COMBINATORIAL OPTIMIZATION IN COMMUNICATION NETWORKS

Combinatorial Optimization

VOLUME 18

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COMBINATORIAL OPTIMIZATION IN COMMUNICATION NETWORKS

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Part III: Combinatorial Optimization in Other Network Applications

Preface

Combinatorial optimization problems arise in all areas of technology, and they certainly have implications for many network problems, for instance, infrastructure deployment, resource management, routing, and QoS provisioning. To advance and promote the theory and applications of combinatorial optimization in communication networks, this volume addresses the unique intersection of the two areas.

There are many combinatorial optimization books on the market and numerous research papers in the literature addressing specific network problems, such as routing optimization, scheduling, and resource allocation. This book is the first to bridge the optimization and networking research communities. To improve the quality and coherence of the book, we carefully selected papers that represent state-of-the-art research, interacted with authors to make every chapter harmonically fit in with the same central theme—combinatorial optimization in communication networks—and finally the book took its current shape.

The book is mainly concerned with network problems that involve one or more combinatorial optimization solution techniques. Having in mind a list of combinatorial optimization methods and a list of network problems, we see a high-degree bipartite graph between them. Two approaches were considered: optimization method oriented (starting from combinatorial optimization methods and finding appropriate network problems as examples) and network problem oriented (focusing on specific network problems and seeking appropriate combinatorial optimization methods to solve them). We finally decided to use the problem-oriented approach, mainly because of the availability of papers: most papers in the recent literature appear to address very specific network problems, and combinatorial optimization comes as a convenient problem solver.

The three editors each bring a different perspective to this book: one is a world-renowned expert in operations research and complexity theory, and has been active in wireless networks, optical networks, and switching networks in recent years, the other two are active in wireless network research, with each having a different focus area. As a result, combinatorial optimization methods in all network technologies are collected in this book, with most papers focused on the wireless networking area.

The book covers a collection of network problems that need combinatorial optimization. Most chapters start from the problem to be addressed, introduce the required background information, describe the combinatorial optimization approach, and some even provide follow-up references for interested readers. It can be used as a handy reference book for research scientists in communication networks and operations research. When used as a textbook, it can be used for a graduate-level network course for specific network problems and their solutions resulting from some optimization techniques, or for a course on combinatorial optimization, using the network problems as real-life examples to enhance student understanding.

Maggie Xiaoyan Cheng
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Introduction

Aims

Combinatorial optimization is concerned with the arrangement, grouping, ordering, or selection of discrete objects from a finite set. Many problems involve seeking a best configuration of a set of parameters to achieve desired objectives. Combinatorial optimization exists everywhere. In communication networks, combinatorial optimization is used in network design and management to meet various operational needs. Classical applications of combinatorial optimization in communication networks trace back to the use of shortest paths in the Internet routing and spanning trees in the bridged LAN configuration. A typical application of combinatorial optimization in networks usually involves mathematical modeling of the problem, with an objective to reduce network deployment cost or operation cost, to provide better quality of service, or to improve the network performance. The recent boom in network research created many new problems that require new insights into their mathematical structures, novel approaches, and efficient solution techniques. The objective of this book is to expose such new findings and advance the theory and applications of combinatorial optimization in communication networks.

Scope

The scope of this book includes some important combinatorial optimization problems arising in optical networks, wireless ad hoc networks, sensor networks, mobile communication systems, and satellite networks.

Example network problems covered include media access control, routing optimization, topology control, resource allocation, and management, QoS provisioning in various wireless networks, light-path establishment in optical networks, etc. The specific combinatorial optimization techniques adopted by the authors are quite diverse, on the other hand. Instead of focusing on the combinatorial optimization techniques and extending them to network problems, most authors adopted an approach of starting from a network problem, analyzing its intrinsic structure, studying its computational complexity, and then developing an efficient solution for it.

Overview

The book includes combinatorial optimization problems in wireless networks, optical networks, and interconnection networks, as well as other network applications. Wireless network research is the major part, partly due to its high availability in recent literature, and partly because all three editors are currently active in this research community. Among the 22 chapters, 12 chapters are dedicated to wireless networks, 4 chapters are dedicated to specific problems in optical networks and interconnection networks, and the remaining 6 chapters are for other more general network applications that are not restricted to one particular network technology. Consequently, the book is divided into three logical parts: part I, wireless networks, part II, optical and interconnection networks, and part III, other network applications.

Part I consists of 12 chapters, all in wireless networks. Chapter 1 introduces topology control algorithms to achieve better energy efficiency and network capacity for both homogeneous and heterogeneous wireless networks. Chapter 2 includes several lines of research in cellular systems, including channel assignment, location management, base station placement, and code division multiple access, and it addresses combinatorial optimization problems in these topics. Chapter 3 discusses the problem of optimally sharing a single server/channel among multiple users/queues in wireless communication systems. Chapter 4 introduces strategies to improve network performance by using multipath routing in ad hoc networks, and specifically addresses the end-to-end multipath routing and N-to-1 multipath routing, and presents the protocols to find multiple paths and policies on the usage of multiple paths. Chapter 5 addresses several optimization problems in ad hoc networks, including the minimum size virtual backbone problem, transmission power control problem, and sensor node localization problem. Chapter 6 introduces stochastic linear programming and its application in proactive resource allocation in heterogeneous wireless networks. Chapter 7 addresses approaches to select working sensors in wireless sensor networks in order to maximize the total lifetime of the network, and meanwhile, satisfying the coverage and connectivity requirements. Chapter 8 addresses QoS provisioning for adaptive multimedia in mobile wireless networks. It introduces an abstract general traffic model and an optimal call admission control scheme that guarantees the QoS requirements and maximizes the utilization. Chapter 9 provides a summary of optimal power assignment algorithms in DS-CDMA networks, introduces a new collision model for DS-CDMA networks, and presents the collision probability and throughput analysis under this model. Chapter 10 introduces information-directed routing that jointly optimizes for maximal information gain and minimal communication cost in sensor networks. Chapter 11 includes call admission and handoff management strategies for multimedia LEO satellite networks. Chapter 12 introduces the time slot allocation problem in MFTDMA (Multi-Frequency Time-Division Multiple Access) satellite networks, in which the throughput is optimized through a linear and integer programming approach.

