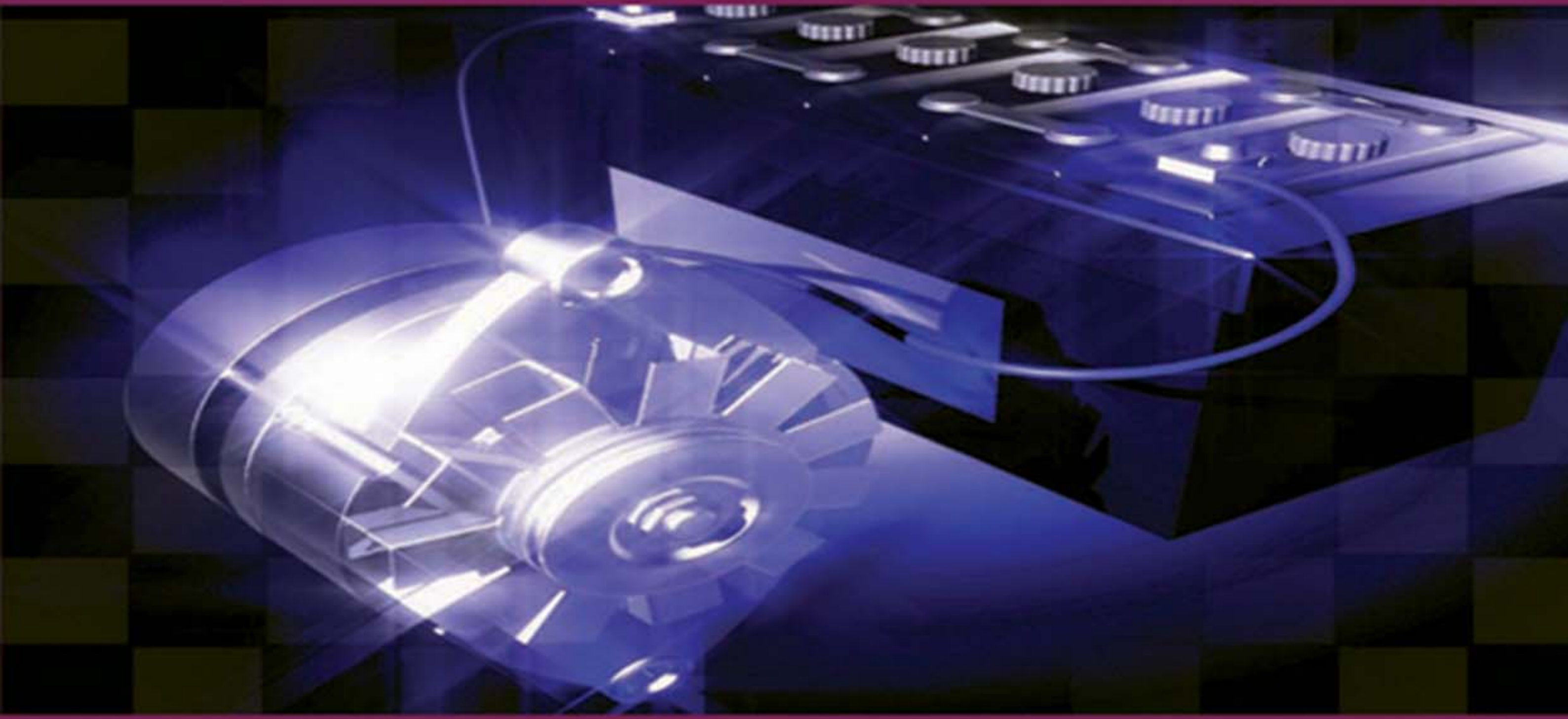

Rotating Electrical Machines

René Le Doeuff
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Table of Contents

Preface

Rotating electrical machines provide the basis of the electromechanical energy conversion and constitute the core of a wide scientific and technological field called “electrical engineering”. This discipline has seen a very important evolution with the extensive development of related fields: power electronics, analogical and digital control techniques, etc. This revolution has led to the generalization of electrical actuators in every industrial area as well as in everyday life. It has also modified the way the machines are used while, at the same time, simplifying their adaptation to new energy sources. Therefore, this evolution has to be taken into account in the teaching of electrical machinery.

