but they address a wide range of physical, chemical, biological, accounting models, to name a few, whereas the present book concentrates exclusively on aeroelastic phenomena.

Nonlinear aeroelasticity has become an increasingly popular research area over the last 30 years. There have been many driving forces behind this development, including faster computers, increasingly flexible structures, automatic control systems for aircraft and other engineering products, new materials, optimisation-based design methods and others. Aeroelasticians have acquired expertise from many different fields in order to address nonlinear aeroelastic problems, mainly nonlinear dynamics, bifurcation analysis, control theory, nonlinear structural analysis and Computational Fluid Dynamics. The main applications of nonlinear aeroelasticity lie in aeronautics and civil engineering but other types of structure are also concerned, such as bridges and wind turbines.

In classical linear aeroelasticity, the relationships between the states of a system and the internal forces acting on them are always assumed to be linear. Force-displacement diagrams for the structure and lift or moment curves for the aerodynamics are always assumed to be linear, while friction is neglected and damping is also linear. As an example, consider a torsional spring that provides a restoring moment M when twisted through an angle ϕ . Figure 1.1a plots experimentally measured values of ϕ and M . Clearly, the function $M(\phi)$ is not linear but, if we concentrate in the range $\phi = [-0.5^\circ \quad 2^\circ]$, the curve is nearly linear and we can curve fit it as the straight line $M = K\phi + M_0$, where K is the linear stiffness of the spring.

Figure 1.1b plots the aerodynamic lift coefficient acting on a wing placed at an angle α to a free stream of speed U , defined as

$$c_l = \frac{l}{1/2\rho U^2 c}$$

where l is the lift force per unit length, ρ is the air density and c is the chord. The curve $c_l(\alpha)$ is by no means linear but, again, if we focus in the range $\alpha = [-5^\circ \quad 10^\circ]$, we can curve fit the lift coefficient as the straight line $c_l = c_{l_a}\alpha + c_{l_0}$, where c_{l_a} is the lift curve slope. An aeroelastic system featuring the spring of Figure 1.1a and the wing of Figure 1.1b will be nonlinear but, if we ensure that ϕ and α never exceed their respective linear ranges for all operating conditions, then we can treat the system as linear and use linear analysis to design it. In nonlinear aeroelasticity, the angles ϕ and α will always exceed their linear ranges and therefore we must use nonlinear analysis, both static and dynamic, in order to design the system.

Nonlinear dynamics is the field of study of nonlinear ordinary and partial differential equations, which in this book model aeroelastic systems. Unlike linear differential equations, nonlinear equations have no general analytical solutions and, in some cases, several different solutions may coexist at the same operating conditions. Furthermore, nonlinear systems can have many more types of solution than linear ones. The operating conditions of an aeroelastic system are primarily the free stream airspeed and the air density (or flight altitude), while the Reynolds number, Mach number and mean angle of attack can also be important. As these system parameters vary, the number and type of solutions of the nonlinear equations of motion can change drastically. The study of the changing nature of solutions as the system parameters are varied is known as bifurcation analysis. In this book we will use almost exclusively local bifurcation analysis, which means that we will identify individual solutions and track their nature and their intersections with other solutions for all the parameter values of interest.

A wide variety of nonlinear aeroelastic phenomena will be investigated, from the galloping of cables to the buckling and flutter of panels in supersonic flow and from stall flutter to

