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Dimensional Analysis for Engineers

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ISSN 2192-4732 ISSN 2192-4740 (electronic)
Mathematical Engineering
ISBN 978-3-319-52026-1 ISBN 978-3-319-52028-5 (eBook)
DOI 10.1007/978-3-319-52028-5

Library of Congress Control Number: 2016963646

Printed on acid-free paper

This Springer imprint is published by Springer Nature
The registered company is Springer International Publishing AG
The registered company address is: Gewerbestrasse 11, 6330 Cham, Switzerland

To our teachers

Preface

At all times, humans have been striving to understand the world that surrounds them and that they live in. We want to understand, why things are happening the way they do. We try to comprehend, we try to understand, we are yearning for insight. From insight we hope to gain foresight, and from understanding we hope to be able to affect and manage our future.

We want to manage for the better, so we need to measure the status before and after we have taken action. We want to make sure by measurement that we have done the right thing. But what is the right measure?

Obviously, the fact that we gain insight is an advantage for us. And the event, when we gain insight, is pleasurable and satisfying for us: "Eureka!" When, after having worked hard and long to solve a difficult problem, after doubt and despair, after struggling to comprehend, when we finally understand: This moment is a most beautiful sensation.

Dimensional analysis is a method that helps to gain insight and that helps to understand physical problems. It is universally valid; it can always be applied; it is always correct. Dimensional analysis shows us the right measure, the correct scaling of a problem, which is always inherent to the problem itself.

This book aims to introduce the reader to the fascinating topic of dimensional analysis through a large number of examples and just as many fundamentals, which are needed to understand the working principles and foundations of the method. The book is organized in four chapters. The first chapter concerns the fundamentals of the method, terms and definitions and the core of the method: the Buckingham- or Pi-Theorem. The second chapter on model theory underlines its importance through brevity. The third chapter collects examples ranging from mechanics, fluid mechanics, thermodynamics, electrodynamics to a variety of physical settings to show the broad range of applications. In the fourth chapter, finally, the application of dimensional analysis to partial differential equations of physical problems is demonstrated, and the power of the method is best shown by concentrating on selected examples, the so-called similarity solutions. The appendix collects some exercises and corresponding solutions for tutorials and self-study.

We, the authors, independently enjoyed our first encounter with dimensional analysis as a universal method that helps understanding common phenomena, physical problems, and through understanding helps to solve engineering tasks at hand. And we certainly appreciated the skills of the teachers, who first showed and taught us the beauty of dimensional analysis. Two of us (Volker Simon and Bernhard Weigand) have greatly benefitted from our teacher Prof. Joseph H. Spurk, to whom we are grateful and express our thankfulness. His lectures and book on dimensional analysis have certainly strongly influenced our work and the way to look at problems and find their solutions.

We hope that our book has the same effect on its readers: It gives them insight into dimensional analysis, supports their understanding of the physical world, and as a consequence the method will be a pleasurable benefit to them.

Many people helped us in all phases of the preparation of this book. We thank very much Roman Frank who helped us with the figures. Many thanks also go to our students for discussions concerning the examples. Finally, we would like to express our thanks to Springer Press for the very good cooperation during the preparation of this manuscript.

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Contents

Symbols

a	Acceleration (m/s^2)
a	Laplacian length (m)
a	Strain rate (1/s)
a_{ij}	dimension matrix (–)
a_s	Velocity of sound (m/s)
A	Area (m^2)
$\mathbf{B}$	Magnetic field (T)
c	Specific heat capacity (J/(kg K))
c	Wave speed (m/s)
C	Dimensional constant (–)
C	Capacitance (F)
d	Number of dimensionless variables (–)
d	Diameter (m)
D	Diffusion coefficient (m^2/s)
D	Diameter (m)
$\mathbf{D}$	Electric displacement field (C/m^2)
E	Energy (J)
E	Modulus of elasticity (N/m^2)
$\mathbf{E}$	Electric field (N/C)
f	Frequency (1/s)
f	Function (–)
F	Force (N)
F	Function (–)
g	Gravitational acceleration (m/s^2)
G	Gravitational constant (m^3/(kg s^2))
h	Heat transfer coefficient (W/(m^2 K))
h	Height, depth (m)
H	Height (m)
H	Enthalpy flux (m^3 K/s)
$\mathbf{H}$	Magnetizing field (A/m)

