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Statistical Tools for Finance and Insurance

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Springer

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Preface

This book is designed for students, researchers and practitioners who want to be introduced to modern statistical tools applied in finance and insurance. It is the result of a joint effort of the Center for Economic Research (CentER), Center for Applied Statistics and Economics (C.A.S.E.) and Hugo Steinhaus Center for Stochastic Methods (HSC). All three institutions brought in their specific profiles and created with this book a wide-angle view on and solutions to up-to-date practical problems.

The text is comprehensible for a graduate student in financial engineering as well as for an inexperienced newcomer to quantitative finance and insurance who wants to get a grip on advanced statistical tools applied in these fields. An experienced reader with a bright knowledge of financial and actuarial mathematics will probably skip some sections but will hopefully enjoy the various computational tools. Finally, a practitioner might be familiar with some of the methods. However, the statistical techniques related to modern financial products, like MBS or CAT bonds, will certainly attract him.

"Statistical Tools for Finance and Insurance" consists naturally of two main parts. Each part contains chapters with high focus on practical applications. The book starts with an introduction to *stable distributions*, which are the standard model for heavy tailed phenomena. Their numerical implementation is thoroughly discussed and applications to finance are given. The second chapter presents the ideas of *extreme value and copula analysis* as applied to multivariate financial data. This topic is extended in the subsequent chapter which deals with *tail dependence*, a concept describing the limiting proportion that one margin exceeds a certain threshold given that the other margin has already exceeded that threshold. The fourth chapter reviews the market in *catastrophe insurance* risk, which emerged in order to facilitate the direct transfer of reinsurance risk associated with natural catastrophes from corporations, insurers, and reinsurers to capital market investors. The next contribution employs *functional data analysis* for the estimation of smooth implied volatility surfaces. These surfaces are a result of using an oversimplified market benchmark model – the Black-Scholes formula – to real data. An attractive approach to overcome this problem is discussed in chapter six, where *implied trinomial trees* are applied to modeling implied volatilities and the corresponding state-price densities. An alternative route to tackling the implied volatility smile has led researchers to develop stochastic volatility models. The relative simplicity and the direct link of model parameters to the market makes *Heston's model* very attractive to front office users. Its application to FX option markets is covered in chapter seven. The following chapter shows how the computational complexity of stochastic volatility models can be overcome with the help of the *Fast Fourier Transform.* In chapter nine the valuation of *Mortgage Backed Securities* is discussed. The optimal prepayment policy is obtained via optimal stopping techniques. It is followed by a very innovative topic of predicting corporate bankruptcy with *Support Vector Machines.* Chapter eleven presents a novel approach to *money-demand modeling* using fuzzy clustering techniques. The first part of the book closes with *productivity analysis* for cost and frontier estimation. The nonparametric Data Envelopment Analysis is applied to efficiency issues of insurance agencies.

The insurance part of the book starts with a chapter on *loss distributions.* The basic models for claim severities are introduced and their statistical properties are thoroughly explained. In chapter fourteen, the methods of simulating and visualizing the *risk process* are discussed. This topic is followed by an overview of the approaches to *approximating the ruin probability* of an insurer. Both finite and infinite time approximations are presented. Some of these methods are extended in chapters sixteen and seventeen, where classical and anomalous *diffusion approximations* to ruin probability are discussed and extended to cases when the risk process exhibits *good and bad periods.* The last three chapters are related to one of the most important aspects of the insurance business – *premium calculation.* Chapter eighteen introduces the basic concepts including the pure risk premium and various safety loadings under different loss distributions. Calculation of a joint premium for a portfolio of insurance policies in the individual and collective risk models is discussed as well. The inclusion of *deductibles* into premium calculation is the topic of the following contribution. The last chapter of the insurance part deals with setting the appropriate level of insurance premium within a broader context of business decisions, including risk transfer through *reinsurance* and the rate of return on capital required to ensure solvability.

Our e-book offers a complete PDF version of this text and the corresponding HTML files with links to algorithms and quantlets. The reader of this book may therefore easily reconfigure and recalculate all the presented examples and methods via the enclosed XploRe Quantlet Server (XQS), which is also available from www.xplore-stat.de and www.quantlet.com. A tutorial chapter explaining how to setup and use XQS can be found in the third and final part of the book.

