

Eleftherios N. Economou

From Quarks to the Universe

A Short Physics Course

Second Edition

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There is, of course, an immense liberating role of Science at a central existential level. It is what Aristotle was saying about “θαυμάζειν”. Science humanizes us, liberating us from our animal instincts, just because it makes us wondering and at the same time desiring to explain... Yet it shows us our limits and our mortality... Thus Science is something immeasurably precious.... Science can help us approach anew the real poetic and mythical dimension of human existence.

C. Castoriadis

The Castoriadis quote is from the Castoriadis and Evangelopoulos book, *Philosophy and Science*, (Editions Eurasia, Athens, 2010)

To Athanasia

Preface to the Second Edition

This book in this second edition has been enlarged (its size now is more than twice that of the first edition) and has been enriched in order to also serve as a senior undergraduate textbook; nevertheless, it retains its main feature of deriving most of the basic formulae governing the behavior of the various structures of the physical world by applying “a little thinking” and employing dimensional considerations.

Explicitly, in each chapter, besides more background information, new sections have been added: One of them includes a summary of the main relevant formulae; another contains many multiple choice questions/statements (their correct answers are given at the end of the book). Finally, there are two more sections in every chapter involving solved and unsolved problems respectively.

Moreover, six new appendices have been added in this new edition: In two of them a summary of the subjects of Electrodynamics of Continuous Media and of Thermodynamics and Statistical Mechanics is presented. These two appendices together with the last three, presenting a list of the required background concepts, formulae, and numbers, make the book to a large degree self-contained. In another new appendix a few basic concepts regarding semiconductor physics are introduced.

As I mentioned before, this book in this second edition may well serve a senior undergraduate course: The students in such a course will be asked to wrap up their basic knowledge and reasoning and apply them to *derive* and *understand* the basic features of the physical world. Of course, as it was stated in the preface to the first edition, graduate students, research scientists, physics teachers and others may find this book intellectually stimulating and entertaining.

I would like thank again my colleague, Prof. V. Charmandaris, for reading the entire text of this second edition and for making many useful suggestions. Of course, whatever misprints or misrepresentations remained are my own responsibility only. I am also grateful to Ms. Maria Dimitriadi for her invaluable help in bringing my manuscript to its final form.

Iraklion

Eleftherios N. Economou

Preface to the First Edition

This short book grew out of lectures presented to different audiences (physics students, physicists, material scientists, engineers) and on various occasions (colloquia and seminars in physics and other departments, conferences, special events). The main purpose of these lectures and, obviously, of the present book is to show that basic formulae concerning the various structures of the *physical world* pop out quickly, if some *basic ideas*, the *universal physical constants*, and *dimensional considerations* are exploited. Of course, as R. Feynman pointed out, “a little thinking has to be applied too”.

The basic ideas include the three cornerstones of science, namely the atomic idea, the wave-particle duality, and the minimization of free energy as the necessary and sufficient condition for equilibrium (these are presented in Chaps. 2, 3, and 4 respectively). These fundamental ideas exhibit their worth when accompanied by the values of the physical constants: the universal ones, \hbar , c , the coupling constants of the four interactions, G , e , g_w , g_s and the masses of the elementary particles, m_p , m_n , m_e , m_w , An important consequence of the atomic idea is that the relevant (for each case) physical constants will appear in the quantities characterizing the various structures of the world either microscopic or *macroscopic*. Combining this last observation—often overlooked—with dimensional analysis, presented in Chap. 5, and “a little thinking”, one can obtain, in several cases, an amazing short-cut derivation of formulae concerning the various structures of Nature from the smallest (baryons and mesons) to the whole Universe, as shown Chaps. 6–13. In each one of these 8 chapters, in parallel with a demonstration of the method just outlined, a *condensed* (sometimes too condensed) introduction to the relevant subject matter together with a few physical remarks are presented.

I must admit that the main fronts on which our scientific horizons are widened, namely the *small*, the *large*, and the *complex* could not be treated even remotely adequately in this short book. Actually the *complex*, as represented by the living matter, was too complex for our simple method; so it was left out completely (however, see the epilogue). The *large* (cosmology) and the *small* (elementary particles) tend to converge to a unified subject (the snake in Fig. 1.1, p.2, is biting

its tail) fed with novel observational data from special instruments mounted usually on satellites, and boosted by high experimental expectations from the Large Hadron Collider. Nevertheless, in these fields there are several open fundamental questions concerning conditions well beyond our present or near future experimental capabilities. This vacuum of confirmed knowledge is filled with new intriguing, imaginative ideas and novel proposed theories (such as supersymmetry, string theory, M-theory, see reference [P1]) which, if established, will radically change our world view. In spite of the wider interest in these ideas and theories and their high intellectual value, I decided for several reasons to restrict myself in the present book to experimentally or observationally tested ideas and theories.

