

Fatigue of Materials and Structures

edited by
Claude Bathias and André Pineau



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Fatigue of Materials and Structures

Fundamentals

Edited by
Claude Bathias
André Pineau

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Foreword

This book, along with the forthcoming publication *Fatigue of Materials and Structures: Application to Damage and Design* (edited by C. Bathias), is the most comprehensive compilation of current approaches in the field of fatigue of materials and structures under repeated loads/loadings of various types. Historic methods, as well as the most recent approaches and current research, are included. Professors Claude Bathias and André Pineau have selected a group of outstanding experts on the various topics in the field for each chapter and have themselves contributed to some chapters dealing with their specialties. These books are great references for anyone wishing to be up-to-date on any topic in this field.

Although the fatigue of materials has been studied for over 150 years, many significant approaches have been developed in the past 100 years. The Coffin-Manson “plastic strain cycling approach” for low cycle failures and later the beginning of the “damage tolerance approach” through the fracture mechanics correlation of crack growth rates were suggested in the 1950s. Indeed these methods were shown to be applicable in the late 1950s but were frequently ignored until the failure of an F-111 aircraft in December 1969. This crash convinced the US Air Force to develop and use damage tolerance methods on every aircraft. The US Federal Aviation Agency was soon applying similar methods to ensure a sufficient crack-growth life in order to set up adequate inspection intervals for critical structural parts. Earlier in the 1960s Westinghouse and others used/applied fatigue crack-growth testing to ensure sufficient life in the case of various power generating systems. Since then,

many novel applications of these newer methods have been developed.

More recently, Bathias and others have shown that the “traditional fatigue limit stress”, below which failures were regarded as not occurring, are unsafe for “very high cycle fatigue” of the order ranging from 10^8 to 10^{10} loading cycles. This evidence was determined using ultrasonic testing at 20 to 30 kHz. This field is still rapidly developing but is thoroughly covered in this book. Further discussion dealing with the historical aspects of fatigue are detailed in the introduction in Chapter of this book.

Each chapter is self-contained on the topic of interest. Each chapter is well referenced in order to provide the reader with a thorough background and to act as a source for deeper study on the topics covered in this book. As a consequence, readers can use this book as a guide to further information on the topics they are interested in. In some cases, the chapters focus on similar topics as they belong to the same general category but are written from different points of view and with different emphasis.

[Chapters 2](#), [3](#), [4](#), [5](#) and [9](#) present the approach of failure cycles from low cycle plastic fatigue to very high cycle behavior, including the effects of notches, hardening mechanisms, etc., with many other variables involved. Within Chapter 9 fatigue crack growth mechanisms are also discussed.

[Chapters 6](#), [7](#), [8](#) and [9](#) (again) deal with fatigue crack growth from small to long cracks with various models. They cover growth laws and their mechanisms. [Chapters 10](#) and [11](#) provide a thorough overview of environmental factors, from aggressive to vacuum effects, leading to the initiation and growth of cracks. While [Chapter 12](#) discusses loading interaction effects for a wide variety of structural applications and the counting methods for these various loading programs and types.

As a veteran of this field, allow me to point out the excellence of the work of some of the outstanding young stars of this field, such as Sylvie Pommier and Thierry Palin-Luc, who contributed to the writing of [Chapters 9](#) and [12](#). Although these volumes present the current state of understanding, this field has many other outstanding young researchers who will develop new approaches as time goes by. However, these volumes stand as a full picture of the current “state of the art” in understanding fatigue phenomena.

Paul C. PARIS
May 2010

Chapter 1

Introduction to Fatigue: Fundamentals and Methodology.

1.1. Introduction to the fatigue of materials¹

1.1.1. Brief history of fatigue: its technical and scientific importance

Experience shows that fracture of structures or machine parts during regular operating conditions are most often due to fatigue. Structural integrity has always been an obstacle to industrial development. Its consequences could be seen during the development of mechanical industry in the 19th century. The industrial revolution, particularly the development of rail transportation, was affected from the start by a certain number of serious accidents, such as the one in Versailles, 1842, where the rupture of an axle caused the death of 60 people [SMI 90]. This death toll is close to that of the two Comet plane crashes that occurred in 1954.

It is known that fatigue damage costs several percent of the gross domestic product of the engineering industry. For

this reason, we can understand the fact that articles and papers about this type of damage are ever increasing. Toth [TOT 01], who recently checked the COMPENDEX data base, found about 10,000 articles on this topic between 1988 and 1993, which comes to 2,000 articles a year.