Part II consists of 4 chapters. Chapter 13 presents a new optimization technique for the lightpath establishment problem in optical networks that considers routing and wavelength assignment jointly. Chapter 14 introduces complexity models for WDM switching networks and provides complexity bounds under different request models. Chapter 15 describes interconnection network models and summarizes the topological properties of the most widely used networks. Chapter 16 includes a brief survey of some bounded degree Cayley networks on their routing, diameter, and fault tolerance properties, and presents an anonymous leader election algorithm in interconnection networks with bounded degree.

Part III consists of 6 chapters. Chapter 17 addresses several optimization techniques including dynamic programming, integer linear programming, Steiner tree construction, clustering, and their applications in routing problems in packet switching networks, WDM optical networks, and wireless ad hoc networks. Chapter 18 introduces an optimal scheduling algorithm that minimizes the ratio of the response time to its service time for on-demand data broadcasts. Chapter 19 includes optimal stream replication and bandwidth allocation problems in simulcasting systems for real-time video distribution. Chapter 20 presents a fast failure recovery scheme in high-speed networks that implements the minimum cost source-based rerouting. Chapter 21 introduces a primal-dual algorithm for the dynamic facility location problem, which has applications in many network problems such as network design, information flow routing, and cache distribution on the Internet. Finally, Chapter 22 presents preliminary work exploring more efficient approaches for hard combinatorial optimization problems that have significant implications for communication networks.

Part I

Combinatorial Optimization in *Wireless Networks*

Chapter 1

Topology Control in Wireless Multihop Networks

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1 Introduction

With the rapid growth of wireless communication infrastructures over the recent years, new challenges have been posed on the system and analysis of wireless mobile ad hoc networks. Energy efficiency [1] and network capacity [2] are among the most important performance metrics, and topology control and management — how to determine the transmission power of each node so as to maintain network connectivity while consuming the minimum possible power and improving the network capacity with spatial reuse — has emerged to be one of the most important issues [1].

Specifically, in a wireless network where every node transmits with its maximal transmission power, the network topology is implicitly built by the routing protocol. In particular, each node keeps a list of neighbor nodes that are within the transmission range. On the other hand, in a topology controlled wireless network, instead of transmitting using the maximum possible power, nodes collaboratively determine their transmission power and define the topology of the wireless network by the neighbor relation under certain criteria. That is, each node has the opportunity of choosing the set of neighbors it would like to communicate with, by adjusting its transmission power. The network topology is thus defined by having each node form its own proper neighbor relation, subject to maintaining network connectivity.

The importance of topology control lies in the fact that it critically affects the system performance in several ways. In addition to reducing energy consumption and improving network capacity, topology control also has an impact on contention for the medium. Collisions can be mitigated as much as possible by choosing the smallest transmission power subject to maintaining network connectivity [3], [4].

Many centralized or localized geometric structures have been used for topology control in wireless networks, for instance, Minimum Spanning Tree (MST) [5], Relative Neighborhood Graph (RNG) [6], Gabriel Graph (GG), Delaunay Triangulation [7], and Yao structure [8], just to name a few. In this chapter, we present several sparse geometric structures that are based on the MST and minimum spanning graph for topology control. We first introduce the *Local Minimum Spanning Tree* (LMST) [9], a localized topology control algorithm for homogeneous wireless networks where the maximum transmission range of each node is the same. Then we present two localized topology control algorithms [10], *Directed Relative Neighborhood Graph* (DRNG) and *Directed Local Spanning Subgraph* (DLSS), for heterogeneous wireless networks where the maximum transmission range of each node may be different. We also consider the fault tolerance issue under topology control. Note that by reducing the number of wireless links in the network, topology control actually decreases the degree of routing redundancy. As a result, the topology thus derived is more susceptible to node failures/departures. To deal with the fault tolerance issue, we introduce a centralized algorithm, the *Fault-Tolerant Global Spanning Subgraph* (FGSS), and a localized algorithm, the *Fault-Tolerant Local Spanning Subgraph* (FLSS) [11]. Both FGSS and FLSS preserve network k-connectivity. Simulation results show that the algorithms discussed in this chapter are not only more energy efficient than existing approaches, but also significantly improve the network capacity.

The rest of this chapter is organized as follows. We define the network model in Section 2. After setting the stage for discussion, we present LMST for homogeneous networks in Section 3, and DRNG and DLSS for heterogeneous networks in Section 4. We then discuss in Section 5 the issue of fault tolerance and introduce FGSS and FLSS. Finally, we present in Section 6 a performance study of all the topology control algorithms discussed in the chapter, and conclude the chapter in Section 7 with several research avenues for future work.

2 Network Model

Let the topology of a multihop wireless network be represented by a simple directed graph $G = (V(G), E(G))$ in the 2-D plane, where $V(G) = \{v_1, v_2, \ldots, v_n\}$ is the set of nodes (vertices) and $E(G)$ is the set of links (edges) in the network. Each node has a unique *id* (such as an IP/MAC address). Here we assume $id(v_i) = i$ for simplicity. Although G is usually assumed to be geometric in the literature, here we only assume that G is a general graph, i.e., $E(G) = \{(u, v) : v \text{ can receive } u\text{'s transmission correctly}\}$. We also assume that the wireless channel is symmetric, and each node is able to gather its own location information via, for example, several lightweight localization techniques for wireless networks [12], [13], [14]. In what follows, we first define the following terms and notations, and then outline the design requirements that one should meet to devise effective topology control algorithms.