The present text is the result of our long teaching and research experience in various universities’ engineering schools, both in France and Algeria. It is intended mainly for Master’s level students enrolled in electrical engineering programs. Its aims consists of providing readers with the essential knowledge of electrical machines, their structures, the ways they can be modeled and their implementation. This basic understanding should allow them to tackle with relative ease the study of transient phenomenon, speed variation and control of drives, and any other special applications.

This methodological approach was first proposed by Professor E. J. Gudefin in Nancy (France) in the 1960s. It is based on matrix representation of the machine equations using instantaneous values of electromagnetic variables.

This modeling approach is particularly suitable for the study of electrical machines fed by static converters; and it is necessary for the analysis of machines in transient regimes or any other study that uses Concordia and Park transformations, etc. It can also be used to establish classic steady state equations of electrical motors. The calculation of the instantaneous electromagnetic torque leads to a simple and convenient representation of the association machine-converter enabling an easy understanding of the continuous energy conversion phenomenon.

The main preliminary knowledge useful for reading this text (electromagnetism, sinusoidal systems, power electronics) is gathered in Chapter 1 (Main Requirements).

General concepts are established in Chapter 2 (Introduction to Rotating Electrical Machines) and are then used for different analyses of conventional machines: Synchronous Machines (Chapter 3), Induction Machines (Chapter 4) and Direct Current Machines (Chapter 5). Many examples describing the use of these machines with and without converters are also presented. Some traditional aspects (e.g. resistive starters, circle diagrams, etc.), which are of very little use today, are still presented because of their historical and pedagogical interest.

To make this book as factual as possible, we have illustrated it with many photographs that have been graciously provided by industrial firms; most of the curves, diagrams and characteristics are those of machines that really exist. Different field distribution plots describing electromagnetic behaviours of machines have been obtained from software codes developed in our research laboratory.

We would like to acknowledge all the individuals and organizations who took part in the realization of this book:

– Our colleagues from Electrical Engineering Department of Polytech’Nantes (France), particularly Professors M.F. Benkhoris and M. Machmoum.

– Professor Bernard Multon, from École Normale Supérieure de Cachan (France).

– Professor Guy Olivier from École Polytechnique de Montréal (Canada).

– The following firms: ECA EN, Converteam and STX France (previously, Aker Yards) for generously providing most of the photographs illustrating this book.

The authors wish to pay a particular tribute to their mentor, the late Professor Emeritus Edmond J. Gudefin (1923-1996).

Chapter 1

Main Requirements

1.1. Introduction

The study of rotating electrical machines is a science which is linked with several other topics. In order to make this book easier to read, we are going to summarize the main results and concepts used later on in this introductory chapter:

– sinusoidal systems;

– electromagnetism;

– power electronics.

1.2. Sinusoidal variables

1.2.1. *Single-phase variables*

1.2.1.1. *Timed expressions*

An x variable, a timed-sinusoidal function, can be written as:

$$x = X\sqrt{2}\cos\omega t$$

where X is the rms (root-mean-square) value and ω is the angular velocity.

1.2.1.2. *Vector representation*

The x variable defined above can be considered to be the projection on an axis of a vector of length $X\sqrt{2}$ rotating anticlockwise at an angular velocity ω (Figure 1.1).

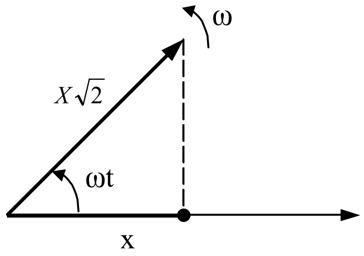

Figure 1.1. *Vector representation of a sinusoidal variable*

1.2.1.3. *Single-phase currents and voltages*

If a sinusoidal single-phase voltage v is applied at a Z impedance terminal, current i in this impedance, at steady state, is also sinusoidal, and can be written:

$$v = V\sqrt{2}\cos\omega t$$

$$i = I\sqrt{2}\cos(\omega t - \varphi)$$

φ being the phase shift between the voltage often chosen as the origin and the current. Conventionally, φ is counted positively when the current is lagging behind the voltage. The instantaneous power supplied to impedance Z is:

$$p = v.i = VI\cos\varphi + VI\cos(2\omega t - \varphi)$$

$$P = VI\cos\varphi \qquad [1.1]$$

is the active power and:

$$P_f = VI\cos(2\omega t - \varphi) \qquad [1.2]$$

is the pulsating power. It must be noted that this variable, which characterizes the fact that the single-phase power supplied to a receiver is time varying, is cancelled with balanced polyphase systems.