According to Schütz [SCH 96], Braithwaite [BRA 1854] introduced the term “metal fatigue” in 1854. Despite this, Lemaitre [LEM 01] reckons that Poncelet mentioned this term during an engineering lecture in Metz as early as 1839, and that Rankine used it in 1843. To gain a better understanding of the work carried by Poncelet and Rankine in this field, we can refer to Timoshenko’s work dealing with the history of the strength of materials [TIM 53]. As a matter of fact, this term has probably been in use for a long time. For instance, Stendhal used it in one of his pieces “Memoirs of a tourist” published in 1838 [STE 1838]. On his way to Civitavecchia, in Italy (where he had been appointed Consul), while crossing the Loire river in La Charité one of the axles of his carriage broke. What he wrote is as follows:

“La Charité — April 13. I was riding through the small town of La Charité, when, as a reminder of the long thoughts I had in the morning about iron diseases, the axle of my carriage suddenly broke down. I have to be blamed: I swore that if I ever had my own carriage, I would get a nice Fourvoirie axle, with six mild steel rods, forged under my own eyes... I checked the iron grain of my axle; it was larger as it has apparently been used for a long time... .”

We should remember that in those times, and for many years during the 19th century, people thought that iron “crystallized” due to mechanical vibrations. The fact that Stendhal, who lived at the same time as Poncelet, already knew what fatigue was, at least in this form, is not surprising. They both campaigned for Napoleon in Russia in 1812 and we can assume that they would have discussed this subject.

Excellent reviews on the history of fatigue have been written, some of them very recently. We can for instance refer to the work of Schutz [SCH 96] which lists more than 550 references, such as Toth [TOT 01], or Schijve [SCH 03].

It is worth noting that some works on this subject have recently been published:

- Bathias and Baillon [BAT 97];
- Bathias and Paris [BAT 05];
- Henaff and Morel [HEN 05];
- Murakami [MUR 02, MUR 03];
- Polak [POL 91];
- Reifsnider [REI 91];
- Schijve [SCH 01];
- Shaniavski [SHA 07]; and
- Suresh [SUR 98].

Here we should mention two regularly published journals that explicitly refer to the fatigue phenomenon: *Fatigue and Fracture of Engineering Materials and Structures* and the *International Journal of Fatigue*. In addition to this, in other countries scientific societies organize lectures and conferences on this subject, such as the ASTM (American Society for Testing and Materials) in the USA and the SF2M (French Society of Metallurgy and Materials) in France.

Table 1.1. *A few stages and main events regarding the history of the fatigue phenomenon*

Year	Event
1842	Meudon railway accident
1858	First publication by Wohler
1860-70	Wohler experiments on smooth and notched axles. Bending and torsion tests — Investigation on the effect mean stress
-1881	Study by Bauschinger which initiated low-cycle fatigue
1910	Basquin law
1913	Stress distribution within notches (Inglis)
1920	Energy balance regarding the propagation of a crack (Griffith)

1930	Stress concentration factor and endurance limit (Peterson)
1937	Neuber concept applied to notches
1939	Statistical approach Weibull law
1945	Miner concept for fatigue damage accumulation
1953-54	Low cycle fatigue. Manson — Coffin law
1954	Comet aircrafts accidents
1956	Introduction of strain energy released rate (Irwin)
1960	Servohydraulic machines
1961	Paris law
1968	Introduction of effective stress intensity factor (Elber)
1988	Aloha B737 accident
1989	DC 10 Sioux City accident
1996	Pensacola accident
1998	ICE. Eschede railway accident
2006	Los Angeles B767 accident

Some memorable stages and events that have marked the history of fatigue are highlighted in [Table 1.1](#). As mentioned earlier, this type of damage has clearly been of great importance during the development of rail transportation. The various ruptures that Wöhler observed in Germany led him to undertake a systematic study of this type of damage.

Along with trains and many other mechanical structures, aircraft were also readily affected by the fatigue phenomenon. The first serious accidents that occurred are those involving two Comet aircraft in 1954. A more recent example was the Aloha accident in 1988, which involved a Boeing 737. The damage was really serious, as we can see in [Figure 1.1](#). This accident was caused by the formation of cracks due to fatigue and corrosion in the assembly rivets area within the fuselage. As a result, numerous studies have been carried out regarding the issue of multiple site damage.

Figure 1.1. *The Aloha Airlines Boeing 737 at Honolulu international airport, Hawaii, following the accident on April 28, 1988*



Another example concerns the MacDonald Douglas DC 10 crash, which occurred in Sioux City in Iowa in 1989 (see [Figure 1.2](#)). The explosion of one of the engines led to this tragic accident. Even more recent was the Pensacola crash, when one of the engines broke apart due to cracking initiation caused by a drilling defect within a fan disk (see [Figure 1.3](#)).

These three examples from the aeronautical industry should not lead people to think that aircrafts as a means of transportation are dangerous and the only means affected by fatigue phenomenon. If we calculate the distance to passenger ratio, flying remains the safest means of transport. Nevertheless, due to its rapid development and despite the work being done on its design, manufacturing and maintenance, we can predict that in about 10 years' time a major aircraft accident is likely to occur every week (see [Figure 1.4](#)). Let us keep in mind that human error is the main cause of accidents involving aircraft. Accidents caused by defects in the materials are still occurring in spite of improved manufacturing processes.

