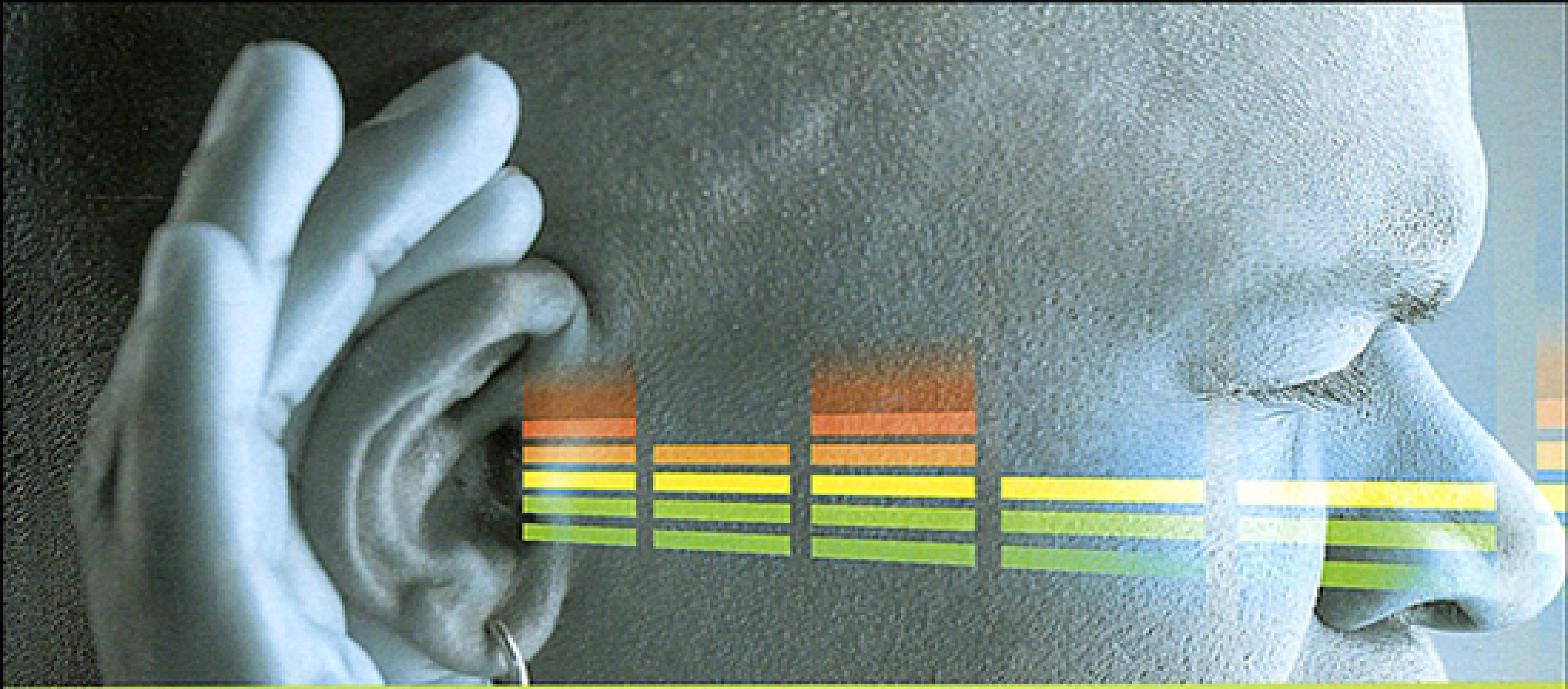

Music and Acoustics

From Instrument to Computer

Philippe Guillaume

ISTE

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From Instrument to Computer

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Series Editor
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To Nicole, Cécile and Olivier

Foreword

How does a tuner achieve such a precise tuning of a piano or an organ using nothing but his ears? Why does the clarinet, though equal in length to the C flute, play one octave lower? What difference is there between the Pythagorean scale and the tempered scale? How can a series of notes seem to rise indefinitely even though it always repeats the same notes? What are the possibilities offered by digital sound? What are its limitations? How can a compression technology such as MP3 achieve a tenfold reduction of a sound file's size without significantly altering it? What is the very simple principle underlying audio synthesis in Yamaha's famous keyboard, the DX7? These are a few examples of the questions we will try to answer.

The goal of this book is to use these questions to give the reader an overview of the nature of musical sound, from its production by traditional musical instruments to sounds obtained by audio synthesis, without trying to be exhaustive however: this book is not meant as a catalogue, but instead, I hope, as a first step that will enable the reader to move on to more specific areas in this field. Musical sound is addressed from a scientific standpoint, and the succession of causes that lead to a specific type of sound are, as much as possible, described in a simplified but precise manner. The fact, for example, that a particular sound is composed of harmonics (strings, pipes, etc.) or of partials (bells, timpani, etc.) finds its causes in the physical laws that govern the behavior of materials, laws that induce mathematical equations, the nature of which leads to a certain characteristic of the produced sound.