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Pavel Čížek, Wolfgang Härdle, and Rafał Weron

Tilburg, Berlin, and Wrocław, February 2005

Part I

Finance

1 Stable Distributions

Szymon Borak, Wolfgang Härdle, and Rafał Weron

1.1 Introduction

Many of the concepts in theoretical and empirical finance developed over the past decades – including the classical portfolio theory, the Black-Scholes-Merton option pricing model and the RiskMetrics variance-covariance approach to Value at Risk (VaR) – rest upon the assumption that asset returns follow a normal distribution. However, it has been long known that asset returns are not normally distributed. Rather, the empirical observations exhibit fat tails. This heavy tailed or leptokurtic character of the distribution of price changes has been repeatedly observed in various markets and may be quantitatively measured by the kurtosis in excess of 3, a value obtained for the normal distribution (Bouchaud and Potters, 2000; Carr et al., 2002; Guillaume et al., 1997; Mantegna and Stanley, 1995; Rachev, 2003; Weron, 2004).

It is often argued that financial asset returns are the cumulative outcome of a vast number of pieces of information and individual decisions arriving almost continuously in time (McCulloch, 1996; Rachev and Mittnik, 2000). As such, since the pioneering work of Louis Bachelier in 1900, they have been modeled by the Gaussian distribution. The strongest statistical argument for it is based on the Central Limit Theorem, which states that the sum of a large number of independent, identically distributed variables from a finite-variance distribution will tend to be normally distributed. However, as we have already mentioned, financial asset returns usually have heavier tails.

In response to the empirical evidence Mandelbrot (1963) and Fama (1965) proposed the stable distribution as an alternative model. Although there are other heavy-tailed alternatives to the Gaussian law – like Student's t, hyperbolic, normal inverse Gaussian, or truncated stable – there is at least one good reason for modeling financial variables using stable distributions. Namely, they are supported by the generalized Central Limit Theorem, which states that stable laws are the only possible limit distributions for properly normalized and centered sums of independent, identically distributed random variables.

Since stable distributions can accommodate the fat tails and asymmetry, they often give a very good fit to empirical data. In particular, they are valuable models for data sets covering extreme events, like market crashes or natural catastrophes. Even though they are not universal, they are a useful tool in the hands of an analyst working in finance or insurance. Hence, we devote this chapter to a thorough presentation of the computational aspects related to stable laws. In Section 1.2 we review the analytical concepts and basic characteristics. In the following two sections we discuss practical simulation and estimation approaches. Finally, in Section 1.5 we present financial applications of stable laws.

1.2 Definitions and Basic Characteristics

Stable laws – also called α-stable, stable Paretian or Lévy stable – were introduced by Levy (1925) during his investigations of the behavior of sums of independent random variables. A sum of two independent random variables having an α-stable distribution with index α is again α-stable with the same index α. This invariance property, however, does not hold for different α's.

The α-stable distribution requires four parameters for complete description: an index of stability $\alpha \in (0,2]$ also called the tail index, tail exponent or characteristic exponent, a skewness parameter $\beta \in [-1,1]$, a scale parameter $\sigma > 0$ and a location parameter $\mu \in \mathbb{R}$. The tail exponent α determines the rate at which the tails of the distribution taper off, see the left panel in Figure 1.1. When $\alpha = 2$, the Gaussian distribution results. When $\alpha < 2$, the variance is infinite and the tails are asymptotically equivalent to a Pareto law, i.e. they exhibit a power-law behavior. More precisely, using a central limit theorem type argument it can be shown that (Janicki and Weron, 1994; Samorodnitsky and Taqqu, 1994):

$$\begin{cases} \lim_{x\to\infty} x^{\alpha}\mathrm{P}(X > x) = C_{\alpha}(1+\beta)\sigma^{\alpha}, \\ \lim_{x\to\infty} x^{\alpha}\mathrm{P}(X < -x) = C_{\alpha}(1+\beta)\sigma^{\alpha}, \end{cases} \tag{1.1}$$