The intended readers of this book are senior undergraduate or graduate students in Physics, Engineering, Applied Mathematics, Chemistry, and Material Science. They may find the book a useful supplement to their courses as a concise overall picture of the physical world. Research physicists, physics teachers, and other scientists may also find this short book intellectually stimulating and entertaining. The required background is no more than a *working* familiarity with the Science/Engineering material taught in the first University year.

I am deeply indebted to my colleague, Prof. V. Charmandaris, for his encouragement during the writing of this book and for reading my entire manuscript and making many useful suggestions. Of course, whatever misprints or misrepresentations remained are my own responsibility only. I am also grateful to Ms. Maria Dimitriadi for her invaluable help in bringing my manuscript to its final form.

Iraklion
January 2011

Eleftherios N. Economou

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Chapter 1

Introduction: The World According to Physics

*Ah, but a man's reach should exceed his grasp.
Or what's a heaven for?*

R. Browning.

Abstract In this introductory chapter the subject matter as well as the methodology of Physics are briefly presented together with a list of the basic equilibrium structures of matter. The main properties of the latter can be obtained by minimizing the internal energy and are expressed in terms of universal physical constants $\hbar, c, e, m_e, m_p \approx m_n \approx u, G$.

1.1 The Nature of Physics

Modern-era Physics started as Natural Philosophy. As this former name implies, Physics is built (and continues to be developed) around the age-old, yet ever-present questions:

- *What the World and its parts are made of? How?*
- *Is there a hidden underlying simplicity in its immense complexity and diversity?*

The first question implies that the subject matter of Physics is the World, both natural and man-made; from its smallest constituent to the whole Universe. In this sense, Physics tends to encompass the other natural sciences (such as Chemistry and Biology) and even Engineering, while at the same time serves also as their foundation. What allows Physics to have this foundational role is its characteristic methodology. The latter is precise and quantitative, yet capable of abstraction (therefore mathematical). It is based on observations and well controlled experiments both as sources of ideas as well as tests for falsification or tentative confirmations of newly proposed and –even– established theories. Moreover, as the second question suggests, the methodology of physics requires the formulation of a few fundamental quantitative relations on which everything else is based. These features of the methodology of Physics account for its role as the foundation of

every other science and engineering, but explain also its limited penetration into very complex, yet very important, parts of the World (such as the molecular and the biological structures). This leaves plenty of space to more specialized sciences such as Chemistry and Biology and, of course, Engineering.

Over the last 50 years or so Physics is actively concerned over another fundamental, age-old, but much more difficult question which stretches its methodology to the limit:

- *How did the World start, how did it evolve, and where is it going?*

Detailed observational data, such as the recession of distant galaxies at a speed proportional to their distance from Earth, the spectral and angular distribution of the Cosmic Microwave Background Radiation, etc, combined with established physical theories, allowed us to reconstruct roughly some of the main events in the history of the Universe. Naturally, other crucial events, including the emergence of life, remain unknown and they are the subject of on-going research. Subject to current theoretical research is also the development of a successful quantum theory of gravity, which is expected to let us describe in a concise manner the very moment of the genesis of the Universe.

1.2 The Subject Matter of Physics

The subject matter of Physics is summarized in Fig. 1.1 and Table 1.1:

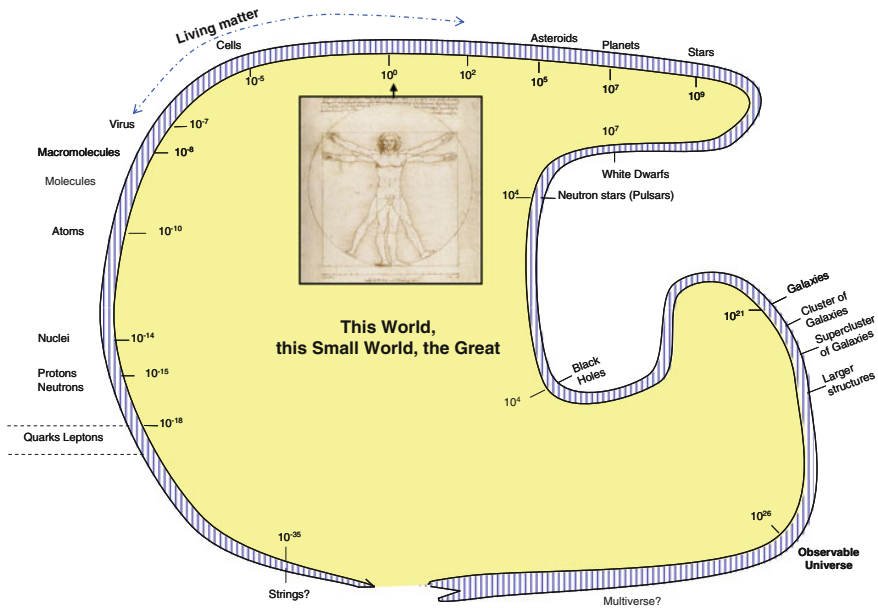


Fig. 1.1 The main structures of matter from the smallest to the largest size (*clockwise*) and the suspected connection of the two extremes (see [1]). The indicated sizes are in meters (see also next page)