Figure 1.2. *DC 10 aircraft crash. Part of a detached engine. Sioux City Airport, July 19, 1989*

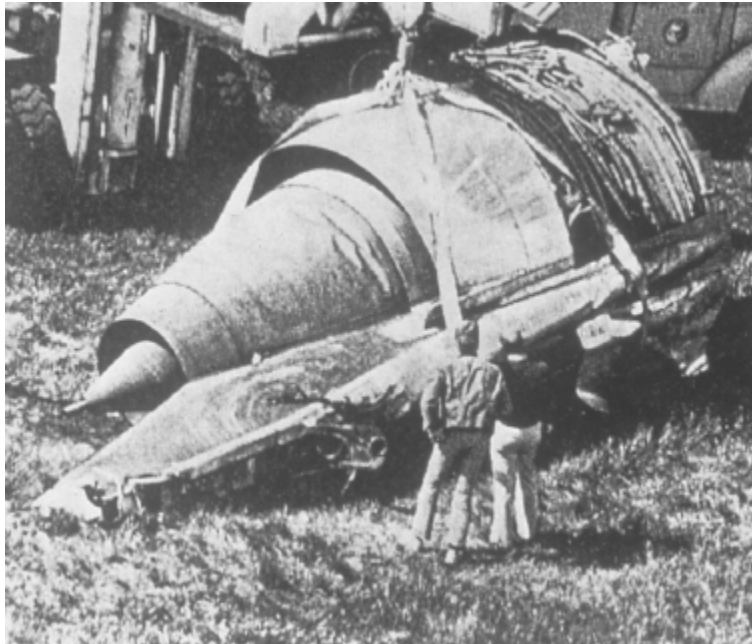
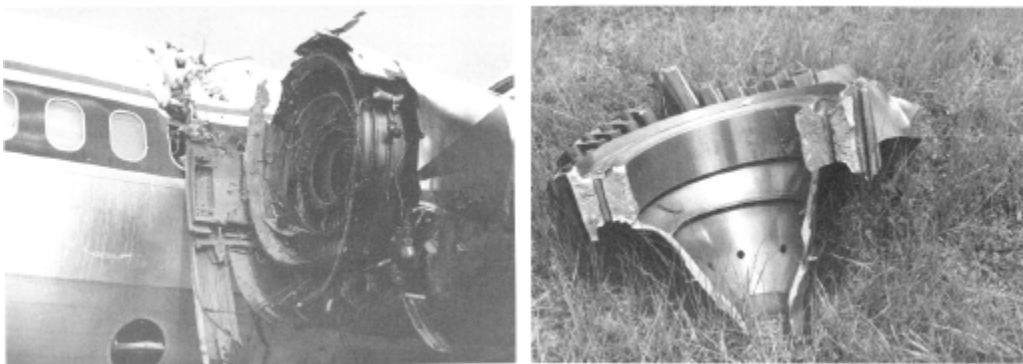


Figure 1.3. *Pensacola Crash (Florida, USA), July 6, 1996, was due to a failure during the take off of a Delta Airlines MD-88 aircraft*



Fatigue also affects many other fields of transport, as shown in [Figure 1.5](#) where cylinder heads of diesel engines subjected to increasing thermo-mechanical loading can break due to thermal and mechanical fatigue cracking if their design is wrong [SAL 07].

Figure 1.4. *Statistical study of the evolution of air traffic and of the number of crashes (MANHIRP, 2001, see also 1001crash.com)*

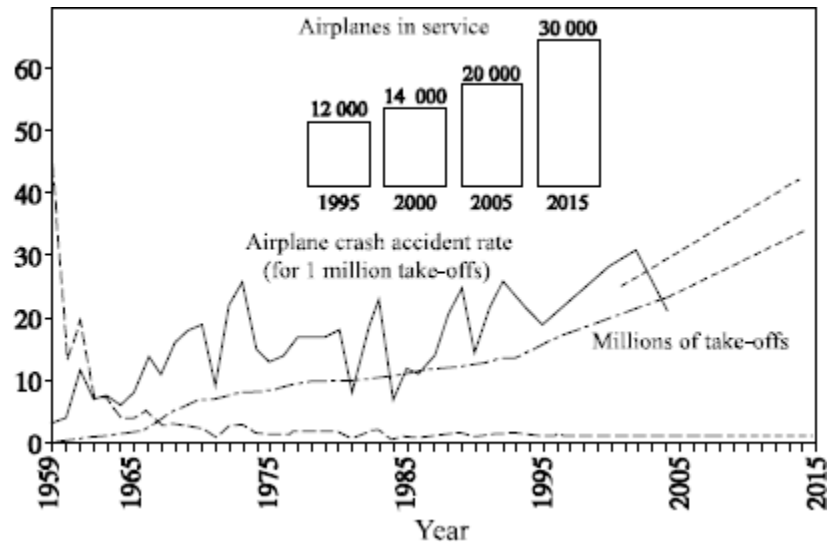
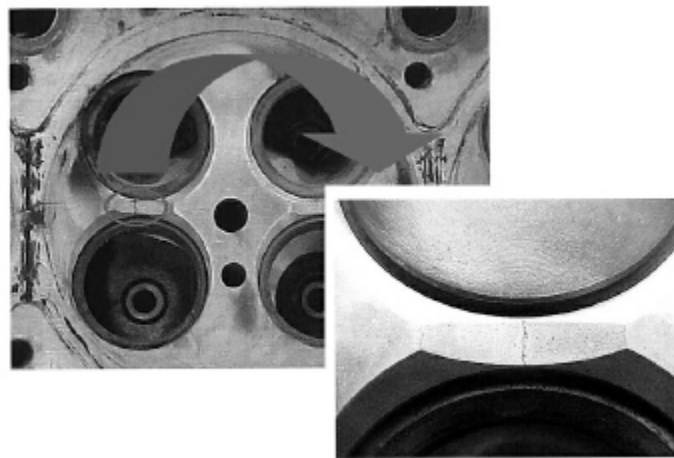