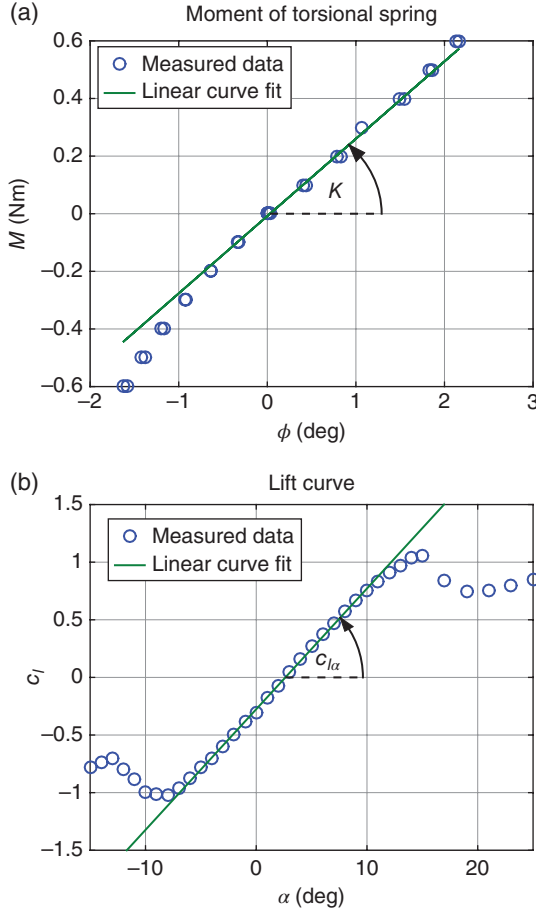


Figure 1.1 Linearised load-displacement diagrams

the limit cycle oscillations of finite wings. We will also briefly discuss transonic aeroelastic phenomena but we will not analyse them in detail because such analysis requires high fidelity computational fluid and structural mechanics and is still the subject of extensive research. The equations of motion treated in this book are exclusively ordinary differential equations; whenever we encounter partial differential equations we will first transform them to ordinary using a series solution. It is hoped that the book will contribute towards the current trend of taking nonlinear aeroelasticity out of the research lab and introducing it into the classroom and in industry.

1.1 Sources of Nonlinearity

Traditionally, a lot of effort has been devoted to designing and building engineering structures that are as linear as possible. Despite this effort, nonlinearity, weak or strong, has always

been present in engineering systems. In recent years, increasing amounts of nonlinearity have been tolerated or even purposefully included in many applications, since nonlinear analysis methods have progressed sufficiently to allow the handling of nonlinearity at the design stage. Furthermore, nonlinearity can have significant beneficial effects, for example in shock absorbers and suspension systems.

In this book we will only consider nonlinearities that are present in aeroelastic systems. Since aeroelasticity is of particular importance to the fields of aeronautics, civil engineering and energy harvesting, we will limit the discussion of nonlinearity to these application areas. The nonlinear functions that are most often encountered in these systems have three main sources:

- the structure,
- the aerodynamics and
- the control system.

The structural nonlinearities of interest occur during the normal operation of the underlying engineering system. Nonlinearities appearing in damaged, cracked, plastically deformed and, in general, off-design systems are beyond the scope of this book. The most common forms of nonlinearity appearing in structures are geometric (caused by large deformations), clearance (i.e. freeplay, contact and other non-smooth phenomena), dissipative (i.e. friction or other nonlinear damping forces) and inertial (of particular interest in rotors and turbomachinery).

Aerodynamic nonlinearities arise from the existence of either unsteady separated flow or oscillating shock waves or a combination of the two (e.g. shock-induced separation). Separation-induced nonlinearity can affect all aeroelastic systems, although bluff bodies such as bridges, towers and cables are always exposed to it. Shock-induced nonlinearity is of interest mostly to the aeronautical industry. It should be noted that aerodynamic nonlinearity is inertial, dissipative and elastic.

Engineering structures are increasingly designed to feature passive and/or active control systems. These systems can either aim to stabilise the structure (e.g. suppress or mitigate unwanted vibrations) or to control it (e.g. aircraft automatic flight control systems). Passive systems can be seen as parts of the structure and therefore included in the structural nonlinearity category (if they are nonlinear). Active systems, however, can feature a number of prescribed and incidental nonlinearities that can be turned off by running the structure in open loop mode. These nonlinear functions are in a category of their own and can take many forms, such as deflection and rate limits on actuators or nonlinear control laws. Furthermore, control actuators always feature a certain amount of freeplay, which is usually strictly limited by airworthiness regulations.

One more source of nonlinearity can be external stores on aircraft that carry them (mainly military aircraft). Stores such as external fuel tanks, bombs and missiles can cause store-induced oscillations, particularly at transonic flight conditions. However, the mechanisms behind these oscillations are still not fully understood and the relevant analyses usually involve computational fluid-structure interaction. Consequently, these phenomena will not be discussed further in this book. Human operator-related nonlinearities (pilot, driver, rider etc.) will not be considered either.

