Introduction to Electric Power and Drive Systems

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PREFACE AND ACKNOWLEDGMENT

TRADITIONALLY, the lead-in to the study of electric power engineering has been a junior-level course on electric machines. However, due to the advent of power electronics and the computer, the present-day electrical engineering student interested in the power area has several areas from which to choose, including electric machines, power electronics, electric drives, and power systems. This has caused some concern as to the most appropriate lead-in course to the power discipline. Although the analysis of electric machinery is fundamental to power systems and electric drives, it may not be the most effective way to introduce the power area to a student who is trying to decide on a career path. As an alternative, some schools have instituted a survey course; however, this often takes the form of a broad and general review lacking the sufficient depth to give the student an indication of the physical concepts and analyses common to the areas or to provide an analytical foundation on which to build in follow-on courses. From the standpoint of the faculty, who must make full use of every credit hour allotted to their area of study, an introductory course that establishes a useful foundation seems appropriate.

This book is an attempt to provide students with an analytical base on which they can build in follow-on courses in electric machines, power electronics, electric drives, and power systems. This is accomplished with only sophomore-level calculus, physics, and basic electric circuits as pre- or co-requisites, allowing the course to be taught early in most engineering disciplines by a professor with either an engineering or physics background. The text is suggested for use in a 3-hour course in electrical engineering at the second-semester sophomore-level or first-semester junior-level in schools with a viable power program. Alternatively, it supports a junior/senior technical elective course in schools without a power program, or in non-electrical engineering disciplines such as mechanical engineering. It is written so that the professor can select the material to fit the school's power program and interests. Although most of the material in Chapter 1 is common to all areas of power and should be covered in its entirety, each of the subsequent chapters contains material fundamental to the areas of electric machines, power electronics, electric drives, and power systems, respectively, along with material that is somewhat more advanced in each of these disciplines. This allows the instructor to choose between a brief and an in-depth coverage of each of the areas. It is not intended that the entirety of the text be covered in a three-credit course.

Electric machines are covered in Chapter 2, focusing on the analysis of the round-rotor permanent-magnet ac machine. This device, when used as an inverter-driven brushless dc motor, has become the low-power electric drive of choice. Since

this is a synchronous machine, the analysis applies, in part, to the synchronous generator that is covered in Chapter 5 on power systems. Tesla's rotating magnetic field is shown to be the key to machine analysis in that it provides a sound basis for a concise analytical derivation of all known mathematical transformations that are used in machine and power system analyses. Moreover, it provides an analytical means of positioning the stator and rotor poles on the phasor diagram, thereby providing a straightforward and instructive illustration of machine operation that adds credence to the approach of analyzing the two-phase devices first and then the three phase. Tesla not only invented the ac machine, his work is instrumental in analyzing and visualizing its operation. An animation described in Appendix C will help the student to visualize Tesla's rotating magnetic field. All animations can be accessed at www.wiley.com/go/krause/electricpower.

Basic power electronic circuit analysis is covered in Chapter 3, including switching-circuit fundamentals, dc-dc converters, ac-dc rectifiers, dc-ac inverters, and a brief introduction to harmonics and distortion. This includes an analysis of the three-phase six-step inverter, which establishes basic relationships between the transistor switching signals and the input (dc) and output (ac) voltages. This inverter is ubiquitous in the electric drives area; as such, its analysis in Chapter 3 is built upon in Chapter 4 on electric drives.

The performance and simulation of an electric drive are covered in Chapter 4. The drive chosen for analysis is the round-rotor permanent-magnet machine described in Chapter 2 powered by the three-phase six-step inverter introduced in Chapter 3. Although this so-called brushless dc drive represents only a small subset of the electric-drives area, it is widely used and its salient features can be presented without becoming too involved. Three methods of brushless dc motor operation – traditional, maximum torque per volt, and maximum torque per ampere – are demonstrated by changing the phase angle and effective value of the applied stator voltages. A direct comparison is made between the simulated performance of the drive with both sinusoidal and six-step applied voltages. The chapter concludes with a straightforward method for simulating the drive system. An animation described in Appendix C will help the student visualize the operation of a brushless dc drive.

