Superconductivity
An introduction

$$\lambda_L = \sqrt{\frac{m}{\mu_0 n_s e^2}} \qquad \phi_0 = \frac{h}{2e} \qquad \xi = \frac{\hbar v_F}{\pi \Delta} \qquad \mathbf{j} = -\frac{1}{\mu_0 \lambda_L^2}\mathbf{A}$$

Philippe Mangin - Rémi Kahn

Springer

Superconductivity

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Philippe Mangin · Rémi Kahn

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 Springer

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PREFACE

This book was written based on original material from Master's level courses for scientists and engineers. Given the lack of elementary texts on the subject, especially in the French language of the original edition, it was intended as an introduction to superconductivity accessible to beginning graduate students or advanced undergraduates. We wished to make a textbook where simple approaches were preferred, assumptions clearly stated, and calculations given in sufficient detail to be followed with ease. A number of the applications of electromagnetism, thermodynamics or quantum physics can make for stimulating exercises for science and engineering students.

In view of the vast state of knowledge of the subject nowadays, this volume can only be an introduction. Other titles in preparation: *Supraconductivité - Matériaux et applications* and *Supraconductivité conventionelle et non-conventionelle*[1] should bring both a broader and a more specialized view of the current state of superconductivity.

The contents of this book have benefitted from the expert advice of our many colleagues in the *Jean Lamour Institute* in Nancy, of the *Léon Brillouin Laboratory* in Saclay and the the French Center for Atomic and Alternative Energies (CEA). The review panel: Jean-Pascal BRISON, Hervé COURTOIS, Thierry KLEIN, Jérôme LESUEUR, Stephane PAILHÈS, Pierre VEDRINE and Georges WAYSAND, brought crucial insight to the content of several chapters. The great pedagogue, José TEIXEIRA, re-read the work in great detail and greatly improved the explanation of several delicate issues.

H. COURTOIS, P. DUBOS, C. GOURDON, V. JEUDY, T. KLEIN, B. PANNETIER, A. PAUTRAT, D. RODITCHEV and J.C. VILLEGIER generously gave us much appreciated advice and were kind enough to allow us to use illustrations from their original publications.

The staff of **Grenoble Sciences** directed by Jean BORNAREL encouraged and stimulated us and did much to create an environment favoring a work of high quality. Laura CAPOLO, Sylvie BORDAGE and Anne-Laure PASSAVANT have, with much patience and good humour produced a remarkably elegant manuscript with careful attention to type-setting and to the thousand details that are so important for a beginning reader.

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1   *Materials and Application* and *Conventional and Unconventional Superconductivity*

<div align="right">

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Chapter 1

INTRODUCTION

1.1 - A history of women and men

Since its discovery in 1911, the history of superconductivity is perhaps one of the most exciting adventures in physics. It was directly responsible for no fewer than five NOBEL prizes:

Heike Kamerlingh ONNES, for the discovery of the phenomenon (1913), John BARDEEN, Leon COOPER and Robert SCHRIEFFER, who provided a microscopic theory (1972), Brian JOSEPHSON and Ivar GIAEVER, whose theoretical and experimental contributions showed effects of quantum coherence and tunnelling (1973), Alex MÜLLER and Johannes Georg BEDNORZ, for the discovery of high temperature superconductors (1987), and Alexei ABRIKOSOV and Vitaly GINZBURG for their extensive work on type II superconductors and the physics of vortices (2003).

Less directly, we can note other winners of this prestigious award who made major contributions to the subject, such as Lev LANDAU (1962), Philip Warren ANDERSON (1967), Pierre-Gilles DE GENNES (1991) and John Michael KOSTERLITZ (2016) and David THOULESS (2016). Besides them, many famous physicists and chemists left their trace in the story. We can cite Walther MEISSNER and Robert OCHSENFELD, the brothers Fritz and Heinz LONDON, Brian PIPPARD, Bern MATTHIAS, Herbert FRÖHLICH, Paul CHU, all of whose names which will come back to us in this book. Finally, numerous women and men have devoted time and enthusiasm to the subject in the past, and continue to do so to this day.

As for the future, it is more than probable that the list of winners will be joined by whoever explains convincingly the mechanisms of what is termed "high-temperature" (High-Tc) superconductivity, or who may discover new materials with critical temperature close to, or even higher than, room temperature.

