APPLIED STOCHASTIC METHODS SERIES

Stochastic Methods for Pension Funds

Pierre Devolder
Jacques Janssen
Raimondo Manca

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Preface

In recent years quantitative finance has become an extraordinary field of research and interest, from an academic point of view as well as for practical applications.

At the same time, the pension issue is clearly a major economic and financial topic for the coming decades, in the context of the well–known longevity risk. The emergence and development of pension schemes in our modern societies can essentially be explained by two factors:

– The individual approach to life: our modern world tends to substitute for large multi–generational families, for an individual model where each person is assumed to be self-supporting, before and after retirement age. As a consequence, personal pension planning, including social security pensions and other incomes, becomes a necessity for everyone

– The longevity syndrome: longevity has increased extraordinarily in recent decades, and nothing indicates a stopping or a reverse in this phenomenon could occur in the next few years. In the past it was usual to retire at 65 and then have a life expectancy of only a few more years. Nowadays, and in the near future, in many countries, it will be quite common for all of us to hope to survive until after 85!

The future of our pension systems is clearly one of the main economic challenges of the world for the coming decades. In the huge majority of countries, collective pension schemes have been created by states as well as by private companies or professional organizations. These systems can use very different tools, techniques, funding approaches or legal forms.

As described in Gronchi [GRO 99], pensions can be classified as a function of the calculation method of the benefits as:

– defined benefit (DB): the pension is a function of salaries and the number of years of service of the retiring person;

– defined contribution (DC): the pension is a function of the contribution that the retiring person paid during his life.

It can also be classified as a function of the financing technique as:

– pay as you go (PAYG): the pensions are paid by the current workers to the previous generation;

– funding: the pensions are generated by the savings accumulated by the contribution of the workers.

Surprisingly few books are devoted to the application of modern stochastic tools to pension analysis.

Therefore, the aim of this book is to fill this gap and to show how recent stochastic methods can be useful for the risk management of pension funds and the computation of market values. Optimal control methods will be especially developed and applied to fundamental problems such as the optimal asset allocation of the fund or the cost spreading of a pension scheme. In these various problems, financial as well as demographic and economic risks will be addressed and modeled.

The layout of this book is as follows: Chapters 1 and 2 present the fundamental issues of the next decades and classical theory of pension funding.

In Chapter 3, we introduce the minimal basic results concerning control theory, both for deterministic and stochastic formulation in order to prepare the models presented in Chapters 7 and 8

Chapter 4 is devoted to concepts of defined contribution and defined benefit pension plans, while Chapter 5 presents some basic definitions on fair and market values for the evaluation of financial flows and stocks in the future – not only to improve the classical concept of present value, but also to fill up the constraints of new IFRS rules as well as Basel II and III rules for banks and Solvency II for insurance companies

Chapter 6 first develops stochastic models for DC and then for DB, in order to measure the various risks faced by the pension fund and to propose eventual solvency buffers in the philosophy of Basel II or Solvency II. In particular, it illustrates the importance of time in the risk assessment of a pension fund.

Chapters 7 and 8 give the main results of the pension theory as various models for the optimal control of the investment strategy and the contribution process for both defined contribution pension schemes and defined benefit pension schemes are presented

Chapter 7 presents – also for DB pension schemes in a stochastic environment – important asymptotic results of evolution of the fund and the contributions

Chapter 9 presents the construction of algorithms for the management of pension fund liabilities necessary for simulation models. Fundamentally we give two approaches to these models, one that is a semi–deterministic approach that we will call the *direct method* and the other based on the Monte Carlo method.

In short, with these last three chapters, it is possible to carefully study the evolution of a pension fund in the future; they constitute the core of this book.

The presentation, in Chapter 10, of discrete time homogeneous semi–Markov processes (DTHSMP), discrete time non–homogeneous (DTNHSMP) semi–Markov processes and the discrete time semi–Markov reward processes (SMRWP), is followed by Chapter 11, the last chapter of this book, which is devoted to another type of stochastic model of pension funds and manpower management study, called generalized non–homogeneous semi-Markov models.

This model is a general, rigorous and tractable stochastic evolution time model for pension funds, called the discrete time non–homogeneous semi-Markov pension fund model, taking into account economic, financial and demographic evolution factors so that it becomes a real–life model using important factors such as *seniority*, *general age dependence*, *rate of inflation* and *salary lines.* It can be particularly useful for the study of private pension fund evolution.

In conclusion, this book presents realistic stochastic models to carefully study the evolution of pension funds in the future, not only from a theoretical point of view but also from a practical point of view, as we present algorithms for the construction of simulation models.

We also present basic concepts in such a way that this book is relatively self-contained. Therefore, the book can be considered as the first textbook in the field of stochastic methods for pension funds and thus, it will be useful for graduate students in economics and actuarial science as well as for managers of pension funds and especially people involved in Solvency II for insurance companies and in Basel II and III for banks

Chapter 1

Introduction: Pensions in Perspective

1.1. Pension issues

1.1.1. *The challenge*

The future of our pension systems is clearly one of the main economic challenges for the world in the coming decades. In the vast majority of countries, collective pension schemes have been created by states as well as by private companies or professional organizations. These systems can use very different tools, techniques, funding approaches or legal forms.