Finally, the power systems area, including transformer connections, the synchronous generator, power factor correction, and the per-unit system, is introduced in Chapter 5. The chapter concludes with a brief discussion of transient stability, provided to illustrate some of the basic challenges faced in the power systems area.

This text covers several power engineering sub-disciplines. It is not surprising, then, that throughout its pages, some differences in notation, variable naming conventions, and analysis methods exist. However, rather than attempt to standardize, we have chosen to recognize and explain any discrepancies as they arise. We have done this so that each chapter may more easily be taught separately from the others and, if the instructor desires it, be incorporated into existing curricula that uses common notation. We hope that this approach will aid the power engineering student to anticipate, understand, and accept these differences.

Obviously, there are other aspects of the power area that would be appropriate for an introductory text; however, the choices we have made seem to be representative

at this time. Nevertheless, as the power area continues to evolve through the twenty-first century, so must an introductory text.

We would like to acknowledge Dr. Brett Robbins of PC Krause and Associates for developing the drawings and formatting the computer traces for the text. We would also like to acknowledge the efforts and assistance of the reviewers, in particular Mohamed El-Hawary, the staff of IEEE Press and John Wiley & Sons, especially Mary Hatcher, Danielle Lacourciere, Victoria Bradshaw, Jeanne Audino and finally the Production Editor Suresh Srinivasan of Aptara for the final typesetting of the manuscript.

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BASIC CONCEPTS

1.1 INTRODUCTION

The twentieth century began with the electric power industry in its infancy; Thomas Edison and Nikola Tesla were locked in battle with Edison advocating direct current (dc) and Tesla alternating current (ac). The century ended with the electric power industry expanding rapidly from the traditional power generation, transmission, and utilization into propulsion of air, ground, and sea transportation. The advent of the computer and the silicon-controlled rectifier in the mid-1900s brought about an expansion of the power area to include the smart grid, microgrids, efficient and robust electric drives, more-electric aircraft, ships, and land vehicles. This growth is likely to continue into the foreseeable future.

Before the advent of the computer, engineers were essentially limited to steady-state analysis and therefore unable to conveniently deal with the analytical challenges of the expanding power industry. This chapter sets forth some of the basic concepts and analysis tools that are part of the present-day power and electric drives area. Although not inclusive, the material covered in this chapter is representative and common to all disciplines of the power area.

1.2 PHASOR ANALYSIS AND POWER CALCULATIONS

Since the early twentieth century, we have lived in an alternating current (ac) world. Thanks to George Westinghouse and Nikola Tesla, power systems are predominately ac; power is generated by large ac generators, transmitted by high voltage transmission lines, and transformed to a low voltage and distributed to homes and factories. The evolution of the ac power system brought about many engineering challenges and, as we look back, it is difficult to comprehend how these problems were solved without a computer. Even steady-state ac-circuit analysis posed a problem until the early 1900s when Charles Stienmetz, who was a less flamboyant colleague of Edison and Tesla, came up with the concept of what is now known as phasors. Some may consider the phasor a casualty of the computer age along with the slide rule. It is, however, still a very useful means for understanding and portraying the steady-state performance of electric machines, power systems, and electric drives. Moreover, the

phasor concept provides a means of visualizing sinusoidal variations from different frames of reference and in Chapter 2 we will find that the voltage and current phasors combined with Tesla's rotating magnetic field provides a straightforward means of analyzing and portraying the steady-state operation of ac machines.

The phasor can be established by expressing a steady-state sinusoidal variable as

$$F_a(t) = F_p \cos \theta_{ef} \tag{1.2-1}$$

where the a subscript is used here to denote sinusoidal quantities. The sinusoidal variations are expressed as cosines, capital letters are used to denote steady-state quantities, and F_p is the peak value of the sinusoidal variation. Generally, F or f represents voltage (V or v) or current (I or i) in circuit analysis, but it could be any sinusoidal variable. For steady-state conditions, θ_{ef} may be written as

$$\theta_{ef}(t) = \omega_e t + \theta_{ef}(0) \tag{1.2-2}$$

where ω_e is the electrical angular velocity in radians/second (2π times the frequency) and $\theta_{ef}(0)$ is the time-zero position of the electrical variable. Substituting (1.2-2) into (1.2-1) yields