1.2 - Experimental signs of superconductivity

1.2.1 - The discovery of superconductivity: the critical temperature

The story begins in Leiden in Holland in the first decade of the 20th century. The research group of H.K. ONNES was unique in having both an almost industrial level of the apparatus needed for the liquefaction of oxygen and, subsequently, of hydrogen,

Heike Kamerlingh ONNES

and to have sufficient quantities of helium [1] to be able to liquify that as well. He succeeded on July 10th, 1908 and was then able to make experiments down to a temperature of 1 K. H.K. ONNES chose to attack one of the great problems that interested the physics community of that age: what is the behavior of the electrical resistance of metals when we approach absolute zero? Does it tend to vanish because of the disappearance of thermal noise? Does it increase because of the localization of free electrons? Does it approach some limiting value determined by impurities, as Augustus MATTHIESSEN had already predicted?

In this project, Gilles HOLST, a student of Heike Kamerlingh ONNES, was given the job of measuring the electrical resistivity of mercury, which can easily be purified by distillation. The measurements were communicated on April 28, 1911 in a short note to the Royal Academy of the Netherlands. It announced "with all reservations" that *the resistivity of mercury apparently disappears just above 4 K*. Superconductivity had just been discovered in its most spectacular form: the total disappearance of electrical resistivity. The resistivity of the metal does not become weak or even very weak, it becomes strictly zero.

Gilles HOLST
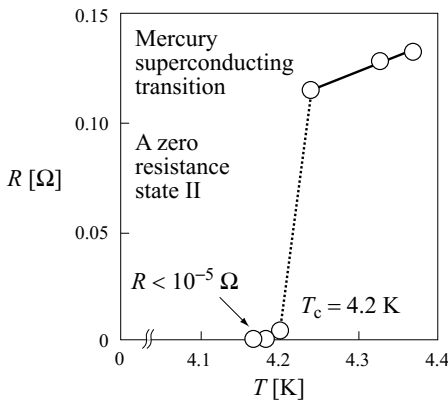
Figure 1.1
The historical evidence showing superconductivity

The original figure showing that mercury loses its resistivity at a temperature just below 4.2 K was published by H. K. ONNES [2]

In this new state of matter, it is possible to make a current flow in a closed circuit, without any generator (other than briefly to set the electrons in motion). Once started, the electrons flow indefinitely with constant speed. At this stage, the first quantity characterizing a superconducting material is the transition temperature T_c (the critical temperature) between the normal and superconducting states.

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1   He brought it from North Carolina in the United States, where the bulk of the world's supplies were to be found at that time.

1.2.2 - The magnetic behavior of superconductors

If the drop to zero of the electrical resistivity of superconductors is the most spectacular phenomenon, their response to a magnetic field was just as unexpected and has turned out to be particularly rich in consequences.

The MEISSNER-OCHSENFELD effect

In 1933, in Berlin, Walther MEISSNER and Robert OCHSENFELD showed that magnetic field **B** is "expelled" from superconductors, that is to say that when subjected to an external magnetic field, they divert the field lines so that the magnetic field vanishes inside [2]. The superconducting material behaves as a perfect diamagnet. [3]

Robert OCHSENFELD
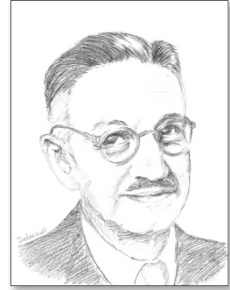
Walther MEISSNER

Critical fields and superconductors of types I and II

Very early on, magnetization measurements showed that the superconducting phase existed in a limited range, not only of temperature but also of magnetic field. After much confusion and conflicting experimental results it was finally the theoretical analysis of A. ABRIKOSOV [4] in 1957 that showed that superconductivity can disappear via two distinct scenarios, thus leading to the classification of superconducting materials into those of type I and of type II.

In a superconductor of type I, the superconductivity vanishes abruptly at a critical value H_c of the field. H_c is always small, with $\mu_0 H_c$ no more than 0.1 tesla. Only pure elemental superconductors (with a few exceptions, such as Niobium), are of type I.