1.2 Origins of Nonlinear Aeroelasticity

Some of the first investigations of nonlinear aeroelasticity concerned stall flutter and started just after WWII. For example, Victory (1943) reported that the airspeed at which wings undergo flutter decreases at high incidence angles, while Mendelson (1948) attempted to model this phenomenon. Rainey (1956) carried out a range of wind tunnel experiments of aeroelastic models of wings and noted the parameters that affect their stall flutter behaviour. It was quickly recognised that, in order to analyse stall flutter, the phenomenon of unsteady flow separation known as dynamic stall needed to be isolated and studied in detail. Bratt and Wight (1945) and Halfman et al. (1951) carried out two of the first experimental studies of the unsteady aerodynamic loads acting on 2D airfoils oscillating at high angles of attack. They were to be followed by a significant number of increasingly sophisticated experiments, covering a wide range of airfoil geometries, Reynolds numbers, Mach numbers and oscillation amplitudes and frequencies. The phenomena of dynamic stall and stall flutter are discussed in Chapter 8.

The effects of structural nonlinearity were first investigated by Woolston et al. (1955, 1957) and Shen (1959). They both set up aeroelastic systems with structural nonlinearity and solved them using analog computers. The systems included 2D airfoils with nonlinear springs, wings with control surfaces and buckled panels in supersonic flow. Such systems have been explored ever since, using increasingly sophisticated mathematical and experimental methods. They are in fact the basis of nonlinear aeroelasticity and will be discussed in detail in the present book. Two-dimensional airfoils with nonlinear springs will be analysed in Chapters 2 to 7, panels in supersonic flow will be presented in Chapter 9 and 3D wings in Chapter 10.

Wind tunnel experiments on nonlinear aeroelastic systems with nonlinear springs have been carried out since the 1980s, notably by McIntosh Jr. et al. (1981); Yang and Zhao (1988); Conner et al. (1997). These works provided both valuable insights into the phenomena that can be encountered in nonlinear aeroelasticity and a basis for the validation of various modelling and analysis methods. The focus of the present book is the application of nonlinear dynamic analysis to nonlinear aeroelasticity. Modelling will be discussed in the last three chapters, as well as in the Appendix.

Shen (1959) was one of the first works to apply the Harmonic Balance method to nonlinear aeroelasticity. This method was first presented in the West by Kryloff and Bogoliuboff (1947) and has since become one of the primary analysis tools for nonlinear dynamic systems undergoing periodic oscillations. We will use several different versions of the Harmonic Balance technique throughout this book.

One of the first studies to apply elements of bifurcation theory to nonlinear aeroelastic systems was carried out by Price et al. (1994). They used stability boundaries, Poincaré sections and bifurcation diagrams to analyse the behaviour of a simple 2D mathematical nonlinear aeroelastic system with structural nonlinearity. Aside from the Hopf bifurcation, they also observed period-doubling bifurcations and chaotic responses. Bifurcation analysis is used throughout the present book but most of the bifurcations typically encountered in nonlinear aeroelasticity are discussed in detail in Chapter 5.

Alighanbari and Price (1996) were the first to use numerical continuation in nonlinear aeroelasticity. Numerical continuation (Allgower and Georg 1990) is a set of mathematical methods for solving nonlinear problems that have static or periodic dynamic solutions. Continuation methods are strongly linked to bifurcation analysis, as they very often start

evaluating solutions at bifurcation points. Such methods will be presented in detail in Chapter 7 and used in all subsequent chapters.

Towards the end of the 1990s, Friedmann (1999) identified nonlinear aeroelasticity as a major research direction in his paper on the future of aeroelasticity. Lee et al. (1999) published a lengthy and authoritative review of past and current nonlinear aeroelastic research, describing all major advances in both understanding and methodologies. A few years later, the nonlinear aeroelasticity chapter by Dowell (2004) provided an extensive description of nonlinear aeroelastic phenomena encountered in flight and in benchmark aeroelastic wind tunnel models and summarised the state of the art.

