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# Parametric and Nonparametric Inference for Statistical Dynamic Shape Analysis with Applications



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# Parametric and Nonparametric Inference for Statistical Dynamic Shape Analysis with Applications

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# Preface

Statistical shape analysis relates to the geometrical study of random objects where location, rotation and scale information can be removed.

The last 20 years have seen a considerable growth in interest in the statistical theory of shape. This has been the result of a synthesis of various disciplines which are interested in measuring, describing and comparing the shapes of objects.

Much work has been done for static or cross-sectional shape analysis, while considerably less research has focused on dynamic or longitudinal shapes. Statistical analysis of dynamic shapes is a problem with significant challenges due to the difficulty in providing the qualitative and quantitative assessment of shape changes over time, across subjects and, eventually, also over groups of subjects.

In this book, we consider specific inferential issues arising from the analysis of dynamic shapes with the attempt to solve the problems at hand using probability models and nonparametric tests. Models are simple to understand and interpret and provide a useful tool to describe the global dynamics of the landmark configurations. However, because of the non-Euclidean nature of shape spaces, distributions in shape spaces are not straightforward to obtain. Here, we consider distributions in the configuration space, with similarity transformations integrated out. This is a simple approach that allows to define models on landmarks themselves giving rise to derived distributions on shapes. The simplest model for a configuration is to assume that the landmarks follow a multivariate Normal distribution about a mean configuration. Various level of generality can also be assumed for the covariance matrix allowing correlations between landmarks and different time points. In this case, it turns out that the distribution which enables inference from configuration onto the shape space is the offset-normal distribution for temporally correlated shapes.

There are also cases of interest in which the use of a model appears problematic and computationally difficult. For example, this is particularly true when the aim of the analysis requires the identification of subsets of landmarks which best describes the dynamics of a whole configuration. A selection of landmarks can enable us to understand and gain information which may not be noticed with a model including all landmarks. To understand whether landmark positions change

significantly over time across subjects and over groups of subjects, we make use of the NonParametric Combination (NPC) tests. The NPC methodology, which allows to build powerful tests in a nonparametric framework, does not require strong underlying assumptions as the traditional parametric competitors and allows to work at a local level to highlight specific areas (*domains*) of a configuration in which we may have systematic differences.

The book has a natural split into two parts, with the first three chapters covering material on the offset-normal shape distribution and the remaining chapters covering the theory of NonParametric Combination (NPC) tests. We have attempted to keep each chapter as self-contained as possible, but some dependencies are of course inevitable. The different chapters offer a collection of applications which are bound together by the theme of this book. They refer to the analysis of the FG-NET (Face and Gesture Recognition Research Network) database with facial expressions. For these data set, it may be desirable to provide a description of the dynamics of the expressions, or testing whether there is a difference between the dynamics of two facial expressions or testing which of the landmarks are more informative in explaining the pattern of an expression.

The book is organized as follows. Chapter 1 is the basic introductory chapter for the rest of the book. It introduces the basic notation and commonly used registration approaches of landmark data on a common coordinate system. In Chap. 2 we assume that the shape data are generated from the induced shape distributions of Gaussian configurations in which the similarity transformations are integrated out. For this probability distribution, we discuss the expectation-maximization (EM) algorithm for parameter estimation. This procedure gives essential results for a likelihood-based approach to statistical inference in shape analysis and provides the basis for making inference in a dynamic setting as described in Chap. 3. This latter chapter, in fact, discusses the difficulties of extending results of Chap. 2 in a dynamic framework. Specifically, it describes the offset-normal shape distributions in a dynamic context and introduces the necessary adjustments of the general update rules of the EM algorithm for general spatio-temporal covariance matrices. Also, in order to represent the shape changes in time and classifying dynamic shapes, it provides a discussion of the use of polynomial regression as well as mixture models. In general, it is shown that the EM approach warrants consideration when modelling the dynamics of shapes. However, unless some model simplifications are assumed, the computational burden of the procedure can limit its use in real applications.

In Chap. 4, we introduce the NonParametric Combination (NPC) methodology of a set of dependent partial tests in the specific context of shape analysis. The basic underlying idea of the methodology is that complex multidimensional testing problems may be reduced to a set of simpler subproblems, each provided with a proper permutation solution. These subproblems can be jointly analysed in order to capture the underlying dependency structure, without specifying the nature of dependence relations among variables. NPC tests are distribution-free and, among good general properties, they enjoy the finite-sample consistency property, thus allowing to obtain efficient solutions for multivariate small sample problems, like

those encountered in shape analysis applications. Solutions for two independent sample problems is shown, along with suitable combination algorithm, and general framework for dealing with longitudinal repeated-measures designs is examined. Chapter 5 provides examples of applications of the methodology to the FG-NET data: in particular solutions allowing to study differences between dynamics of facial expressions or to identify landmarks that are more involved in the dynamics will be presented. Finally an NPC solution for assessing shape asymmetry in dynamic data is also presented.

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**Part I**  
**Offset Normal Distribution**  
**for Dynamic Shapes**