Figure 1.5. *Cracking within the cylinder head of a diesel engine [SAL 07]*



1.1.2. Definitions

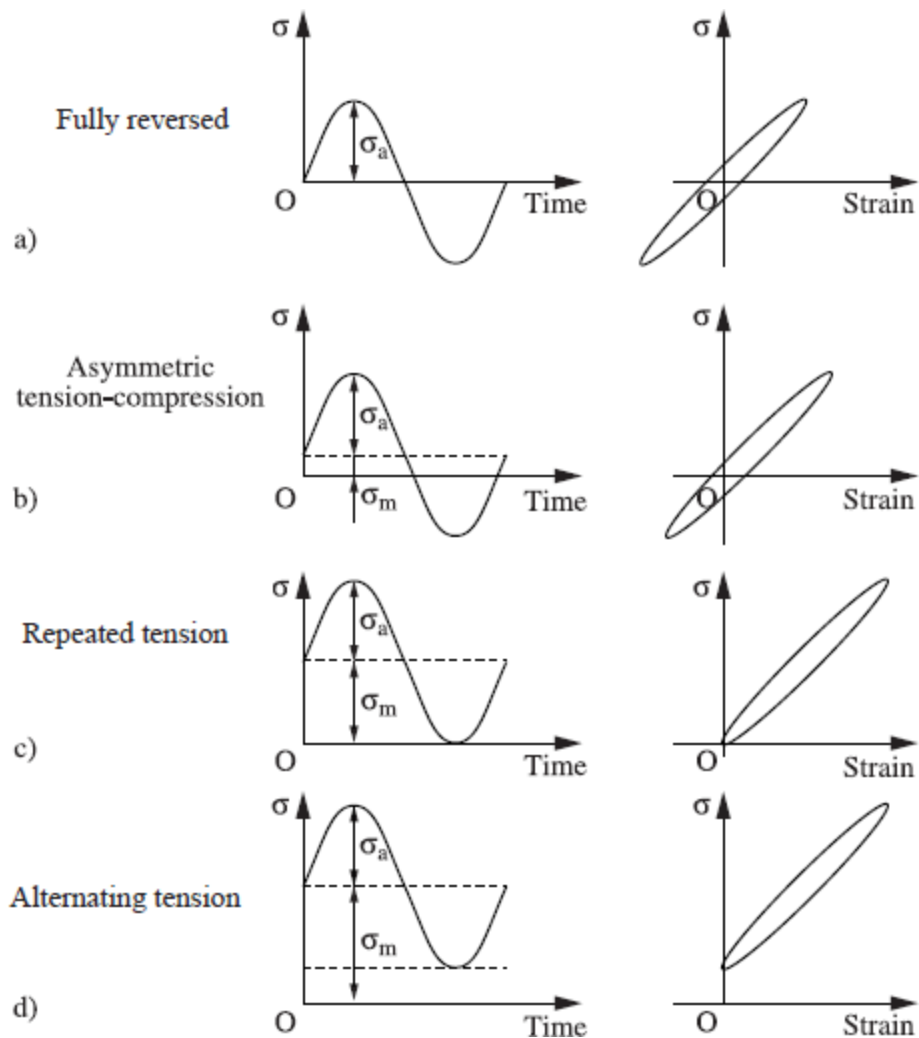
Fatigue or *fatigue damage* refers to the modification of the properties of materials due to the application of stress cycles whose repetition can lead to fracture.

Uniaxial loading is defined as *the amplitude of the maximum stress* during a cycle σ_{\max} . The *stress ratio* R is the ratio between the *minimum stress* σ_{\min} and the *maximum stress* σ_{\max} , that is to say $R = \sigma_{\min}/\sigma_{\max}$. We sometimes have to distinguish the *alternating component*

σ_a from the mean stress σ_m . Thus, depending on the relative values of these two components, we can differentiate the tests under different stresses (see [Figure 1.6](#)), such as:

- fully reversed: $\sigma_m = 0$, $R = -1$;
- asymmetrically reversed: $0 < \sigma_m < \sigma_a$, $-1 < R < 0$;
- repeated: $R = 0$;
- alternating tension: $\sigma_m > \sigma_a$, $0 < R < 1$.