From a historical point of view, the concept of an organized pension institution is relatively new in human evolution and can be seen essentially as a creation of the 20th Century. Of course for centuries, societies have accepted the idea of solidarity between generations and the fact that at a given age people must stop working and then receive other forms of income to survive. But in the traditional way of life, this solidarity was guaranteed by familial or tribal assistance without need of complicated collective systems. For instance, it was common, in this context, to have families with 3 (or even 4) different generations living together in the same house and sharing a global income. At the same time, the mean longevity was not as important and the number of retirement years was limited in general.

The emergence and development of pension schemes in our modern societies can essentially be explained by two factors:

– the “individual approach” of life: our modern world tends to replace large “multi–generational” families by an individual model where each person is assumed to be self–supporting, before and after retirement age. As a consequence, personal pension planning, including social security pensions and other incomes, becomes a necessity for everyone;

– the “longevity syndrome”: longevity has increased extraordinarily in recent decades and nothing indicates that a stopping or reversing of this phenomenon could occur in the next few years. In the past it was usual to retire at 65 and then have a life expectancy of only a few years. Nowadays, and in the near future, in a lot of countries, it will be quite common for all of us to hope to survive until after 85!

This longevity evolution is a major driving force in motivating the development of various pension schemes under different forms. But at the same time it induces a huge challenge in terms of financial sustainability. Indeed, for the first time in history, we can simultaneously observe two major demographic evolutions: a continuous increase in longevity leading to an expected increase in the proportion of retirees and a decrease of fertility rates leading to a decrease of active workers.

As a consequence, we will live in increasingly ageing societies, with fewer contributors and more beneficiaries.

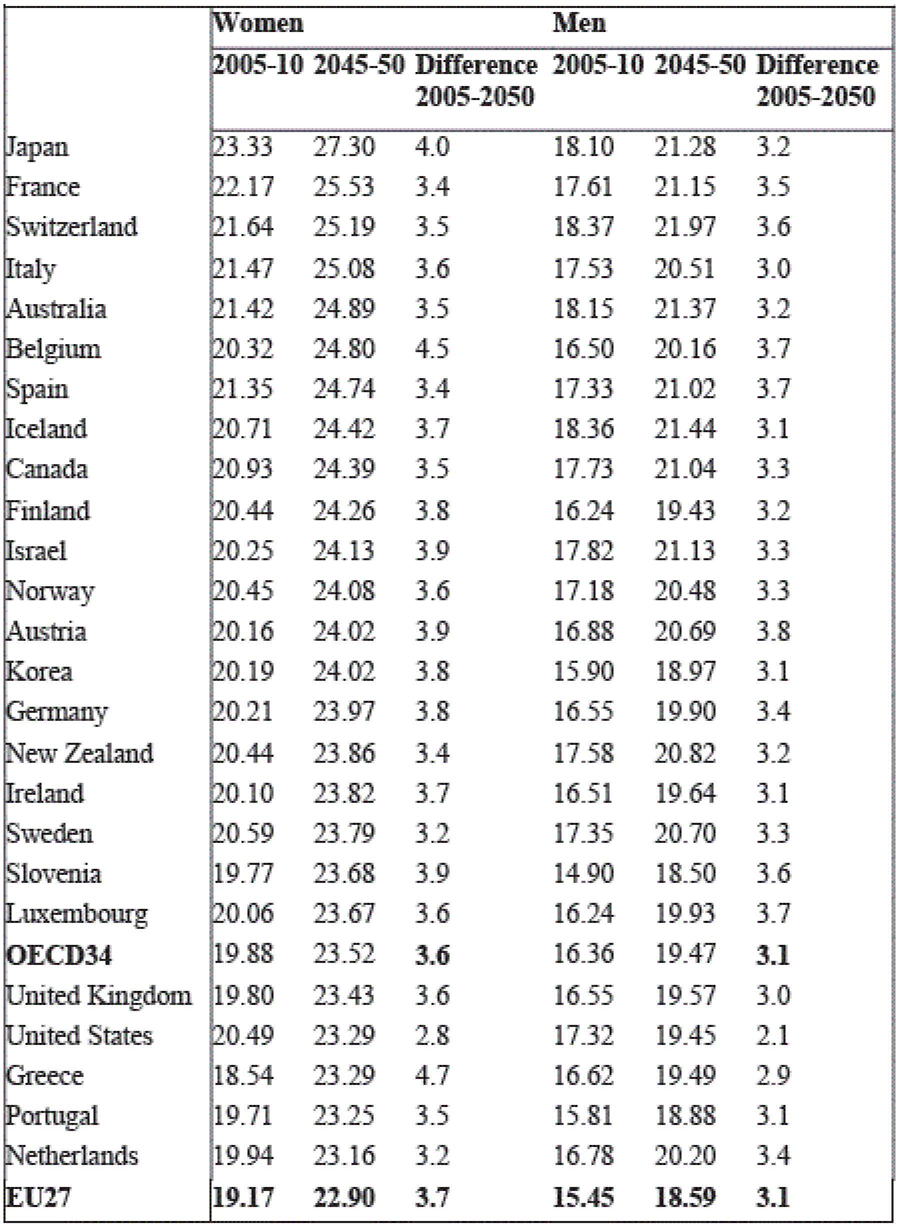
1.1.2. *Some figures*

In order to give a global view of the importance of the pension challenge and its worldwide coverage, we present some international demographical and financial figures [OEC 11].

