ROBUST CONTROL

Theory and Applications

KANG-ZHI LIU
YU YAO

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Kang-Zhi Liu

Professor, Chiba University, Japan

Yu Yao

Professor, Harbin Institute of Technology, China

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This book is dedicated to our parents and families.

Contents

The sections and chapters marked by $*$ require relatively high level mathematics and may be skipped in the first reading, which does not affect the understanding of the main body.

Preface

This textbook evolved from our course teachings both at Chiba University and at Harbin Institute of Technology (HIT). It is intended for use in graduate courses on advanced control method. Many case studies are included in the book, hence it is also useful for practicing control engineers.

Robust control aims at conducting realistic system design in the face of model uncertainty. Since its advent in the 1980s, it has received extensive attention and have been applied to many real-world systems such as steel making process, vehicles, mass storage devices, and so on.

In this book, our ultimate goal is to summarize comprehensively the major methods and results on robust control, and provide a platform for readers to study and apply robust control. We also intend to provide a shortcut to the frontier of robust control research for those who are interested in theoretical research. Further, we hope to achieve the following goals:

1. Easy to understand: presenting profound contents in the most concise manner;
2. Showing the essence of robust control clearly and thoroughly;
3. Emphasizing engineering implications while keeping the theoretic rigor;
4. Self-contained: reducing to the lowest level the necessity of referring to other sources.

In order to realize the preceding objectives, whenever we introduce a new content, we first use a simple example or the concept that the readers are familiar with to illustrate the engineering background and idea clearly. Then we show the simplest proof known to the authors. All examples in this book stem from engineering practice and have clear backgrounds. It is hoped that this will help the readers learn how to apply these knowledge. For each important result, a rigorous proof is provided. To be self-contained, a quite complete coverage on linear algebra and convex analysis is provided, including proofs.

The book falls into three parts: mathematical preliminaries, fundamentals of linear systems and robust control. Chapters 2 and 3 summarize the knowledge of linear algebra, convex analysis, and LMI, which are required in order to understand the robust control theory completely. Chapter 4 reviews the basics of linear systems. Chapter 5 analyzes the performance specifications of reference tracking, disturbance rejection and fast response. Chapter 6 treats the stabilization of a given plant by using state feedback and observer. Further, Chapter 7 shows how to use a simple formula to parameterize all controllers that can stabilize a given plant, and analyze the structure of the closed-loop system, including the two-degree-of-freedom system. Chapter 8 provides several important results that bridge the time domain and frequency domain characteristics of signals/systems, especially the KYP lemma that lays the foundation for solving the robust control problem in the time domain. Chapter 9 describes some of the main results on the Riccati equation. Chapter 10 discusses the limitation of feedback control, so as to provide a guide for setting up reasonable performance specifications as well as the design of easy-to-control systems. Chapter 11 discusses the model uncertainty, including its classification, the features of different types, and the modeling methods. Chapter 12 is about the robust control analysis based on the gain information of uncertainty. The central issue is how to use the nominal model and the bounding function of uncertainty gain to derive the conditions for robust stability and robust performance. Based on Lyapunov stability theory and quadratic common Lyapunov functions, Chapter 13 establishes the robust stability conditions for systems with parameter uncertainty, and discusses Lur'e systems as well as passive systems. Chapter 14 presents the so-called IQC (integral quadratic constraint) theory. Chapters 15 and 16 handle, respectively, the $\mathcal{H}_2$ and $\mathcal{H}_\infty$ control theory and their applications. Chapter 17 is devoted to the robustness analysis and design of systems with multiple uncertainties. Chapter 18 covers systems with parameter uncertainty. Chapter 19 focuses on the parametric systems and shows how to improve the transient response by placing the closed-loop poles in a suitable region. Chapter 20 treats systems with known time-varying parameters. A higher performance can be achieved through the gain-scheduling of the controller, that is, letting the controller parameters vary together with the plant parameters. Finally, Chapter 21 describes how to use the positive real property of an uncertain system to realize robust control. Its essence is to make use of the phase information of uncertainty.

This book is an extended version of our textbook written in Chinese and published by Science Press, Beijing. Many staffs and students at HIT involved in the translation. Meanwhile, students at Chiba University helped us in doing the application researches and numerical designs. All case studies included in the book were conducted by our students. Without their efforts, this book will not be as it is.