$$F_a(t) = F_p \cos[\omega_e t + \theta_{ef}(0)] \tag{1.2-3}$$

Now, Euler's Identity is

$$e^{j\alpha} = \cos \alpha + j \sin \alpha \tag{1.2-4}$$

and since we are expressing the sinusoidal variation as a cosine, (1.2-3) may be written as

$$F_a(t) = \text{Re}\left\{F_p e^{j[\omega_e t + \theta_{ef}(0)]}\right\} \tag{1.2-5}$$

where Re is the shorthand for "real part of." Equations (1.2-3) and (1.2-5) are equivalent. Let us rewrite (1.2-5) as

$$F_a(t) = \text{Re}\left\{F_p e^{j\theta_{ef}(0)} e^{j\omega_e t}\right\} \tag{1.2-6}$$

We need to take a moment to define what is referred to as the root-mean-square (rms) of a sinusoidal variation. In particular, the rms value is defined as

$$F = \left(\frac{1}{T} \int_0^T F_a^2(t)\,dt\right)^{\frac{1}{2}} \tag{1.2-7}$$

where F is the rms value of $F_a(t)$ and T is the period of the sinusoidal variation. It is left to the reader to show that the rms value of (1.2-3) is $F_p/\sqrt{2}$. Therefore, we can express (1.2-6) as

$$F_a(t) = \text{Re}[\sqrt{2}F e^{j\theta_{ef}(0)} e^{j\omega_e t}] \tag{1.2-8}$$

By definition, the phasor representing $F_a(t)$, which is denoted with a raised tilde, is

$$\tilde{F}_a = F e^{j\theta_{ef}(0)} \tag{1.2-9}$$

which is a complex number. The reason for using the rms value as the magnitude of the phasor will be addressed later in this section. Equation (1.2-6) may now be written as

$$F_a(t) = \text{Re}\left[\sqrt{2}\tilde{F}_a e^{j\omega_e t}\right] \tag{1.2-10}$$

A shorthand notation for (1.2-9) is

$$\tilde{F}_a = F\underline{/\theta_{ef}(0)} \tag{1.2-11}$$

Equation (1.2-11) is commonly referred to as the *polar form* of the phasor. The *Cartesian* form is

$$\tilde{F}_a = F\cos\theta_{ef}(0) + jF\sin\theta_{ef}(0) \tag{1.2-12}$$

When using phasors to calculate steady-state voltages and currents, we think of the phasors as being stationary at $t = 0$; however, we know that a phasor is related to the instantaneous value of the sinusoidal quantity it represents. Let us take a moment to consider this aspect of the phasor and thereby, give some physical meaning to it. From (1.2-4), we realize that $e^{j\omega_e t}$ is a line of unity length rotating counterclockwise at an angular velocity of ω_e. Therefore, backing up for a minute

$$\sqrt{2}\,\tilde{F}_a e^{j\omega_e t} = \sqrt{2}Fe^{j\theta_{ef}(0)}e^{j\omega_e t} \tag{1.2-13}$$

is a line with a constant amplitude of $\sqrt{2}F$ rotating counterclockwise in the real-imaginary plane at an angular velocity of ω_e with a time-zero displacement from the positive real axis of $\theta_{ef}(0)$. Since $\sqrt{2}F$ is the peak value of the sinusoidal variation, the instantaneous value of $F_a(t)$ expressed as a cosine is the real part of (1.2-13). In other words, the real projection of the phasor $\tilde{F}_a$ rotating counterclockwise at ω_e is the instantaneous value of $F_a(t)/\sqrt{2}$. Thus, with $\theta_{ef}(0) = 0$ in (1.2-3)

$$F_a(t) = \sqrt{2}F\cos\omega_e t \tag{1.2-14}$$

the phasor representing (1.2-14) is

$$\tilde{F}_a = Fe^{j0} = F\underline{/0°} = F + j0 \tag{1.2-15}$$

For

$$F_a(t) = \sqrt{2}F\sin\omega_e t$$
$$= \sqrt{2}F\cos(\omega_e t - 90°) \tag{1.2-16}$$

the phasor is

$$\tilde{F}_a = Fe^{-j\pi/2} = F\underline{/-90°} = 0 - jF \tag{1.2-17}$$

We will use degrees and radians interchangeably when expressing phasors. Although there are several ways to arrive at (1.2-17) from (1.2-16), it is helpful to ask yourself where the rotating phasor must be positioned at time-zero so that, when it rotates counterclockwise at ω_e, its real projection is $(1/\sqrt{2})F_p\sin\omega_e t$. It follows that a phasor of amplitude F positioned at $90°$ represents $-\sqrt{2}F\sin\omega_e t$.