Figure 1.2 shows that voltage is changed into current through a similitude of ratio Z and angle φ.

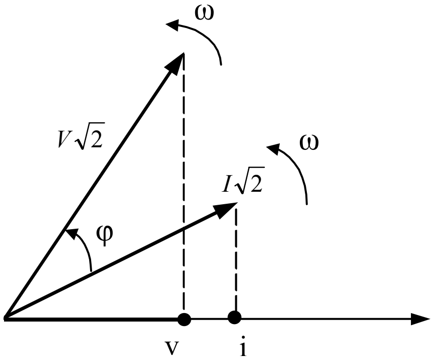

Figure 1.2. *Vector representation of sinusoidal current and voltage*

1.2.1.4. *Complex representation*

Complex numbers are very useful to represent the previous similitude and vector $\vec{V}$ will thus be associated with complex number $\overline{V}$, as well as complex number $\overline{I}$ with vector $\vec{I}$. They can then be written as follows:

$$\overline{V} = Ve^{j\omega t}$$

$$\overline{I} = Ie^{j(\omega t - \varphi)}$$

Complex impedance $\overline{Z}$ is also defined by ratio:

$$\overline{Z} = \frac{\overline{V}}{\overline{I}} = \frac{V}{I} e^{j\varphi} = Z e^{j\varphi}$$

It will be set down:

$$Z \cos \varphi = R$$

$$Z \sin \varphi = X$$

R and X respectively being the resistance and the reactance expressed in Ohms.

$\overline{S}$ is also introduced:

$$\overline{S} = \overline{VI}^* = VIe^{j\varphi} = VI \cos \varphi + jVI \sin \varphi = P + jQ \qquad [1.3]$$

$\overline{S}$ is the apparent power expressed in volt-amperes (VA). Q is the reactive power expressed in volt-amperes reactives (VA_r).

1.2.2. *2-phase voltages and currents*

A 2-phase voltage system is defined by two voltages in quadrature:

$$v_\alpha = V\sqrt{2} \cos(\omega t)$$

$$v_\beta = V\sqrt{2} \sin(\omega t)$$

if it is loaded onto a symmetrical impedance it leads to a balanced 2-phase current system:

$$i_\alpha = I\sqrt{2} \cos(\omega t - \varphi)$$

$$i_\beta = I\sqrt{2} \sin(\omega t - \varphi)$$

There is no pulsating power and the instantaneous power is constant:

$$p = 2VI\cos\varphi = P$$

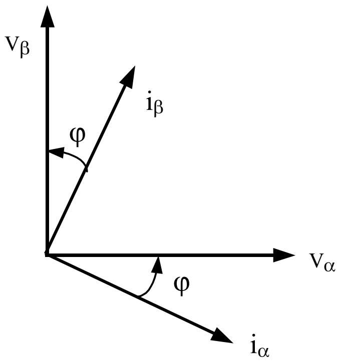

Figure 1.3. *2-phase currents and voltages*

The complex representation can also be introduced:

$$\overline{V}_\alpha = V \qquad \overline{V}_\beta = jV$$

$$\bar{I}_\alpha = Ie^{-j\varphi} \quad I_\beta = jI^{-j\varphi}$$

with the expressions of the active and reactive powers:

$$P = 2VI\cos\varphi$$

$$Q = 2VI\sin\varphi$$

1.2.3. *Balanced 3-phase sinusoidal systems*

1.2.3.1. *Time expressions*

A balanced 3-phase voltage system is composed of three voltages with the same frequency, with the same amplitude and phase shifted by a third of a period with respect to the others. It is thus written as a time expression:

$$v_a = V\sqrt{2}\cos\omega t$$

$$v_b = V\sqrt{2}\cos\left(\omega t - \frac{2\pi}{3}\right)$$

$$v_c = V\sqrt{2}\cos\left(\omega t + \frac{2\pi}{3}\right)$$

If this voltage system is connected to a symmetrical load (with a circulating impedance matrix), it leads to a balanced current system (Figures 1.4 and 1.5):

$$i_a = I\sqrt{2}\cos(\omega t - \varphi)$$

$$i_b = I\sqrt{2}\cos\left(\omega t - \frac{2\pi}{3} - \varphi\right)$$

$$i_c = I\sqrt{2}\cos\left(\omega t + \frac{2\pi}{3} - \varphi\right)$$

with the vector representation shown in Figure 1.4.