This book is intended for any reader interested in sound and music, and with a basic scientific background: students, teachers, researchers, people who work in a scientific or technical field. It describes and relies on concepts of acoustics, mathematics, psychoacoustics, computer science and signal processing, but only to the extent that this is useful in describing the subject. In order to broaden its reach, it was written in such a way that the reader may understand sound phenomena with simple analytical tools and the smallest possible amount of required knowledge. Those who teach this material will find diverse and motivating study problems, and students will find ideas for different kinds of 'projects' they may encounter in their undergraduate and graduate studies. In the end, my greatest wish would be to succeed in sharing with the reader the pleasure I find in understanding the basic mechanisms underlying the manifestation and the perception of the sound and music phenomenon.

After an introduction to acoustics, a bit of music theory, and a study of sounds and their representation in Chapter 1, we will discuss vibrational modes and the timbre of a few typical instruments in Chapter 2, and in Chapter 3, we will relate this with the question of scales and tuning systems. After wandering off into psychoacoustics in Chapter 4, and using the opportunity to discover a beautiful acoustic illusion, we will discuss several aspects of digital sound in Chapters 5 and 6: sampling, compression technology based on the properties of hearing (such as the widely known MP3 format), sound effects (vibrato, reverberation, the Leslie effect) and synthesized sounds, such as for example those produced using the Chowning technique, made popular by DX7 synthesizers.

For further development, each chapter ends with the following:

–study problems, to explore certain themes, or to study them further in depth. For the reader's information, the difficulty and the amount of work required are indicated with stars: (*) means easy, (**) is average and (***) is difficult;

–practical applications meant to be carried out on a computer, where the reader will create different kinds of sounds and play them on a crude synthesizer, experimenting on the phenomena described in the book, as well as put his or her hearing to the test, and practice his or her scales! Practical instructions relevant to these applications are given at the end of the first chapter.

Website. A website is available to illustrate the book. It contains many examples of sounds, as well as the programs used to generate them. It also contains the programs and sound files necessary to perform the practical applications, along with the answers. The address of the website is:

www-gmm.insa-toulouse.fr/~guillaum/AM/

Throughout the book, it will be referred to simply as the AM website.

Reading advice. The chapters were written in a particular, logical order, and the concept and methods developed in a given chapter are assumed to be understood in the chapters that follow. For example, the approach used to go from the wave equation to the Helmholtz equation, which is detailed in Chapter 1, will not be explained again when studying the vibrations of sonorous bodies in Chapter 2. However, you can also browse through it in any other order, referring if necessary to the previous chapters, and using the cross-references and the index to easily find where a given concept was discussed. Finally, because some phenomena are easier heard than explained, listening to the website's audio examples should shed light on any areas that may still be unclear!

Philippe GUILLAUME

Chapter 1

Sounds

Sound and air are closely related: it is common knowledge that the Moonians (the inhabitants of the Moon) have no ears! This means we will begin our study of sound with the physics of its traveling medium: air. Sounds that propagate through our atmosphere consist of a variation of the air's pressure $p(x, y, z, t)$ according to position in space x, y and z and to the time t. It is these variations in pressure that our ears can perceive. In this chapter, we will first study how these sounds propagate as waves. We will then describe a few different types of sounds and various ways of representing them. Finally, we will explain the concept of filtering, which allows certain frequencies to be singled-out.

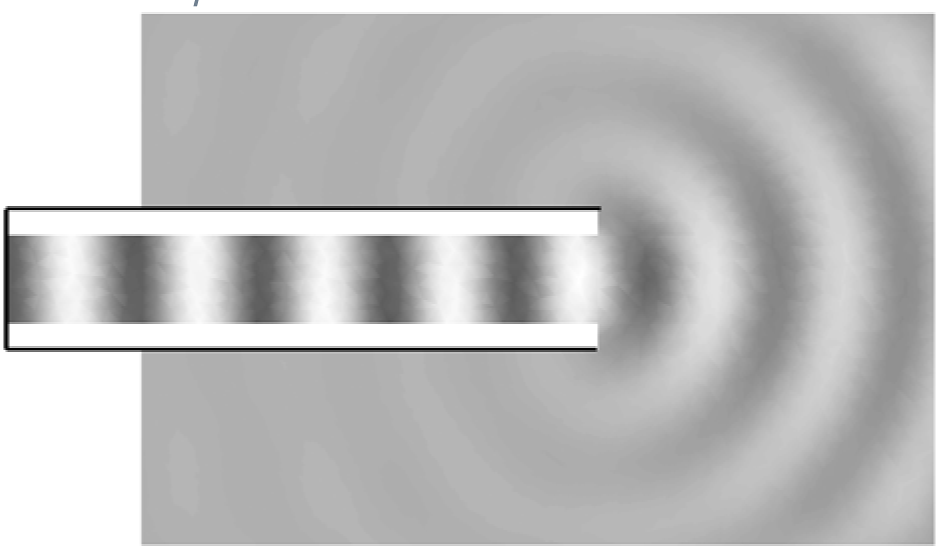
1.1. Sound propagation

The propagation of a sound wave can occur in any direction, and depends on the obstacles in its path. We will essentially be focusing on *plane waves*, that is to say waves that only depend on one direction of space. We will assume that this direction is the x-axis, and therefore that the pressure $p(x, y, z, t)$ is independent of y and z. Hence it can simply be denoted by $p(x, t)$. This type of function represents a plane wave propagating through space, but also a sound wave inside a tube (see Figure 1.1), such as for example the one propagating through an organ pipe.