In a type II superconductor, there is no discontinuity to be seen, but rather a gradual weakening of the magnetic response starting from a lower critical magnetic field H_{c1}. Complete suppression of superconductivity occurs only when the field reaches an upper critical value H_{c2} which can be very high ($\mu_0 H_{c2}$ may be several tens of, or even a hundred, teslas). Superconducting compounds and alloys are all of type II.

1.2.3 - Critical current

As well as the temperature and magnetic field, a finite density of electrical current also destroys superconductivity when it exceeds some critical value. We shall see

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2 W. MEISSNER, R. OCHSENFELD (1933) *Naturwissenschaffen* **21**, 787.

3 W. MEISSNER and R. OCHSENFELD interpreted their result as seeing "a possible analogy to ferromagnetism"; this will be taken up by the LONDON brothers. It is true that W. HEISENBERG had just provided a "microscopic" quantum theory based on interactions between the spins of closely neighbouring electrons.

4 A.A. ABRIKOSOV (1957) *Sov. Phys. JETP* **5**, 1974.

that in type I superconductors, the critical current density is intrinsically related to the field H_c, while in type II superconductors the critical current depends strongly on the metallurgy and, more generally, on the microstructure of the material. The mechanisms determining the critical currents are very different in the two types of superconductor. We note that wires of Nb-Ti (the type II superconducting alloy most used in the manufacture of magnets) can survive current densities of 1000, even 5000 A mm^{-2}, without generating any heat.

1.2.4 - The isotope effect [5]

This is a more discrete effect, which could have passed unnoticed, but fundamental to the understanding of superconductivity: for a given material it was noticed that the transition temperature T_c is linear in the inverse of the square root of the atomic mass. This observation shows that it is not only the electronic structure that is to be considered, but that atomic vibrations, whose frequency is inversely proportional to the square root of the atomic mass, are actively involved.

1.2.5 - JOSEPHSON currents and flux quantization

The very specific nature of the superconducting state appeared without question in 1962 in a paper by Brian JOSEPHSON which shocked the scientific community; [6] according to this brilliant young thesis student, an electrical current may flow between two bulk superconductors separated by a thin insulating layer, *even when there is no potential difference between the two*. Even more surprising, his theory predicted that applying a constant potential leads to the appearance of an alternating current between the two superconductors.

Brian JOSEPHSON

These predictions, based on a kind of BOSE condensation of the superconducting charges and on the fixing of the phases of the associated wave-functions, were immediately confirmed by experiment. Formation of a macroscopic quantum state, which implies quantification of the magnetic field flux, is the basis of the development of an ultra-sensitive technique for measuring magnetic fields (SQUID) as well as a multitude of other subtle effects.

Ivar GIAEVER demonstrated the existence of a tunnelling effect between a superconductor and a normal metal separated by a thin insulating barrier.
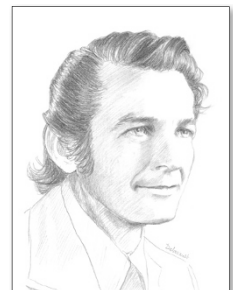
Ivar GIAEVER

5   Isotope Effect: The transition temperature T_c depends on the isotope, *i.e.* on the atomic mass. Now the mass determines the vibrational frequency of the atomic lattice.

6   B.D. JOSEPHSON (1962) *Phys. Lett.* **1**, 251.

1.3 - Phenomenological models

Despite its spectacular features, superconductivity proved to be one of the most difficult problems that the physicists of the first half of the 20th century had to deal with. The community had to wait 44 years (1911-1955) before finally a satisfactory theory (BCS) was published by John BARDEEN, Leon COOPER and Robert SCHRIEFFER.

The initial attempts to explain this "super-conductivity" were all trying to consider the perfect conductivity as a limiting case of metallic conductivity. As only classical models of electrical conduction were available (BOHR's model of the atom dates from 1913) such efforts were in vain. In 1922 Albert EINSTEIN, whose assistance H.K. ONNES had called for in clarifying the question, admitted:

...With our considerable ignorance of complicated quantum mechanic systems, we are far from being able to formulate these ideas in a comprehensive theory. We can only attack the problem experimentally...

With no more success, the most brilliant minds of this era, from EINSTEIN to FEYNMAN, including SCHRÖDINGER and many others, agonized over this phenomenon with the aim of finding an acceptable microscopic theory.