1.1.2.1. *Longevity at 65*

The following table compares in various countries, for men and women, the life expectancy at 65 in 2005 and that projected for 2050. It shows that everywhere our longevity is expected to continue increasing in the coming years by more than 3 years. For the OECD34, a man aged 65 in 2045 is expected to survive until 84 (88 for a woman).

Table 1.1. *Life expectation at 65 in 2005 and 2045 (source: OECD 2011)*



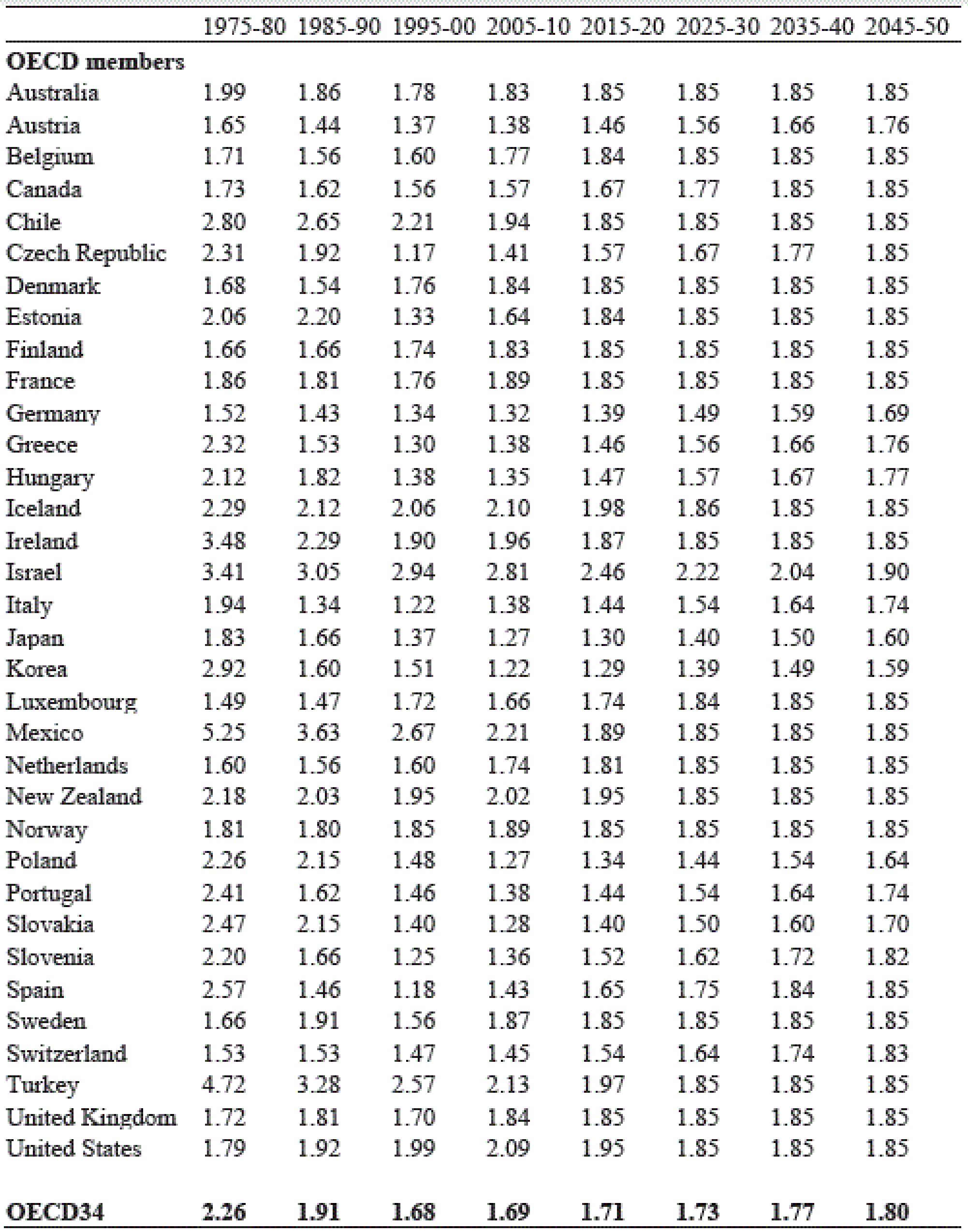
	Women			Men		
	2005-10	2045-50	Difference 2005-2050	2005-10	2045-50	Difference 2005-2050
Japan	23.33	27.30	4.0	18.10	21.28	3.2
France	22.17	25.53	3.4	17.61	21.15	3.5
Switzerland	21.64	25.19	3.5	18.37	21.97	3.6
Italy	21.47	25.08	3.6	17.53	20.51	3.0
Australia	21.42	24.89	3.5	18.15	21.37	3.2
Belgium	20.32	24.80	4.5	16.50	20.16	3.7
Spain	21.35	24.74	3.4	17.33	21.02	3.7
Iceland	20.71	24.42	3.7	18.36	21.44	3.1
Canada	20.93	24.39	3.5	17.73	21.04	3.3
Finland	20.44	24.26	3.8	16.24	19.43	3.2
Israel	20.25	24.13	3.9	17.82	21.13	3.3
Norway	20.45	24.08	3.6	17.18	20.48	3.3
Austria	20.16	24.02	3.9	16.88	20.69	3.8
Korea	20.19	24.02	3.8	15.90	18.97	3.1
Germany	20.21	23.97	3.8	16.55	19.90	3.4
New Zealand	20.44	23.86	3.4	17.58	20.82	3.2
Ireland	20.10	23.82	3.7	16.51	19.64	3.1
Sweden	20.59	23.79	3.2	17.35	20.70	3.3
Slovenia	19.77	23.68	3.9	14.90	18.50	3.6
Luxembourg	20.06	23.67	3.6	16.24	19.93	3.7
OECD34	19.88	23.52	**3.6**	16.36	19.47	**3.1**
United Kingdom	19.80	23.43	3.6	16.55	19.57	3.0
United States	20.49	23.29	2.8	17.32	19.45	2.1
Greece	18.54	23.29	4.7	16.62	19.49	2.9
Portugal	19.71	23.25	3.5	15.81	18.88	3.1
Netherlands	19.94	23.16	3.2	16.78	20.20	3.4
EU27	**19.17**	**22.90**	**3.7**	**15.45**	**18.59**	**3.1**