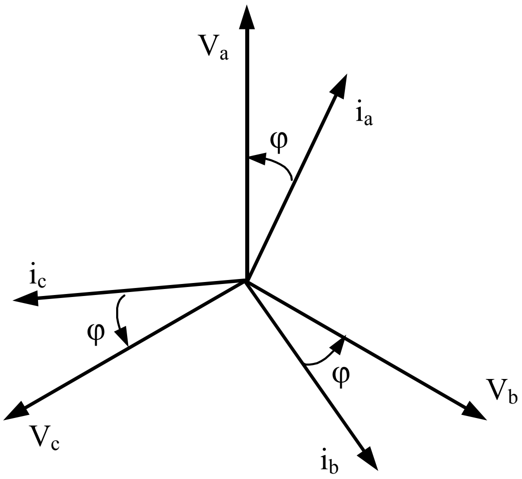

Figure 1.4. *3-phase currents and voltages*

A zero pulsating power is then obtained and the instantaneous power is constant and equal to the active power:

$$p = 3VI\cos\varphi$$

1.2.3.2. *Associated complex notations*

Complex vectors are associated with balanced voltage and current systems:

$$\left[\overline{V}\right] = \overline{V}\begin{bmatrix} 1 \\ a^2 \\ a \end{bmatrix} \quad \text{and} \quad \left[\overline{I}\right] = \overline{I}\begin{bmatrix} 1 \\ a^2 \\ a \end{bmatrix} \qquad [1.4]$$

where 1, a and a^2 are the cube roots of the unit: $a = e^{j2\pi/3}$, $a^2 = e^{j4\pi/3}$.

If the 3-phase voltage system is applied to a load characterized by a circulating impedance matrix $[\overline{Z}]$ (Figure 1.5) such as:

$$\left[\overline{Z}\right] = \begin{bmatrix} \overline{z} & \overline{z}' & \overline{z}'' \\ \overline{z}'' & \overline{z} & \overline{z}' \\ \overline{z}' & \overline{z}'' & \overline{z} \end{bmatrix}$$

the expression $[\overline{V}] = [\overline{Z}][\overline{I}]$ leads to the phase equation:

$$\overline{V} = \overline{Z}\overline{I}$$

in which:

$$\overline{Z} = (\overline{z} + a^2\overline{z}' + a\overline{z}'') \qquad [1.5]$$

is the impedance of the load.

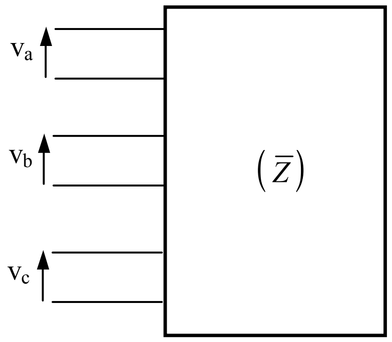

Figure 1.5. *Balanced 3-phase load*

This happens as if each of the three phases was loaded with a $\overline{Z}$ impedance decoupled from the other two (it is in fact a diagonalization of the impedance matrix that has $\overline{Z}$ as an eigenvalue). In those conditions balanced 3-phase systems can be dealt with as independent and decoupled single-phase systems.

1.2.4. *Unbalanced 3-phase sinusoidal systems: Fortescue symmetrical components*

Voltage and current systems may be unbalanced (different amplitudes depending on the phases or phase-shifts different from 2π/3). Expressions [1.4] and [1.5] are no longer valid and 3-phase equations cannot be replaced by single-phase equations. Generally, the analysis of these systems is very difficult.

However there is a system class, fortunately quite commonplace in electrical engineering, for which there is a mathematical simplification. They are the devices described by a circulating impedance matrix $[\overline{Z}]$ and to which dissymmetrical external conditions are imposed. It can be demonstrated that matrix $[\overline{Z}]$ has three eigenvectors:

$$\begin{bmatrix}1\\1\\1\end{bmatrix},\ \begin{bmatrix}1\\a^2\\a\end{bmatrix}\ \text{and}\ \begin{bmatrix}1\\a\\a^2\end{bmatrix}$$

respectively associated with the three eigenvalues:

$$\overline{Z}_0 = \overline{z} + \overline{z}' + \overline{z}'' \qquad [1.6]$$

$$\overline{Z}_d = \overline{z} + a^2\overline{z}' + a\overline{z}'' \qquad [1.7]$$

$$\overline{Z}_i = \overline{z} + a\overline{z}' + a^2\overline{z}'' \qquad [1.8]$$