1.1.1. *A look at the physical models*

The propagation of sound through air is governed by the wave equation (see page 21), an equation we will come across several times since it also determines the movement of sound waves in the vibrating parts (strings, membranes, tubes...) of many instruments. In the following paragraphs, we will see that, in the case of air, this equation is inferred from three fundamental equations of continuum mechanics.

Figure 1.1. *Pressure waves in a tube open at its right end, with pressure imposed at the other end*



Along with the pressure $p(x, t)$, we rely on two other variables to describe the state of air: its density $\rho(x, t)$, and the *average* speed $v(x, t)$ of the air molecules set in motion by the sound wave, which is not to be confused with the norm of the *individual* speed of each molecule due to thermal agitation, the magnitude of which is close to that of the speed of sound, denoted by c. In the case of the plane wave that we are studying, the air moves in a direction parallel to the Ox-axis, and both the speed υ, and the pressure are independent of y and z. In the absence of an atmospheric perturbation, υ varies around the average value 0, and p and ρ vary around their average values ρ_0 and ρ_0 (see section 1.1.2), that is to say, their values in the equilibrium state: silence.

1.1.1.1. *Mass conservation*

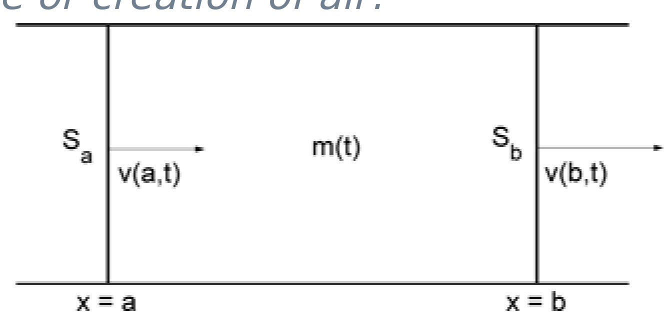
In a *fixed* section of space, bounded by a cylinder with its axis parallel to the Ox axis and the two surfaces S_a and S_b, with respective x-coordinates a and b and areas S (see Figure 1.2), the variation of the air mass $m(t)$ is due to the amount of air going through the two surfaces. Nothing goes through the other interfaces, because the speed is parallel to the Ox axis. The air mass located inside the section is

$$m(t) = S\int_a^b \rho(x,t)\,dx,$$

and the variation of the air mass per unit of time is the derivative of $m(t)$, denoted by $m'(t)$. The incoming *flux* through S_a, that is to say, the amount of air entering the section per unit of time, is equal to $S\rho(a, t)v(a, t)$. As for the incoming flux through S_b, it is equal to $-S\rho(b, t)v(b, t)$, the change of sign being due to the fact that we are calculating the balance of what is *entering* the section (and not of what is going from left to right). The total flux is therefore

$$\Phi(t) = S[\rho(a,t)v(a,t) - \rho(b,t)v(b,t)].$$

Figure 1.2. *Mass balance in the air section: there is no disappearance or creation of air!*



The fact that the total flux $\Phi(t)$ is the derivative of the mass $m(t)$,

$$\Phi(t) = m'(t),$$

can be expressed, if ∂t denotes the partial derivative with respect to t, by

$$S[\rho(a,t)v(a,t) - \rho(b,t)v(b,t)] = S\int_a^b \partial_t \rho(x,t)\,dx.$$

If we divide by $b - a$ and if $b - a$ tends to 0 (calculation of the derivative with respect to the first argument), then after dividing both sides of the equation by x (who was on parole, confined between a and b):

$$(1.1)\quad -\partial_x(\rho(x,t)v(x,t)) = \partial_t \rho(x,t).$$

The linear acoustics hypothesis consists of assuming that the variations with respect to the equilibrium state are small, hence the use of the parameter ε, assumed to be 'small':

$$v(x,t) = \varepsilon v_1(x,t), \quad \rho(x,t) = \rho_0 + \varepsilon \rho_1(x,t).$$

If we substitute these two expressions in (1.1), and if we neglect ε^2, we get the conservation of mass equation, also called *continuity equation*:

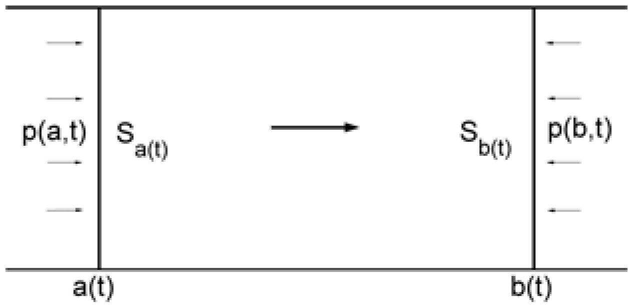
$$(1.2)\quad \partial_t \rho_1(x,t) + \rho_0 \partial_x v_1(x,t) = 0.$$

1.1.1.2. *The Euler equation*

We are now going to observe an amount of air as it moves: the section of air contained between the surfaces $S_a(t)$ and $S_{b(t)}$, with x-coordinates $a = a(t)$ and $b = b(t)$, respectively (see Figure 1.3), which follow the average movement of the air molecules; their derivatives are therefore such that

$$a'(t) = v(a,t) \text{ and } b'(t) = v(b,t).$$

Figure 1.3. *The air section shown above is migrating. Its acceleration results from the pressure forces applied to the two surfaces $S_{a(t)}$ and $S_{b(t)}$*



The external force applied through the surface $S_{a(t)}$ to the air section is equal to $S\ p(a,\ t)$, and the one applied through the surface $S_{b(t)}$ is equal to $-S\ p(b,\ t)$. For the other interfaces, the forces cancel each other out since p is independent of y and z. We now write Newton's second law of motion $F = d(mv)/dt$:

$$S[p(a,t) - p(b,t)] = \frac{d}{dt}\left(S\int_{a(t)}^{b(t)} \rho(x,t)\,v(x,t)\,dx\right)$$

$$= S\left(\rho(b,t)\,v(b,t)\,b'(t) - \rho(a,t)\,v(a,t)\,a'(t) + \int_{a(t)}^{b(t)} \partial_t(\rho(x,t)\,v(x,t))\,dx\right)$$

$$= S\left(\rho(b,t)v^2(b,t) - \rho(a,t)v^2(a,t) + \int_{a(t)}^{b(t)} \partial_t(\rho(x,t)\,v(x,t))\,dx\right).$$

If we divide by $b - a$ and by S, and if $b - a$ tends to 0, this leads us to:

$$-\partial_x p(x,t) = \partial_x(\rho(x,t)v^2(x,t)) + \partial_t(\rho(x,t)v(x,t)).$$

If we still assume that variations with respect to the equilibrium state are small, with

$$p(x,t) = p_0 + \varepsilon p_1(x,t),$$

we get, by neglecting the ε^2 terms and those of higher order, the *Euler equation*:

(1.3) $$-\partial_x p_1(x,t) = \rho_0 \partial_t v_1(x,t).$$

1.1.1.3. *The state equation*

By assuming that there are no heat transfers from one air section to the other or with the outside, or in other words that compression and expansion are *adiabatic* (a hypothesis confirmed by experiment if these effects are fast enough), the *state equation* expresses the fact that pressure variations are proportional to variations in density:

(1.4) $p_1(x,t) = c^2\rho_1(x,t).$

This equation also means that air has an elastic behavior: it acts like a spring. A constant *c* has appeared, we will see later that it represents the speed of sound. If we substitute this equation in (1.2), we find another expression for the state equation:

(1.5) $\partial_t p_1(x,t) + c^2\rho_0\partial_x v_1(x,t) = 0.$

1.1.2. *The wave equation*

We now have at our disposal all the tools necessary to describe the movement of sound waves through air. If we differentiate the state equation (1.5) with respect to time and the Euler equation (1.3) with respect to *x*, we get

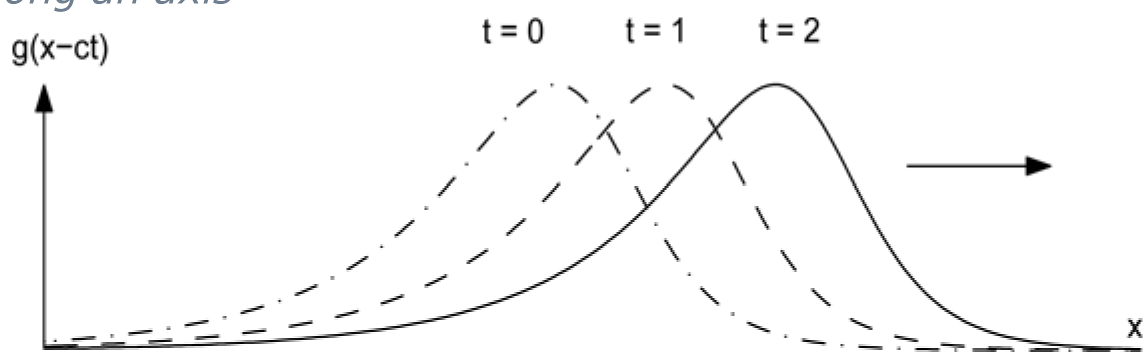
$$\partial_{t^2} p_1(x,t) = -c^2\rho_0\partial_{tx} v_1(x,t),$$

$$\partial_{x^2} p_1(x,t) = -\rho_0\partial_{tx} v_1(x,t).$$

The expression ∂_{t^2} indicates two differentiations with respect to time, ∂_{tx} indicates one differentiation with respect to time and another with respect to *x*, and so on. All we have to do now is compare these two equations to obtain the *wave equation*:

(1.6) $\partial_{t^2} p_1(x,t) = c^2\partial_{x^2} p_1(x,t).$

Figure 1.4. *Three snapshots of a traveling plane wave along an axis*



A mathematical analysis of this equation shows that the general solution is of the form

$$p_1(x,t) = g(x - ct) + h(x + ct).$$

The function *g*(*x* — *ct*) keeps a constant value in the case of a point in motion, the trajectory of which is such that *x* — *ct* = constant (such a trajectory is called a *characteristic trajectory*); thus *g*(*x* — *ct*) represents a *traveling* wave propagating along the *x*-axis at the *speed of sound c* from left to right (Figure 1.4 shows the usual orientation for the axis). Likewise, the function *h*(*x* + *ct*) is constant at the points with *x*-coordinates such that *x*+*ct* = constant, and in that case represents a traveling wave propagating at the speed *c* from right to left. For air at a temperature *T* expressed in Kelvin (with 32°F = 0°C = 273 K), the approximate values for the speed of sound, the density and the atmospheric pressure (in *pascals* and in *bars*) are

$$c = 20\sqrt{T}, \quad \rho_0 = \frac{353}{T}, \quad p_0 = 1.013\,10^5 \text{ Pa} = 1.013 \text{ bar at } 0°\text{C},$$

$$c = 330 \text{ m/s at } 0°\text{C}, c = 340 \text{ m/s at } 16°\text{C}.$$

For example, the functions

$$u_+(x,t) = \sin(kx - 2\pi ft),$$

$$u_-(x,t) = \sin(kx + 2\pi ft),$$

with $k = 2\pi f/c$, are solutions to the wave equation. They are periodic with respect to variables of time and space. The space period

$$\lambda = \frac{2\pi}{k} = \frac{c}{f}$$

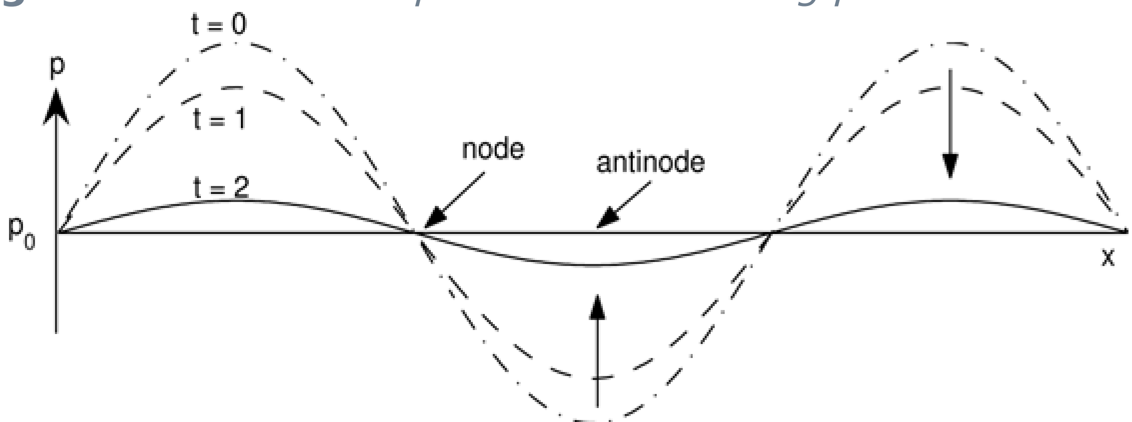
is called the *wavelength*. It is one of the most elementary forms of musical sound, with a pitch, or a *frequency f*, measured in *hertz* (1 Hz = 1 s^{-1}), a unit named after physicist H. R. Hertz, and with a *timbre* (the sound's 'color') similar to that of a recorder (a type of flute).

These two functions u_+ and u_- propagate in opposite directions. Adding the two leads to an interesting wave, also a solution to the wave equation:

$$p_1(x,t) = \sin(kx - 2\pi ft) + \sin(kx + 2\pi ft)$$
$$= 2\sin(kx)\cos(2\pi ft).$$

As you can see, for all points $x = n\pi/k$, $n \in \mathbb{Z}$ (the set of integers), for which $\sin(kx) = 0$, the pressure $p = p_0 + \varepsilon p_1$ is constant and equal to p_0: these points are called vibration *nodes*, whereas for points $x = (n + 1/2)\pi/k$, $n \in \mathbb{Z}$, the pressure $p(x, t) = p_0 \pm 2\varepsilon \cos(2\pi ft)$ undergoes its maximum amplitude variations: these points are called *antinodes*. Such waves are referred to as standing waves (see Figure 1.5).

