

2ND EDITION

# ADVANCED PETROLEUM RESERVOIR SIMULATION

M. R. Islam

M.E. Hossain

S. Hossein Mousavizadegan

Shabbir Mustafiz

Jamal H. Abou-Kassem

 Scrivener  
Publishing

WILEY



# Advanced Petroleum Reservoir Simulation

**Scrivener Publishing**

100 Cummings Center, Suite 541J  
Beverly, MA 01915-6106

*Publishers at Scrivener*

Martin Scrivener (martin@scrivenerpublishing.com)  
Phillip Carmical (pcarmical@scrivenerpublishing.com)

# **Advanced Petroleum Reservoir Simulation**

**Towards Developing Reservoir Emulators**

Second Edition

**M. R. Islam, M. E. Hossain,  
S. H. Moussavizadegan, S. Mustafiz,  
J. H. Abou-Kassem**



**WILEY**

Copyright © 2016 by Scrivener Publishing LLC. All rights reserved.

Co-published by John Wiley & Sons, Inc. Hoboken, New Jersey, and Scrivener Publishing LLC, Salem, Massachusetts.  
Published simultaneously in Canada.

No part of this publication may be reproduced, stored in a retrieval system, or transmitted in any form or by any means, electronic, mechanical, photocopying, recording, scanning, or otherwise, except as permitted under Section 107 or 108 of the 1976 United States Copyright Act, without either the prior written permission of the Publisher, or authorization through payment of the appropriate per-copy fee to the Copyright Clearance Center, Inc., 222 Rosewood Drive, Danvers, MA 01923, (978) 750-8400, fax (978) 750-4470, or on the web at [www.copyright.com](http://www.copyright.com). Requests to the Publisher for permission should be addressed to the Permissions Department, John Wiley & Sons, Inc., 111 River Street, Hoboken, NJ 07030, (201) 748-6011, fax (201) 748-6008, or online at <http://www.wiley.com/go/permission>.

**Limit of Liability/Disclaimer of Warranty:** While the publisher and author have used their best efforts in preparing this book, they make no representations or warranties with respect to the accuracy or completeness of the contents of this book and specifically disclaim any implied warranties of merchantability or fitness for a particular purpose. No warranty may be created or extended by sales representatives or written sales materials. The advice and strategies contained herein may not be suitable for your situation. You should consult with a professional where appropriate. Neither the publisher nor author shall be liable for any loss of profit or any other commercial damages, including but not limited to special, incidental, consequential, or other damages.

For general information on our other products and services or for technical support, please contact our Customer Care Department within the United States at (800) 762-2974, outside the United States at (317) 572-3993 or fax (317) 572-4002.

Wiley also publishes its books in a variety of electronic formats. Some content that appears in print may not be available in electronic formats. For more information about Wiley products, visit our web site at [www.wiley.com](http://www.wiley.com).

For more information about Scrivener products please visit [www.scrivenerpublishing.com](http://www.scrivenerpublishing.com).

Cover design by Kris Hackerott

***Library of Congress Cataloging-in-Publication Data:***

ISBN 978-1-119-03851-1

Printed in the United States of America

10 9 8 7 6 5 4 3 2 1

*Authors would like to dedicate this book to their teacher and 'grand teacher', Professor S.M. Farouq Ali, Encana/Petroleum Society Chair Professor at the University of Calgary.*