1.1.2.2. *Fertility rates*

The following table compares, in various countries, the evolution of the fertility rate between 1975 and 2050. It illustrates the persistent low level of this rate (less than 2.1, which is often seen as the necessary rate in order to guarantee the strict renewal of the population). For OECD34, this rate is expected to stay around 1.7.

Total fertility rates. 1975–2050

Table 1.2. *Evolution of the fertility rates 1975–2050 (source: OECD 2011)*

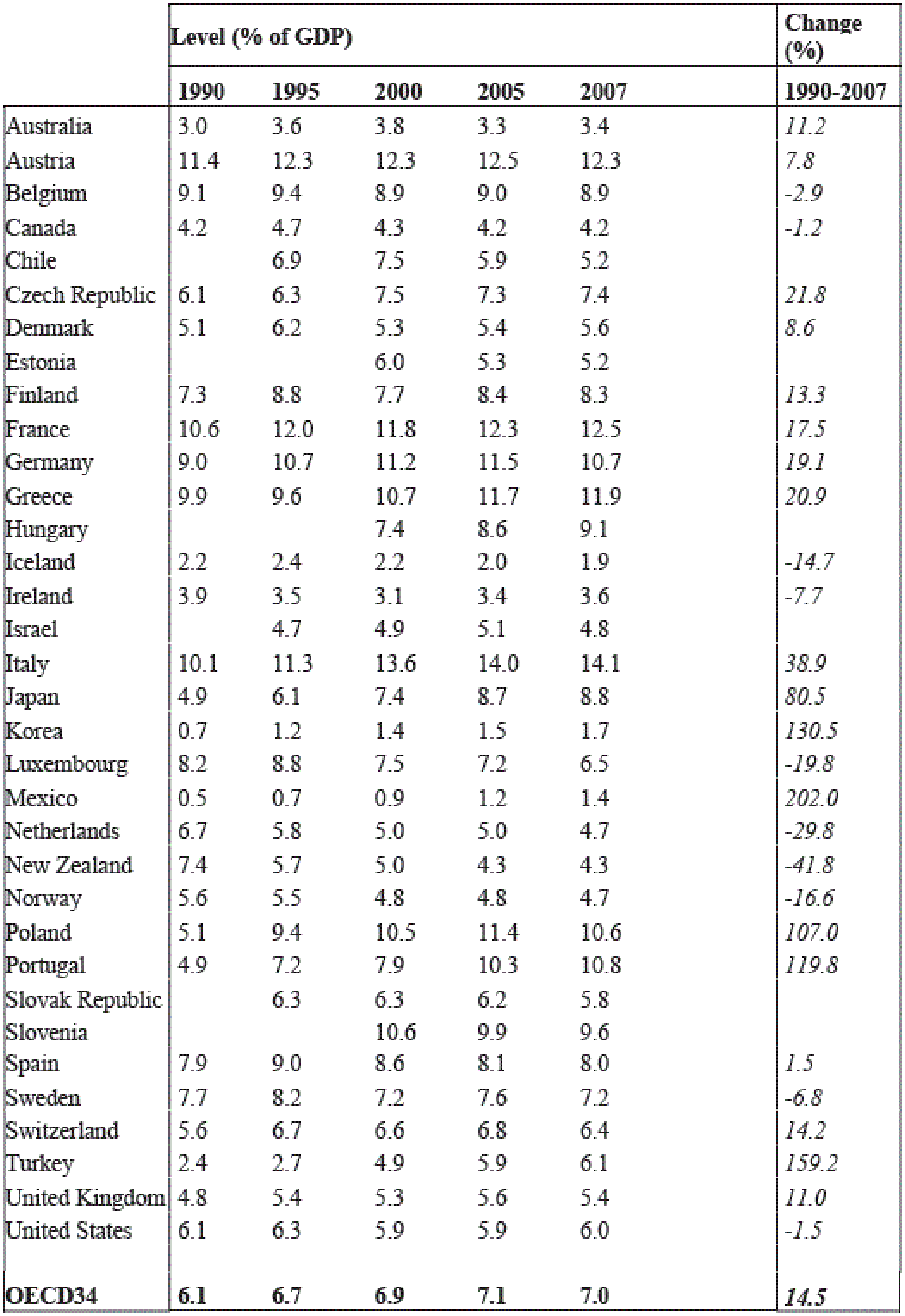


	1975-80	1985-90	1995-00	2005-10	2015-20	2025-30	2035-40	2045-50
OECD members								
Australia	1.99	1.86	1.78	1.83	1.85	1.85	1.85	1.85
Austria	1.65	1.44	1.37	1.38	1.46	1.56	1.66	1.76
Belgium	1.71	1.56	1.60	1.77	1.84	1.85	1.85	1.85
Canada	1.73	1.62	1.56	1.57	1.67	1.77	1.85	1.85
Chile	2.80	2.65	2.21	1.94	1.85	1.85	1.85	1.85
Czech Republic	2.31	1.92	1.17	1.41	1.57	1.67	1.77	1.85
Denmark	1.68	1.54	1.76	1.84	1.85	1.85	1.85	1.85
Estonia	2.06	2.20	1.33	1.64	1.84	1.85	1.85	1.85
Finland	1.66	1.66	1.74	1.83	1.85	1.85	1.85	1.85
France	1.86	1.81	1.76	1.89	1.85	1.85	1.85	1.85
Germany	1.52	1.43	1.34	1.32	1.39	1.49	1.59	1.69
Greece	2.32	1.53	1.30	1.38	1.46	1.56	1.66	1.76
Hungary	2.12	1.82	1.38	1.35	1.47	1.57	1.67	1.77
Iceland	2.29	2.12	2.06	2.10	1.98	1.86	1.85	1.85
Ireland	3.48	2.29	1.90	1.96	1.87	1.85	1.85	1.85
Israel	3.41	3.05	2.94	2.81	2.46	2.22	2.04	1.90
Italy	1.94	1.34	1.22	1.38	1.44	1.54	1.64	1.74
Japan	1.83	1.66	1.37	1.27	1.30	1.40	1.50	1.60