We are indebted to Prof. Stephen Boyd and Prof. Kemin Zhou. Their books on convex analysis and small-gain approach to robust control have a significant influence on the writing of this book. Further, we would like to thank our supervisors, Prof. Tsutomu Mita and Prof. Zicai Wang, who guided us into the wonderful field of control engineering.

Kang-Zhi Liu
Yu Yao

List of Abbreviations

For simplicity, frequently used words are abbreviated as follows.

ARE	algebraic Riccati equation
BMI	bilinear matrix inequality
BIBO	bounded-input bounded-output
EVP	eigenvalue problem
GEVP	generalized eigenvalue problem
HDD	hard disk drive
IQC	Integral quadratic constraint
LFT	linear fractional transformation
LMI	linear matrix inequality
LPV	linear parameter varying
MIMO	multi-input multi-output
PSS	power system stabilizer
RTP	rapid thermal processing
SISO	single-input single-output
SVD	singular value decomposition
1-DOF	1-degree-of-freedom
2-DOF	2-degree-of-freedom
i.e.	that is
w.r.t.	with respect to
iff	if and only if

Notations

The notations used in this book are listed below.

$:=$	defined as
$\Leftrightarrow$	equivalent to
$\in$	belong to
$\forall$	for all
$\subset$	included in
$\emptyset$	empty set
δ_{ij}	Dirac index, satisfying $\delta_{ii} = 1, \delta_{ij} = 0(i \neq j)$
$\mathbb{R}$	field of real numbers
$\mathbb{C}$	field of complex numbers
$\mathbb{R}^n$	n-dimensional space of real vectors
$\mathbb{C}^n$	n-dimensional space of complex vectors
$\mathbb{R}^{m\times p}$	set of real matrices with m rows and p columns
$\mathbb{C}^{m\times p}$	set of complex matrices with m rows and p columns
$\mathbb{F}$	In stating a result which is true for both real and complex variables, $\mathbb{F}$ is used to present both.
sup	Supremum (i.e., least upper bound), may be regarded as the maximum $\max$ in engineering.
inf	Infimum (i.e., greatest lower bound), may be regarded as the minimum $\min$.
$\max\limits_{i}$	maximum w.r.t. all variables i
$\bar{x}$	conjugate of $x = a + jb$, namely $\bar{x} = a - jb$.
$\Re(x)$, $\Im(x)$	real and imaginary parts of a complex number x.
arg (x)	phase angle of a complex number x
$\hat{u}(s) = \mathcal{L}[u(t)]$	Laplace transform of time function $u(t)$
$\hat{u}(j\omega) = \mathcal{F}[u(t)]$	Fourier transform of time function $u(t)$
$\dot{u}(t), \ddot{u}(t)$	First and second derivatives of $u(t)$ about time t
$A = (a_{ij})$	matrix with a_{ij} as its element in the ith row and jth column
$X^* = \bar{X}^T$	Conjugate transpose of complex matrix X