Table 1.1 Levels of the structure of matter (see also [2])

Level of the structure of matter	Size (in m)	Constituents	Interaction(s) responsible for the structure
Quarks	$<10^{-18}$	It seems to be elementary	–
Electron	$<10^{-18}$	It seems to be elementary	–
Proton	10^{-15}	u, u, d quarks	Strong, weak, E/M
Neutron	10^{-15}	u, d, d quarks	Strong, weak, E/M
Nuclei	10^{-15} – 10^{-14}	Protons, neutrons	Strong, E/M, weak
Atoms	10^{-10}	Nucleus, electrons	E/M
Molecules	$>10^{-10}$	Atoms and/or ions and electrons	E/M
Solids (primitive cell)	$>10^{-10}$	Atoms and/or ions and electrons	E/M
Cells	$\geq 10^{-6}$	Molecules	E/M
Biological entities (e.g., <i>Homo sapiens</i>)	10^{-8} – 10^2 (10^0)	Molecules, cells, tissues, organs, microbes	E/M
Planets	10^6 – 10^7	Solids, liquids, gases	E/M, gravitational
Stars, Sun	10^8 – $10^{12}, 10^9$	Electrons, nuclei, ions, photons	Gravitational, strong, weak, E/M
White dwarfs	10^7	Nuclei, electrons	Gravitational
Neutron stars	10^4	Neutrons and some protons and electrons	Gravitational
Astrophysical black holes	10^4	?	Gravitational
Galaxies	10^{21}	Stars, ordinary and dark matter, photons, neutrinos	Gravitational
Observable universe	10^{26}	Galaxies, dust, dark matter, dark energy	Gravitational, others?

1.3 Various Branches of Physics

In concluding these introductory remarks regarding the subject matter of Physics, we present below some of the various branches of Physics and their correspondence and/or overlap with more specialized sciences as well as some examples of the impact of Physics on important technologies (Table 1.2):

Table 1.2 Connection of branches of Physics with Technologies and other Sciences and Mathematics

Mathematics	Elementary particle physics Nuclear physics Atomic and molecular physics Condensed matter physics	Chemistry, material science
	Biophysics	Biology
	Geophysics	Geology
	Atmospheric and space physics	Meteorology, global climate
	Astrophysics, cosmology	Astronomy
Technology	E/M waves, lasers	Telecommunications
	Solid state devices Integrated circuits Magnetic devices	Computers
	X-rays γ -rays Magnetic resonance (MRI) Positron annihilation (PET)	Medical technologies

1.4 The Main Points of This Book: Basic Ideas Applied to Equilibrium Structures of Matter¹

1. Out of the elementary matter-particles presented in Chapter 2, Table 2.1 only the *up quark* (u) and the *down quark* (d) make up the proton consisting of two u 's and one d and the neutron consisting of two d 's and one u . *Electrons* (e^-) are trapped around nuclei, made of protons and neutrons, to form atoms.
2. The constituents of all composite equilibrium structures are mutually attracted and self-trapped because of one or more of the four interactions presented in Table 2.2.
3. The interactions, which tend to continuously squeeze the composite structures, are counterbalanced by the pressure due to the perpetual motion of the constituents microscopic particles. This motion is of quantum nature and stems from the uncertainty principle aided by the exclusion principle (if more than two fermions (see Sect. 3.3) are involved). In other words, equilibrium of composite systems is established when the squeezing pressure of the interactions is exactly balanced by the expanding pressure of the quantum perpetual motion of the constituent particles. The equality of pressures is a consequence of the general principle of the minimization of the internal energy U (under conditions of negligible external pressure and temperature). Thus:

¹Section 1.4 summarizes the content of this book. It may be useful for the reader to return to this section at later times.

Equilibrium of composite structures \Leftrightarrow Minimum of internal energy U . Internal energy $U =$ Internal potential energy $E_P +$ internal kinetic energy E_K . Internal quantum kinetic energy of N identical fermions:

$$E_K = 2.87 \frac{\hbar^2 N^{5/3}}{m V^{2/3}} = 1.105 \frac{\hbar^2 N^{5/3}}{m R^2} \text{ non-relativistic}$$

Internal quantum kinetic energy of N identical fermions:

$$E_K = 2.32 \frac{\hbar c N^{4/3}}{V^{1/3}} = 1.44 \frac{\hbar c N^{4/3}}{R} \text{ extreme-relativistic}$$

4. The combination of the first and the second law of thermodynamics leads to the following relation: The so-called Gibbs free energy, $G \equiv U + PV - TS$, under conditions of constant pressure and temperature, is always decreasing during the system's path towards equilibrium and reaches its minimum value when equilibrium is established. G reduces to U when PV and TS are negligible.
5. Dimensional analysis is a powerful method for producing physics formulae. It requires the identification of the parameters and/or the universal physical constants on which a quantity X may depend. Then the formula for X is a product (with the same dimensions as X) of appropriate powers of usually three of those parameters/physical constants times a function of their dimensionless combinations.