The three above-written impedances are respectively called zero phase-sequence impedance, forward impedance and backward impedance. A transformation matrix can be built from the three eigenvectors:

$$[S] = \begin{bmatrix}1 & 1 & 1\\1 & a^2 & a\\1 & a & a^2\end{bmatrix} \qquad [1.9]$$

called "Fortescue's matrix". Its backward matrix is:

$$[S]^{-1} = \frac{1}{3}\begin{bmatrix}1 & 1 & 1\\1 & a & a^2\\1 & a^2 & a\end{bmatrix} \qquad [1.10]$$

If three variables composing an unbalanced 3-phase system are named G_a, G_b and G_c (voltages, currents, flux, etc.), then the homologous variables G_0, G_d and G_i can be defined by:

$$\begin{bmatrix}\overline{G}_0\\\overline{G}_d\\\overline{G}_i\end{bmatrix} = [S]^{-1}\begin{bmatrix}\overline{G}_a\\\overline{G}_b\\\overline{G}_c\end{bmatrix} \qquad [1.11]$$

with, of course, the opposite transition expression:

$$\begin{bmatrix} \overline{G}_a \\ \overline{G}_b \\ \overline{G}_c \end{bmatrix} = [S] \begin{bmatrix} \overline{G}_0 \\ \overline{G}_d \\ \overline{G}_i \end{bmatrix} \qquad [1.12]$$

This last expression shows that if only the forward (positive phase-sequence) part exists, the above-mentioned balanced 3-phase system (Figure 1.4) will be found:

$$\begin{bmatrix} \overline{G}_a \\ \overline{G}_b \\ \overline{G}_c \end{bmatrix} = \overline{G}_d \begin{bmatrix} 1 \\ a^2 \\ a \end{bmatrix}$$

only if the backward (negative phase-sequence) part is not zero:

$$\begin{bmatrix} \overline{G}_a \\ \overline{G}_b \\ \overline{G}_c \end{bmatrix} = \overline{G}_i \begin{bmatrix} 1 \\ a \\ a^2 \end{bmatrix}$$

A balanced 3-phase system is obtained, also known as "backward (negative phase-sequence)", for which components b and c exchange their roles.

Finally, if only $\overline{G}_0$ is different from zero:

$$\begin{bmatrix} \overline{G}_a \\ \overline{G}_b \\ \overline{G}_c \end{bmatrix} = \overline{G}_0 \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

This is an expression defining a zero phase-sequence 3-phase system in which the three components are identical.

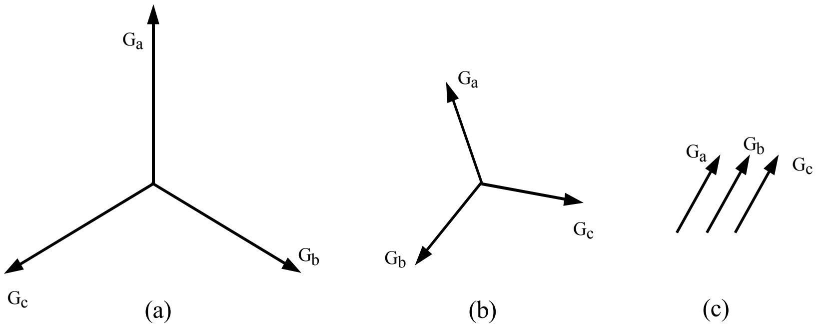

Figure 1.6. *Systems: a) forward (positive phase-sequence); b) backward (negative phase-sequence); c) zero phase-sequence*

This approach therefore consists of replacing an unbalanced 3-phase system with the superposition of three different balanced 3-phase systems of different natures: forward, backward and zero phase-sequence, which can be studied separately and easily.

Matrix equation:

$$[\overline{V}] = [\overline{Z}][\bar{I}]$$

in which $[\overline{V}]$ and $[\bar{I}]$ represent voltage and current unbalanced systems, can be divided into:

$$\overline{V}_0 = \overline{Z}_0 \bar{I}_0$$

$$\overline{V}_d = \overline{Z}_d \bar{I}_d$$

$$\overline{V}_i = \overline{Z}_i \bar{I}_i$$

$\overline{Z}_0, \overline{Z}_d$ and $\overline{Z}_i$ are respectively the impedances of the device in zero phase-sequence, forward and backward modes.