Figure 1.5. *Three snapshots of a standing plane wave*



1.1.3. *The Helmholtz equation*

In physics, a wave containing only one frequency, *i.e*. of the form

$$p_1(x,t) = \varphi(x)\exp(2i\pi ft)$$

where φ can also be a complex function[1] and where $f \in \mathbb{R}$ (set of real numbers), is said to be harmonic. The real and imaginary parts of such a wave are also harmonic. Functions of the form

$$p_1(x,t) = \varphi(x)\psi(t) \quad (1.7)$$

are said to be *separated variable* functions. Additionally, if φ is real, the wave is referred to as a *standing* wave: except for a real multiplicative factor $\varphi(x)$, all points simultaneously undergo the same variation in pressure $\psi(t)$.

If we substitute equation (1.7) in (1.6), we get, after dividing by $\varphi(x)\psi(t)$,

$$\frac{\psi''(t)}{\psi(t)} = c^2 \frac{\varphi''(x)}{\varphi(x)}.$$

This expression cannot vary, since the term on the left depends only on time, and the one on the right depends only on x. Hence it is a constant, which will be denoted by $-(2\pi f)^2$, where f is an arbitrary real number[2]. Thus, on the one hand, we get

$$\psi''(t) + (2\pi f)^2 \psi(t) = 0,$$

the general solution of which is

$$\psi(t) = A\exp(2i\pi ft) + B\exp(-2i\pi ft).$$

If $B = 0$ or $a = 0$, the wave is harmonic with frequency $\pm f$. On the other hand, if we define $k = 2\pi f/c$, called the *wavenumber*, we obtain the homogenous *Helmholtz equation*:

(1.8) $$\varphi''(x) + k^2\varphi(x) = 0,$$

the general solution of which is

$$\varphi(x) = \alpha\exp(ikx) + \beta\exp(-ikx).$$

Thus, the harmonic pressure waves with frequency f are of the form

$$p_1(x,t) = [\alpha\exp(ikx) + \beta\exp(-ikx)]\exp(2i\pi ft),$$

where the constants $a, \beta \in \mathbb{C}$ (set of complex numbers) are determined by the conditions imposed at the interfaces with objects. As for standing harmonic waves with frequency f, they are of the form

$$p_1(x,t) = \alpha\sin(k(x - x_0))\exp(2i\pi ft),$$

where x_0 is one of the vibration nodes.

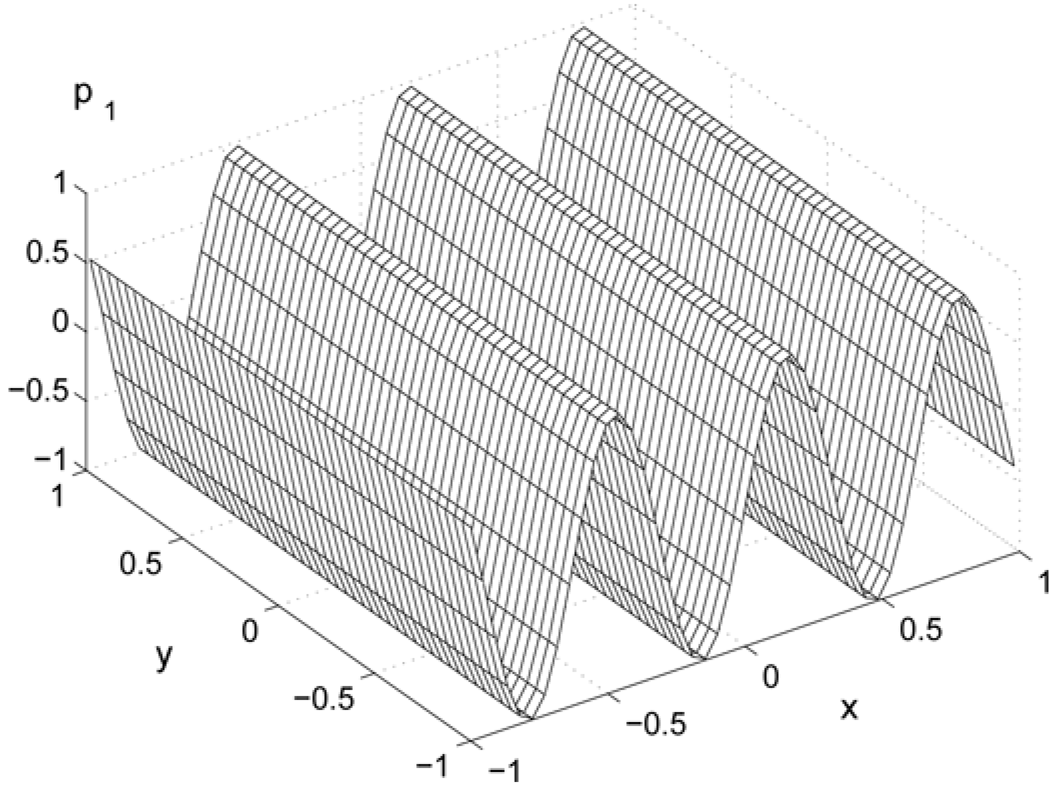
If we follow the same process (*i.e.* start with three fundamental equations), we come to the conclusion that, in the general case, when the waves are not necessarily plane waves, the pressure is a solution to the three dimensional wave equation