Korea	2.92	1.60	1.51	1.22	1.29	1.39	1.49	1.59
Luxembourg	1.49	1.47	1.72	1.66	1.74	1.84	1.85	1.85
Mexico	5.25	3.63	2.67	2.21	1.89	1.85	1.85	1.85
Netherlands	1.60	1.56	1.60	1.74	1.81	1.85	1.85	1.85
New Zealand	2.18	2.03	1.95	2.02	1.95	1.85	1.85	1.85
Norway	1.81	1.80	1.85	1.89	1.85	1.85	1.85	1.85
Poland	2.26	2.15	1.48	1.27	1.34	1.44	1.54	1.64
Portugal	2.41	1.62	1.46	1.38	1.44	1.54	1.64	1.74
Slovakia	2.47	2.15	1.40	1.28	1.40	1.50	1.60	1.70
Slovenia	2.20	1.66	1.25	1.36	1.52	1.62	1.72	1.82
Spain	2.57	1.46	1.18	1.43	1.65	1.75	1.84	1.85
Sweden	1.66	1.91	1.56	1.87	1.85	1.85	1.85	1.85
Switzerland	1.53	1.53	1.47	1.45	1.54	1.64	1.74	1.83
Turkey	4.72	3.28	2.57	2.13	1.97	1.85	1.85	1.85
United Kingdom	1.72	1.81	1.70	1.84	1.85	1.85	1.85	1.85
United States	1.79	1.92	1.99	2.09	1.95	1.85	1.85	1.85
OECD34	**2.26**	**1.91**	**1.68**	**1.69**	**1.71**	**1.73**	**1.77**	**1.80**

1.1.2.3. *Public pension expenditure*

The following table compares, in various countries, the evolution of the public pension expenditure between 1990 and 2007 (in % of the GDP). Absolute levels and evolutions can be quite different from one country to another, depending on the importance of social security and the eventual financial measures already taken by some states (for instance Sweden or Luxemburg). But from a global point of view, the increasing trend seems to be clear. For OECD34, nowadays public pension expenditures represent 7% of the GDP.