Thirteen years later, there has been a significant increase in the research and application of nonlinear aeroelasticity. Transonic aeroelastic phenomena, the highly flexible structures of High Altitude Long Endurance aircraft, aeroelastic tailoring, gust loads acting on nonlinear aircraft, wind turbine aeroelasticity and high-fidelity fluid structure interaction have all become major areas of research. Major national and international research projects have addressed such issues and the results are slowly starting to be applied in industry. Given this wealth of activity in the field, it was felt that an introductory text in nonlinear aeroelasticity is missing from the literature. It is hoped that the present book will come to fill this gap, providing a basis for understanding nonlinear aeroelastic phenomena and methodologies on relatively simple systems and preparing the reader for more advanced work in state-of-the-art applications.

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2

Nonlinear Dynamics

2.1 Introduction

This chapter will introduce the subject of nonlinear dynamics and will discuss some of its most important concepts. The analysis will focus on dynamic systems with a single degree of freedom, such as the linear harmonic oscillator and the galloping oscillator. Although the concepts addressed in this chapter can be found in other textbooks on nonlinear dynamics, the context is aeroelastic. Furthermore, the information presented here will be used in later chapters in order to analyse more realistic aeroelastic systems with many degrees of freedom and various nonlinearities.

Some of the important concepts to be discussed are common to both linear and nonlinear dynamical systems. Examples are fixed points, the phase plane, response trajectories and stability. As linear systems have analytical solutions, they will be preferred to nonlinear ones for the introduction of such concepts. The early parts of the chapter are therefore mostly devoted to linear dynamics; readers already familiar with the subject should read these sections as a revision but also as a familiarisation with the terminology that will be encountered throughout the book.

The main part of the chapter concerns nonlinear dynamics. Concepts such as multiple solutions, bifurcations and limit cycle oscillations will be introduced on simple nonlinear systems. The emphasis is not on the calculation of solutions of the equations of motion; such calculations will be presented in the next chapter. The focus here is on the characterisation of the types of response that can be observed and on simple methods for qualitative or approximate analysis.

2.2 Ordinary Differential Equations

In this chapter, we will examine nonlinear Ordinary Differential Equations (ODE) of the form

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}, \mathbf{q}) \quad (2.1)$$

where $\mathbf{x}(t)$ is the $n \times 1$ vector of system states, t is time, $\mathbf{f}$ is a $n \times 1$ vector of nonlinear functions, $\mathbf{q}$ is a $m \times 1$ vector of system parameters and the overdot denotes differentiation with respect

to time. The states $\mathbf{x}$ are functions of time, t , while the parameters $\mathbf{q}$ are constants. The overdot denotes differentiation with respect to time, that is $\dot{\mathbf{x}} = d\mathbf{x}/dt$. Notice that the system described by equation 2.1 is autonomous, that is, there is no external excitation force. The equations are completed by a set of initial conditions $\mathbf{x}(0) = \mathbf{x}_0$.

Any linear or nonlinear unforced ODE can be written in the form of equation 2.1. To demonstrate this fact, consider the equation of motion of the damped linear harmonic oscillator

$$m\ddot{y} + d\dot{y} + ky = 0 \quad (2.2)$$

where $y(t)$ is the oscillator's instantaneous displacement, m the mass, d the linear damping coefficient and k the linear stiffness coefficient. The system defined by equation 2.2 has a single degree of freedom (DOF), the displacement of the oscillator, y . By defining $x_1 = \dot{y}$ and $x_2 = y$, the equation of motion becomes

$$\dot{x}_1 = -\frac{d}{m}x_1 - \frac{k}{m}x_2 \quad (2.3)$$

$$\dot{x}_2 = x_1 \quad (2.4)$$

Defining $\mathbf{q} = [m \ d \ k]^T$, $\mathbf{x} = [x_1 \ x_2]^T$, where T denotes transposition, we obtain

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}, \mathbf{q}) = \left[-\frac{d}{m}x_1 - \frac{k}{m}x_2 \ x_1 \right]^T$$

which is an expression of the form of equation 2.1, whereby the functions $\mathbf{f}$ are linear. Notice that the second order and first order formulations of the linear harmonic oscillator are equivalent. The variables $x_1(t)$ and $x_2(t)$ are known as the *system states* and denote the system's velocity and displacement responses, respectively.