Definition 2.1 (Visible Neighborhood.) *The visible neighborhood N_u^V is the set of nodes that node u can reach by using the maximum transmission power; i.e., $N_u^V = \{v \in V(G) : (u, v) \in E(G)\}$. For each node $u \in V(G)$, let $G_u^V = (V(G_u^V), E(G_u^V))$ be the induced subgraph of G such that $V(G_u^V) = N_u^V$.*

Definition 2.2 (Weight Function.) *Given two edges $(u_1, v_1), (u_2, v_2) \in E(G)$ and the Euclidean distance function $d(\cdot, \cdot)$, the weight function $w : E \mapsto R$ satisfies:*

$$\begin{aligned}
& w(u_1, v_1) > w(u_2, v_2) \\
\Leftrightarrow\quad & d(u_1, v_1) > d(u_2, v_2) \\
or\quad & (d(u_1, v_1) = d(u_2, v_2) \ \&\& \ \max\{id(u_1), id(v_1)\} > \max\{id(u_2), id(v_2)\}) \\
or\quad & (d(u_1, v_1) = d(u_2, v_2) \ \&\& \ \max\{id(u_1), id(v_1)\} = \max\{id(u_2), id(v_2)\} \\
& \quad \&\& \ \min\{id(u_1), id(v_1)\} > \min\{id(u_2), id(v_2)\}).
\end{aligned}$$

The weight function w ensures that two edges with different end-nodes have different weights. As most of the topology control algorithms introduced below are executed by each node in a decentralized manner, the weight function defined above is used to guarantee a unique outcome of the topology control algorithms. Also note that $w(u, v) = w(v, u)$.

Definition 2.3 (Neighbor Set.) *Node v is an* out-neighbor *of node u (and u is an* in-neighbor *of v) under an algorithm ALG, denoted $u \xrightarrow{ALG} v$, if and only if there exists an edge (u, v) in the topology generated by the algorithm.*

In particular, we use $u \rightarrow v$ to denote the neighbor relation in G. $u \overset{ALG}{\longleftrightarrow} v$ if and only if $u \xrightarrow{ALG} v$ and $v \xrightarrow{ALG} u$. The out-neighbor set *of node u is $N^{out}_{ALG}(u) = \{v \in V(G) : u \xrightarrow{ALG} v\}$, and the* in-neighbor set *of u is $N^{in}_{ALG}(u) = \{v \in V(G) : v \xrightarrow{ALG} u\}$.*

Definition 2.4 (Degree.) *The* out-degree *of a node u under an algorithm ALG, denoted $deg^{out}_{ALG}(u)$, is the number of out-neighbors of u; i.e., $deg^{out}_{ALG}(u) = |N^{out}_{ALG}(u)|$. Similarly, the* in-degree *of a node u, denoted $deg^{in}_{ALG}(u)$, is the number of in-neighbors; i.e., $deg^{in}_{ALG}(u) = |N^{in}_{ALG}(u)|$.*

Definition 2.5 (Topology.) *The topology generated by an algorithm ALG is a directed graph $G_{ALG} = (E(G_{ALG}), V(G_{ALG}))$, where $V(G_{ALG}) = V(G)$, and $E(G_{ALG}) = \{(u,v) : u \xrightarrow{ALG} v, u, v \in V(G_{ALG})\}$.*

Definition 2.6 (Radius.) *The radius, R_u, of node u is defined as the distance between node u and its farthest neighbor (in terms of Euclidean distance), i.e, $R_u = \max_{v \in N^{out}_{ALG}(u)}\{d(u,v)\}$.*

Definition 2.7 (Connectivity.) *For any topology generated by an algorithm ALG, node u is said to be* connected to *node v (denoted $u \Rightarrow v$) if there exists a path ($p_0 = u, p_1, \ldots, p_{m-1}, p_m = v$) such that $p_i \xrightarrow{ALG} p_{i+1}, i = 0, 1, \ldots, m-1$, where $p_k \in V(G_{ALG}), k = 0, 1, \ldots, m$. It follows that $u \Rightarrow v$ if $u \Rightarrow p$ and $p \Rightarrow v$ for some $p \in V(G_{ALG})$.*

Definition 2.8 (BiDirectionality.) *A topology generated by an algorithm ALG is* bidirectional, *if for any two nodes $u, v \in V(G_{ALG})$, $u \in N^{out}_{ALG}(v)$ implies $v \in N^{out}_{ALG}(u)$.*

Definition 2.9 (Bidirectional Connectivity.) *For any topology generated by an algorithm ALG, node u is said to be* bidirectionally connected to *node v (denoted $u \Leftrightarrow v$) if there exists a path ($p_0 = u, p_1, \ldots, p_{m-1}, p_m = v$) such that $p_i \overset{ALG}{\longleftrightarrow} p_{i+1}, i = 0, 1, \ldots, m-1$, where $p_k \in V(G_{ALG}), k = 0, 1, \ldots, m$. It follows that $u \Leftrightarrow v$ if $u \Leftrightarrow p$ and $p \Leftrightarrow v$ for some $p \in V(G_{ALG})$.*

Definition 2.10 (Addition and Removal.) *The* Addition *operation adds an extra edge (v,u) into G_{ALG} if $(u,v) \in E(G_{ALG})$ and $(v,u) \notin E(G_{ALG})$. The* Removal *operation deletes any edge $(u,v) \in E(G_{ALG})$ if $(v,u) \notin E(G_{ALG})$.*

Both *Addition* and *Removal* operations attempt to create a bidirectional topology by either converting unidirectional edges into bidirectional ones or removing all unidirectional edges. The resulting topology after *Addition* or *Removal* is alway bidirectional, if the transmission range for each node is the same. If the transmission range for each node is not the same, the result of *Removal* is still bidirectional, but the result of *Addition* may not be bidirectional.