Table 1.3. *Evolution of the public pension expenditure in % of GDP (source: OECD 2011)*



	Level (% of GDP)					**Change (%)**
	1990	**1995**	**2000**	**2005**	**2007**	**1990-2007**
Australia	3.0	3.6	3.8	3.3	3.4	*11.2*
Austria	11.4	12.3	12.3	12.5	12.3	*7.8*
Belgium	9.1	9.4	8.9	9.0	8.9	*-2.9*
Canada	4.2	4.7	4.3	4.2	4.2	*-1.2*
Chile		6.9	7.5	5.9	5.2	
Czech Republic	6.1	6.3	7.5	7.3	7.4	*21.8*
Denmark	5.1	6.2	5.3	5.4	5.6	*8.6*
Estonia			6.0	5.3	5.2	
Finland	7.3	8.8	7.7	8.4	8.3	*13.3*
France	10.6	12.0	11.8	12.3	12.5	*17.5*
Germany	9.0	10.7	11.2	11.5	10.7	*19.1*
Greece	9.9	9.6	10.7	11.7	11.9	*20.9*
Hungary			7.4	8.6	9.1	
Iceland	2.2	2.4	2.2	2.0	1.9	*-14.7*
Ireland	3.9	3.5	3.1	3.4	3.6	*-7.7*
Israel		4.7	4.9	5.1	4.8	
Italy	10.1	11.3	13.6	14.0	14.1	*38.9*
Japan	4.9	6.1	7.4	8.7	8.8	*80.5*
Korea	0.7	1.2	1.4	1.5	1.7	*130.5*
Luxembourg	8.2	8.8	7.5	7.2	6.5	*-19.8*
Mexico	0.5	0.7	0.9	1.2	1.4	*202.0*
Netherlands	6.7	5.8	5.0	5.0	4.7	*-29.8*
New Zealand	7.4	5.7	5.0	4.3	4.3	*-41.8*
Norway	5.6	5.5	4.8	4.8	4.7	*-16.6*
Poland	5.1	9.4	10.5	11.4	10.6	*107.0*
Portugal	4.9	7.2	7.9	10.3	10.8	*119.8*
Slovak Republic		6.3	6.3	6.2	5.8	
Slovenia			10.6	9.9	9.6	
Spain	7.9	9.0	8.6	8.1	8.0	*1.5*
Sweden	7.7	8.2	7.2	7.6	7.2	*-6.8*
Switzerland	5.6	6.7	6.6	6.8	6.4	*14.2*
Turkey	2.4	2.7	4.9	5.9	6.1	*159.2*
United Kingdom	4.8	5.4	5.3	5.6	5.4	*11.0*
United States	6.1	6.3	5.9	5.9	6.0	*-1.5*
OECD34	**6.1**	**6.7**	**6.9**	**7.1**	**7.0**	***14.5***

1.1.2.4. *Assets of pension funds*

The following table gives the pension assets (in % of the GDP and in USD) for 2009, in various countries. These figures illustrate the importance of the pension fund assets invested in the financial markets. For OECD34, these assets represent two–thirds of the GDP.

Table 1.4. *Importance of the assets of the pension funds in % of GDP and in US dollars (source: OECD 2011)*

	Pension funds	
	% of GDP	USD
OECD members		
Australia	82.3	808,224
Austria	4.9	18,987
Belgium	3.3	16,677
Canada	62.9	806,350
Chile	65.1	106,596
Czech Republic	4.6	11,332
Denmark	43.3	133,980
Estonia	6.9	1,371
Finland	76.8	182,286
France	0.8	21,930
Germany	5.2	173,810
Greece	0.0	63
Hungary	13.1	16,886
Iceland	118.3	14,351
Ireland	44.1	100,278
Israel	46.9	95,257
Italy	4.1	86,818
Japan	25.2	1,042,770
Korea	2.2	29,632
Luxembourg	2.2	1,171
Mexico	7.5	107,135
Netherlands	129.8	1,028,077
New Zealand	11.8	13,755
Norway	7.3	27,852

Poland	13.5	58,143
Portugal	13.4	30,441
Slovak Republic	4.7	4,640
Slovenia	2.6	1,266
Spain	8.1	118,056
Sweden	7.4	35,307
Switzerland	101.2	496,957
Turkey	2.3	14,017
United Kingdom	73.0	1,589,409
United States	67.6	9,583,968
OECD 34	**67.6**	**16,777,792**

1.2. Pension scheme

After having seen the worldwide importance of the ageing trend and the unavoidable pension challenge for the coming years we present, in this section, general concepts used in pension theory.

1.2.1. *Definition*

A pension scheme can be defined as a systematic and organized mechanism, prescribed by law or by a convention, in order to provide after retirement regular incomes to a well defined category of people. This broad definition is based on the following keywords:

– *Systematic*: the benefits offered by a pension scheme are supposed to be defined by an objective rule and are not given “*à la carte*”. Two people in exactly the same situation will receive the same pension amount.