$\mathrm{He}(X)$	$\mathrm{He}(X) = X + X^*$
$F^{\sim}(s)$	$F^{\sim}(s) = F^T(-s)$
$\mathrm{diag}(a_1, \cdots, a_n)$	diagonal matrix with a_i as its ith diagonal element
$\det(A)$	determinant of square matrix A
$\lambda_i(A)$	the ith eigenvalue of matrix A
$\sigma(A)$	set of eigenvalues $\{\lambda_1, \cdots, \lambda_n\}$ of matrix A
$\rho(A)$	spectral radius $\max_i \lvert\lambda_i(A)\rvert$ of matrix A
$\sigma_i(A)$	the ith singular value of matrix A. Be cautious not to confuse with the eigenvalue set $\sigma(A)$.
$\mathrm{rank}(A)$	rank of matrix A
$A_\perp$	orthogonal matrix of matrix A $(m \times n)$, i.e., a matrix with the maximum rank among all matrices U satisfying $AU = 0$ $(m \leq n)$ or $UA = 0$ $(m \geq n)$.
$\mathrm{Tr}(A)$	trace of $A = (a_{ij}) \in \mathbb{C}^{n \times n}$, i.e., the sum of its diagonal elements $\sum_{i=1}^n a_{ii}$
$\mathrm{Im}A$	$\{y \in \mathbb{C}^n \mid y = Ax, x \in \mathbb{C}^m\}$, image of A.
$\mathrm{Ker}\ A$	$\{x \mid Ax = 0, x \in \mathbb{C}^n\}$, kernel space of A.
$A \otimes B$	Kronecker product
$A \oplus B$	Kronecker sum
$\mathrm{vec}(A)$	vector formed by sequentially aligning the columns of matrix A starting from the first column
$A^\dagger$	pseudo-inverse of matrix A
$A > 0 (\geq 0)$	positive definite (positive semidefinite) matrix
$\lVert x\rVert$	norm of vector x
$\lVert A\rVert$	norm of matrix A
$\langle u, v\rangle$	inner product of vectors u, v.
$\langle A, B\rangle$	inner product of matrices A, B.
$\mathrm{span}\{u_1, \cdots, u_k\}$	space spanned by the vector set $u_1, \cdots, u_k$
$\mathbf{conv}\ C$	convex hull of set C
$x \prec y$	inequality of vectors (x, y), satisfying $x_i < y_i$.
$\frac{\partial f(x)}{\partial x}$	first-order partial derivative of function $f(x)$ whose transpose is the gradient $\nabla f(x)$ of $f(x)$.
$\frac{\partial^2 f(x)}{\partial x^2}$	second-order partial derivative of $f(x)$ whose transpose is the Hessian matrix $\nabla^2 f(x)$ of $f(x)$.
$\mathcal{F}_\ell(M, X)$	lower linear fractional transformation
$\mathcal{F}_u(M, X)$	upper linear fractional transformation
$f_1(t) * f_2(t)$	convolution integral of $f_1(t)$ and $f_2(t)$
$\mathbf{dom}\ f$	domain of function f
$f'(x)$	derivative of function $f(x)$
$\mathrm{Res}_{s_i} \hat{f}(s)$	residue of complex function $\hat{f}(s)$ at the point s_i

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Introduction

Mathematical model is indispensable in the simulation of physical systems or the design of control systems. The so-called natural science is, in fact, the systematic and categorized study of various kinds of physical, chemical, and other natural phenomena. Various models are used to describe and reproduce the observed phenomena. The models used in engineering are usually described as differential equations, difference equations, or statistical data. It may be said that the design of modern control systems is essentially based on the models of systems.

However, a mathematical model cannot describe the physical phenomenon of a system perfectly. Even if it could, the model would be unnecessarily complicated which makes it difficult to capture the main characteristic of the system. In particular, this is often the case in engineering practice. In almost all cases, a system is not isolated from the outside world. Instead, it is constantly influenced by the surrounding environment. It is difficult to describe the external influence by models. This means that there is inevitably a gap between the actual system and its mathematical model. This gap is called as the *model uncertainty*. The purpose of robust control is to extract the characteristics of model uncertainty and apply this information to the design of control system, so as to enhance the performance of the actual control system to the limit.

1.1 Engineering Background of Robust Control

Here, we give some specific examples to illustrate the model uncertainty.

Example 1.1 *Hard disk (refer to Figure 1.1(a)) is commonly used as the data storage device for computers. The frequency response of a hard disk drive (HDD in short hereafter) is shown in Figure 1.1(b) as the dotted line. The approximate model only considers the rigid body and is described as a double integrator (the solid line). Since the arm is very thin, the drive has numerous resonant modes in the high-frequency band, and these resonant modes vary with the manufacturing error in mass production. So it is difficult to obtain the exact model. Therefore, we have to design the control system based on the model of rigid body.*