# Contents

<b>Preface</b>	<b>xv</b>
<b>1 Introduction</b>	<b>1</b>
1.1 Summary	1
1.2 Opening Remarks	2
1.3 The Need for a Knowledge-Based Approach	2
1.4 Summary of Chapters	5
<b>2 Reservoir Simulation Background</b>	<b>7</b>
2.1 Essence of Reservoir Simulation	8
2.2 Assumptions Behind Various Modeling Approaches	10
2.2.1 Material Balance Equation	11
2.2.2 Decline Curve	12
2.2.3 Statistical Method	13
2.2.4 Analytical Methods	15
2.2.5 Finite-Difference Methods	16
2.2.6 Darcy's Law	19
2.3 Recent Advances in Reservoir Simulation	19
2.3.1 Speed and Accuracy	19
2.3.2 New Fluid-Flow Equations	21
2.3.3 Coupled Fluid Flow and Geo-Mechanical Stress Model	26
2.3.4 Fluid-Flow Modeling Under Thermal Stress	29
2.4 Memory Models	31
2.4.1 Thermal Hysteresis	32
2.4.2 Mathematical and Numerical Models	32
2.5 Future Challenges in Reservoir Simulation	33
2.5.1 Experimental Challenges	33
2.5.2 Numerical Challenges	35
2.5.2.1 Theory of Onset and Propagation of Fractures due to Thermal Stress	35
2.5.2.2 Viscous Fingering during Miscible Displacement	36
	vii

<b>3</b>	<b>Reservoir Simulator-Input/Output</b>	<b>39</b>
3.1	Input and Output Data	40
3.2	Geological and Geophysical Modeling	42
3.3	Reservoir Characterization	45
3.3.1	Representative Elementary Volume, REV	46
3.3.2	Fluid and Rock Properties	49
3.3.2.1	Fluid Properties	49
3.3.3	Rock Properties	54
3.4	Upscaling	58
3.4.1	Power Law Averaging Method	59
3.4.2	Pressure-Solver Method	60
3.4.3	Renormalization Technique	62
3.4.4	Multiphase Flow Upscaling	63
3.5	Pressure/Production Data	65
3.6	Phase Saturations Distribution	66
3.7	Reservoir Simulator Output	68
3.8	History Matching	70
3.8.1	History-Matching Formulation	72
3.8.2	Uncertainty Analysis	75
3.8.2.1	Measurement Uncertainty	76
3.8.2.2	Upscaling Uncertainty	78
3.8.2.3	Model Error	79
3.8.2.4	The Prediction Uncertainty	80
3.9	Real-Time Monitoring	81
<b>4</b>	<b>Reservoir Simulators: Problems, Shortcomings, and Some Solution Techniques</b>	<b>85</b>
4.1	Multiple Solutions in Natural Phenomena	87
4.1.1	Knowledge Dimension	90
4.2	Adomian Decomposition	104
4.2.1	Governing Equations	106
4.2.2	Adomian Decomposition of Buckley-Leverett Equation	108
4.2.3	Results and Discussions	111
4.3	Some Remarks on Multiple Solutions	114
<b>5</b>	<b>Mathematical Formulation of Reservoir Simulation Problems</b>	<b>117</b>
5.1	Black Oil Model and Compositional Model	119
5.2	General Purpose Compositional Model	120
5.2.1	Basic Definitions	120
5.2.2	Primary and Secondary Parameters and Model Variables	122

5.2.3	Mass Conservation Equation	125
5.2.4	Energy Balance Equation	128
5.2.5	Volume Balance Equation	133
5.2.6	The Motion Equation in Porous Medium	134
5.2.7	The Compositional System of Equations and Model Variables	139
5.3	Simplification of the General Compositional Model	141
5.3.1	The Black Oil Model	141
5.3.2	The Water Oil Model	143
5.4	Some Examples in Application of the General Compositional Model	146
5.4.1	Isothermal Volatile Oil Reservoir	146
5.4.2	Steam Injection Inside a Dead Oil Reservoir	148
5.4.3	Steam Injection in Presence of Distillation and Solution Gas	150
<b>6</b>	<b>The Compositional Simulator Using Engineering Approach</b>	<b>155</b>
6.1	Finite Control Volume Method	156
6.1.1	Reservoir Discretization in Rectangular Coordinates	157
6.1.2	Discretization of Governing Equations	158
6.1.2.1	Components Mass Conservation Equation	158
6.1.2.2	Energy Balance Equation	166
6.1.3	Discretization of Motion Equation	168
6.2	Uniform Temperature Reservoir Compositional Flow Equations in a 1-D Domain	170
6.3	Compositional Mass Balance Equation in a Multidimensional Domain	175
6.3.1	Implicit Formulation of Compositional Model in Multidimensional Domain	178
6.3.2	Reduced Equations of Implicit Compositional Model in Multidimensional Domain	180
6.3.3	Well Production and Injection Rate Terms	183
6.3.3.1	Production Wells	183
6.3.3.2	Injection Wells	185
6.3.4	Fictitious Well Rate Terms (Treatment of Boundary Conditions)	186
6.4	Variable Temperature Reservoir Compositional Flow Equations	190
6.4.1	Energy Balance Equation	190