Next, we apply the general principles presented above to several basic equilibrium structures; the main properties of them are expressed in terms of universal physical constants.

(i) *Nuclei* consist of Z protons and N neutrons, i.e. of $A = Z + N$ nucleons. Both the strong interactions and the Coulomb interactions contribute to the potential energy: $E_P = -\frac{1}{2}AN_{nn}\mathcal{V}_s + \frac{1}{2}\sum_{ij}e^2/r_{ij}$, where N_{nn} is the number of nearest neighbors of a nucleon, $-\mathcal{V}_s$ is the strong interaction between a pair of nearest neighbor nucleons and the double summation in the Coulomb term is over all Z protons. The nuclei's radius R is proportional to the 1/3 power of A and the kinetic energy is the sum of the kinetic energy of the protons and that of the neutrons

$$E_K = 1.105 \frac{\hbar^2 Z^{5/3}}{m_p R^2} + 1.105 \frac{\hbar^2 N^{5/3}}{m_n R^2}$$

(ii) *Atoms* consist of a single nucleus (of Z protons) and Z electrons trapped around it by Coulomb interactions: $E_P = -\sum_i Ze^2/r_i + \frac{1}{2}\sum_{ij}e^2/r_{ij}$. The ground state energy of the electrons is found approximately by calculating the single electron atomic orbitals (see Sect. 10.3) and the corresponding energy levels; the lower ones of the latter are fully populated by the electrons, within the restrictions imposed by Pauli's principle, starting from that of lowest energy until all Z electrons are exhausted.

(iii) *Molecules* are consisting of atoms (or cations and anions) of practically unlimited combinations held together by Coulomb interactions. The molecular orbitals are approximately expressed as linear combinations of atomic orbitals or hybridized atomic orbitals.

(iv) In *solids* and *liquids* a huge number of atoms and/or molecules are coming in contact under the action of Coulomb interactions. In metals the main kinetic energy is that of *detached* electrons given approximately by $E_K = 2.87 \frac{\hbar^2 N_e^{5/3}}{m_e V^{2/3}}$, while the potential energy is of Coulomb nature and of the form $E_P = -N_a E_o \gamma / \bar{r}$, where N_a is the total number of atoms, $E_o = e^2 / a_B$, $a_B \equiv \hbar^2 / m_e e^2$ is the so-called Bohr radius, \bar{r} is connected to the volume by the relation $\frac{4\pi}{3} (\bar{r} a_B)^3 = (V / N_a)$, $\gamma \approx 0.56 \zeta^{4/3} + 0.9 \zeta^2$, and ζ is the valence. For semiconductors and insulators, as for molecules, a linear combination of atomic (hybridized or not) orbitals turns out to be a more convenient way of studying them.

(v) *Planets* are spherical objects of mass $M = N_v u$ and radius R , where $N_v = A_W N_a \approx 10^{49} - 10^{55}$ is the total number of nucleons within all nuclei, A_W is the average atomic weight, and $u \equiv \frac{1}{12} m(C^{12})$. In planets both the Coulomb interaction as well as the gravitational one, $E_G = -a_G G M^2 / R$, contribute to the potential energy, while the kinetic energy is due mainly to the electrons as in solids and liquids. The spherical shape of moons, planets and stars is a consequence of the long range character of the gravitational interaction which becomes appreciable as a result of the huge mass involved.

(vi) Dead stars are of three types: *White dwarfs* ($M < 1.4 M_S$), *Neutron stars* ($1.4 M_S \leq M < 3 M_S$), and *Black Holes* ($3 M_S < M$), where M_S is the present mass of the Sun. White dwarfs have a radius comparable to that of Earth, i.e. about 100 times smaller than that of their typical previous phase as active stars. Because of this large compression all electrons have been detached from the parent atoms; thus the white dwarf consists of electrons and bare nuclei. The kinetic energy is mainly that of electrons, as in the case of metals, and the potential energy is the gravitational one. Minimization of the total energy gives the radius as a function of the mass: $R = 1.42 \frac{\hbar^2}{G u^2 m_e N_v^{1/3}}$, $M = N_v u$. If the mass of the white dwarf keeps increasing, the kinetic energy of the electrons tends to the extreme relativistic limit which is of the form A/R , i.e. similar to the gravitational one, $-B/R$. Thus when $B \geq A$ the white dwarf will collapse to a neutron star. Hence, the equality $B = A$ gives the collapse critical value, which is $N_{v,cr1} = 0.77 \left(\frac{\hbar c}{G u^2} \right)^{3/2}$ corresponding to $1.4 M_S$. After the collapse, the electrons will be forced within the nuclei, which in turn lose their identity; thus a neutron star consists mainly of neutrons ($N_n \approx 0.934 N_v$) and of a small percentage of protons and electrons ($N_p = N_e \approx 0.066 N_v$). The potential energy in a neutron star is that of gravity as given before. The kinetic energy is mainly that of neutrons and to a very small degree that of protons (both of which are non-relativistic) plus that of electrons (which is extreme relativistic). Minimizing the total energy with respect to R we obtain, $R = 3.16 \left(\frac{\hbar^2}{G m_n^3 N_v^{1/3}} \right) \approx 10$ km. This