Requirements for effective topology control algorithms. An effective topology control algorithm should meet several requirements [9]

(1) The algorithm should preserve network connectivity (or k-connectivity for the purpose of fault tolerance). This is the fundamental requirement of topology control.

(2) Because there is usually no central authority in a multihop wireless network, each node has to make its own decision based on the information collected from the network. That is, the topology control algorithm should be distributed.

(3) As the network topology may change as a result of mobility and/or node failure, a topology control algorithm may have to be executed multiple times in response to mobility or network dynamics. It is thus desirable that the algorithm depend only on the information collected *locally* so as to reduce the control message overhead and the delay incurred in topology management.

(4) It is desirable if all the links in the network topology induced by the topology control algorithm are bidirectional. Bidirectional links guarantee the existence of reverse paths, and facilitate link-level acknowledgment [3] and handshaking mechanisms, such as the floor acquisition mechanism *request-to-send/clear-to-send* (RTS/CTS).

3 Topology Control in Homogeneous Networks

In this section, we consider a homogeneous wireless network where every node has the same transmission range $d_{\max}$. As a result, the network topology G becomes an undirected graph. The homogeneity assumption has been commonly used in most of the topology control algorithms in the literature, perhaps except for [10, 15]. We now introduce a localized topology control algorithm, *Local Minimum Spanning Tree* (LMST) [9], for homogeneous networks.

3.1 LMST: Local Minimum Spanning Tree

The proposed algorithm consists of three steps: information collection, topology construction, and construction of topology with only bidirectional links.

The information needed by each node u in the topology construction process is the information of all nodes in its visible neighborhood, N_u^V. This can be obtained by having each node broadcast periodically a Hello message using its maximal transmission power. The information contained in a Hello message should at least include the *id* and the position of the node. The time interval between two broadcasts of Hello messages depends on the level of node mobility, and can be determined using a probabilistic model in [9].

After obtaining the neighborhood information, each node u builds its local minimum spanning tree T_u that spans all the nodes within its visible neighborhood N_u^V. The time it takes to build the MST varies from $O(m \log n)$ (the original Prim's algorithm [16]) to almost linear of m (the optimal algorithm [17]), where n is the number of vertices and m is the number of edges. Node v is a neighbor of node u if and only if (u, v) is a link on the local MST built by u. The network topology under LMST is all the nodes in V and their individually perceived neighbor relations. Note that the topology is *not* a simple superposition of all local MSTs.

Definition 3.1 (LMST.) *In the Local Minimum Spanning Tree (LMST), node v is a neighbor of node u, denoted $u \xrightarrow{LMST} v$, if and only if $(u, v) \in E(T_u)$. That is, v is a neighbor of u if and only if v is on u's local MST T_u, and is one hop away from u.*

Because the neighbor relation is determined locally by each node, some links in the final topology may be unidirectional; i.e., $u \xrightarrow{LMST} v$ does not necessarily imply $v \xrightarrow{LMST} u$. An example is given in Figure 1, where $d(u, v) = d < d_{\max}$, $d(u, w_4) < d_{\max}$, $d(u, w_i) > d_{\max}, i = 1, 2, 3$, and $d(v, w_j) < d_{\max}, j = 1, 2, 3, 4$. Because $N_u^V = \{u, v, w_4\}$, $u \xrightarrow{LMST} v$ and $u \xrightarrow{LMST} w_4$. Also $N_v^V = \{u, v, w_1, w_2, w_3, w_4\}$; hence $v \xrightarrow{LMST} w_1$. Here link (u, v) is unidirectional. We can apply either *Addition* or *Removal* to obtain a bidirectionally connected topology.

Definition 3.2 (Topology G_{ALG}^+.) *The topology G_{ALG}^+ generated by an algorithm ALG is an undirected graph $G_{ALG}^+ = (V(G_{ALG}^+), E(G_{ALG}^+))$, where $V(G_{FLSS}^+) = V(G_{ALG})$, and $E_{ALG}^+ = \{(u, v) : (u, v) \in E(G_{ALG})$ or $(v, u) \in E(G_{ALG})\}$.*

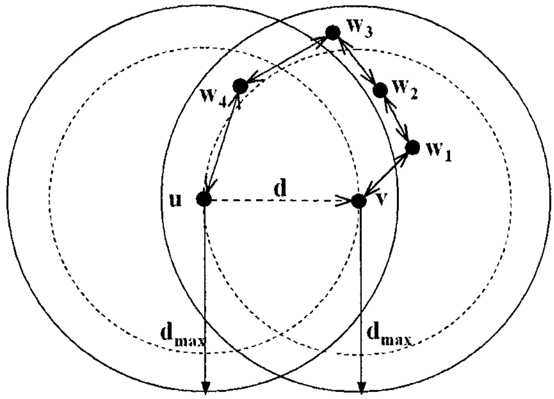


Figure 1: Links in the topology derived by LMST may be unidirectional.

Definition 3.3 (Topology G^-_{ALG}.) *The topology G^-_{FLSS} generated by an algorithm ALG is an undirected graph $G^-_{ALG} = (V(G^-_{ALG}), E(G^-_{ALG}))$, where $V(G^-_{ALG}) = V(G_{ALG})$, and $E^-_{ALG} = \{(u,v) : (u,v) \in E(G_{ALG})$ and $(v,u) \in E(G_{ALG})\}$.*

3.2 Properties of LMST

In this section, we prove several desirable properties of LMST, including its connectivity and degree bound. The following lemma is important to the proof of connectivity.