$\mathbf{I}$	Electric current (A)
J	Second moment of area (m^4)
$\mathbf{J}$	Electric current density (A/m^2)
k	Thermal conductivity (W/(m K))
k_B	Boltzmann constant (J/K)
k_j	Dimension exponent (–)
K	Kinematic momentum (m^4/s^2)
L	Length (m)
L	Inductivity (H)
m	Number of base quantities (–)
m	Mass (kg)
M	Shaft torque (Nm)
M	Scale factor (–)
M	Molecular weight (kg)
M	Planet mass (kg)
n	Volume specific number ($1/m^3$)
n	Natural frequency (1/s)
n	Rotational speed (1/s)
n	Number of physical quantities (–)
p	Physical quantity
p	Pressure (N/m^2)
P	Power (W)
P	Force (N)
q	Electric charge (C)
$\dot{Q}$	Heat flux (W)
r	Radial distance (m)
r	Rank of dimension matrix (–)
R	Radius (m)
R	Resistance (Ω)
R	Gas constant (J/(kg K))
R_{m}	Universal gas constant (J/(mol K))
t	Time (s)
t	Thickness (m)
T	Temperature (K)
u,v,w	Velocity (m/s)
U,V	Velocity (m/s)
v	Specific volume (m^3/kg)
v	Fluctuation velocity (m/s)
$\mathbf{v}$	Velocity vector (m/s)
$\dot{V}$	Volume flow (m^3/s)
w	Deformation (m)

W	Resistance force (N)
x	Numerical value of physical quantity (–)
x,y,z	Coordinates (m)
X	Base unit (–)

Greek Letter Symbols

α	Thermal diffusivity (m^2/s)
α	Angle (–)
α	Scale factor (–)
β	Thermal expansion coefficient (1/K)
γ	Angle of attack (–)
Γ	Circulation (m^2/s)
δ	Boundary layer thickness (m)
ε	Restitution coefficient (–)
ε	Permittivity (F/m)
η	Dynamic viscosity (kg/(m s))
η	Non-dimensional coordinate (–)
ϑ	Temperature (K)
θ	Temperature (K)
λ	Wave length (m)
μ	Volume fraction ratio (–)
μ	Permeability (N/A^2)
ν	Kinematic viscosity (m^2/s)
Π	Dimensionless product (–)
ρ	Density (kg/m^3)
ρ	Volumetric charge density (C/m^3)
σ	Surface tension (N/m)
σ	Strain (N/m^2)
σ	Electric conductivity (S/m)
ϕ	Electric potential (V)
Ψ	Stream function (m^2/s)
ω	Angular velocity (1/s)
Ω	Rotational speed (1/s)

Subscripts

0	Initial state
0	In vacuum
m	Maximum values
G	Gas
W	Wall
∞	Ambient, at large distance

Definition of Non-dimensional Numbers

$C_D = F_D/(\rho A U^2)$	Drag coefficient
$C_L = F_L/(\rho A U^2)$	Lift coefficient
$c_M = 4M/(\rho \omega^2 R^5)$	Torque coefficient
$c_w = W/(\rho\, U^2\, L^2)$	Drag coefficient
$\mathrm{Fr} = U^2/(g\, L)$	Froude number
$Re = Ud/v$	Reynolds number
$Re_\omega = \rho \omega R^2/\eta$	Rotational Reynolds number
$\mathrm{Pr} = \eta c/k = v/a$	Prandtl number
$Pe = \mathrm{Re}\ \mathrm{Pr}$	Peclet number

Chapter 1
Some Fundamentals of Dimensional Analysis

1.1 Some Preliminary Remarks

Why can't a mouse in the size of an elephant stand without breaking its legs? And why can an ant carry multiples of its own weight while humans are barely able to carry more than their own weight? Is this reasoning somehow connected to the fact that the legs of a mouse are "relatively thin" or the weight of the elephant is "relatively large"? One would need to ask: "Relative to what?" Obviously, it appears rather meaningless to claim a length to be "relatively short" or a weight to be "relatively heavy" without defining a suitable scale. But what constitutes an appropriate scale and how may it be found?

In natural and engineering sciences, the common way to measure is by comparison to some agreed standard. For instance, lengths are compared in terms of the unit "meter" and one would say: "This length measures 5 m." This means that the regarded length is five times as long as 1 m. Just as well one might compare the same length with the "millimeter" scale and state: "This length measures 5000 mm." By specifying a numerical value and the applied unit, the considered physical quantity is uniquely described. In practice, units are usually chosen, such that the numerical values can be conveniently used in calculations. However, depending on the applied unit, different values are obtained, to describe exactly the same physical property, in our example the length. Now, on the one hand, physical events are obviously entirely independent of the unit of measure that we arbitrarily have chosen. On the other hand, in order to quantify a physical quantity we are forced to pick some comparative measure.

The way out of this dilemma is to simply compare the regarded length to another length scale that is relevant to the problem of interest. Defining a length through its ratio to some problem related length scale not only provides a unique numerical value, but it also becomes independent of whether one chooses to apply "millimeters" rather than "meters". These considerations are of course also true for all other physical quantities and their respective units. Thus, only the comparison of

V. Simon et al., *Dimensional Analysis for Engineers*,
Mathematical Engineering, DOI 10.1007/978-3-319-52028-5_1