radius is smaller than that of a white dwarf by a factor of the order of $m_e/m_n \approx 10^{-3}$. As the mass of a neutron star increases the kinetic energy of both the neutrons and the protons tends to become extreme relativistic, i.e. of the form again A/R . Thus, when $A = B$ the neutron star will collapse to a black hole. The condition $A = B$ gives now $N_{\text{ver}2} \approx 1.6 \left(\frac{\hbar c}{G m_n^2} \right)^{3/2}$ which corresponds to about $3M_S$.

Both the minimum and the maximum mass of *active stars* can be expressed in terms of physical constants: $N_{\text{v,min}} \approx 0.2 \left(\frac{u}{m_e} \right)^{3/4} \left(\frac{e^2}{G u^2} \right)^{3/2}$, $N_{\text{v,max}} \approx 100 \left(\frac{\hbar c}{G u^2} \right)^{3/2}$.

The *Universe*, according to observational data as well as the general theory of relativity, is expanding in the sense that the distance R between two distant points is increasing at a rate \dot{R} proportional to R , $\dot{R}/R = H$, where H is the so-called Hubble constant. The basic equation obeyed by the ratio \dot{R}/R is the following: $\left(\frac{\dot{R}}{R} \right)^2 = \frac{8\pi G \varepsilon}{3 c^2}$, where ε , the total average energy density of the Universe, consists of several contributions: $\varepsilon = \varepsilon_{\text{ph}} + \varepsilon_{\text{v}} + \varepsilon_{\text{b}} + \varepsilon_{\text{dm}} + \varepsilon_{\text{de}}$. The first term refers to photons (proportional to $1/R^4$), the second to neutrinos, the third to baryons (proportional to $1/R^3$), the fourth to dark matter (proportional to $1/R^3$), and the fifth one to the dark energy (which seems to be a constant independent of R).

References

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Part I

Three Key-Ideas and a Short-Cut

Summary

First Idea: The Atomic Structure of the Cosmos

Everything consists of indivisible microscopic particles. As a result the properties of each system depend on the properties and on the motions of these elementary particles and on their interactions.

Second Idea: Everything is a Waveparticle (\Rightarrow QM)

A waveparticle either elementary or composite is of particle nature in its very fabric but nevertheless moves as a wave. As a result it is subject to Quantum Laws. Mutual attractive interactions lead to confinement; the latter, according to Quantum laws, produces perpetual motion which prevents collapse and establishes equilibrium.

Third Idea: Equilibrium Corresponds to Minimum Energy

To be more precise, it corresponds to minimum free energy. This minimum implies the equality of the compressive pressure of the interactions with the expansive pressure of the perpetual motion dictated by the Quantum laws, and therefore equilibrium.

The Short-Cut

Physical quantities depend, at least in principle, on a few universal constants and on some parameters. Dimensional considerations and a little thinking may allow us to find what this dependence is.

Chapter 2

The Atomic Idea

If, in some cataclysm, all of scientific knowledge were to be destroyed, and only one sentence passed on to the next generation of creatures, what a statement would contain the most information in the fewest words? I believe it is the atomic hypothesis that all things are made of atoms—little particles that move around in perpetual motion, attracting each other when they are a little distance apart, but repelling upon being squeezed into one another. In that one sentence, there is an enormous amount of information about the world, if just a little imagination and thinking are applied.

R.P. Feynman, *The Feynman Lectures on Physics*.

Abstract In this chapter we introduce the elementary particles from which all things are made of as well as the interactions which bring the particles together. The interactions are transmitted by indivisible particle-like entities. The so-called Feynman diagrams, describing in a vivid pictorial way the various interactions, are also presented.

2.1 Introduction

According to the atomic idea everything is made of indivisible elementary particles (to be called here *m-particles*¹) which attract each other and are self-trapped without collapsing because they move perpetually. Thus they form, in a hierarchical way, composite stable structures of ever increasing size and complexity.

For example, the matter inside and around us is made from only three kinds of indivisible elementary particles: The *electron*, the so-called *up quark*, and the so-called *down quark*. Two up quarks and one down quark are self-trapped through the strong nuclear interactions to form the *proton*. Two down quarks and one up quark are self-trapped through the strong nuclear interactions to form the *neutron*.

¹m stands for matter.