Lemma 3.4 *For any edge $(u,v) \in E(G)$, $u \Leftrightarrow v$ in G_{LMST}.*

Proof. Let all the edges $(u,v) \in E(G)$ be sorted in the ascending order of the weight; i.e., $w(u_1,v_1) < w(u_2,v_2) < \cdots < w(u_l,v_l)$, where l is the total number of edges in G. We prove by induction.

1. *Basis*: The first edge (u_1,v_1) satisfies $w(u_1,v_1) = \min_{(u,v)\in E(G)} \{w(u,v)\}$. Because the shortest edge is always on the local MST, we have $u_1 \overset{LMST}{\longleftrightarrow} v_1$, which means $u_1 \Leftrightarrow v_1$.

2. *Induction*: Assume the hypothesis holds for all edges $(u_i,v_i), 1 \le i < k$, we prove $u_k \Leftrightarrow v_k$ in G_{LMST}. If $u_k \overset{LMST}{\longleftrightarrow} v_k$, then $u_k \Leftrightarrow v_k$. Otherwise, without loss of generality, assume $u_k \nrightarrow v_k$. In the local topology construction of u_k, before edge (u_k,v_k) is inspected, there must already exist a path $p = (w_0 = u_k, w_1, w_2, \ldots, w_{m-1}, w_m = v_k)$

from u_k to v_k, where $(w_i, w_{i+1}) \in E(T_{u_k}), i = 0, 1, \ldots, m-1$. Because edges are inserted in the ascending order of the weight, we have $w(w_i, w_{i+1}) < w(u_k, v_k)$. Applying the induction hypothesis to each pair $(w_i, w_{i+1}), i = 0, 1, \ldots, m-1$, we have $w_i \Leftrightarrow w_{i+1}$. Therefore, $u_k \Leftrightarrow v_k$. □

Lemma 3.4 shows that, for any edge $(u, v) \in E(G)$, either $u \xleftrightarrow{LMST} v$, or u and v are bidirectionally connected to each other in G_{LMST} via links of smaller weight.

Theorem 3.5 (Connectivity of LMST.) *If G is connected, then G_{LMST}, G^+_{LMST} and G^-_{LMST} are all connected.*

Proof. We only need to prove that G^-_{LMST} preserves the connectivity of G, for $E(G^-_{LMST}) \subseteq E(G_{LMST}) \subseteq E(G^+_{LMST})$. Suppose G is connected. For any two nodes $u, v \in V(G)$, there exists at least one path $p = (w_0 = u, w_1, w_2, \ldots, w_{m-1}, w_m = v)$ from u to v, where $(w_i, w_{i+1}) \in E(G), i = 0, 1, \ldots, m-1$. Because $w_i \Leftrightarrow w_{i+1}$ by Lemma 3.4, we have $u \Leftrightarrow v$ in G_{LMST}. Because p is bidirectional in G_{LMST}, the removal of unidirectional links does not affect the existence of p. Therefore, $u \Leftrightarrow v$ in G^-_{LMST}; i.e., G^-_{LMST} preserves the connectivity of G. □

It has been observed that any minimum spanning tree of a finite set of points in the plane has a maximum node degree of six [18]. We prove this property independently for LMST. We only need to prove the degree bound for G^+_{LMST} because the degree bound of G_{LMST} or G^-_{LMST} can only be lower.

Lemma 3.6 *Given three nodes $u, v, p \in V(G)$ satisfying $w(u, p) < w(u, v)$ and $w(p, v) < w(u, v)$, then $u \nrightarrow v$ and $v \nrightarrow u$ in G^+_{LMST}.*

Proof. We only need to consider the case where $(u, v) \in E(G)$ because $(u, v) \notin E(G)$ would imply $u \nrightarrow v$ and $v \nrightarrow u$. Consider the local topology construction of u and v. Before we insert (u, v) into T_u or T_v, the two edges (u, p) and (p, v) have already been processed because $w(u, p) < w(u, v)$ and $w(p, v) < w(u, v)$. Thus $u \Leftrightarrow p$ and $p \Leftrightarrow v$ by Lemma 3.4, which means $u \Leftrightarrow v$. Therefore, (u, v) should be inserted into neither T_u nor T_v; i.e., $u \nrightarrow v$ and $v \nrightarrow u$ in G^+_{LMST}. □

Before stating the next corollary, we give the definition of the *Relative Neighborhood Graph* (RNG).

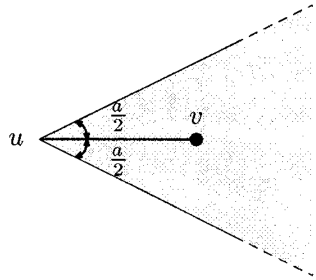


Figure 2: The definition of $cone(u, \alpha, v)$.

Definition 3.7 (Neighbor Relation in RNG.) *For RNG [19], [20], $u \overset{RNG}{\longleftrightarrow} v$ if and only if there does not exist a third node p such that $w(u,p) < w(u,v)$ and $w(p,v) < w(u,v)$. Or equivalently, there is no node inside the shaded area in Figure 3(a).*

The following corollary is a byproduct of Lemma 3.6.

Corollary 3.8 *The topology by LMST is a subgraph of the topology by RNG; i.e., $G_{LMST} \subseteq G_{RNG}$.*

Definition 3.9 *A $cone(u, \alpha, v)$ is the unbounded shaded region shown in Figure 2.*

Theorem 3.10 (Degree Bound) *The out-degree of a node in G^+_{LMST} is bounded by six; i.e., $deg^{out}_{LMST+}(u) \leq 6, \forall u \in V(G^+_{LMST})$.*

Proof. First we prove by contradiction that if $v \in N^{out}_{LMST+}(u)$, then there cannot exist any other node $w \in N^{out}_{LMST+}(u)$ that lies inside $Cone(u, 2\pi/3, v)$. Assume that such a node w exists; then $\angle wuv < \pi/3$. If $w(u,w) > w(u,v)$, then $\angle uvw > \pi/3 > \angle wuv$. We have $w(u,w) > w(v,w)$, which implies $u \nrightarrow w$ by Lemma 3.6. If $w(u,w) < w(u,v)$, then $\angle uwv > \pi/3 > \angle wuv$. We have $w(u,v) > w(v,w)$, which implies $u \nrightarrow v$ by Lemma 3.6. Both scenarios contradict the assumption that $v, w \in N^{out}_{LMST+}(u)$.