6.4.2	Implicit Formulation of Variable Temperature Reservoir Compositional Flow Equations	194
6.5	Solution Method	197
6.5.1	Solution of Model Equations Using Newton's Iteration	198
6.6	The Effects of Linearization	203
6.6.1	Case 1: Single Phase Flow of a Natural Gas	203
6.6.2	Effect of Interpolation Functions and Formulation	210
6.6.3	Effect of Time Interval	210
6.6.4	Effect of Permeability	212
6.6.5	Effect of Number of Gridblocks	214
6.6.6	Spatial and Transient Pressure Distribution Using Different Interpolation Functions	214
6.6.7	CPU Time	218
6.6.8	Case 2: An Oil/water Reservoir	220
<b>7</b>	<b>Development of a New Material Balance Equation for Oil Recovery</b>	<b>239</b>
7.1	Summary	239
7.2	Introduction	241
7.3	Mathematical Model Development	243
7.3.1	Permeability Alteration	243
7.3	Porosity Alteration	244
7.4	Pore Volume Change	246
7.4.1	A Comprehensive MBE with Memory for Cumulative Oil Recovery	247
7.5	Numerical Simulation	250
7.5.1	Effects of Compressibilities on Dimensionless Parameters	251
7.4.2	Comparison of Dimensionless Parameters Based on Compressibility Factor	252
7.4.3	Effects of $M$ on Dimensionless Parameter	253
7.4.4	Effects of Compressibility Factor with $M$ Values	255
7.4.5	Comparison of Models Based on RF	255
7.4.6	Effects of $M$ on MBE	257
7.5	Conclusions	258
	Appendix Chapter 7: Development of an MBE for a Compressible Undersaturated Oil Reservoir	259

<b>8</b>	<b>State-of-the-art on Memory Formalism for Porous Media</b>	
	<b>Applications</b>	<b>271</b>
8.1	Summary	271
8.2	Introduction	272
8.3	Historical Development of Memory Concept	273
8.3.1	Constitutive Equations	274
8.3.2	Application of Memory in Diffusion in Porous Media	274
8.3.3	Definition of Memory	277
8.4	State-of-the-art Memory-Based Models	277
8.5	Basset Force: A History Term	284
8.6	Anomalous Diffusion: A memory Application	287
8.6.1	Fractional Order Transport Equations and Numerical Schemes	288
8.7	Future Trends	297
8.8	Conclusion	298
<b>9</b>	<b>Modeling Viscous Fingering During Miscible Displacement in a Reservoir</b>	<b>301</b>
9.1	Improvement of the Numerical Scheme	302
9.1.1	The Governing Equation	303
9.1.2	Finite Difference Approximations	305
9.1.2.1	Barakat-Clark FTD Scheme	305
9.1.2.2	DuFort-Frankel Scheme	307
9.1.3	Proposed Barakat-Clark CTD Scheme	307
9.1.4	Accuracy and Truncation Errors	309
9.1.5	Some Results and Discussion	309
9.1.6	Influence of Boundary Conditions	316
9.2	Application of the New Numerical Scheme to Viscous Fingering	317
9.2.1	Stability Criterion and Onset of Fingering	318
9.2.2	Base Stable Case	318
9.2.3	Base Unstable Case	324
9.2.4	Parametric Study	330
9.2.4.1	Effect of Injection Pressure	331
9.2.4.2	Effect of Overall Porosity	335
9.2.4.3	Effect of Mobility Ratio	336
9.2.4.4	Effect of Longitudinal Dispersion	341
9.2.4.5	Effect of Transverse Dispersion	343
9.2.4.6	Effect of Aspect Ratio	347