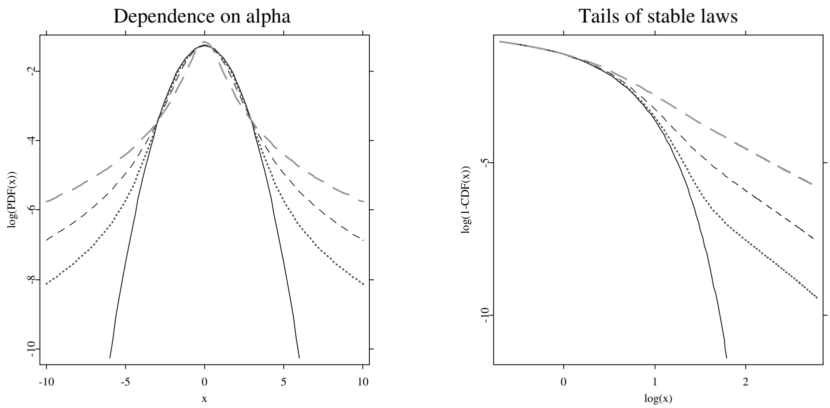

Figure 1.1: *Left panel*: A semilog plot of symmetric ($\beta = \mu = 0$) α-stable probability density functions (pdfs) for $\alpha = 2$ (black solid line), 1.8 (red dotted line), 1.5 (blue dashed line) and 1 (green long-dashed line). The Gaussian ($\alpha = 2$) density forms a parabola and is the only α-stable density with exponential tails. *Right panel*: Right tails of symmetric α-stable cumulative distribution functions (cdfs) for $\alpha = 2$ (black solid line), 1.95 (red dotted line), 1.8 (blue dashed line) and 1.5 (green long-dashed line) on a double logarithmic paper. For $\alpha < 2$ the tails form straight lines with slope $-\alpha$.

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where:

$$C_\alpha = \left(2 \int_0^\infty x^{-\alpha} \sin(x) dx \right)^{-1} = \frac{1}{\pi} \Gamma(\alpha) \sin \frac{\pi\alpha}{2}.$$

The convergence to a power-law tail varies for different α's and, as can be seen in the right panel of Figure 1.1, is slower for larger values of the tail index. Moreover, the tails of α-stable distribution functions exhibit a crossover from an approximate power decay with exponent $\alpha > 2$ to the true tail with exponent α. This phenomenon is more visible for large α's (Weron, 2001).

When $\alpha > 1$, the mean of the distribution exists and is equal to μ. In general, the pth moment of a stable random variable is finite if and only if $p < \alpha$. When the skewness parameter β is positive, the distribution is skewed to the right,

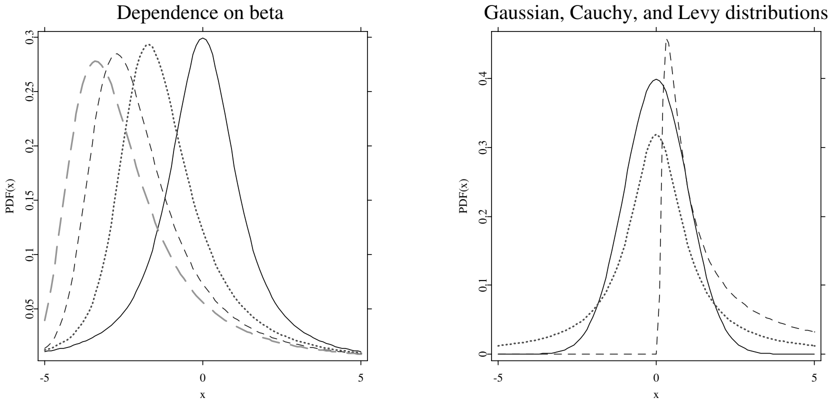

Figure 1.2: *Left panel*: Stable pdfs for $\alpha = 1.2$ and $\beta = 0$ (black solid line), 0.5 (red dotted line), 0.8 (blue dashed line) and 1 (green long-dashed line). *Right panel*: Closed form formulas for densities are known only for three distributions – Gaussian ($\alpha = 2$; black solid line), Cauchy ($\alpha = 1$; red dotted line) and Levy ($\alpha = 0.5, \beta = 1$; blue dashed line). The latter is a totally skewed distribution, i.e. its support is $\mathbb{R}_+$. In general, for $\alpha < 1$ and $\beta = 1$ (-1) the distribution is totally skewed to the right (left).

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i.e. the right tail is thicker, see the left panel of Figure 1.2. When it is negative, it is skewed to the left. When $\beta = 0$, the distribution is symmetric about μ. As α approaches 2, β loses its effect and the distribution approaches the Gaussian distribution regardless of β. The last two parameters, σ and μ, are the usual scale and location parameters, i.e. σ determines the width and μ the shift of the mode (the peak) of the density. For $\sigma = 1$ and $\mu = 0$ the distribution is called standard stable.