Consider any node $u \in V(G^+_{LMST})$. Put the nodes in $N^{out}_{LMST+}(u)$ in order such that for the ith node w_i and the jth node w_j ($j > i$), $w(u, w_j) > w(u, w_i)$. We have proved that $\angle w_i u w_j \geq \pi/3$; i.e., node w_j cannot reside inside $Cone(u, 2\pi/3, w_i)$. Therefore, node u cannot have any neighbor other than node w_i inside $Cone(u, 2\pi/3, w_i)$. By induction on the rank of nodes in $N^{out}_{LMST+}(u)$, the maximal number of neighbors that u can have is no greater than six; i.e., $deg^{out}_{LMST+}(u) \leq 6$. □

4 Topology Control in Heterogeneous Networks

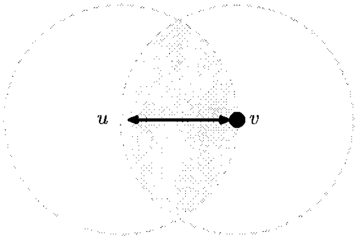
As mentioned in Section 3, most of the topology control algorithms in the literature assume homogeneous networks. However, this may not always hold in practice due to various reasons. First, even devices of the same type may have slightly different transmission ranges. Second, there exist devices of dramatically different capabilities in the same network. Third, the transmission range may be time-varying or affected by environmental stimuli.

In this section, we consider a heterogeneous wireless network where the maximum transmission range of each node may be different. In this case the network topology G becomes a directed graph. Let $r_{\min}$ and $r_{\max}$ be the smallest and the largest transmission ranges among all nodes in the network, respectively. We first show that most of the topology control algorithms that are devised under the homogeneity assumption cannot be directly applied to heterogeneous networks. Then we introduce two localized topology control algorithms, *Directed Relative Neighborhood Graph* (DRNG) and *Directed Local Spanning Subgraph* (DLSS).

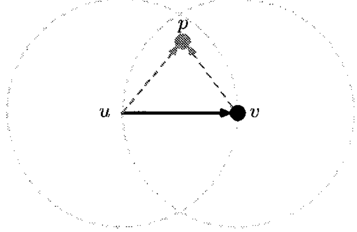
4.1 Motivations

In this section, we give several examples that show topology control algorithms devised under the homogeneity assumption, e.g., CBTC [21], RNG [6], and LMST, may render disconnectivity in heterogeneous networks [10], thus motivating the need for new topology control algorithms for heterogeneous networks.

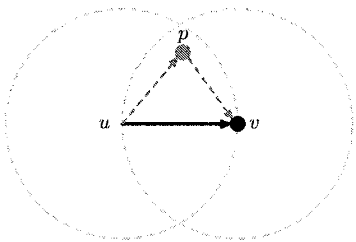
CBTC and RNG. Two of the other well-known topology control algorithms for homogeneous networks are *Cone-Based Topology Control* (CBTC) [21] and *Relative Neighborhood Graph RNG* [6]. CBTC(α) is a two-phase algorithm in which each node finds the minimum power p such that transmitting with p ensures that it can reach some node in every cone of degree α. The algorithm has been analytically shown to preserve network connectivity if $\alpha < 5\pi/6$. It has also ensured that every link between nodes is bidirectional. Several optimizations to the basic algorithm are also discussed, which include: (i) a *shrink-back* operation can be added at the end to allow a boundary node to broadcast with less power, if doing so does not reduce the cone coverage; (ii) if $\alpha < 2\pi/3$, asymmetric edges can be removed while maintaining the network connectivity; and (iii) if there exists an edge from u to v_1 and from u to v_2, respectively, the longer edge can be removed while preserving connectivity, as long as $d(v_1, v_2) < \max\{d(u, v_1), d(u, v_2)\}$.



(a) *Relative Neighborhood Graph*



(b) *Modified Relative Neighborhood Graph* (MRNG)



(c) *Directed Relative Neighborhood Graph* (DRNG)

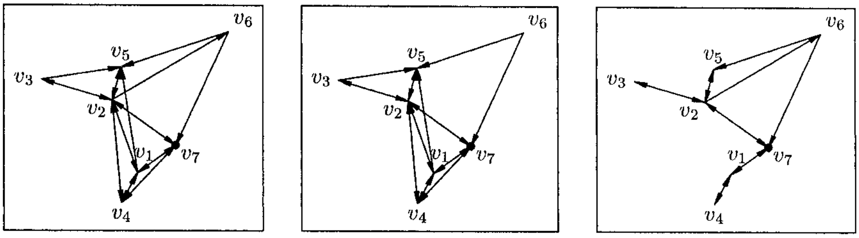
Figure 3: The definitions of RNG, MRNG, and DRNG.

To facilitate the introduction of RNG, we give the definition of the *Relative Neighborhood Graph* (RNG) below.

Definition 4.1 (Neighbor Relation in RNG.) *For RNG [19],[20], $u \overset{RNG}{\longleftrightarrow} v$ if and only if there does not exist a third node p such that $w(u,p) < w(u,v)$ and $w(p,v) < w(u,v)$. Or equivalently, there is no node inside the shaded area in Figure 3(a).*

The notion of RNG is proposed in [6] to facilitate topology initialization of wireless networks. Based on local knowledge, each node makes decisions to derive the network topology based on RNG. The network topology thus derived has been reported to exhibit good overall performance in terms of power usage, interference, and reliability.