(1.9) $\partial_{t^2} p_1(x,y,z,t) = c^2 \Delta p_1(x,y,z,t)$

where $\Delta = \partial_x^2 + \partial_y^2 + \partial_z^2$ is called the *Laplacian*, and the Helmholtz equation becomes

$$\Delta\varphi(x,y,z) + k^2\varphi(x,y,z) = 0.$$

Figure 1.6. *A harmonic plane wave. It propagates along the Ox-axis (2D section) without any damping*



For example, *spherical harmonic waves*, produced by a punctual source assumed to be placed at the origin, are of the type (with $r = \sqrt{x^2 + y^2 + z^2}$):

$$p_1(x,y,z,t) = \alpha \frac{\exp(ikr - 2i\pi ft)}{r}.$$

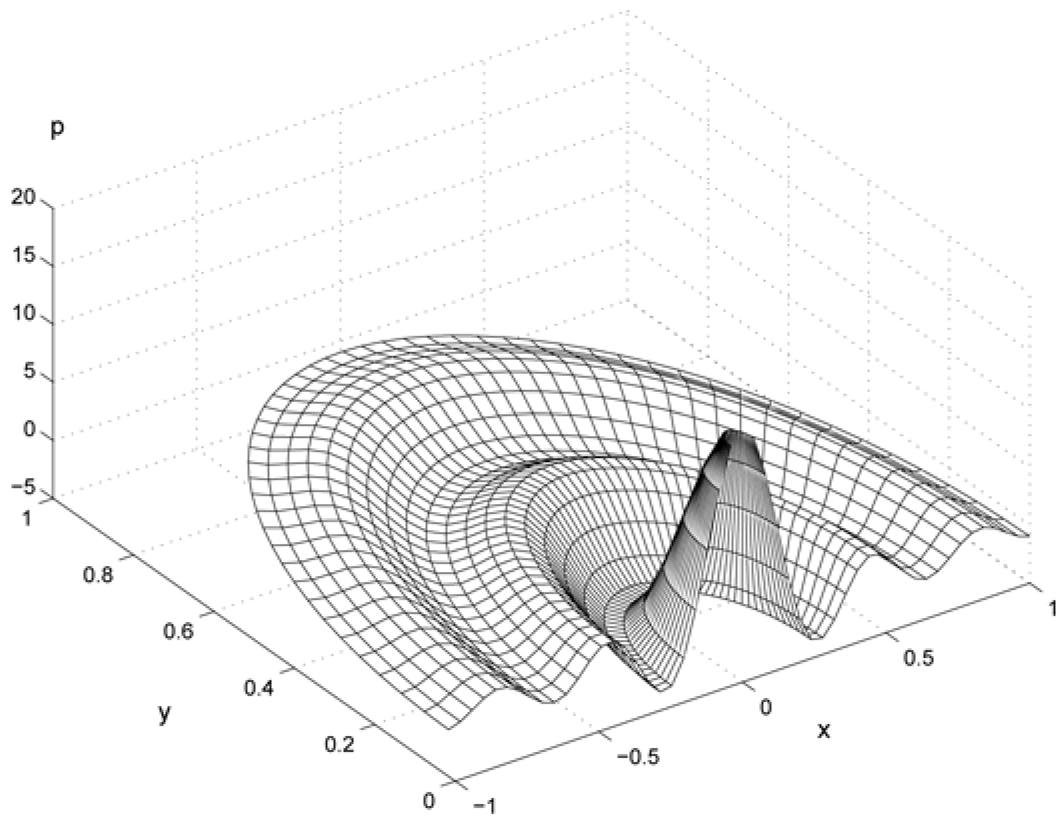
These waves are called spherical because, for a set value of t, given a sphere with its center at the origin, the pressure is the same at every point on the sphere. Note that these are not standing waves.

1.1.4. *Sound intensity*

Earlier in this chapter, we denoted the pressure (in the case of a plane wave) by $p(x, t) = p_0 + \varepsilon p_1(x, t)$ where p_0 is the pressure in the equilibrium state, or average pressure. The difference $p(x, t) — p_0$ is called the *acoustic pressure* p_a:

$$p_a(x,t) = p(x,t) - p_0.$$

Figure 1.7. *A spherical harmonic wave (2D section). It decreases as 1/r*



In order to set air in motion, a certain amount of energy had to be provided. The propagation of the air deformation corresponds to the propagation of the initial energy. The term *sound intensity* (or acoustic intensity) refers to the mean power — with respect to time — of the acoustic wave per unit of surface. It is measured in watts/m^2, depends on the point where it is measured, *x*, and is obtained from the formula

$$I(x) = \frac{1}{T}\int_0^T p_a(x,t)v(x,t)\,dt$$

where the time scale *T* depends on the context. This integral can be equal to zero if for example p_a and *v* are in quadrature (a difference in phase equal to $\pi/2$). In the case of a traveling plane wave $p_a(x, t) = g(x - ct)$, the calculations based on the Euler equation (1.3) and the state equation (1.5) lead to $v(x, t) = p_a(x, t)/c\rho_0$, hence