In other words, we are viewing a sinusoidal variation as the real projection in the real-imaginary plane of a rotating line equal in magnitude to the positive peak value ($\sqrt{2}F$) of the variation and rotating at the electrical angular velocity of the sinusoidal variation. Since we are dealing with a steady-state variation, we can stop the rotation at any time and view it as a fixed line, but knowing full well that it, in fact, represents a sinusoidal variation and to represent the sinusoidal variation we must rotate it counterclockwise at ω_e and take the real projection. Please understand that if we ran at ω_e in unison with the rotating $\sqrt{2}F$ line it would appear as a constant to us; therefore, in viewing a sinusoidal variation in this manner it would appear to us as a constant. This is no different than stopping the phasor at some arbitrary time-zero; but realizing that it actually represents a sinusoidal variation. We will talk more about this important aspect as we go along; in particular, see Example 1A.

In order to show the facility of the phasor in the analysis of steady-state performance of ac circuits and devices, it is useful to consider a series circuit consisting of a resistance, an inductance L, and a capacitance C. Thus, using uppercase letters to indicate steady-state variables

$$V_a = RI_a + L\frac{dI_a}{dt} + \frac{1}{C} \int I_a dt \tag{1.2-18}$$

Throughout the text, we will use either R or r to represent resistance. For steady-state operation, let

$$V_a = \sqrt{2}V \cos[\omega_e t + \theta_{ev}(0)] \tag{1.2-19}$$

$$I_a = \sqrt{2}I \cos[\omega_e t + \theta_{ei}(0)] \tag{1.2-20}$$

where we have dropped the functional notation and the subscript a helps to distinguish the instantaneous value from the rms value of the steady-state variables. The steady-state voltage equation may be obtained by substituting (1.2-19) and (1.2-20) into (1.2-18), whereupon we can write

$$\sqrt{2}V \cos[\omega_e t + \theta_{ev}(0)] = R\sqrt{2}I \cos\left[\omega_e t + \theta_{ei}(0)\right]$$
$$+ \omega_e L \sqrt{2}I \cos\left[\omega_e t + \tfrac{1}{2}\pi + \theta_{ei}(0)\right]$$
$$+ \frac{1}{\omega_e C} \sqrt{2}I \cos\left[\omega_e t - \tfrac{1}{2}\pi + \theta_{ei}(0)\right] \tag{1.2-21}$$

The second term on the right-hand side of (1.2-21), which is $L\frac{dI_a}{dt}$, can be written

$$\omega_e L \sqrt{2}I \cos\left[\omega_e t + \tfrac{1}{2}\pi + \theta_{ei}(0)\right] = \omega_e L \text{Re}\left[\sqrt{2}I e^{j\frac{1}{2}\pi} e^{j\theta_{ei}(0)} e^{j\omega_e t}\right] \tag{1.2-22}$$

Since $\tilde{I}_a = I e^{j\theta_{ei}(0)}$, from (1.2-21), we can write

$$L\frac{dI_a}{dt} = \omega_e L e^{j\frac{1}{2}\pi} \tilde{I}_a \tag{1.2-23}$$

Since $e^{j\frac{1}{2}\pi} = j$, (1.2-23) may be written

$$L\frac{dI_a}{dt} = j\omega_e L\tilde{I}_a \qquad (1.2\text{-}24)$$

If we follow a similar procedure, we can show that

$$\frac{1}{C}\int I_a dt = -j\frac{1}{\omega_e C}\tilde{I}_a \qquad (1.2\text{-}25)$$

Differentiation of a steady-state sinusoidal variable rotates the phasor counterclockwise by $\frac{1}{2}\pi$ or j; integration rotates the phasor clockwise by $\frac{1}{2}\pi$ or $-j$.