Counterexamples. As shown in Figure 4, the network topology derived under $CBTC(\frac{2}{3}\pi)$ [21] (without optimization) may not preserve connectivity in

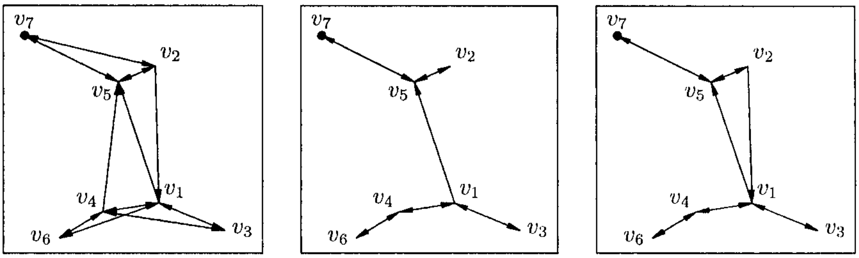


(a) Original topology (without topology control) is strongly connected

(b) Topology by $CBTC(\frac{2\pi}{3})$ without optimization is not strongly connected: there is no path from v_2 to v_6

(c) Topology by DLSS is strongly connected

Figure 4: An example that shows $CBTC(\frac{2\pi}{3})$ may render disconnectivity in heterogeneous networks. There is no path from v_2 to v_6 due to the loss of edge (v_2, v_6), which is discarded by v_2 because v_5 and v_7 have already provided the necessary coverage.



(a) Original topology (without topology control) is strongly connected

(b) Topology by RNG is not strongly connected: there is no path from v_5 to v_1

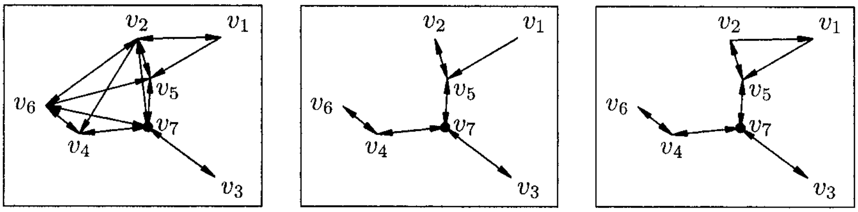
(c) Topology by DLSS is strongly connected

Figure 5: An example that shows RNG may render disconnectivity in heterogeneous networks. There is no path from v_5 to v_1 due to the loss of edge (v_2, v_1), which is discarded because $|(v_5, v_1)| < |(v_2, v_1)|$, and $|(v_5, v_2)| < |(v_2, v_1)|$.

a heterogeneous network. (The arrows in the figure indicate the direction of the links.) Similarly, as shown in Figure 5, the network topology derived under RNG may be disconnected in a heterogeneous network. As RNG is defined for undirected graphs only, we may modify its definition for directed graphs.

Definition 4.2 (MRNG.) *For* Modified Relative Neighborhood Graph *(MRNG), $u \xrightarrow{MRNG} v$ if and only if there does not exist a third node p such that $w(u,p) < w(u,v), d(u,p) \le r_u$ and $w(p,v) < w(u,v), d(v,p) \le r_v$ (Figure 3(b)).*

In spite of the modification, as shown in Figure 6, the topology derived under MRNG may still be disconnected in a heterogeneous network.



(a) Original topology (without topology control) is strongly connected

(b) Topology by MRNG is not strongly connected: there is no path from v_5 to v_1

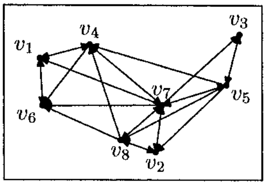
(c) Topology by DLSS is strongly connected

Figure 6: An example that shows MRNG may render disconnectivity in heterogeneous networks. There is no path from v_5 to v_1 due to the loss of edge (v_2, v_1), which is discarded because $|(v_2, v_5)| < |(v_2, v_1)|$, and $|(v_1, v_5)| < |(v_2, v_1)|$.

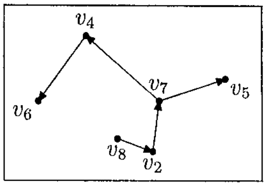
One possible extension to LMST is for each node to build a local *directed* minimum spanning tree [22], [23], [24] and keep only neighbors within one hop. Unfortunately, as shown in Figure 7, the resulting topology does not preserve strong connectivity. In the next subsection, we elaborate on how to revise LMST and RNG so that strong connectivity is preserved in heterogeneous networks.

4.2 Localized Algorithms: DRNG and DLSS

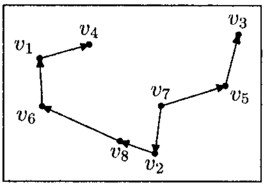
In this section, we present two localized topology control algorithms, *Directed Relative Neighborhood Graph* (DRNG) and *Directed Local Spanning Subgraph* (DLSS), for heterogeneous networks [10]. Both algorithms are composed of three steps: information collection, topology construction, and construction of topology with only bidirectional links. The first and the last steps are essentially the same as those described in Section 3.1. Therefore, we only



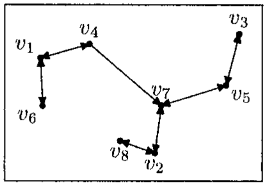
(a) Original topology (without topology control) is strongly connected



(b) The local directed MST rooted at v_8



(c) The local directed MST rooted at v_7



(d) The resulting topology is not strongly connected: there is no path from v_7 to v_4

Figure 7: An example that shows the algorithm in which each node builds a local directed minimum spanning tree and only keeps the one-hop neighbors may result in disconnectivity.

elaborate on the step of topology construction here. Essentially, instead of building a directed local minimum spanning tree (as in LMST) or using MRNG (as in RNG-based topology control) to define the neighbor relation, a node will use the following definition.