Figure 1.6. *Different cases of fatigue stresses: load-time; force-strain*



Plastic deformations occur with low-cycle fatigue (see [Chapter 4](#)). Usually, the fatigue phenomenon occurs without

any general plastic deformation, which makes it less likely to be noticed. Nevertheless this phenomenon occurs with a localized plastic deformation around pre-existing defects within the materials, at the notches of the structures, or at the tips of a crack when it has already been initiated.

For multi-axial loading, which will be presented in [BAT 10], the definition of a strain amplitude is much more subtle, especially when loading is not proportional.

Fatigue is rarely perfectly cyclic (of constant amplitude and frequency), as shown in [Figure 1.6](#). In many cases (thermal engines, bridges, etc.), loads have variable amplitudes and frequencies. These kinds of loads are examined in detail in [Chapter 12](#).

Theoretically, fatigue damage only depends on the number of cycles and not on their frequency. As a matter of fact in most cases frequency does have a consequence. This is the case when environmental and visco-plasticity effects at high temperatures are involved (see [Chapters 10](#) and [11](#)).

In general, *lifetime* is measured using the number of cycles to failure, N_F . When N cycles have occurred ($N < N_F$) a given damage is accumulated and has to be evaluated. This allows us to determine the residual lifetime of the structure and the management of its operation, such as the timing between aircraft inspections.

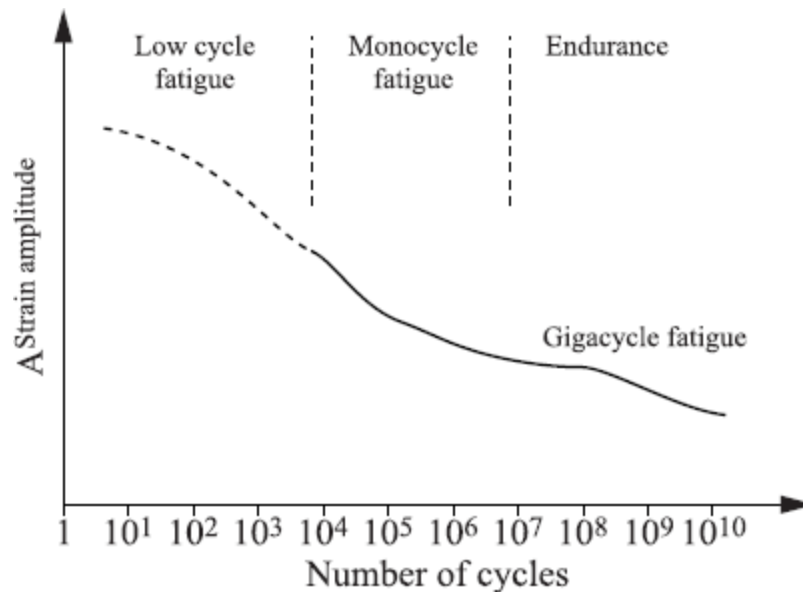
We define *endurance* as the strength capability of components and entire structures before fatigue develops.

Thus, in general, fatigue occurs as soon as time dependent forces are involved. As a consequence, fatigue damage is characterized by its danger, which is basically that fracture can occur at low cycle stresses that in most cases are lower than the tensile strength and even lower than the elastic limit of the material.

1.1.3. *Endurance diagrams*

The easiest fatigue test consists of subjecting each specimen to periodic loading cycles, most frequently sinusoidal, with a maximum amplitude J_a , along with a constant frequency. The number of cycles is measured once rupture starts to occur (N_F). We then obtain a curve which looks like the one plotted in [Figure 1.7](#).

Figure 1.7. *Wöhler curve and definition of the various endurance areas*



With this curve, known as the Wöhler curve, SN curve (*stress-number of cycles*) or endurance curve, we can differentiate four different regions:

- with a high stress, we get *low cycle plastic fatigue*. Within this region, studied later on in [Chapter 4](#), fracture occurs after a relatively low number of cycles (10² to 10⁴) along with a significant plastic deformation. This type of damage has been studied since the 1950s, following Manson [MAN 52] and Coffin [COF 54] who introduced the Coffin-Manson law;
- with a lower stress there is a fatigue region where endurance is limited. Within this region, fracture occurs for a given number of cycles. The lower the stress amplitude, the

higher the number of cycles. The region of limited endurance is presented in [Chapter 2](#);

- an endurance region, which has been considered as an infinite lifetime, or safety region, corresponding to what is called the endurance limit. For steels, this region is reached after 10^6 to 10^7 cycles. In reality metal alloys have no real endurance limit. This has led us to consider the fourth (gigacycle) region in the past 10 years;

- a region corresponding to the *gigacycle* fatigue, which is significant for a given number of applications. Within this region, studied in [Chapter 5](#), we often see that the “endurance limit” still decreases when the number of fracture cycles increases.