The steady-state voltage equation given by (1.2-21) can now be written in phasor form as

$$\tilde{V}_a = \left[R + j(\omega_e L - \frac{1}{\omega_e C})\right]\tilde{I}_a \qquad (1.2\text{-}26)$$

We can express (1.2-26) compactly as

$$\tilde{V}_a = Z\tilde{I}_a \qquad (1.2\text{-}27)$$

where the impedance, Z, is a complex number; it is not a phasor. It may be expressed as

$$Z = R + j(X_L - X_C) \qquad (1.2\text{-}28)$$

where $X_L = \omega_e L$ is the inductive reactance and $X_C = \frac{1}{\omega_e C}$ is the capacitive reactance. We should be careful here. Some prefer to write (1.2-28) as $R + jX$ where X is $X_L + X_C$ and let X_C be negative. This is essentially a matter of choice and does not change the end result. We will deal primarily with X_L and not X_C, therefore, this will have little impact on our work; nevertheless, since some authors will use a negative X_C we should make the reader aware of this difference.

It is appropriate to discuss the notation that will be used throughout the text. When an equation is written with the variables in lowercase letters it is valid for transient and steady state. If the variables are written with uppercase letters as in (1.2-18) the equation is a function of time and valid for instantaneous steady-state conditions. Equations (1.2-26) and (1.2-27) are phasor equations representing steady-state sinusoidal variables and are written in uppercase letters with an over tilde.

Power and Reactive Power

The instantaneous steady-state power is

$$\begin{aligned} P &= V_a I_a \\ &= \sqrt{2}V\cos\left[\omega_e t + \theta_{ev}(0)\right]\sqrt{2}I\cos\left[\omega_e t + \theta_{ei}(0)\right] \end{aligned} \qquad (1.2\text{-}29)$$

where V and I are rms values. After some manipulation, we can write (1.2-29) as

$$P = VI\cos[\theta_{ev}(0) - \theta_{ei}(0)] + VI\cos[2\omega_e t + \theta_{ev}(0) + \theta_{ei}(0)] \qquad (1.2\text{-}30)$$

The instantaneous steady-state power given by (1.2-30) varies about an average value at a frequency of $2\omega_e$. That is, the second term of (1.2-30) has a zero average value and the average power P_{ave} may be written

$$P_{\text{ave}} = |\tilde{V}_a|\,|\tilde{I}_a|\cos[\theta_{ev}(0) - \theta_{ei}(0)] \tag{1.2-31}$$

where $|\tilde{V}_a|$ and $|\tilde{I}_a|$ are V and I, respectively which are the magnitudes of the phasors (rms value); $\theta_{ev}(0) - \theta_{ei}(0)$ is referred to as the *power factor angle* φ_{pf}, and $\cos[\theta_{ev}(0) - \theta_{ei}(0)]$ is the *power factor*. Power is in watts. If current is assumed positive in the direction of voltage drop then (1.2-31) is positive if power is consumed and negative if power is generated. It is interesting to point out that in going from (1.2-29) to (1.2-30), the coefficient of the two right-hand terms is $\frac{1}{2}(\sqrt{2}V\sqrt{2}I)$ or one-half the product of the peak values of the sinusoidal variables. Therefore, it was considered more convenient to use the rms values for the phasors, whereupon average steady-state power could be calculated by the product of the magnitude of the voltage and current phasors as given by (1.2-31).

The reactive power is defined as

$$Q = |\tilde{V}_a|\,|\tilde{I}_a|\sin[\theta_{ev}(0) - \theta_{ei}(0)] \tag{1.2-32}$$

The unit of Q is var (volt-ampere reactive). An inductance is said to absorb reactive power where the current lags the voltage by 90° and Q is positive and supplied by a capacitor where the current leads the voltage by 90° and Q is negative. Actually, Q is a measure of the interchange of energy stored in the electric (capacitor) and magnetic (inductor) fields. However, unlike instantaneous real power, the average value of instantaneous reactive power is zero. We will talk more about reactive power when we get to Chapter 5.