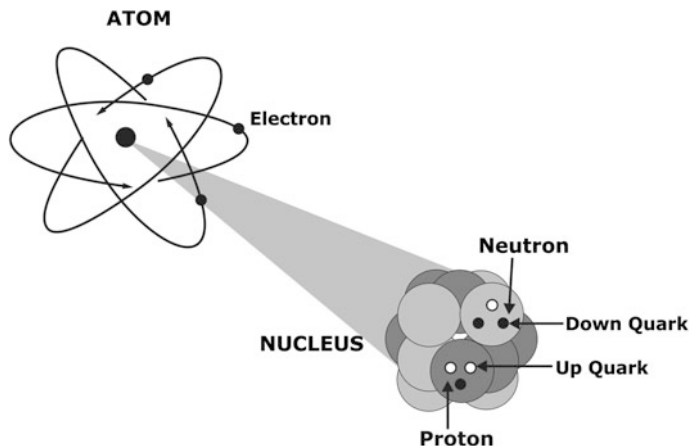


Fig. 2.1 Up and down quarks form protons and neutrons which in turn combine to form atomic nuclei. The latter trap around them electrons to form atoms or ions

Protons and neutrons in various combinations are self-trapped through residual strong nuclear interactions to form the various atomic *nuclei*. The latter attract electrostatically (by Coulomb's law) and trap around them electrons to form *atoms* (when the number of trapped electrons is equal to the number of protons in the nucleus, see Fig. 2.1) or *ions* (when the number of trapped electrons is not equal to the number of protons in the nucleus). Atoms or ions are combined together through electrostatic interactions to form molecules of a huge variety of sizes and shapes. Figure 1.1 and Table 1.1 in the previous chapter show how this hierarchical way of building larger and larger stable structures continues.

To accurately substantiate the atomic idea we must answer in a systematic way the following questions:

- What are the various kinds of *m*-particles (see Footnote 1) and what are their properties?
- What kind of interactions lead to their mutual attraction?²
- What counterbalances this attraction and establishes equilibrium?

In this chapter we provide some answers to questions (a) and (b); (c) will be examined in the next chapter.

The atomic idea is the decisive step towards the sought-after underlying simplicity in the World; this is so, because the immense complexity and diversity of the World can be deduced in principle from the *properties* and the *microscopic motions*

²It turns out that the interactions are actually transmitted through indivisible elementary quantities to be named here (for distinguishing them from the *m*-particles) *interaction-carrying-particles* (*ic*-particles). The *ic*-particles, in addition to mediating forces, are capable of making certain transformations of the *m*-particles among each other; thus the indestructibility of elementary *m*-particles (assumed by Demokritos) or *ic*-particles is of questionable validity.

Table 2.1 The established elementary m -particles are of two types (leptons and quarks) and of three families (1st, 2nd, 3rd) [2]

Leptons					Quarks				
	Symbol, name	Mass $\times c^2$ (MeV)	Electric charge	Spin	Symbol, name	Mass $\times c^2$ (MeV)	Electric charge	Spin	Baryon number
1st	ν_e , e-neutrino	$10^{-7}?$	0	1/2	u , up quark	1.5–4.5	2/3	1/2	1/3
	e^- , electron	0.511	-1	1/2	d , down quark	5–8.5	-1/3	1/2	1/3
2nd	ν_μ , μ -neutrino	$10^{-7}?$	0	1/2	c , charm quark	1000–1400	2/3	1/2	1/3
	μ , muon	105.7	-1	1/2	s , strange quark	80–155	-1/3	1/2	1/3
3rd	ν_τ , τ -neutrino	$10^{-7}?$	0	1/2	t , top quark	174,000	2/3	1/2	1/3
	τ , tau	1777	-1	1/2	b , bottom quark	4000–4500	-1/3	1/2	1/3

Only three of the m -elementary particles (e^- , u , d) are participating in the structure of ordinary matter. Neutrinos are too light and weakly interacting to be bound inside ordinary matter. Particles of the 2nd and 3rd family, having excessive rest energy, are metastable and decay to particles of the 1st family. More exotic particles, which have not been directly observed yet, may account for the so-called dark matter and, possibly, for the so-called dark energy; the existence of these invisible components of the Cosmos is supported by astrophysical and cosmological observations (see also [2]). The neutrino masses are arbitrary guesses

(under the influence of interactions) of all twelve kinds of elementary m -particles presented in Table 2.1. Actually, out of the 12 kinds of elementary particles included in Table 2.1, only the three mentioned before (electrons, up quarks, and down quarks) do participate in the structure of the known ordinary matter. The neutrinos are too light and interact too weakly to be trapped inside the matter. The remaining eight kinds of elementary particles of Table 2.1 are too unstable to be part of existing long-lived structures.

As shown in Table 2.1, all m -particles have spin³ 1/2. This is an important characteristic property which distinguishes the m -particles from the ic-particles; the latter have integer spin, as shown in Table 2.2. In Table 2.1 each rest mass is multiplied by c^2 to be given in energy units ($1 \text{ MeV} = 10^6 \text{ eV} = 1.6022 \times 10^{-13} \text{ J}$). The rest mass of each neutrino is not zero, but its value has not been determined yet. The numbers given in Table 2.1 are arbitrary guesses consistent with established upper limit: $(m_{\nu_e} + m_{\nu_\mu} + m_{\nu_\tau})c^2 < 0.23 \text{ eV}$.