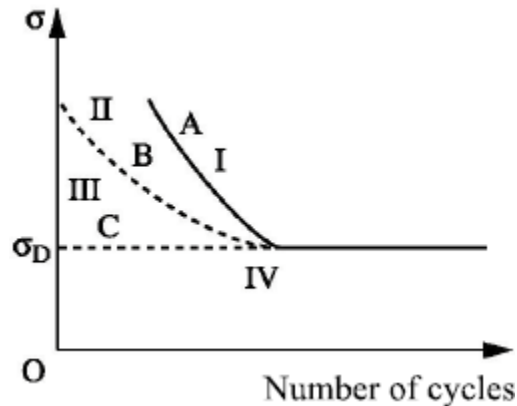
1.2. Mechanisms of fatigue damage

1.2.1. *Introduction/background*

On the Wöhler curve (see [Chapter 2](#)) we can see four stages, as shown in [Figure 1.8](#), where, in contradiction with what is described in the previous figure, an endurance limit σ_d is highlighted for demonstration purposes. The upper region I corresponds to conditions in which specimens are broken. The lower region IV corresponds to the cases of unbroken specimens, where curve A separates both regions. Within the area directly below curve A, we can see two new regions located above the endurance limit: region III corresponding to the initiation of a crack, and region II associated with the propagation of this crack, the number of corresponding cycles being N_p . We can also see that

initiation N_i represents the main part of the lifetime when the number of cycles to failure N_F , is high.

Figure 1.8. *Wöhler SN curve (A) and number of crack initiation cycles (B)*



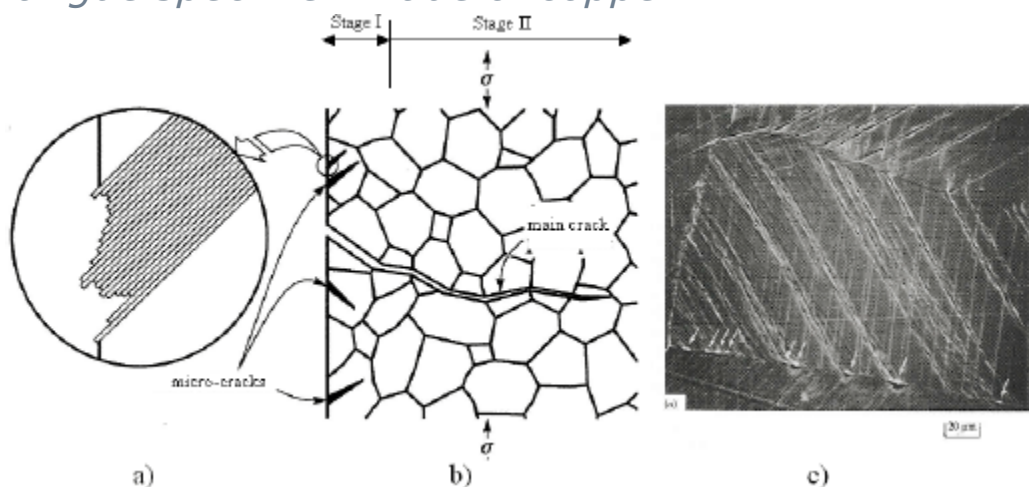
Numerous fatigue damage indicators, in addition to crack initiation and propagation, have been studied, such as electrical resistivity. For a decade, infrared thermography has been used, providing researchers with encouraging results [DOU 04, LUO 95, LUO 98]. However, it is still too early to know whether this method can provide reliable results, and especially whether it can be used to speed up the determination of endurance curves.

1.2.2. *Initiation of fatigue cracks*

Since the first observations were made using optical microscopy in 1903 by Ewing and Humphrey [EWI 03], the initiation of fatigue cracks has been widely studied. In the mid-1970s the articles published by Thompson and Wadworth [THO 58] and by Laird and Duquette [LAI 71] enabled a review to be written on this subject. Since then, significant efforts have been devoted to this stage of fatigue damage.

Work carried out by Forsyth [FOR 51, FOR 53] showed that fatigue damage is mainly surface related. On the polished surface of the specimens, we can observe steps due to the formation of localized deformation bands, known as *persistent slip bands*. These bands are formed on the sliding planes with a maximum resolved shear stress. The mechanisms by which these bands form are presented in [Chapter 4](#). Topography of the surface reveals the formation of *intrusions* and *extrusions*, as shown in [Figure 1.9](#).