From here on in, the term *system response* will be used to denote the form of $\mathbf{x}(t)$ for all times starting from $t = 0$, up to $t \rightarrow \infty$. The objective of nonlinear dynamics is the calculation of the response and its evolution as the initial conditions and parameters change values. It should be stressed that, unlike linear systems, nonlinear equations can have multiple solutions. A *global analysis* of a nonlinear dynamic system consists in the calculation of all the solutions of the system at all parameter values of interest. In contrast, a *local analysis* follows one solution of the system as the parameters vary.

The system response is the solution of the complete equations of motion 2.1. The static solutions, or fixed points, of the system are the solutions of

$$\mathbf{f}(\mathbf{x}, \mathbf{q}) = \mathbf{0} \quad (2.5)$$

or, equivalently,

$$\dot{\mathbf{x}} = \mathbf{0} \quad (2.6)$$

The fixed points are denoted by $\mathbf{x}_F$ and are constant in time. Clearly, as $\mathbf{f}(\mathbf{x}_F, \mathbf{q}) = \mathbf{0}$ and $\dot{\mathbf{x}}_F = \mathbf{0}$, fixed points are also solutions of equations 2.1. In some cases, the fixed points can represent the steady-state response of the system, that is, the value reached by $\mathbf{x}(t)$ as $t \rightarrow \infty$. In other cases, a general system response will never subside to any one of the fixed points. Finally, if $\mathbf{f}(\mathbf{x}, \mathbf{q}) = \mathbf{0}$ has no real solutions, there will be no fixed points.

2.3 Linear Systems

In order to demonstrate some basic concepts of dynamic behaviour we will first consider the solution of the damped linear harmonic oscillator of equations 2.3 and 2.4. The equations can be written in matrix form as

$$\dot{\mathbf{x}} = \mathbf{A}(\mathbf{q})\mathbf{x} \quad (2.7)$$

where

$$\mathbf{A} = \begin{pmatrix} -\frac{d}{m} & -\frac{k}{m} \\ 1 & 0 \end{pmatrix} \quad (2.8)$$

The solution of equations of the form of 2.7 subject to the initial conditions $\mathbf{x}(0) = \mathbf{x}_0$ are well known but will be derived here in detail because similar mathematical treatments will be used for the analysis of nonlinear systems in later chapters.

We will first evaluate the fixed points of the damped linear harmonic oscillator, that is the solution of

$$\mathbf{A}\mathbf{x}_F = \mathbf{0} \quad (2.9)$$

For a general $\mathbf{A}$, these equations have one solution, $\mathbf{x}_F = \mathbf{0}$. Therefore, autonomous linear systems have a single fixed point, the origin. As mentioned earlier, $\mathbf{x}(t) = \mathbf{x}_F$ is a static solution of the complete equations of motion 2.7. Furthermore, if $\mathbf{x}_0 = \mathbf{x}_F = \mathbf{0}$, the system response will be $\mathbf{x}(t) = \mathbf{0}$ for all times.

In order to evaluate solutions of equations 2.7 from non-trivial initial conditions, we will try to separate the variables $\mathbf{x}$ and t . To achieve this separation, we will make use of the matrix exponential function; the matrix exponential of $\mathbf{A}$ is defined as

$$e^{\mathbf{A}} = \mathbf{I} + \mathbf{A} + \frac{\mathbf{A}^2}{2!} + \frac{\mathbf{A}^3}{3!} + \dots = \sum_{k=0}^{\infty} \frac{\mathbf{A}^k}{k!} \quad (2.10)$$

where $\mathbf{I}$ is the unit matrix of the same size as $\mathbf{A}$. This series always converges for a matrix $\mathbf{A}$ with finite entries. Arnold (1992) discusses in detail the definition and properties of the matrix exponential.