It is useful to also introduce here the concept of lepton and baryon number: All leptons, such as electrons and neutrinos, have lepton number 1 and baryon number 0; all quarks have lepton number 0 and baryon number 1/3 so that protons, neutrons and other composite entities consisting of three quarks to have baryon number 1. The composite particles that have baryon number 1 are also called *baryons*.

³Spin times the universal constant \hbar ($\hbar = 1.0545716 \times 10^{-34} \text{ Js}$) gives the intrinsic angular momentum of each m -particle or ic-particle.

The electric charge e of proton is taken as the unit of electric charge ($e = 1.6021765 \times 10^{-19}$ C).

Returning to the main point of this section, we have to stress that the atomic idea implies that the properties of macroscopic equilibrium matter ought to be expressed, at least in principle, in terms of a few numbers characterizing the elementary particles and their interactions. This is true as long as the considered properties of macroscopic matter do not depend on some uncontrolled initial conditions.

2.2 The Elementary Particles of Matter

We must point out that for each m-particle of Table 2.1 there is an m-antiparticle which is denoted with a bar above the symbol of the corresponding particle. So $\bar{\nu}_e$ is the antiparticle of the e-neutrino and \bar{d} is the antiparticle of the down quark. For historical reasons, the antiparticle of electron is denoted as e^+ and it is called *positron*. Each antiparticle has exactly the same rest mass and spin but exactly the opposite electric charge, the opposite lepton number, and the opposite baryon number of the corresponding particle. Note that antiparticles do not participate in the structure of matter, since a pair of same particle/antiparticle sharing the same volume is annihilated by giving rise to two photons⁴ and/or, rarely, a pair of neutrinos/antineutrinos. However, antiparticles can be produced artificially and their properties have been determined experimentally in spite of their short life time.

Quarks have never been observed as isolated free particles; they have been seen always as constituents of composite entities such as baryons (e.g., protons or neutrons) or *mesons*.⁵ This is the reason for which the rest mass of the quarks as free particles, especially the lighter ones, is not known accurately.

It must also be pointed out that quarks besides their electric charge, carry another type of charge which will be called here *color-charge*, or c-charge (although there is no relation with the usual color). The c-charge is the emitter and the absorber of the strong nuclear force, the same way that the electric charge is the emitter and the absorber of the electromagnetic (E/M) force (see Table 2.2). In contrast to the electric charge, which is of one type, the c-charge is of three different types termed as *red* (R), *green* (G), and *blue* (B). The opposite charges of these three types of c-charges are called *antired*, *antigreen*, and *antiblue* and are denoted by \bar{R} , \bar{G} , \bar{B} , respectively. Each quark carries an R , or a G , or a B , c-charge. Each antiquark carries an \bar{R} , or \bar{G} , or \bar{B} , c-anticharge. The combination of a pair of quark/antiquark

⁴The medical diagnostic method known as Positron Emission Tomography (PET) is based on the introduction of positron-emitting substances to the body of the patient; the positrons, subsequently, by meeting electrons annihilate giving rise to two photons of opposite direction and the same energy each (equal to the rest mass of electrons, 0.511 MeV).

⁵*Mesons* are composite short-lived entities consisting of one quark and one antiquark. Baryons and mesons are collectively called *hadrons*.

of the form $R\bar{R}$, or $G\bar{G}$, or $B\bar{B}$ has zero c-charge (in other words, can be considered as colorless). Similarly colorless is the combination of three quarks of c-charges R , G , and B respectively or of three antiquarks of c-anticharges \bar{R} , \bar{G} , \bar{B} respectively. In view of this property the notation R , G , and B for the three types of c-charge is not unreasonable, since it is reminiscent of Newton's disc, where the equal combination of red, green, and blue real colors produces white. Following this analogy with real colors, we could say that an antiblue c-charge, which is equivalent to the RG combination of c-charges, could be presented as a "yellow" c-charge [2].

We should stress once more that out of the twelve elementary m -particles of Table 2.1 and the corresponding twelve m -antiparticles, only three (the electron, the up quark, and the down quark) participate in the structure of all ordinary matter surrounding us and in us.