During all this time, however, experiments and results were accumulating. While lacking a credible microscopic model, physicists made several phenomenological models based on *ad hoc* equations with varying degrees of intuitive appeal or success, each describing a set of experimental results.

1.3.1 - LONDON theory

LONDON theory is one of the most noteworthy phenomenological theories. Following the work of W. MEISSNER and R. OCHSENFELD, Fritz LONDON and his younger brother Heinz recognized the condition **B** = **0** as the fundamental property of the superconducting state. The challenge was no longer to explain the perfect conductivity but the perfect diamagnetism, *i.e.* the state where the system responds to an external magnetic field by developing a super-current which generates a field in response that is equal, but opposite to, the applied field. In face of this challenge, the LONDON brothers invented the radically new concept of the existence of a macroscopic quantum state. Proceeding by analogy with LANGEVIN's model for the magnetic susceptibility of atoms, they made the hypothesis that *the bulk superconductor can be considered as a single, enormous, diamagnetic atom* [7].

Fritz LONDON

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7   F. LONDON (1960) *Superfluids: Vol I Macroscopic Theory of Superconductivity* 2nd edition, Dover, New York; (2005) *Une conception nouvelle de la supra-conductibilité*, re-edition Hermann.

Heinz LONDON

With the equations that bear his name, Fritz LONDON presented the first satisfactory electromagnetic model in 1933, [8] which described in a precise way the MEISSNER effect of the expulsion of magnetic fields **B**. It also showed that if the magnetic field really vanishes at the heart of the superconductor it does not do so near the surface, but penetrates a distance λ_L called the LONDON penetration depth. This was the first characteristic length scale that emerged for the superconducting state.

In the wake of this result, and in order to obtain numerical values for the penetration depth closer to the experimental results, Brian PIPPARD generalized LONDON's equations to include non-local effects, thereby introducing the coherence length, which was to become the second characteristic length of superconductivity. Inherent to superconductivity and reduced by the impurities, the coherence length leads also to a renormalisation of the LONDON penetration depth, renamed simply penatration depth λ.

Brian PIPPARD

1.3.2 - The thermodynamic approach

In parallel with LONDON's work, Cornelius Jacobus GORTER and Hendrik CASIMIR developed a thermodynamic approach to superconductivity in which they considered the normal-superconducting transition as a proper phase transition. They constructed a phenomenological model of two fluids, one superconducting and the other normal, with both intensive and extensive variables, an internal energy, thermodynamic functions and potentials; in short all the tools of thermodynamics. The success was all the greater as it described the superconducting/normal phase transition and, by integrating the negative surface energy associated with the penetration depth, it opened up the path towards an understanding of type II superconductivity.

1.3.3 - GINZBURG-LANDAU Theory

We have to wait until 1950, however, to see an approach associating electromagnetism and thermodynamics as was proposed by Vitaly GINZBURG and Lev LANDAU [9]. By generalizing the LANDAU model of phase transitions, they proposed a set of equations (the GINZBURG-LANDAU equations) to describe the behavior of the order parameter of the transition, vanishing in the normal phase and non-vanishing in the superconducting phase. With Alexei ABRIKOSOV
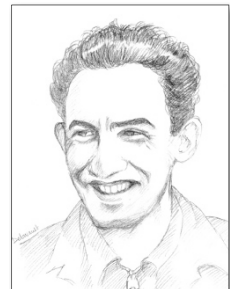
Vitaly GINZBURG

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8   F. LONDON (1934) *Phys. Rev.* **45**, 379.

9   V. GINZBURG & L. LANDAU (1950) *Zh. Eksp. Teor. Fiz.* **20**, 1064.

and Lev GORKOV, they assisted in the birth of the GLAG (GINZBURG-LANDAU-ABRIKOSOV-GORKOV) theory.