(1.10) $$I(x) = \frac{1}{Tc\rho_0}\int_0^T p_a^2(x,t)\,dt.$$

In the case of a harmonic wave, for example $p_a(x, t) = p_\alpha \sin(kx - 2\pi ft)$, we get, for $T = 1/f$,

$$I(x) = \frac{p_\alpha^2}{2c\rho_0} \simeq \frac{p_{\text{eff}}^2}{415},$$

with $p_{\text{eff}} = p_\alpha/\sqrt{2}$, a formula often used when calculating the intensity. In the case of a spherical wave $p_a(x, y, z, t) = p_a \sin(kr - 2\pi ft)/r$, the calculation for a high enough value of *r* leads to

$$I(x,y,z) \simeq \frac{p_\alpha^2}{2c\rho_0} \times \frac{1}{r^2}.$$

Therefore the intensity of a sound originating from a punctual source (in the absence of damping) is inversely proportional to the square of the distance from that source.

The *hearing threshold* is approximately

$$I_0 = 10^{-12}\ \text{W/m}^2,$$

the normal level for a conversation is 1.210^{-5} W/m^2 and the *pain threshold* is about 1 W/m^2. We will see in Chapter 4 that these threshold values depend on certain parameters, particularly frequency. Notice the impressive value for the ear's dynamic range, 10^{12}! Rather than W/m^2, the preferred unit is the bel (named after Alexander Graham Bell, a professor at a school for the hearing impaired, and the inventor of the telephone) or the *decibel*, a dimensionless unit that measures the tenth of the base 10 logarithm of a ratio to a given threshold, the hearing threshold for example. If the sound intensity is denoted by L_I, then this can be expressed as:

$$L_I = 10 \log \frac{|I|}{I_0} \text{ dB}.$$

As a consequence, the hearing threshold is set at 0 dB, the pain threshold is 120 dB, and for a conversation, it is equal to 70 dB. Note that at some rock concerts, the intensity sometimes exceeds 140 dB!

Small question: what happens to a symphonic orchestra when the number of violins is multiplied by 10?

Answer (see section 1.6.3): a 10 dB increase in the sound level. In other words, going from 1 to 10 violins leads to the same volume increase as when going from 10 to 100 violins! This is an example of Fechner's law, named after the German physiologist Gustav Fechner: *a sensation varies proportionally to the logarithm of the stimulus* (see [LEI 80], but also [ZWI 81] for a more moderate point of view, discussed in Chapter 4).

1.2. Music theory interlude

Before we go any further, it may be necessary to brush up on a few elementary concepts of music theory and the relevant vocabulary. A musical note is characterized by three main parameters: its pitch, its duration and its intensity. Here we will be focusing on the pitch. The pitch is directly related to the note's frequency[3]: low frequencies correspond to *low-pitched* sounds, and high frequencies to *high-pitched* sounds. The reference frequency for a musician is the A at 440 Hz, the note made by a *tuning fork*, and also the note used for most dial tones.

1.2.1. *Intervals, octave*

In music theory, the distance between two different notes is called an *interval*. When our ears estimate the interval between two notes, what affects their perception is the *ratio* of their frequencies, and not the *difference* in frequencies. This is another example of Fechner's law, previously mentioned in regards to intensity: our sensation of pitch varies proportionally to the logarithm of the frequency (this law does not apply to extremely high-pitched and low-pitched sounds, as we will see in Chapter 4). For example, the two musical intervals [110 Hz, 220 Hz] and [220 Hz, 440 Hz] are perceived as *equal* because the frequency ratios are equal: 220/110 = 440/220 = 2, whereas in the mathematical sense, the second interval is twice as long as the first: 440–220 = 2 × (220–110). The interval between two notes, in the case where their frequency ratio is equal to two, is called an *octave*.

1.2.2. *Scientific pitch notation*

The sounds produced by two notes one octave apart from each other are very similar (we will see why in section 2.1.2), to the point that they are referred to as the same note. The frequency 880 Hz, for example, one octave above the A of a tuning fork, will also produce an A, but at a higher pitch. In order to tell them apart, we will use the scientific pitch notation: the 440 Hz A will be denoted by A4, the next one at 880 Hz will be denoted by A5, followed by A6 at 1, 760 Hz. Likewise, in the other direction we have A3 at 220 Hz, then A2 at 110 Hz, and so on. The same goes for other notes, number 4 being used for the notes found between the C at 261.6 Hz and the B at 493.9 Hz, all of which are located near the middle of a piano keyboard.