This method, called “Fortescue’s symmetric components”, is very convenient for studying and calculating unbalanced sinusoidal 3-phase systems. It is also noticeable that it can be used to study non-sinusoidal balanced 3-phase systems. Indeed it can be demonstrated that 3k rank harmonics create zero phase-sequence systems, that 3k + 1 rank harmonics create forward systems and that 3k – 1 rank harmonics create backward systems.

1.3. Electromagnetism

1.3.1. *Primary laws*

1.3.1.1. *Maxwell's equations*

Considering the industrial frequencies used in power systems, displacement currents $\frac{\partial \vec{D}}{\partial t}$ are neglected, and Maxwell's equations can be written as follows:

$$r\vec{o}t\,\vec{E} = -\frac{\partial \vec{B}}{\partial t} \qquad [1.13]$$

$$r\vec{o}t\,\vec{H} = \vec{J} \qquad [1.14]$$

$$div\,\vec{D} = \rho \qquad [1.15]$$

$$div\,\vec{B} = 0 \qquad [1.16]$$

E [V/m] and H [A/m] are respectively electric and magnetic fields. D [C/m^2] and B [T] are the electric flux density and the magnetic flux density. J [A/m^2] is the current density, and ρ [C/m^3] the volume charge density. Equation [1.13] illustrates the coupling between E and B, whereas equation [1.14] leads to $div\,\vec{J} = 0$ (i.e. nodal rule). Equations [1.13] to [1.16] are valid in any fixed or mobile axis systems provided the different variables are measured and their derived values are calculated in the same coordinate system. Considering the speeds encountered in power systems, H, B and J are unchanged in a reference frame moving at speed $\vec{v}$. Only the electric field is modified as follows:

$$\vec{E}' = \vec{E} + \vec{v} \wedge \vec{B} \quad [1.17]$$

1.3.1.2. *Ampere's theorem*

Equation [1.14] leads to:

$$\oint_{c} \vec{\mathrm{H}}\, \mathrm{d}\vec{\mathrm{l}} = \sum_{j} \mathrm{i}_{j} \quad [1.18]$$

The magnetic field circulation on a closed circuit (C) is equal to the algebraic sum of the embraced currents.

1.3.1.3. *Faraday's law*

Let's consider circuit (C) in Figure 1.7, first assumed to be fixed. Equation [1.13] leads to:

$$e = \oint_{C} \vec{\mathrm{E}}\, \mathrm{d}\vec{\mathrm{l}} = -\frac{\partial\phi}{\partial t} = -\frac{d\phi}{dt} \quad [1.19]$$

ϕ is the magnetic flux through circuit (C)'s surface S and e is the induced electromotive force (emf) on (C)'s terminals. This is a transformation emf.

If (C) is moving at speed $\vec{v}$, equation [1.17] gives:

$$e == -\frac{d\phi}{dt} = \oint_{C} \vec{\mathrm{E}}\, \mathrm{d}\vec{\mathrm{l}} + \oint_{C} (\vec{v} \Lambda \vec{B})\, d\vec{l} = e_T + e_V \quad [1.20]$$

The induced emf at (C)'s terminals is the sum of the transformation emf e_T and of emf e_V, also called speed emf or emf due to the cut-flux.

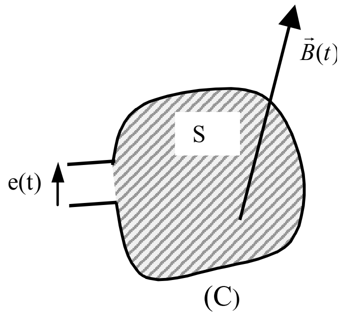

Figure 1.7. *Production of an emf at the circuit's terminals*

1.3.2. *Materials and magnetic circuits*

When a material is subjected to a magnetic field H, each dV element gains a magnetic moment able to oppose or add itself to H. Those magnetic moments can be considerable for ferromagnetic materials. Magnetic flux density within the material is written:

$$\vec{B} = \mu_0 \vec{H} + \mu_0 \vec{M}$$

$\mu_0 \vec{H}$ is the flux density which would have been created into free space, and $\vec{M}$ [A/m] is the magnetization. This is noted:

$$\vec{M} = \chi \vec{\mathrm{H}}$$

The magnetic susceptibility χ usually varies in a very complex way with the field and leads to a B(H) expression presenting a hysteresis (Figure 1.8). Figure 1.8 shows the remanence flux density Br, the coercive field Hc and the initial magnetization characteristics.