This formulation, essentially derived from pure intuition, proved to be very powerful in practice. The equations imply not only the MEISSNER effect and the penetration depth of the magnetic field, but also lead to the appearance of the second characteristic length. This coherence length, ξ, can be interpreted as the minimal distance required for any variation in the density of superconducting carriers.
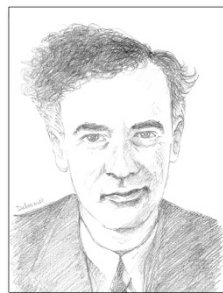
Lev LANDAU

1.3.4 - Vortices
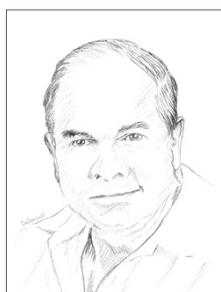
Alexei ABRIKOSOV

Lev GORKOV

By solving, towards the middle of the 1950's, the GINZBURG-LANDAU equations, A. ABRIKOSOV showed that the sign of the surface energy for a normal-superconducting interface depends on the ratio between the characteristic lengths for penetration of a magnetic field λ (the penetration depth) and for coherence ξ:

» in materials where $\xi < \lambda$ the interface surface energy turns negative when the material is subject to a field exceeding H_{c1}, which leads the system to develop internal "lines of normal phase" (or vortices), thus explaining the decrease in magnetization density of type II superconductors above this field H_{c1};

» in materials where $\xi > \lambda$ the interface surface energy is always positive and the material remains uniformly superconducting until the transition to the normal state. When the field reaches the critical value H_c the transition occurs simultaneously throughout the material.

The nature, type I or II, of the superconductor thus depends on the relative values of these two lengths.
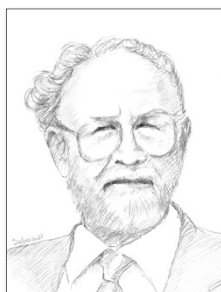
Daniel CRIBIER
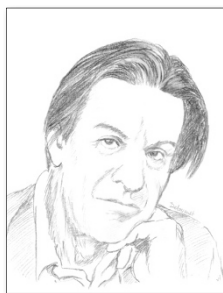
Pierre-Gilles DE GENNES

Daniel CRIBIER was the first experimentalist to show explicitly the existence of a vortex lattice, following the suggestion of Pierre-Gilles DE GENNES, the most prominent leader of the *Orsay "School" of Physics*.

1.4 - The microscopic BCS theory

Despite the advances facilitated by the GINZBURG-LANDAU theory, at the begin-

ning of the 1950's superconductivity remained as mysterious
as ever with respect to its microscopic origins. Things started
to move faster, however, with the arrival of two fundamental
results: the proof of the rôle of phonons and the appearance
of COOPER pairs.

While the involvement of lattice vibrations (phonons), seen
via the isotope effect, was well known, it was only in 1953
and the calculations of Herbert FRÖHLICH [10] that the idea
emerged of an attractive interaction, *via* the phonons, between
two electrons of opposite velocities and spins (*phonon drag*).

Herbert FRÖHLICH

The following year, Leon COOPER [11] showed that if two electrons of opposite wave-
vector and spin on the FERMI surface feel an attractive interaction, they will form a
bound pair whose energy is less than the sum of the kinetic energy of the individual
particles. Such a pair is called a "COOPER pair".

Starting from these results, John BARDEEN, Leon COOPER
and Robert SCHRIEFFER developed the BCS theory, for which

they were to receive the NOBEL
prize in 1972. [12] They described
the collective behavior of COOPER
pairs by exploiting many-body tech-
niques of calculation. With a very
limited number of parameters they
reproduced, explained and quanti-
fied most experimental results: the
MEISSNER effect, the electrodynam-
ic behavior, the coherence length,
the critical field, the critical tem-

John BARDEEN

Leon COOPER

perature, specific heat, thermodynamic properties… They
showed that the charge carriers are definitely not individual
electrons but COOPER pairs, and that a minimal energy Δ
must be applied to the system in order to create the first ex-
cited states in the form of single electrons (quasi-particles).
This energy Δ, called the superconducting gap, plays a cen-
tral role in superconductivity.
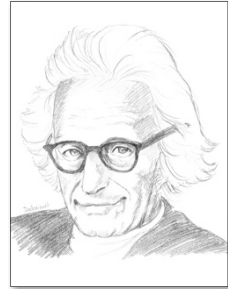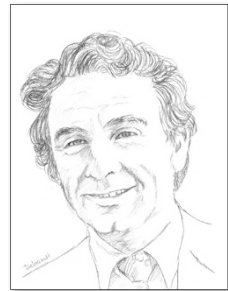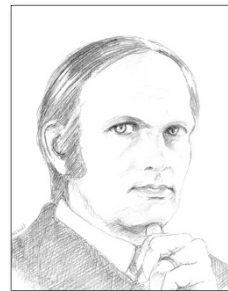
Robert SCHRIEFFER