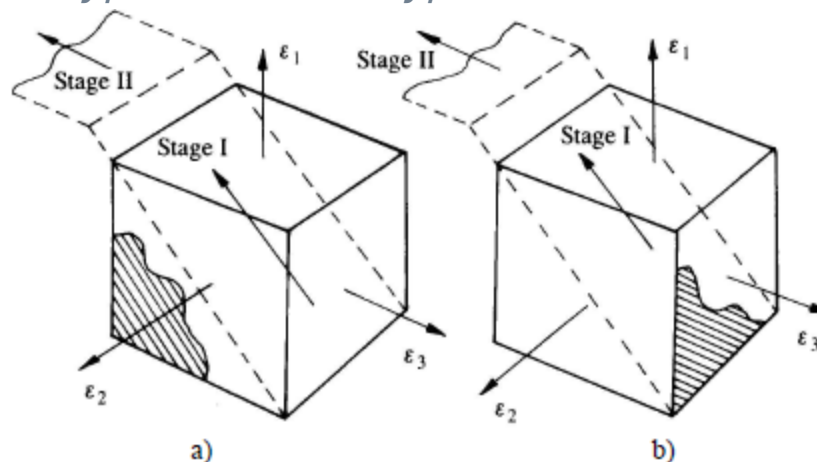
Figure 1.9. *a) Initiation of micro-cracks due to the sliding of alternate planes and to the formation of intrusions and extrusions at the free surface (cross-section); b) formation of a main crack from micro-cracks; c) characteristic formation of stage I intrusions and extrusions at the surface of a fatigue specimen made of copper*



During a uniaxial test on polycrystalline specimens, these bands, which will lead to the formation of *stage I* micro-cracks, appear at a 45 degree angle to the tensile axis. Only a few grains are involved in the formation of these bands. Orientation of the persistent bands and of the stage I cracks is significant not only in the case of a uniaxial loading (tension or torsion), but also in the case of a multi-axial loading where the directional characteristic of fatigue damage is essential. Brown and Miller [BRO 79, MIL 91] have introduced the really useful notion of type A and B

facets regarding multi-axial loading, as shown in [Figure 1.10](#). As expected, type B facets, whose slip vector goes into the material, are usually more dangerous than type A facets, whose slip direction is tangent to the free surface of the specimen. Unfortunately, few studies have been carried out regarding this directional characteristic that occurs at the start of fatigue damage. We can nevertheless list some studies, such as the one carried out by Jacquelin [JAC 85].

Figure 1.10. *Directional characteristic of fatigue damage. The significance of the orientation of the strain field compared to the surface plane and free surface plane (cross hatched); a) type A facet; b) type B facet*



Intrusions and extrusions, associated with persistent slip bands and the micro-propagation of stage I cracks, extend over a distance of the order of the grain size. Indeed, as soon as this micro-crack of significant crystallographic nature hits the first grain boundary it branches off, following a stage II course and then propagates perpendicularly to the direction of the maximum principal stress.

The definition of initiation remains ambiguous as it depends on the chosen scale. We usually define this damage stage as corresponding to the number of cycles, N_i , that have to be applied before the crack branches to stage II. Here, the corresponding distance is similar to the size of a grain. To the best of our knowledge, this is the most

plausible definition. The most commonly accepted definition, which corresponds to the failure of a specimen or to the reduction of the maximum tensile stress by a certain amount (for example 5%), is not accurate enough.

1.2.3. *Propagation of fatigue cracks*

Stage II crack propagation according, that is to say in mode I, has been studied frequently since the early work by Paris and Erdogan in 1963 [PAR 63]. When these cracks are long enough, the rate of crack propagation, as presented in [Chapter 4](#), can be described using Paris' law:

$$[1.1] \quad \frac{da}{dN} = C (\Delta K)^m$$

where a stands for the length of the crack and ΔK for the variation of the stress intensity factor K , whereas C and m are constants, depending on the material. The loading ratio R is also important. The various laws proposed are listed in [Chapter 4](#).

A region between stage I and the long crack stage is known as the *short crack* stage. It has been extensively studied since Pearson published his work on the topic in 1975 [PEA 75]. He proved that short cracks propagate faster than long cracks at the same apparent value, ΔK . This short-crack phenomenon is significant and is presented in [Chapter 7](#). It increasingly appears that the particular behavior of short cracks, or at least those termed “physically short”, can mainly be explained by their tri-dimensional aspect and by the crack closure phenomenon [LIN 95, PIN 86] concept introduced by Elber in 1970 [ELB 70].