– *Organized*: a pension scheme must be initiated by a sponsor (a state, a private company, etc.) who is the guarantor of the continuity of the system.

– *By law*: when the sponsor is a state or an assimilated national or international institution (social security systems).

– *By convention*: when the sponsor is a private company or employer (occupational pension schemes).

– *Regular incomes*: the typical form of the benefits offered by a pension scheme is a lifetime annuity but sometimes other forms of benefits may exist (for instance, a lump sum paid at retirement age or temporary annuity).

– *Category*: the affiliates to the scheme are generally members of an objective category.

A pension scheme is therefore by definition a collective vehicle with precise rules, even if it may concern only a few people. Apart from this concept, individuals can also buy, during their active life, individual financial products or life insurances in order to increase their standard of living after retirement. This idea to accumulate different forms of pension arrangements, from the social security system to individual saving accounts, is a basic concept of the modern pension theory, sometimes known as the multi–pillar philosophy.

1.2.2. *The four dimensions of a pension scheme*

When considering setting up a pension scheme, various fundamental choices arise. In order to choose a specific pension scheme, we must first understand the alternatives that arise and measure the benefits and disadvantages of different systems.

We can roughly summarize the strategy of choosing a pension scheme in four basic questions to be answered:

– Dimension 1: for whom? Affiliates: who is affected by the plan?

– Dimension 2: by whom? Sponsor: who organizes and manages the system?

– Dimension 3: how much? Benefits: how much to provide?

– Dimension 4: how? Funding: how can we finance these pension benefits?

1.2.2.1. *The dimension of the affiliates*

The first question to ask is who is the population to be covered by the pension plan.

The table below shows some community types encountered in practice, showing the large variety in terms of magnitude:

	POPULATION	EXAMPLE
1	The entire population of a country	Minimum pension income for all the citizens of a country
2	One category of workers of a country	Scheme for the civil servants
3	Members of a profession	Pension fund for accountants
4	One category of workers from one economic sector	Blue collar workers of the chemical industry
5	One category of workers of one private company	Workers of a company aged 25 or more
6	A key person of one company	CEO of the company

1.2.2.2. *The dimension of the sponsor: public or private*

The sponsor is the legal entity at the origin of the scheme. The table below shows some classical sponsors encountered in practice. Clearly the dimensions of the affiliates and of the sponsor are correlated.

	SPONSOR	EXAMPLE
1	A State	General social security scheme
2	A region	Scheme for the public servants of one region

3	A public company	Scheme for a national electricity company
4	A group of private companies	Common scheme for the chemical industry
5	A company	Company XY
6	A professional body	Association of Notaries

1.2.2.3. *The dimension of the benefits: DB or DC*

There are two main logics in designing the benefits of a pension scheme:

– defined benefits pension scheme (DB schemes): “we say what we want to receive as benefits at retirement age”;

– defined contributions pension scheme (DC schemes): “we say what we want to pay as contributions during the active life”.

In a *DB pension scheme* , the pension incomes to receive after retirement are explicitly defined by the rules of the plan and are generally related to the salaries and the years of service of the affiliate. For instance, a pension plan could provide, after retirement age, fixed at 65, a pension income corresponding to 65% of the average of the last 5 salaries for 40 years of service in the company.

In a *DC pension scheme* , the contributions to be paid by the employer and/or the employee are explicitly defined by the rules of the plan and constitute the only liability of the sponsor. The benefits will be generated just by the accumulation of these contributions. For instance, a pension plan could provide a contribution of 5% of the salaries paid each year by the employer and 1% paid by the employee.

Clearly the sharing of risks between the affiliates and the sponsor are quite different in the two philosophies. This point will be illustrated in section 1.3.

1.2.2.4. *The dimension of the funding: pay as you go or funding*

In a DB pension scheme where benefits have been defined, we must finance these liabilities. In particular, we must compute a level of contributions in order to have an actuarial equilibrium between incomes and outcomes; this equilibrium is based on a long–term approach and not on a pure accounting point of view.

More precisely, the actuary will check on a certain time horizon and on a given population, the actuarial equivalence between the present value of the contributions and the present value of the benefits.

There are a lot of different actuarial techniques in order to finance a DB scheme depending on the way these discounted values are computed (definition of the time horizon and the population concerned). A major distinction is generally made between the pay as you go mechanism and funding techniques.

In a *pay as you go mechanism* (PAYG), the contributions paid one year by the active affiliates are directly used in order to pay the benefits to the retired people. Time horizon is typically one year. As a consequence, everybody is paying for somebody else! In particular, there are no (or nearly no) reserves in such a scheme.

The first condition in order to apply this technique is of course to be sure of the continuity of the sponsor. If one day, for one reason or another, the system must stop, clearly there is a big problem for the active affiliates! So in practice this technique can only be used for social security purposes. The ageing of the population, which was previously mentioned, is clearly a major concern for PAYG schemes.