Example 1A. Phasor analysis.   The parameters of a series RLC circuit are $R = 6\,\Omega$, $L = 20\,\text{mH}$, $C = 1 \times 10^3\,\mu\text{F}$. The 60 Hz applied voltage is $V_a = 155.6\cos\omega_e t$. Calculate $\tilde{I}_a, P_{\text{ave}}, Q$ and draw the phasor diagram and the sinusoidal variations as viewed running at counterclockwise with the phasor representing maximum V_a ($\sqrt{2}V_s\underline{/0°}$). From the expression of V_a

$$\tilde{V}_a = 110\underline{/0°}\ \text{V} \tag{1A-1}$$

Now, $\omega_e = 2\pi f = 2\pi \times 60 = 377\,\text{rad/s}$ and

$$Z = R + j(X_L - X_C)$$

$$= R + j\left(\omega_e L - \frac{1}{\omega_e C}\right)$$

$$= 6 + j\left(377 \times 20 \times 10^{-3} - \frac{1}{377 \times 1 \times 10^{-3}}\right) = 7.73\underline{/39.1°}\ \Omega \tag{1A-2}$$

$$\tilde{I}_a = \frac{\tilde{V}_a}{Z} = \frac{110\underline{/0°}}{7.73\underline{/39.1°}} = 14.2\underline{/-39.1°}\ \text{A} \tag{1A-3}$$

$$P_{\text{ave}} = |\tilde{V}_a|\,|\tilde{I}_a|\cos\phi_{pf} \tag{1A-4}$$

where

$$\phi_{pf} = \theta_{ev}(0) - \theta_{ei}(0)$$
$$= 0 - (-39.1°) = 39.1° \tag{1A-5}$$
$$P_{ave} = 110 \times 14.2 \cos 39.1°$$
$$= 1212.2\,\text{W} \tag{1A-6}$$
$$Q = |\tilde{V}_a|\,|\tilde{I}_a| \sin \phi_{pf}$$
$$= 110 \times 14.2 \sin 39.1° = 985.1\,\text{vars} \tag{1A-7}$$

The phasor diagram is shown in Fig. 1A-1.
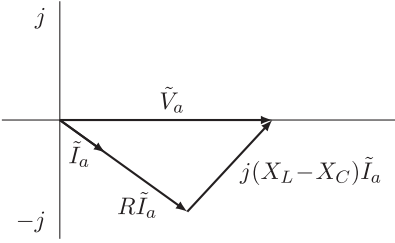
Figure 1A-1   Phasor diagram.

Fig. 1A-2 shows the "stationary" waveforms of the voltages viewed with 360° vision while running at ω_e with the counterclockwise rotating phasor representing V_a. The current I_a is not shown; however, $\sqrt{2}\,|\tilde{I}_a|$ would be in phase with the voltage $R\sqrt{2}\,|\tilde{I}_a|$. Also, the type of diagram shown in Fig. 1A-2, is not convenient to portray that the real part of the rotating phasor is the instantaneous value of the sinusoidal variation. Rotating the phasor diagram given in Fig. 1A-1 best illustrates that feature of a phasor.

Figure 1A-2   Voltage waveforms of *RLC* circuit viewed with 360° vision while rotating with phasors at ω_e counterclockwise.

SP1.2-1. Express the instantaneous steady-state power for Example 1A. [Substitute into (1.2-30)]

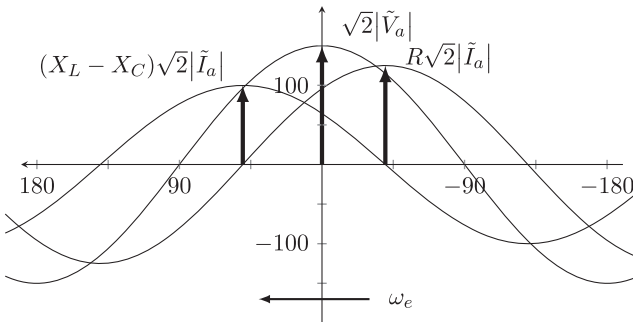**SP1.2-2.** Redraw the phasor diagram shown in Fig. 1A-1 showing $jX_L\tilde{I}_a$ and $-jX_C\tilde{I}_a$ as individual voltages. [Show $jX_L\tilde{I}_a$ and then from the terminus of $jX_L\tilde{I}_a$ show $-jX_C\tilde{I}_a$]