1.2.1 Characteristic Function Representation

Due to the lack of closed form formulas for densities for all but three distributions (see the right panel in Figure 1.2), the α-stable law can be most

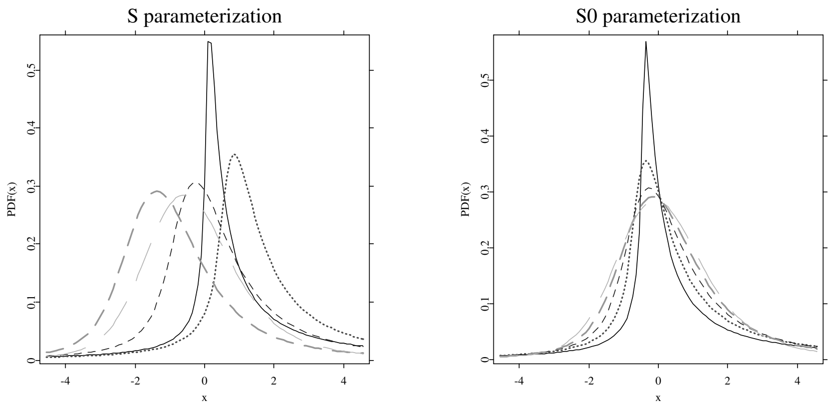

Figure 1.3: Comparison of S and S^0 parameterizations: α-stable pdfs for $\beta = 0.5$ and $\alpha = 0.5$ (black solid line), 0.75 (red dotted line), 1 (blue short-dashed line), 1.25 (green dashed line) and 1.5 (cyan long-dashed line).

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conveniently described by its characteristic function $\phi(t)$ – the inverse Fourier transform of the probability density function. However, there are multiple parameterizations for α-stable laws and much confusion has been caused by these different representations, see Figure 1.3. The variety of formulas is caused by a combination of historical evolution and the numerous problems that have been analyzed using specialized forms of the stable distributions. The most popular parameterization of the characteristic function of $X \sim S_\alpha(\sigma, \beta, \mu)$, i.e. an α-stable random variable with parameters α, σ, β, and μ, is given by (Samorodnitsky and Taqqu, 1994; Weron, 2004):

$$\ln \phi(t) = \begin{cases} -\sigma^\alpha |t|^\alpha \{1 - i\beta \text{sign}(t) \tan \frac{\pi\alpha}{2}\} + i\mu t, & \alpha \neq 1, \\ \\ -\sigma |t| \{1 + i\beta \text{sign}(t) \frac{2}{\pi} \ln |t|\} + i\mu t, & \alpha = 1. \end{cases} \tag{1.2}$$

For numerical purposes, it is often advisable to use Nolan's (1997) parameterization:

$$\ln \phi_0(t) = \begin{cases} -\sigma^\alpha |t|^\alpha \{1 + i\beta \text{sign}(t) \tan \frac{\pi\alpha}{2} [(\sigma|t|)^{1-\alpha} - 1]\} + i\mu_0 t, & \alpha \neq 1, \\ -\sigma|t|\{1 + i\beta \text{sign}(t) \frac{2}{\pi} \ln(\sigma|t|)\} + i\mu_0 t, & \alpha = 1. \end{cases} \tag{1.3}$$

The $S_\alpha^0(\sigma, \beta, \mu_0)$ parameterization is a variant of Zolotariev's (M)-parameterization (Zolotarev, 1986), with the characteristic function and hence the density and the distribution function jointly continuous in all four parameters, see the right panel in Figure 1.3. In particular, percentiles and convergence to the power-law tail vary in a continuous way as α and β vary. The location parameters of the two representations are related by $\mu = \mu_0 - \beta\sigma \tan \frac{\pi\alpha}{2}$ for $\alpha \neq 1$ and $\mu = \mu_0 - \beta\sigma \frac{2}{\pi} \ln \sigma$ for $\alpha = 1$. Note also, that the traditional scale parameter σ_G of the Gaussian distribution defined by:

$$f_G(x) = \frac{1}{\sqrt{2\pi}\sigma_G} \exp\left\{-\frac{(x-\mu)^2}{2\sigma_G^2}\right\}, \tag{1.4}$$

is not the same as σ in formulas (1.2) or (1.3). Namely, $\sigma_G = \sqrt{2}\sigma$.