10 H. FROHLICH (1950) *Phys. Rev.* **79**, 845; (1952) *Proc. Roy. Soc.* **A215**, 291; (1954) *Adv. Phys.* **3**, 325.

11 L.N. COOPER (1956) *Phys. Rev.* **104**, 1189.

12 J. BARDEEN, L.N. COOPER & J.R. SCHRIEFFER (1957) *Phys. Rev.* **108**, 1175.

1.5 - Tunnelling effects

Coming shortly after BCS, studies of tunnelling effects between two bulk superconductors separated by an insulating barrier shed light on two processes:

» the JOSEPHSON effect, which corresponds to the transport of "whole" COOPER pairs from one bulk superconductor to the other. This transfer is controlled by the phase difference between the coherent condensates formed by the COOPER pairs of each superconductor;

» tunnelling across the insulating barrier by individual electrons (or more exactly by quasi-particles) formed by the unbinding of COOPER pairs. [13] This gives rise to many spectacular effects, including ANDREEV-SAINT-JAMES reflections [14] predicted independently by Alexander ANDREEV in Moscow and Daniel SAINT-JAMES in Paris, and provides vital information on the gaps, densities of state, the intensities of electron-phonon coupling and so on…

1.6 - A great diversity of superconducting materials

For a long time, the only known superconductors were metals or metal alloys. As their critical temperatures did not exceed 9.2 K for Niobium (the pure element with the highest T_c) or 23.2 K for the metallic compound Nb_3Ge discovered in 1973, there was a certain waning of interest in superconductors. Nonetheless some activity continued. It was rewarded by an initial success constituted by the discovery, in 1980 by Denis JÉROME and his group, of purely organic superconductors [15] whose critical temperatures were not very high, but this proved that superconductivity was not just restricted to metals and metal alloys.

Then came the real revolution at the hands of Alex MÜLLER and Georg BEDNORZ. Convinced that the most promising candidates for high critical temperatures would be found more in the direction of oxides, they managed to synthesize a material with Lanthanum (La), Barium (Ba), Copper (Cu) and Oxygen (O). On January 27th 1986 they observed a rapid decrease in the resistivity at around 30 K which they interpreted as signalling the presence of a superconducting phase.
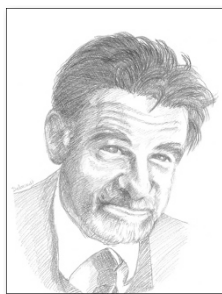
Alex MÜLLER

Georg BEDNORZ

13 I. GIAEVER (1960) *Phys. Rev. Lett.* **5**, 147.

14 A. ANDREEV (1964) *Zh. Eksperim. i. Teor. Fiz.* **46**, 1823 (*Soviet Physics JETP* **19**, 1228)
    D. SAINT-JAMES (1964) *J. de Physique* **25**, 899.

15 D. JÉROME *et al.* (1980) *J. Phys. Lett.* (Paris) **41**, L45.

Just like H. K. ONNES and G. HOLST in an earlier age, they
made sure that the results were reproducible and with the cau-
tion of their predecessors, submitted to the journal *Zeitschrift
für Physik* [16] an article with the title *Possibility of a high
temperature superconductor in the system Ba-La-Cu-O*. This
started a race to find similar compounds with a transition tem-
perature that would be even higher. This was how shortly af-
ter, the compound Y-Ba-Cu-O, whose critical temperature T_c
of 93 K is therefore above that of liquid nitrogen, was dis-
covered by Paul CHU. [17] It was then the turn of more com-
plex compounds with T_c = 110 K (1988), 128 K (1991), 138 K (1993)! All have in
common the existence of CuO_2 planes carrying the superconductivity, whence their
generic name of "cuprates".

Paul CHU

More recently a multitude of new superconducting materials have been discovered:
in 1991 the family of doped fullerenes doped with alkaline elements ($T_c \leq 40$ K), [18]
in 2001 the metallic compound MgB_2 (T_c = 39 K), [19] in 2008 the iron-based family
(pnictides) ($T_c \leq 55$ K) [20] and in 2015 the coumpound SH_3, the critical temperature
of which reached 203K under a pressure of 155GPa. [21]

1.7 - "Unconventional" superconductors

As a good number of the new compounds possess electronic structures which are
very different from those of metals and alloys, many researchers have questioned the
very nature of the new superconductors and whether the model of BCS is appropri-
ate for them.