SP1.2-3. We know that $P_{ave} = |\tilde{I}|^2 R$, does Q equal $|\tilde{I}_a|^2 X_L - |\tilde{I}_a|^2 X_C$? [Yes]

SP1.2-4. If $\tilde{V} = 1\underline{/0°}$ V and $\tilde{I} = 1\underline{/180°}$ A in the direction of the voltage drop, calculate Z and P_{ave}. Is power generated or consumed? [$(-1 + j0)$ ohms, 1 watt, generated]

SP1.2-5. Express the instantaneous power for 60 Hz voltage, $\tilde{V}_a = 1\underline{/0°}$ V, applied to a resistive circuit, $\tilde{I}_a = 1\underline{/0°}$ A. [$1 + \cos 754t$]

SP1.2-6. Repeat SP1.2-5 for (a) an inductance, $\tilde{I}_a = I_L\underline{/-90°}$ A and (b) a capacitance, $\tilde{I}_a = I_C\underline{/90°}$ A. [(a) $I_L \cos(754t - 90°)$, (b) $I_C \cos(754t + 90°)$]

1.3 ELEMENTARY MAGNETIC CIRCUITS

Electric machines and transformers, which are the backbone of the power industry, are electromagnetic systems. Therefore, magnetic circuits and magnetic coupling play a major role in power and drives systems and it is necessary to establish the principles of magnetic systems sufficiently to convey the basic operation of the electromagnetic devices considered in later chapters. We will attempt to do this without becoming too involved.

An elementary magnetic circuit is shown in Fig. 1.3-1. It consists of a ferromagnetic member (core) with a coil of wire of N turns wound on it and an air gap of length x. The ferromagnetic member could be iron, nickel, cobalt, or steel, for example. The voltage equation of the electric circuit may be written

$$v = ri + e \tag{1.3-1}$$

where r is the total resistance of the circuit, v is the source voltage, e is the voltage induced in the coil according to Faraday's Law, and i is the current flowing in the circuit. The current flowing through the coil causes a magnetomotive force (mmf), which produces flux in the magnetic circuit denoted as Φ_m and Φ_l in Fig. 1.3-1, much as an electromotive force (emf) or source voltage produces current in an electric circuit.
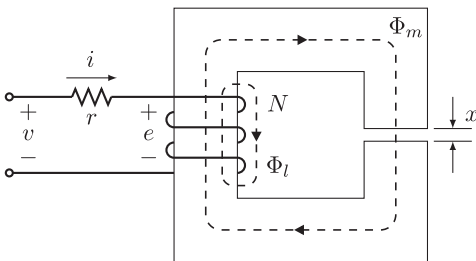

Figure 1.3-1   Elementary magnetic circuit with one air gap.

There are arrows associated with the dashed lines representing the flux paths in Fig. 1.3-1. These arrows indicate the assumed positive direction of flux which is determined from the assumed positive direction of current by the so called "right-hand" rule. If you grasp the coil with your right hand with your fingers in the assumed direction of positive current flow around the coil, your thumb will point in the direction of positive flux. Or, imagine grasping a turn of the coil with thumb in the assumed direction of positive current. If you ungrasp your fingers they will point in the direction of positive flux.

The total flux, Φ, that travels through (links) all of the turns N is

$$\Phi = \Phi_l + \Phi_m \tag{1.3-2}$$

where Φ_l is the equivalent flux that links all the turns of the coil but does not traverse the ferromagnetic member and Φ_m is referred to as the magnetizing flux that transverses the ferromagnetic member and links all of the turns of the coil. The leakage flux Φ_l is shown by one streamline in Fig. 1.3-1, it represents the aggregate of the flux that occurs around each wire of the coil, traveling partially in the ferromagnetic material and partially in air.