1.2.2 Stable Density and Distribution Functions

The lack of closed form formulas for most stable densities and distribution functions has negative consequences. For example, during maximum likelihood estimation computationally burdensome numerical approximations have to be used. There generally are two approaches to this problem. Either the fast Fourier transform (FFT) has to be applied to the characteristic function (Mittnik, Doganoglu, and Chenyao, 1999) or direct numerical integration has to be utilized (Nolan, 1997, 1999).

For data points falling between the equally spaced FFT grid nodes an interpolation technique has to be used. Taking a larger number of grid points increases accuracy, however, at the expense of higher computational burden. The FFT based approach is faster for large samples, whereas the direct integration method favors small data sets since it can be computed at any arbitrarily chosen point. Mittnik, Doganoglu, and Chenyao (1999) report that for $N = 2^{13}$ the FFT based method is faster for samples exceeding 100 observations and slower for smaller data sets. Moreover, the FFT based approach is less universal – it is efficient only for large α's and only for pdf calculations. When computing the cdf the density must be numerically integrated. In contrast, in the direct integration method Zolotarev's (1986) formulas either for the density or the distribution function are numerically integrated.

Set $\zeta = -\beta \tan \frac{\pi\alpha}{2}$. Then the density $f(x;\alpha,\beta)$ of a standard α-stable random variable in representation S^0, i.e. $X \sim S^0_\alpha(1,\beta,0)$, can be expressed as (note, that Zolotarev (1986, Section 2.2) used yet another parametrization):

- when $\alpha \neq 1$ and $x > \zeta$:

$$f(x;\alpha,\beta) = \frac{\alpha(x-\zeta)^{\frac{1}{\alpha-1}}}{\pi \mid \alpha - 1 \mid} \int_{-\xi}^{\frac{\pi}{2}} V(\theta;\alpha,\beta) \exp\left\{-(x-\zeta)^{\frac{\alpha}{\alpha-1}} V(\theta;\alpha,\beta)\right\} d\theta, \tag{1.5}$$

- when $\alpha \neq 1$ and $x = \zeta$:

$$f(x;\alpha,\beta) = \frac{\Gamma(1+\frac{1}{\alpha})\cos(\xi)}{\pi(1+\zeta^2)^{\frac{1}{2\alpha}}},$$

- when $\alpha \neq 1$ and $x < \zeta$:

$$f(x;\alpha,\beta) = f(-x;\alpha,-\beta),$$

- when $\alpha = 1$:

$$f(x;1,\beta) = \begin{cases} \frac{1}{2|\beta|} e^{-\frac{\pi x}{2\beta}} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} V(\theta;1,\beta) \exp\left\{-e^{-\frac{\pi x}{2\beta}} V(\theta;1,\beta)\right\} d\theta, & \beta \neq 0, \\ \frac{1}{\pi(1+x^2)}, & \beta = 0, \end{cases}$$

where

$$\xi = \begin{cases} \frac{1}{\alpha}\arctan(-\zeta), & \alpha \neq 1, \\ \frac{\pi}{2}, & \alpha = 1, \end{cases}$$

and

$$V(\theta;\alpha,\beta) = \begin{cases} (\cos\alpha\xi)^{\frac{1}{\alpha-1}} \left(\frac{\cos\theta}{\sin\alpha(\xi+\theta)}\right)^{\frac{\alpha}{\alpha-1}} \frac{\cos\{\alpha\xi+(\alpha-1)\theta\}}{\cos\theta}, & \alpha \neq 1, \\ \frac{2}{\pi}\left(\frac{\frac{\pi}{2}+\beta\theta}{\cos\theta}\right) \exp\left\{\frac{1}{\beta}(\frac{\pi}{2}+\beta\theta)\tan\theta\right\}, & \alpha = 1, \beta \neq 0. \end{cases}$$

The distribution $F(x;\alpha,\beta)$ of a standard α-stable random variable in representation S^0 can be expressed as:

- when $\alpha \neq 1$ and $x > \zeta$:

$$F(x;\alpha,\beta) = c_1(\alpha,\beta) + \frac{\operatorname{sign}(1-\alpha)}{\pi} \int_{-\xi}^{\frac{\pi}{2}} \exp\left\{-(x-\zeta)^{\frac{\alpha}{\alpha-1}} V(\theta;\alpha,\beta)\right\} d\theta,$$

where

$$c_1(\alpha,\beta) = \begin{cases} \frac{1}{\pi}\left(\frac{\pi}{2} - \xi\right), & \alpha < 1, \\ 1, & \alpha > 1, \end{cases}$$