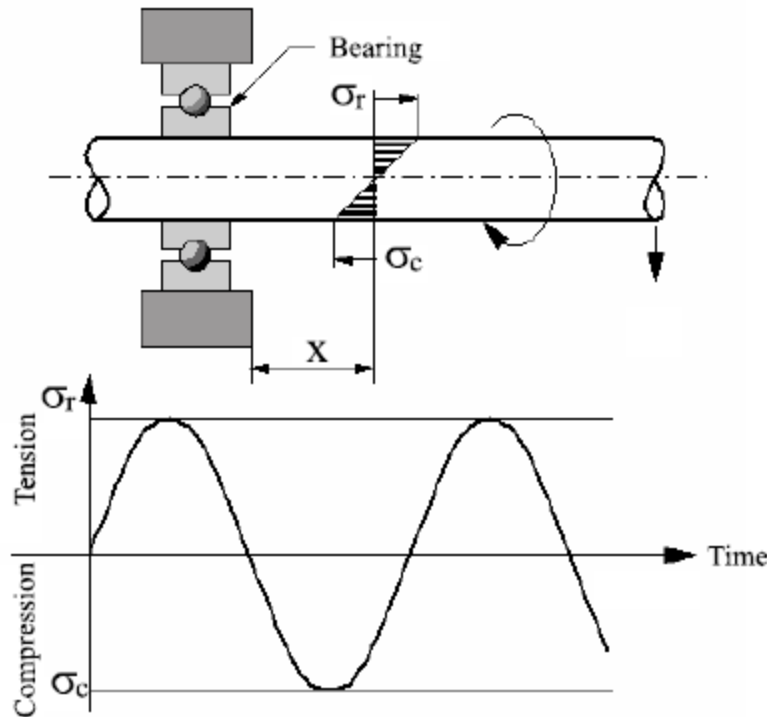
Paris' law is purely phenomenological. Since then, some authors have tried to develop and improve this law using the properties of materials. These authors have developed what we call a local approach to fatigue crack. The principle

of this approach is to start from the crack tip stress-strain field ahead of the crack-tip (see [Chapters 8](#) and [9](#)) and then introduce a fatigue failure criterion. The first model of this type was proposed by McClintock in 1963 [CLI 63], who assumed that the crack propagates in successive stages under the effect of low-cycle fatigue-type damage. The law thus formulated by McClintock considers that the exponent m in Paris' law [\[1.1\]](#) is equal to four and a non-propagation crack threshold ΔK_{th} is involved. A second model was also suggested by McClintock in 1967 [CLI 67], due to the observations of Pelloux [PEL 64] and McMillan [MIL 67]. These authors showed that fatigue failure surfaces were covered with striations — one striation corresponding to one cycle — at least in a certain region of crack propagation rates (0.01 to 1 $\mu\text{m}/\text{cycle}$). McClintock then related crack growth rate per cycle, being the distance between the striations, to the blunting at the crack tip (see [Chapter 6](#)). This model then considers that the slope m of Paris' law is equal to two. In practice, we can observe that the value of this exponent goes from two to six in the case of most materials. Since McClintock developed his models, others have been proposed (see [Chapters 6](#), [8](#) and [9](#) for more information).

1.3. Test systems

The most commonly used method to obtain endurance curves is the rotating bending or plane bending test (see [Figure 1.11](#)). The machines used for these tests allow for frequencies close to 20 Hz.

Figure 1.11. *Principle of rotary bending tests*



These machines are simple to use and relatively inexpensive. We can also use these machines to perform some traction/compression tests. The advantage of this is that the effect of a constant stress through the section of the specimen can be observed. The choice of specimen and loading mode is significant because a size effect is involved. Thus, with similar conditions, and especially with a similar cyclic stress, the tension-compression endurance is lower than that calculated in the case of rotary bending, itself being lower than that measured using plane bending.

Determination of an endurance curve takes a long time. A failure test with $N_F = 10^7$ cycles performed at 20 Hz lasts around six days. In addition, the test results are scattered. This is why testing methods were developed very early on to determine the endurance limit of metallic materials at higher frequencies (see [Chapter 2](#)). In the case of polymers and elastomers, the frequency has to be further reduced, otherwise significant over-heating is observed.

Resonance machines have been developed in order to perform higher frequency tests on metallic materials. Some specific machines can enable much higher frequency tests (close to 20 KHz) and thus allow us to study the region of gigacycle fatigue (see [Figure 1.7](#)). [Chapter 5](#) deals with this particular topic.

All of these tests have been carried out with a load or imposed strain within the elastic region. Low-cycle fatigue tests, however, are performed under strain control, meaning that an extensometer system has to be used. The servo-hydraulic machines were developed in the mid-1960s for that purpose. A description of these tests, specimens and of extensometers can be found in [Chapter 4](#).

More specific and less common machines have to be used in order to carry out multi-axial fatigue tests. Tension-torsion fatigue tests on thin tubes are well developed. The introduction of an internal/external pressure within the traction-torsion stressed tubes allows the degree of multi-axiality to be increased. These machines are less commonly found in practice, especially the ones allowing a range of temperatures to be studied. The downside (or the advantage, depending on the context) of these tests, is that the reference of the main stresses is not fixed. This issue can be overcome if a machine that can exert independent forces along three orthogonal directions (XYZ) is used. Only one machine of this type can be found in France. Biaxial machines (XY), however, are easier to use in order to test relatively thin materials.

1.4. Structural design and fatigue

Since Wöhler, engineers have developed new mechanical parts that are capable of resisting fatigue. The improvement