On the other hand, PAYG systems are relatively protected against financial and inflation risks.

In a *funding approach* , the contributions paid by one generation are invested on the financial market and will be used by the same generation later upon retirement.

The time horizon can be very long and there are significant reserves. This technique can be used for any kind of sponsor. In case of stopping of the system, reserves exist in order to protect the rights of the affiliates. This method seems to be better adjusted against demographic risk (even if the longevity risk is also present as it will be explained in section 1.3). But inflation and market risks will also greatly affect the scheme. In particular, the search for a good investment strategy for the accumulated reserves is crucial.

This distinction between PAYG and funding is particularly meaningful for a DB pension scheme. But it can be also applied to a DC plan. When looking at a DC plan, it seems natural to associate a classical saving mechanism of individual accounts and therefore a funding approach. But DC schemes can also be based on pure PAYG techniques like notional accounts or point systems for social security purposes.

1.3. Pension and risks

Pension liabilities are exposed to a large variety of risks and the influence of these risks on the level of the benefits and on the funding mechanism may vary greatly from one scheme to another. For a given pension scheme, we must, for each kind of risk, answer two basic questions:

- Is the risk affecting the pension scheme and how much?
- If so, who is affected (the sponsor or/and the affiliate)?

We propose breaking the various risks down into two main categories: demographic risks and financial risks.

1.3.1. *Demographic risks*

By definition, a pension scheme is a collective vehicle covering a population.

Not surprisingly, the evolution of the affiliates and the beneficiaries is quite important. Among the demographic risks affecting the population of a pension fund, we have:

– the longevity risk: risk caused by the (unexpected) increase in the duration of human life; in a pension scheme this risk will typically increase the number of years of service of the benefits;

– the renewal risk: risk caused by a decrease in the number of entrances in the population; in a pension scheme this risk will typically decrease the amount of contributions;

– the lapse risk: risk caused by the number of people going out of the pension fund before retirement age (disability, dismissal, etc.).

1.3.2. *Financial risks*

In a pension scheme, we can also observe different financial risks:

– the market risk: risk caused by the variation of the market value of the financial assets underlying the pension benefits (stocks, investment funds, real estate, etc.);

– the interest rate risk: risk caused by the effect of a variation of the interest rates on the valuation of the assets and the liabilities of the pension scheme;

– the credit risk: risk caused by the change of rating of a counterparty or by a partial or total default of a debtor;

– the inflation risk: risk caused by the effect of inflation on the level of benefits to be paid by the fund.

1.3.3. *Impact of the risks on various kinds of pension schemes*

We will shortly analyze the effect of all these risks on 4 important kinds of pension schemes illustrated by the following table:

	DB	*DC*
FUNDING	*1*	*3*
PAYG	*2*	*4*

1.3.3.1. *DB funding scheme*

If we consider a DB scheme using a funding technique where contributions are (mainly) paid by the sponsor (for instance an occupational scheme organized by a private company for its employees), clearly a lot of risks are supported by this sponsor (the company) and the affiliates (the employees) have nearly no risk. The main risks for the sponsor are the different financial risks and the longevity risk. Market and inflation risks are particularly crucial in this combination. The contributions computed actuarially and to be paid by the sponsor can largely fluctuate depending on the level of indexation of the benefits (inflation risk) and on the financial return achieved on assets (market and interest rate risks). For a pension scheme where benefits are paid by annuities after retirement, longevity risk has also a big influence on the actuarial valuation.

1.3.3.2. *DB PAYG scheme*

If we consider a DB scheme using a PAYG technique (for instance a social security scheme organized by a State for its civil servants), where contributions are (mainly) paid by the sponsor (the State), the main risk taken by the sponsor is now the demographic renewal risk. In a pure PAYG system without any assets, market, inflation and interest rates have no effect.

1.3.3.3. *DC funding scheme*

If we consider a DC scheme using a funding technique where contributions are paid by the sponsor and/or the affiliates (for instance an occupational scheme based on pension individual saving accounts), a lot of risks are now supported by the affiliates themselves. The only liability for the sponsor is to pay the fixed level of contributions. The main risks for the affiliates are all the financial risks (bad returns on the saving account of the affiliate or a significant inflation) and the longevity risk. To switch from a DB scheme to a DC scheme does not imply a removal of the risks but just their transfer from the sponsor to the affiliates!

1.3.3.4. *DC PAYG scheme*

The logic is the same if we consider a DC scheme using a PAYG mechanism (for instance a social security system based on notional accounts). The level of contributions to be paid is fixed but the level of benefits to pay to the retired people can fluctuate due to the influence of all the demographic risks (longevity and renewal risks).

1.3.4. ***The time horizon of a pension scheme***

Apart from the typology of risks, another main characteristic of pension liabilities is the long time horizon. A young worker aged 20 and entering into a pension fund where benefits are paid by annuities after retirement age, can stay in this fund for more than 60 years!!! Pension governance is definitively a long term business, by essence