- when $\alpha \neq 1$ and $x = \zeta$:

$$F(x;\alpha,\beta) = \frac{1}{\pi}\left(\frac{\pi}{2} - \xi\right),$$

- when $\alpha \neq 1$ and $x < \zeta$:

$$F(x;\alpha,\beta) = 1 - F(-x;\alpha,-\beta),$$

- when $\alpha = 1$:

$$F(x;1,\beta) = \begin{cases} \frac{1}{\pi}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \exp\left\{-e^{-\frac{\pi x}{2\beta}} V(\theta;1,\beta)\right\} d\theta, & \beta > 0, \\ \frac{1}{2} + \frac{1}{\pi}\arctan x, & \beta = 0, \\ 1 - F(x,1,-\beta), & \beta < 0. \end{cases}$$

Formula (1.5) requires numerical integration of the function $g(\cdot)\exp\{-g(\cdot)\}$, where $g(\theta;x,\alpha,\beta) = (x-\zeta)^{\frac{\alpha}{\alpha-1}} V(\theta;\alpha,\beta)$. The integrand is 0 at $-\xi$, increases monotonically to a maximum of $\frac{1}{e}$ at point θ^* for which $g(\theta^*;x,\alpha,\beta) = 1$, and then decreases monotonically to 0 at $\frac{\pi}{2}$ (Nolan, 1997). However, in some cases the integrand becomes very peaked and numerical algorithms can miss the spike and underestimate the integral. To avoid this problem we need to find the argument θ^* of the peak numerically and compute the integral as a sum of two integrals: one from $-\xi$ to θ^* and the other from θ^* to $\frac{\pi}{2}$.

1.3 Simulation of α-stable Variables

The complexity of the problem of simulating sequences of α-stable random variables results from the fact that there are no analytic expressions for the inverse F^{-1} of the cumulative distribution function. The first breakthrough was made by Kanter (1975), who gave a direct method for simulating $S_\alpha(1,1,0)$ random variables, for $\alpha < 1$. It turned out that this method could be easily adapted to the general case. Chambers, Mallows, and Stuck (1976) were the first to give the formulas.

The algorithm for constructing a standard stable random variable $X \sim S_\alpha(1,\beta,0)$, in representation (1.2), is the following (Weron, 1996):

- generate a random variable V uniformly distributed on $(-\frac{\pi}{2},\frac{\pi}{2})$ and an independent exponential random variable W with mean 1;
- for $\alpha \neq 1$ compute:

$$X = S_{\alpha,\beta} \cdot \frac{\sin\{\alpha(V + B_{\alpha,\beta})\}}{\{\cos(V)\}^{1/\alpha}} \cdot \left[\frac{\cos\{V - \alpha(V + B_{\alpha,\beta})\}}{W}\right]^{(1-\alpha)/\alpha}, \quad (1.6)$$

  where

$$\begin{aligned} B_{\alpha,\beta} &= \frac{\arctan(\beta \tan \frac{\pi\alpha}{2})}{\alpha}, \\ S_{\alpha,\beta} &= \left\{1 + \beta^2 \tan^2\left(\frac{\pi\alpha}{2}\right)\right\}^{1/(2\alpha)}; \end{aligned}$$

- for $\alpha = 1$ compute:

$$X = \frac{2}{\pi}\left\{\left(\frac{\pi}{2} + \beta V\right)\tan V - \beta \ln\left(\frac{\frac{\pi}{2} W \cos V}{\frac{\pi}{2} + \beta V}\right)\right\}. \quad (1.7)$$

Given the formulas for simulation of a standard α-stable random variable, we can easily simulate a stable random variable for all admissible values of the parameters α, σ, β and μ using the following property: if $X \sim S_\alpha(1,\beta,0)$ then

$$Y = \begin{cases} \sigma X + \mu, & \alpha \neq 1, \\ \sigma X + \frac{2}{\pi}\beta\sigma \ln \sigma + \mu, & \alpha = 1, \end{cases} \quad (1.8)$$

is $S_\alpha(\sigma,\beta,\mu)$. It is interesting to note that for $\alpha = 2$ (and $\beta = 0$) the Chambers-Mallows-Stuck method reduces to the well known Box-Muller algorithm for generating Gaussian random variables (Janicki and Weron, 1994). Although many other approaches have been proposed in the literature, this method is regarded as the fastest and the most accurate (Weron, 2004).