From an experimental point of view, the zero resistivity, the MEISSNER effect, the pen-
etration depth, the critical fields H_{c1} and H_{c2}, the flux quantum, the JOSEPHSON effects
are all similar. In contrast, for some cases the gap is either anisotropic or multiple; in
others, such as the cuprates, heavy fermions and certain organic compounds, the con-
ducting phases coexists with magnetic fluctuations, [22] or even coexists with a mag-
netically ordered phase, two properties considered hitherto as being incompatible.

From the point of view of theory, the COOPER pair remains the fundamental com-
ponent even if in some cases it seems to be constituted by two electrons of the same

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16 A. MÜLLER & G. BEDNORZ (1986) *Zeitschrift für Physik* **4**, 189.

17 M.K. WU *et al.* (1987) *Phys. Rev. Lett.* **58**, 908.

18 A.F. HEBARD *et al.* (1991) *Nature* **350**, 600;
    M.J. ROSSEINSKY *et al.* (1991) *Phys. Rev. Lett.* **66**, 2830.

19 J. NAGAMATSU *et al.* (2001) *Nature* **410**, 63.

20 Y. KAMIHARA *et al.* (2008) *J. Am. Chem. Soc.* **130**, 3296.

21 A.P. DROZDOV *et al.* (2015) *Nature* **525**,73.

22 See for example D. MANSKE (2004) *Theory of unconventional superconductors*, Springer.

spin (triplet superconductivity) in contrast to BCS where they are of opposite spin (singlet superconductivity). Perhaps more importantly, the pairing mechanism mediated by phonons is called into question; several theoretical models have been proposed with pairing mechanisms including magnetic fluctuations [22] as numerous experimental results [23] in cuprates and other systems would seem to suggest.

Jean ROSSAT-MIGNOT

In any case, a large number of the new superconductors differ, in one way or another, from the BCS model and intensive research is in progress to clarify the situation.

Jean ROSSAT-MIGNOT showed, by the inelastic scattering of neutrons, the occurrence of anti-ferromagnetic fluctuations in the superconductivity state of the cuprates. He showed in particular that the intensity of the antiferromagnetic resonance peak decreases with temperature, vanishing at T_c.

1.8 - Numerous spectacular applications

Thanks to their extraordinary properties, superconductors have never ceased to inspire. Many applications, linked to their perfect electrical conductivity, the expulsion of magnetic fields and to coherence effects have been dreamed up, and for a good number of them, implemented.

Thus the immense majority of the apparatus for Magnetic Resonance Imaging (MRI) are equipped with superconducting coils. Electromagnets with superconducting windings are familiar to scientific laboratories and are crucial to the operation of large instruments, such as the enormous hadron collider LHC of CERN or the future *tokamak* of the ITER project. On a more experimental level, motors, transformers and transmission lines and current limiters with superconducting wires have been designed and tested. A number of magnetically levitated trains function with superconducting coils and superconducting bearings are being developed.

Relying on the properties of JOSEPHSON junctions, the SQUID is an extremely sensitive probe of magnetic fields. Already the instrument of choice for ultra-sensitive measurements of magnetization, its use in medicine is under development in order to detect the electromagnetic activity of several different organs of the human body.

In a host of other contexts, superconductivity is also used in astrophysics (as ultra-sensitive particle detectors) in the engineering large particle detectors (for intense magnetic fields in very large volumes), in the resonant cavities of particle accelerators… Computers based on JOSEPHSON junctions using the flux quantum to store information have been tested with the idea of making ultra-rapid machines with low energy consumption.

23 J. ROSSAT-MIGNOD *et al.* (1991) *Physica C* **86,** 91.

The search for materials combining both extreme superconducting parameters (high transition temperatures, critical currents and critical fields) and satisfactory physico-chemical and mechanical properties of ductility and resistance, is currently very active in university laboratories as well as in industry. Advances in this will certainly lead to even more applications and extension of existing one on a greater scale.