According to the hysteresis cycle, "soft" materials can be distinguished from "hard" materials. Soft materials (electrical steel, solid steel, etc.) are characterized by a narrow cycle. Br and Hc are weak. Hc is around 50 to

70 A/m, whereas Br is below 0.1 T. Hard materials (permanent magnets) have a wide cycle. The coercive field Hc is held between 200 and 1,000 kA/m while Br is held between 0.3 T and 1.2 T.

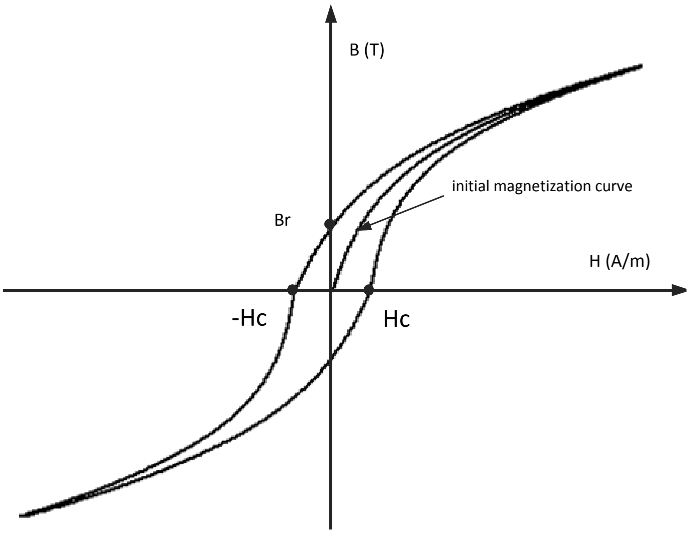

Figure 1.8. *Hysteresis cycle*

1.3.2.1. *Soft ferromagnetic materials*

As the hysteresis cycle is narrow, only the initial magnetization curve is taken into consideration. If the material is characterized by a constant χ:

$$\vec{B} = \mu_0 (1+\chi)\vec{H} = \mu_0 \mu_R \vec{H} = \mu \vec{H}$$

$\mu_R = 1 + \chi$ is the relative permeability. μ is the material's permeability [H/m]. The ferromagnetic materials are characterized by $\chi \approx \mu_R >> 1$. The magnetic materials have a susceptibility close to zero. This is negative ($\chi \approx -10^{-5}$) for the diamagnetic materials and positive ($\chi = 10^{-3}$) for the paramagnetic materials.

1.3.2.1.1. Saturation

The magnetic permeability of ferromagnetic materials depends on the applied field:

$$\vec{B} = \mu(B)\vec{H}$$

Figure 1.9 shows the initial magnetization curve as well as the relative permeability in terms of the magnetic field of a steel frequently used in electrical machines.

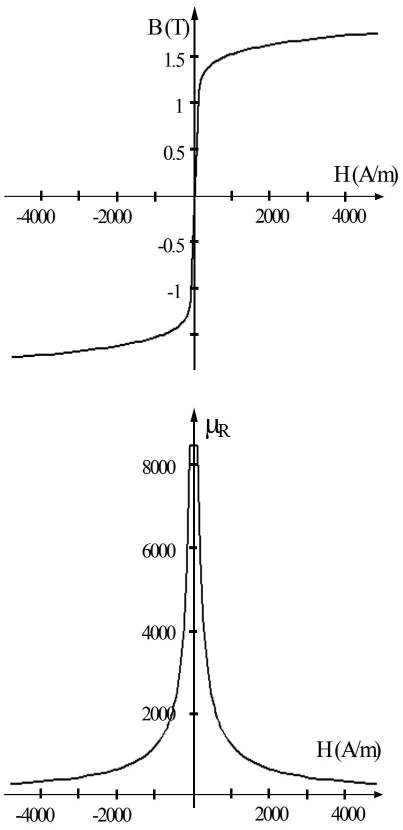

Figure 1.9. *Initial magnetization curve and variation of the relative permeability of FeV 400-50 HA steel in terms of the field*