2.3 The Interactions and Their Elementary Interaction-Carrying-Particles

There are four basic established interactions: The gravitational, the electromagnetic (EM), the weak (nuclear), and the strong (nuclear). All forces among m -particles are mediated through interactions as follows: An m -particle emits an ic -particle which in turn is absorbed by another m -particle; this causes a force between the two m -particles. Sometimes the emission or the absorption of an ic -particle may transform an m -particle to another m -particle. An ic -particle besides disappearing by being absorbed, may disappear by creating a pair of m -particle/ m -antiparticle. All these elementary processes are subject to certain conservation laws, meaning that specific quantities must be conserved, i.e., must be the same before and after each of these elementary processes. The electric charge is such a quantity, so that the sum of the electric charges of the particles entering an elementary process must be the same as the sum of the electric charges of the particles coming out of this process. Other conserved quantities, besides the electric charge, are the momentum or the angular momentum, the color-charge, the baryon number, the lepton number (separately for each of the three families⁶). The energy must also be conserved *overall*, meaning that the total energy of the initial particles entering a *physical* process must be the same as the energy of the final particles, after this process is over. (For additional comments concerning energy conservation, see Sect. 6.1). Some interactions, such as the strong one, conserve additional quantities as well. The conservation laws impose severe restrictions regarding which physical processes are allowable and which are forbidden.

In Table 2.2 we present some properties of the four interactions and of their elementary ic -particles.

⁶Neutrinos of a given family (see Table 2.1) as they travel after leaving a reaction exhibit a transformation from one family to another; e.g. an e -neutrino may change to a μ -neutrino and vice versa.

Table 2.2 The four basic interactions and their interaction-carrying-particles [2]

Name	Dimensionless strength	Range (m)	Type	ic-particle name-symbol	Rest mass (MeV)	e-charge	Spin	Emitters/absorbers
Gravitational	$a_G \equiv \frac{Gm_p^2}{\hbar c} = 5.9 \times 10^{-39}$	∞	Attractive	Graviton?	0	0	2	All m - and ic-particles
Electromagnetic	$\alpha \equiv \frac{e^2}{\hbar c} = \frac{1}{137}$	∞^a	Attractive or repulsive	Photon, γ	0	0	1	Electrically-charged particles
Weak “nuclear”	$a_w \equiv \frac{g_w^2}{\hbar c} \frac{\sqrt{2}m_p^2}{m_w^2} \approx 10^{-5}$	10^{-18}	$u \rightleftharpoons d + \dots$	Vector Bosons } Z^0 W^+ W^-	91,000	0	1	All m -particles, photons, vector bosons
					80,000	1	1	
					80,000	-1	1	
Strong “nuclear”	$a_s \equiv \frac{g_s^2}{\hbar c} \approx 1$	10^{-15}	Attractive or repulsive	Gluons, g	0	0	1	Quarks and gluons

^aE/M interaction being either attractive or repulsive lead to the formation of neutral systems for which their residual E/M interaction becomes short range of the order of a few Angstroms

The physical symbols appearing in the second column of this table are as follows: G is the gravitational constant, m_p is the rest mass of the proton, \hbar is the reduced Planck's constant, "trade mark" of Quantum Mechanics, c is the velocity of light in vacuum, g_w is the strength of the weak interaction, m_w is the mass of the W^+ vector boson, and g_s is the strength of the strong interaction. Numerical values of some of these quantities are given in Table I.1 in Appendix I. The range r_o of each interaction is connected to the rest mass m of the corresponding ic-particle through the formula $r_o = \hbar/mc$. The strong interaction to be discussed in Chap. 7, is an apparent exception.

There are several points to be noticed in Table 2.2: the strength of the gravitational interaction is unimaginably weaker than any other interaction. To stress this point take into account that when you lift an object, such as a chair, the residual electric force generated by your muscles overcomes the gravitational force exercised on the chair by the whole Earth. Nevertheless, gravity, in spite of its extreme weakness compared to the other three forces, becomes significant and eventually dominates in systems, such as planets and stars, consisting of a very large number N of m -particles. The reason is that each of these N m -particles interacts gravitationally with every other as a result of gravity's long range character. Moreover, each of the $N(N-1)/2$ contributions to the gravitational self-energy of the system is of the same sign because of gravity's always attractive nature.

Neither the gravitational waves nor their ic-particles, the gravitons, have been detected yet experimentally, because they carry such a minute amount of energy. Finally, one must keep in mind that all particles, both the m - and the ic-ones are emitters and absorbers of the gravitational force, since all of them possess energy and, hence, relativistic mass.

The E/M interaction has also some unique and very important features: First, it dominates the structure of matter from the scale of atoms all the way to that of an asteroid, i.e., it reigns over 15 orders of magnitude from 10^{-10} to 10^5 m. Second, its ic-particle, the photon, is the only one that, besides mediating a force, can also travel free in space carrying over long distances both energy and information. No other ic-particle can do that: Gravitons are too weak to be noticed at least with present day technology; vector bosons decay too fast; and gluons are trapped forever within baryons, mesons, or any other composite quark structure. Third, photons can be easily emitted and detected. These unique features account for the dominant role of photons in the World, including their biological and technological role. Photons deserve further examination; so we shall return to them in Chap. 6.

2.4 Feynman Diagrams

The interactions, besides mediating forces, exhibit other functions as well: They transform particles, they create pairs of particles/antiparticles, etc. All these functions are better presented by the so-called Feynman diagrams which provide a vivid