1.9 - Superconductivity in the history of mankind

We should not forget that superconductivity is also a history of people. Several of the protagonists were directly involved in the drama and dark moments of the 20th century: the LONDON brothers, as German Jews, had to flee to live as exiles in England; Lev SHUBNIKOV was executed on the basis of falsified documents during the great Stalinist purges; the issue of the Soviet physics Journal *Zh. Eksperim. i. Teor. Fiz.*, published during the worst period of McCarthyism and the communist witch-hunt, and which included the famous article by GINZBURG and LANDAU, was thrown into the sea by the New York dock workers when it arrived by boat, and their work remained unknown until years later.

Following the "rediscovery" of superconductivity with the materials with high superconducting temperatures, several historical and popular works have been published. Amongst them, we note *The Cold Wars: A History of Superconductivity* [24]

which follows the scientific paths pursued, the successes and the blind alleys, but also relates the story of the people who took part in the advancement of our scientific knowledge.

As specialist in low temperature physics, SHUBNIKOV worked on type II superconductors in particular. The intermediate phase between the completely MEISSNER phase and the normal state of type II superconductors now bears his name.
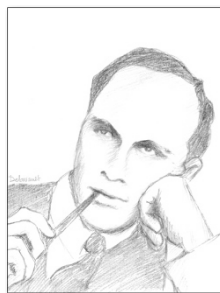
Lev SHUBNIKOV

24 J. MATRICON & G. WAYSAND (1994) *The Cold Wars: A History of Superconductivity*, Rutgers University Press, translated from the French edition (1994) of *La guerre du froid : une histoire de la supraconductivité*, Seuil.

Chapter 2

LONDON THEORY

The first phenomenological approach to the electromagnetic behavior of superconductors was published in 1935 by the brothers Fritz and Heinz LONDON.[1] Fascinated by the results of MEISSNER and OCHSENFELD,[2] who had shown that within the superconductor the magnetic field **B** vanishes irrespective of the thermal and magnetic history of the sample, they understood that the key feature of superconductivity is not the disappearance of the electrical resistance but rather the expulsion of the magnetic flux.

Now MAXWELL's equations, that govern electromagnetism, turn out to be incapable of including this effect. The most they can show is that, in a perfect conductor, the magnetic field cannot *vary* with time; to include the *exclusion* of the magnetic field, we must add some additional ingredient. This is precisely what was done by F. and H. LONDON, with the equations now bearing their name.

2.1 - MAXWELL's equations

The theory of electromagnetism is based on the equations of MAXWELL in their bulk form: [3]

$$\boldsymbol{\nabla} \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \quad \text{(M1)} \quad \boldsymbol{\nabla} \cdot \mathbf{E} = \frac{\rho}{\varepsilon_0} \quad \text{(M2)}$$

$$\boldsymbol{\nabla} \cdot \mathbf{B} = 0 \quad \text{(M3)} \quad \boldsymbol{\nabla} \times \mathbf{B} = \mu_0 \mathbf{j} + \varepsilon_0 \mu_0 \frac{\partial \mathbf{E}}{\partial t} \quad \text{(M4)}$$

$$(2.1a)$$

where ρ and **j** represent respectively, the total charge and total current densities of the system, including polarization charges, displacement currents and AMPÈRE currents.

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1   F. LONDON & H. LONDON (1935) *Proc. Roy. Soc. (London) A* **155**, 71.

2   W. MEISSNER & R. OCHSENFELD (1933) *Naturwissenschaften* **21**, 787.

3   In this book we will use the symbol $\boldsymbol{\nabla}$ for the gradient operator. Alternative notations that are common in the literature are:
$\boldsymbol{\nabla} U = \mathbf{grad}\, U$; $\boldsymbol{\nabla} \cdot \mathbf{A} = \operatorname{div} \mathbf{A}$; $\boldsymbol{\nabla} \times \mathbf{A} = \mathbf{curl}\, \mathbf{A} = \mathbf{rot}\, \mathbf{A}$; $\boldsymbol{\nabla} \cdot \boldsymbol{\nabla} U \equiv \boldsymbol{\nabla}^2 U = \Delta U$; $\boldsymbol{\nabla}^2 \mathbf{A} = \Delta \mathbf{A}$
(vector with components ΔA_x, ΔA_y, ΔA_z).