Equation 2.7 is written as

$$\dot{\mathbf{x}} - \mathbf{A}\mathbf{x} = \mathbf{0}$$

and pre-multiplied by the matrix exponential of $-\mathbf{A}t$, that is, $e^{-\mathbf{A}t}$, giving

$$e^{-\mathbf{A}t}\dot{\mathbf{x}} - e^{-\mathbf{A}t}\mathbf{A}\mathbf{x} = \mathbf{0} \quad (2.11)$$

Using definition 2.10, it is straightforward to show that

$$\frac{d}{dt}(e^{-\mathbf{A}t}) = -\mathbf{A}e^{-\mathbf{A}t} = -e^{-\mathbf{A}t}\mathbf{A}$$

Consequently, the left hand side of equation 2.11 is the time derivative of $e^{-\mathbf{A}t}\mathbf{x}$, so that

$$\frac{d}{dt}(e^{-\mathbf{A}t}\mathbf{x}) = \mathbf{0}$$

As the left hand side is only a function of time, this equation can be integrated in time from 0 to t , yielding

$$\int_0^t \frac{d}{dt} (e^{-At} \mathbf{x}) dt = \mathbf{0} \quad (2.12)$$

Consequently, the general solution becomes

$$e^{-At} \mathbf{x}(t) - \mathbf{x}_0 = \mathbf{0}$$

or, after pre-multiplying both sides by e^{At} ,

$$\mathbf{x}(t) = e^{At} \mathbf{x}_0 \quad (2.13)$$

Note that this general solution includes the fixed point; if we set $\mathbf{x}_0 = \mathbf{0}$, then $\mathbf{x}(t) = \mathbf{0}$ for all times.

Expression 2.13 is the complete general solution of equation 2.7 but contains a matrix exponential. A more useful version of the solution can be obtained by eigenvalue decomposition. Consider the decomposition of matrix $\mathbf{A}$ into

$$\mathbf{A} = \mathbf{V}\mathbf{L}\mathbf{V}^{-1}$$

where $\mathbf{V}$ is the matrix containing the eigenvectors of $\mathbf{A}$ as its columns and $\mathbf{L}$ is a diagonal matrix containing the eigenvalues of $\mathbf{A}$ in its diagonal. Now we note that

$$\begin{aligned} \mathbf{A}^2 &= \mathbf{V}\mathbf{L}\mathbf{V}^{-1}\mathbf{V}\mathbf{L}\mathbf{V}^{-1} = \mathbf{V}\mathbf{L}^2\mathbf{V}^{-1} \\ \mathbf{A}^3 &= \mathbf{A}^2\mathbf{A} = \mathbf{V}\mathbf{L}^2\mathbf{V}^{-1}\mathbf{V}\mathbf{L}\mathbf{V}^{-1} = \mathbf{V}\mathbf{L}^3\mathbf{V}^{-1} \\ &\vdots \\ \mathbf{A}^j &= \mathbf{V}\mathbf{L}^j\mathbf{V}^{-1} \end{aligned}$$

Substituting these results in the definition of the matrix exponential 2.10, and noting that $\mathbf{I} = \mathbf{V}\mathbf{V}^{-1}$, we get

$$e^{\mathbf{A}} = \mathbf{V}e^{\mathbf{L}}\mathbf{V}^{-1}$$

Furthermore, the properties of the eigenvalue decomposition dictate that the eigenvectors of $\mathbf{A}t$ are equal to the eigenvectors of $\mathbf{A}$, while the eigenvalues of $\mathbf{A}t$ are equal to $\mathbf{L}t$. Therefore,

$$e^{\mathbf{A}t} = \mathbf{V}e^{\mathbf{L}t}\mathbf{V}^{-1} \quad (2.14)$$

For a system with n states, taking advantage of the fact that $\mathbf{L}$ is diagonal, equation 2.13 becomes

$$\mathbf{x}(t) = \sum_{i=1}^n \mathbf{v}_i e^{\lambda_i t} b_i \quad (2.15)$$

where $\mathbf{v}_i$ is the i th eigenvector of $\mathbf{A}$ (i.e. the i th column of $\mathbf{V}$), λ_i is its i th eigenvalue (i.e. the i th element of the diagonal of $\mathbf{L}$) and b_i is the i th element of the vector $\mathbf{b} = \mathbf{V}^{-1}\mathbf{x}_0$.

The eigenvalues of matrix $\mathbf{A}$ are the solutions to $\det(\mathbf{I}\lambda - \mathbf{A}) = 0$, which is a polynomial equation of order equal to the number of states n . This equations is usually referred to as the