Definition 4.3 (DRNG.) *For* Directed Relative Neighborhood Graph *(DRNG),* $v \xrightarrow{DRNG} u$ *if and only if* $v \in N_u^V$ *and there does not exist another node* $p \in N_u^V$ *such that* $w(u,p) < w(u,v)$ *and* $w(p,v) < w(u,v), d(p,v) \leq r_p$ *(see Figure 3(b)).*

Definition 4.4 (DLSS.) *For* Directed Local Spanning Subgraph *(DLSS),* $v \xrightarrow{DLSS} u$ *if and only if* $(u,v) \in E(S_u)$*, where* S_u *is the output of DLSS(u) (Figure 8). Hence node* v *is a neighbor of node* u *if and only if node* v *is on node* u*'s directed local spanning graph* S_u*, and is one hop away from node* u*.*

Procedure: DLSS(u)
Input: G_u^V, the induced subgraph of G that spans the visible neighborhood of u;
Output: $S_u = (V(S_u), E(S_u))$, the local spanning subgraph of G_u^V;
begin
$V(S_u) := V$, $E(S_u) := \emptyset$;
Sort all edges in $E(G_u^V)$ in the ascending order of weight
for each edge (u_0, v_0) in the order
  if u_0 is not connected to v_0 in S_u
    $E(S_u) := E(S_u) \cup \{(u_0, v_0)\}$;
  endif
end
end

Figure 8: DLSS Algorithm.

DRNG and DLSS are natural extensions of RNG and LMST for heterogeneous networks, respectively. Conceptually, in DLSS instead of computing a directed local MST that minimizes the *total* cost of all the edges in the subgraph (Section 4.1), each node computes a directed local subgraph (Figure 10) that minimizes the *maximum* cost among all edges in the subgraph.

4.3 Properties of DRNG and DLSS

In this section, we discuss several desirable properties of DRNG and DLSS by presenting a sequence of lemmas and theorems. In particular, Lemma 4.5, Theorem 4.6, and Lemma 4.7 can be proved by keeping in mind that G is a directed graph and following the same line of argument in Section 3.1. Note that we can only prove that $u \Rightarrow v$ in Lemma 4.5, because $u_k \rightarrow v_k$ does not guarantee that $v_k \rightarrow u_k$. Theorem 4.6 can be proved with the use of Lemma 4.5, in the same fashion Theorem 3.5 was proved with the use of Lemma 3.4.

Lemma 4.5 *For any edge $(u, v) \in E(G)$, we have $u \Rightarrow v$ in G_{DLSS}.*

Theorem 4.6 (Connectivity of DLSS.) *G_{DLSS} preserves the connectivity of G, i.e., G_{DLSS} is strongly connected if G is strongly connected.*

Lemma 4.7 *Given three nodes $u, v, p \in V(G_{DLSS})$ satisfying $w(u,p) < w(u,v)$ and $w(p,v) < w(u,v)$, $d(p,v) \leq r_p$, then $u \nrightarrow v$ in G_{DLSS}.*

By leveraging Theorem 4.6 to prove that DRNG preserves strong connectivity, we first prove the following lemma.

Lemma 4.8 *The edge set of G_{DLSS} is a subset of the edge set of G_{DRNG}, i.e., $E(G_{DLSS}) \subseteq E(G_{DRNG})$.*

Proof. We prove by contradiction. For any edge $(u, v) \in E(G_{DLSS})$, assume $(u, v) \notin E(G_{DRNG})$. From the definition of $DRNG$, there must exist a third node p such that $w(u,p) < w(u,v), d(u,p) \leq r_u$ and $w(p,v) < w(u,v), d(p,v) \leq r_p$. By Lemma 4.7, $u \not\rightarrow v$ in G_{DLSS}; i.e., $(u, v) \notin E(G_{DLSS})$. □

The following theorem that proves DRNG preserves strong connectivity is a direct result of Theorem 4.6 and Lemma 4.8.

Theorem 4.9 (Connectivity of DRNG) *If G is strongly connected, then G_{DRNG} is also strongly connected.*

Let $Disk(u, r)$ denote the disk of radius r, centered at node u. Then the following lemma is a direct result of the definition of DRNG.

Lemma 4.10 *Given three nodes $u, v, p \in V(G_{DRNG})$ satisfying $w(u,p) < w(u,v)$ and $w(p,v) < w(u,v)$, $d(p,v) \leq r_p$, then $u \not\rightarrow v$ in G_{DRNG}.*

Now to derive the in-degree bound, we state the following corollary (which is a direct result of Lemma 4.7 and Lemma 4.10).

Corollary 4.11 *If v is an out-neighbor of u in G_{DLSS} or G_{DRNG}, and $d(u,v) \geq r_{min}$, then u can not have any other out-neighbor inside $Disk(v, r_{min})$.*

Based on the above corollary, the following two theorems that give the in-degree bound of G_{DLSS} and G_{DRNG} can be proved by following the same line of arguments found in Theorem 3.10.

Theorem 4.12 *For any node u in G_{DLSS} or G_{DRNG}, the number of out-neighbors that are inside $Disk(u, r_{min})$ is at most 6.*

Theorem 4.13 (In Degree Bound.) *The in-degree of any node in G_{DLSS} or G_{DRNG} is bounded by 6.*

To derive the out-degree bound, we prove the following theorem.

Theorem 4.14 (Out-Degree Bound.) *The out-degree of any node in G_{DLSS} or G_{DRNG} is bounded by a constant that depends only on r_{max} and r_{min}.*