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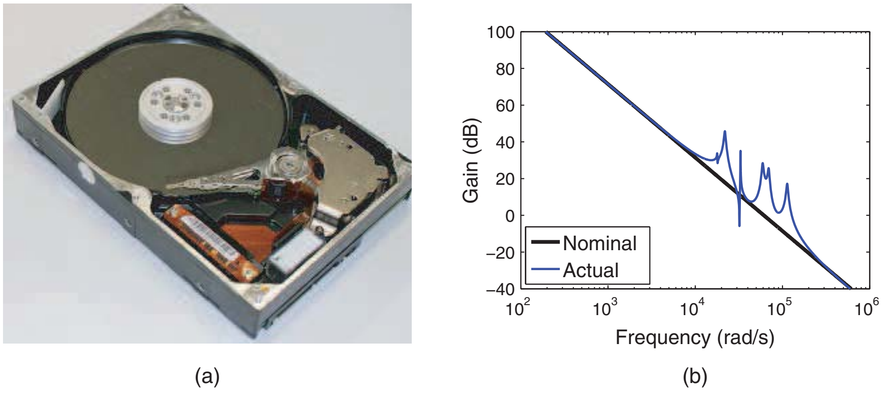

Figure 1.1 Hard disk drive and its frequency response (a) Photo, (b) Bode plot

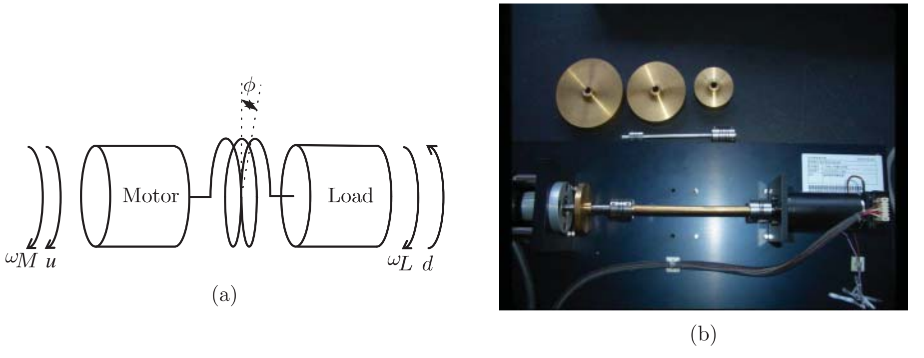

Figure 1.2 Motor drive system (a) Two-mass–spring system, (b) Motor drive

Example 1.2 *The system in Figure 1.2(a) is called* two-mass–spring system*. As shown in Figure 1.2(b), this is essentially a motor drive system in which the motor and load are connected through a shaft. The purpose is to control the load speed indirectly by controlling the motor inertia moment. There are many such systems around us. Typical examples are the DVD drive in home electronics, the rolling mill in steel factory, and so on.*

Let J_M be the inertia moment of the motor, k the spring constant of the shaft, J_L the inertia moment of the load, and D_M and D_L the viscous friction coefficients of the motor and the load. In addition, the motor speed is ω_M, the load speed is ω_L, the torsional angle of the shaft is ϕ, and the motor torque is u. As the direct measurement of load speed is difficult, we usually measure the motor speed ω_M only. The equations of moment balance as well as the speed equation are

$$\begin{aligned} J_L\dot{\omega}_L + D_L\omega_L &= k\phi + d \\ \dot{\phi} &= \omega_M - \omega_L \\ J_M\dot{\omega}_M + D_M\omega_M + k\phi &= u. \end{aligned}$$

Here, d denotes the torque disturbance acting on the load. Select the state vector as $x = [\omega_L \;\; \phi \;\; \omega_M]^T$, the following state equation is obtained easily:

$$\dot{x} = \begin{bmatrix} -\frac{D_L}{J_L} & \frac{k}{J_L} & 0 \\ -1 & 0 & 1 \\ 0 & -\frac{k}{J_M} & -\frac{D_M}{J_M} \end{bmatrix} x + \begin{bmatrix} \frac{1}{J_L} \\ 0 \\ 0 \end{bmatrix} d + \begin{bmatrix} 0 \\ 0 \\ \frac{1}{J_M} \end{bmatrix} u \tag{1.1}$$

$$y = [0 \quad 0 \quad 1]x. \tag{1.2}$$

However, in practice the motor load is rather diverse. Specifically, the load inertia moment J_L and the spring constant k of the shaft change over a wide range. This will undoubtedly affect the performance of the motor drive system.