The concept of magnetic poles can be used to advantage to explain the operation of electromechanical devices. The poles can be established by considering the magnetic circuit shown in Fig. 1.3-1. In particular, to locate the north and south poles of an electromechanical device for the assumed positive direction of coil current, place yourself on the member that has the coil (windings). Use the right-hand rule to establish the direction of positive flux Φ_m for the assumed positive direction of coil current; where positive flux issues from the member with the coil, into the air is the assumed north pole. The south pole is where the positive flux returns from the air to the member with the coil. In the case of the magnetic circuit shown in Fig. 1.3-1, with the assumed direction of positive current a north pole exists over the upper face of the air gap and a south pole over the lower face.

A magnetically linear circuit behaves much as a resistive electric circuit. According to Ohm's Law, the current i in a resistive circuit is equal to the applied electromotive force (emf) or applied voltage divided by the resistance r. In the case of a magnetic circuit, the total flux Φ is equal to the magnetomotive force (mmf), which is in ampere turns Ni, divided by the equivalent reluctance $\mathcal{R}$ of the magnetic circuit. Thus, in (1.3-2)

$$\Phi_l = \frac{Ni}{\mathcal{R}_l} \tag{1.3-3}$$

$$\Phi_m = \frac{Ni}{\mathcal{R}_i + \mathcal{R}_g} \tag{1.3-4}$$

where $\mathcal{R}_l$ is the reluctance of the leakage flux path, $\mathcal{R}_i$ is the reluctance of the ferromagnetic member, and $\mathcal{R}_g$ is the reluctance of the air gap. Although the leakage reluctance is generally determined by test or an involved calculation, $\mathcal{R}_i$ and $\mathcal{R}_g$ may be calculated from

$$\mathcal{R} = \frac{\ell}{\mu_r \mu_0 A} \tag{1.3-5}$$

where ℓ is the length of the flux path, A is the cross sectional area of the flux path, μ_0 is the permeability of free space ($4\pi \times 10^{-7}$ Wb/A $\cdot$ m or H/m), and μ_r is the permeability relative to free space. The unit of reluctance is (henry^{-1}) or H^{-1}. In the case of the air gap the relative permeability is considered to be unity ($\mu_{rg} \approx 1$), for the ferromagnetic member typical μ_{ri} values vary from 500 to 4000 depending upon the type of ferromagnetic material. It follows that the reluctance of the ferromagnetic member is much less than the reluctance of either the air gap or the part of the leakage path that is in air. In fact, the reluctance of the ferromagnetic member is generally neglected when an air gap is present as in the case of an electric machine. The magnetic equivalent circuit shown in Fig. 1.3-2 may be helpful to visualize the flux paths of the magnetic system shown in Fig. 1.3-1.
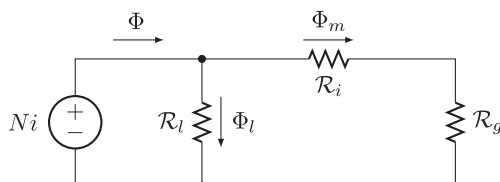

Figure 1.3-2   Magnetic equivalent circuit for magnetic system shown in Fig. 1.3-1.

Before proceeding, there are a couple of things that we should talk about. We have arrived at (1.3-3) and (1.3-4) by exploiting the similarities between a resistive electric circuit and a linear magnetic circuit. Although this approach is straight forward and easy to follow, let us take a minute to apply basic laws of magnetically linear systems that we learned in early physics courses to justify (1.3-3) and (1.3-4). Ampere's Law states that the line integral of the field intensity (field strength), H, about a closed path is equal to the net current enclosed within the closed path of integration. Now, for a two dimensional magnetically linear system

$$B = \mu H \tag{1.3-6}$$

where B is the flux density, μ is the permeability, and H is the field strength. If we assume the flux, Φ, is uniform over the cross-sectional surface area of the magnetic path then

$$B = \frac{\Phi}{A} \tag{1.3-7}$$

where A is the cross-sectional area. Thus, H may be expressed

$$H = \frac{\Phi}{A\mu} \tag{1.3-8}$$

Now, if H is integrated over the closed path that encloses the total turns of the coil and assuming H is the same over this path, then the magnetomotive force, mmf, or Ni is

$$Ni = \int_0^\ell \frac{\Phi}{A\mu} d\xi \tag{1.3-9}$$