As illustrated by these examples, the mathematical model of a system always contains some uncertain part. Nevertheless, we still hope that a control system, designed based on a model with uncertainty, can run normally and has high performance. To achieve this goal, it is obvious that we should make use of all available information about the uncertainty. For example, in the loop shaping design of the classical control, a very important principle is to ensure that the high-frequency gain of controller rolls off sufficiently to avoid exciting the unmodeled high-frequency dynamics of the plant, so that the controller can be successfully applied to the actual system. In addition, the open-loop transfer function needs also to have sufficient gain margin and phase margin so as to ensure that the closed-loop system is insensitive to the model uncertainty in the low- and middle-frequency domain. This implies that the information about model uncertainty has already been used *indirectly* in system designs based on the classical control methods. Of course, indirect usage of model uncertainty has only a limited effect. So, the mission of robust control theory is to develop an effective, high-performance design method that makes use of the uncertainty information *fully and directly* for systems with uncertainty.

Next, we use an example to show the influence of model uncertainty. It will be illustrated that if the model uncertainty is neglected in the system design, the controller may have trouble when applied to the actual system. In the worst case, even the stability of the closed-loop system may be lost.

Example 1.3 *Consider a system containing both rigid body and a resonant mode (a simplified dynamics of HDD)*

$$\tilde{P}(s) = \frac{1}{s^2} - \frac{0.1 \times \omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}, \quad \zeta = 0.02, \omega_n = 5.$$

In the control design, only the rigid body model $P(s) = 1/s^2$ is considered, and the resonant mode $-0.1 \times \omega_n^2/(s^2 + 2\zeta\omega_n s + \omega_n^2)$ is ignored. In the stabilization design of the rigid body model, we hope that the closed-loop system response is fast enough while the overshoot is as small as possible. So the damping ratio of the closed-loop system is set as $1/2$ and the natural

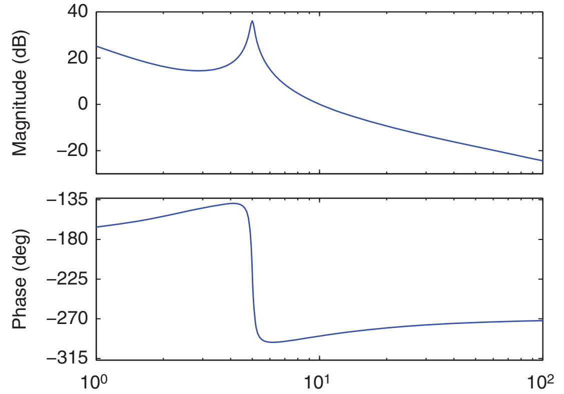

Figure 1.3 Bode plot of open-loop transfer function

frequency set as 4 [rad/s] (slightly lower than the natural frequency of the resonant mode). The corresponding characteristic polynomial is

$$p(s) = s^2 + 4s + 16.$$

The following proportional derivative (PD) compensator is able to achieve the goal:

$$K(s) = 4(s + 4).$$

However, when this controller is applied on the actual system $\tilde{P}(s)$, we find that the characteristic polynomial of the actual closed-loop becomes

$$\tilde{p}(s) = s^4 - 5.8s^3 + 1.8s^2 + 103.2s + 400$$

which obviously has unstable roots. In fact, the closed-loop poles are xx

$$5.2768 \pm j3.8875, \quad -2.3768 \pm j1.9357,$$

in which two of them are unstable. That is, the closed-loop system not only fails to achieve performance improvement but also losses the stability. This phenomenon is known as spillover in vibration control. The reason for the spillover is clear from Figure 1.3, the Bode plot of the open-loop transfer function $\tilde{P}K$. Since the response speed of the closed loop is designed too fast for the rigid body model, the roll-off of the controller gain is not sufficient near $\omega_n = 5$ rad/s, the natural frequency of the resonant mode, which excites the resonant mode.

1.2 Methodologies of Robust Control

In this section, various descriptions of model uncertainty and the corresponding robust control methods are briefly illustrated for single-input single-output systems.

1.2.1 Small-Gain Approach

Suppose that the system model is $P(s)$ while the actual system is $\tilde{P}(s)$. If no information is known about the structure of the actual system $\tilde{P}(s)$, the simplest method is to regard the difference between them as the model uncertainty $\Delta(s)$, that is,

$$\tilde{P}(s) = P(s) + \Delta(s). \tag{1.3}$$

Then, the closed-loop system can be transformed equivalently into the system of Figure 1.4 in which

$$M(s) = \frac{K}{1 + PK}$$

is the closed-loop transfer function about $P(s)$ and should be stable.

As for the description of the characteristics of $\Delta(s)$, a straightforward method is to use the frequency response. In general, the frequency response of $\Delta(s)$ varies within a certain range, such as

$$0 \leq |\Delta(j\omega)| \leq |W(j\omega)| \ \ \forall \omega. \tag{1.4}$$

Here $W(s)$ is a known transfer function whose gain specifies the boundary of the uncertainty $\Delta(s)$. When $\Delta(s)$ is a stable transfer function,

$$|M(j\omega)\Delta(j\omega)| < 1 \ \ \forall \omega \quad \Leftrightarrow \quad |M(j\omega)W(j\omega)| < 1 \ \ \forall \omega \tag{1.5}$$

guarantees the stability of the closed-loop system for all uncertainties $\Delta(s)$. This condition is known as the *small-gain condition*. The reason for the stability of the closed-loop system is clear. The open-loop transfer function in Figure 1.4 is $L(s) = M(s)\Delta(s)$ and is stable. $L(j\omega)$ does not encircle the critical point $(-1, j0)$ when the small-gain condition is satisfied, so the Nyquist stability condition holds. Here, the information of uncertainty gain is used.

Chapter 12 describes in detail the robust analysis based on the small-gain condition, while Chapters 16 and 17 provide the corresponding robust design methods.

1.2.2 Positive Real Method

On the contrary, in (1.3) if the phase angle $\arg \Delta(j\omega)$ of the uncertainty $\Delta(s)$ changes in a finite range, particularly when

$$-90^\circ \leq \arg \Delta(j\omega) \leq 90^\circ \ \ \forall \omega \quad \Leftrightarrow \quad \Re[\Delta(j\omega)] \geq 0 \ \ \forall \omega, \tag{1.6}$$

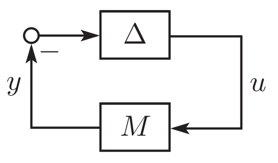

Figure 1.4 Closed-loop system with uncertainty

the following condition guarantees the stability of the closed-loop system for all the uncertainties $\Delta(s)$:

$$-90^\circ < \arg M(j\omega) < 90^\circ \ \ \forall\omega \quad \Leftrightarrow \quad \Re[M(j\omega)] > 0 \ \ \forall\omega. \tag{1.7}$$

This is because the open-loop transfer function $L(s) = M(s)\Delta(s)$ is stable and the phase angle is never equal to $\pm 180^\circ$. As such, $L(j\omega)$ does not encircle the critical point $(-1, j0)$, and the Nyquist stability condition is met.

For all frequencies ω, a transfer function satisfying $\Re[G(j\omega)] \geq 0$ is called a *positive real function*. So the previous condition is named as the *positive real condition*. General positive real condition is described detailedly in Section 13.4, and the robust design based on positive real condition is discussed in Chapter 21.

1.2.3 Lyapunov Method

For systems with known dynamics but uncertain parameters, such as the two-mass–spring system, methods based on quadratic Lyapunov function are quite effective. For example, consider an autonomous system

$$\dot{x} = Ax, \quad x(0) \neq 0.$$

If the quadratic Lyapunov function

$$V(x) = x^T P x, \quad P > 0$$

satisfies the strictly decreasing condition

$$\dot{V}(x) < 0 \quad \forall x \neq 0,$$

then $V(x)$ converges to zero eventually since it is bounded below. As $V(x)$ is positive definite, $V(x) = 0$ implies that $x = 0$. Further, the strictly decreasing condition is equivalent to the inequality on the matrix A:

$$A^T P + PA < 0.$$

Even if some parameters of the matrix A take value in a certain range, the stability of the system is secured so long as there exists a matrix $P > 0$ satisfying this inequality.

Refer to Chapter 13 for the analysis on such uncertain systems and Section 18.3, Chapter 19, and Chapter 20 for the design methods.

1.2.4 Robust Regional Pole Placement

For a system with parameter uncertainty, we can place the poles of the closed-loop system in some region of the complex plane, such as the region shown in Figure 1.5, so as to guarantee the quality of transient response. Detailed design conditions and design methods are given in Chapter 19.