9.2.5	Comparison of Numerical Modeling Results with Experimental Results	350
9.2.5.1	Selected Experimental Model	350
9.2.5.2	Physical Model Parameters	350
9.2.5.3	Comparative Study	351
9.2.5.4	Concluding Remarks	355
<b>10</b>	<b>An Implicit Finite-Difference Approximation of Memory-Based Flow Equation in Porous Media</b>	<b>359</b>
10.1	Summary	359
10.2	Introduction	360
10.3	Background	361
10.4	Theoretical Development	364
10.4.1	Mass Conservation	365
10.4.2	Composite Variable, $\eta$	366
10.4.3	Implicit Formulation	367
10.6	Numerical Simulation	369
10.7	Results and Discussion	370
10.8	Conclusion	381
<b>11</b>	<b>Towards Modeling Knowledge and Sustainable Petroleum Production</b>	<b>383</b>
11.1	Essence of Knowledge, Science, and Emulation	384
11.1.1	Simulation vs. Emulation	384
11.1.2	Importance of the First Premise and Scientific Pathway	386
11.1.3	Mathematical Requirements of Nature Science	388
11.1.4	The Meaningful Addition	392
11.1.5	“Natural” Numbers and the Mathematical Content of Nature	394
11.2	The Knowledge Dimension	397
11.2.1	The Importance of Time as the Fourth Dimension	398
11.3	Aphenomenal Theories of Modern Era	400
11.3.1	Examples of Linearization and Linear Thinking	408
11.3.2	The Knowledge-Based Cognition Process	409
11.4	Towards Modeling Truth and Knowledge	412
11.5	The Single-Parameter Criterion	413
11.5.1	Science Behind Sustainable Technology	413

11.5.2	A New Computational Method	415
11.5.3	Towards Achieving Multiple Solutions	420
11.6	The Conservation of Mass and Energy	422
11.6.1	The Avalanche Theory	423
11.6.2	Aims of Modeling Natural Phenomena	428
11.6.3	Challenges of Modeling Sustainable Petroleum Operations	430
11.6.4	The Criterion: The Switch that Determines the Direction at a Bifurcation Point	433
11.6.4.1	Some Applications of the Criterion	436
11.7	The Need for Multidimensional Study	442
11.8	Assessing the Overall Performance of a Process	445
11.9	Implications of Knowledge-Based Analysis	452
11.9.1	A General Case	452
11.9.2	Impact of Global Warming Analysis	455
11.9.3	Examples of Knowledge-based Simulation	458
<b>12</b>	<b>Reservoir Simulation of Unconventional Reservoirs</b>	<b>465</b>
12.1	Introduction	465
12.2	Material Balance Equations	466
12.3	New Fluid Flow Equations	476
12.4	Coupled Fluid Flow and Geo-mechanical Stress Model	478
12.5	Fluid Flow Modeling under Thermal Stress	480
12.6	Challenges of Modeling Unconventional Gas Reservoirs	481
12.7	Comprehensive Modeling	489
12.7.1	Governing Equations	489
12.7.2	Darcy's Model	490
12.7.3	Forchheimer's Model	491
12.7.4	Modified Brinkman's Model	494
12.7.5	The Comprehensive Model	496
<b>13</b>	<b>Final Conclusions</b>	<b>501</b>
	<b>References and Bibliography</b>	<b>505</b>
	<b>Appendix A</b>	<b>545</b>
	<b>Index</b>	<b>569</b>





## Preface

The Information Age is synonymous with an overflow, a superflux, of “information”. Information is necessary for traveling the path of knowledge, leading to the truth. Truth sets one free; freedom is peace.

Yet, here a horrific contradiction leaps out to grab one and all by the throat: of all the characteristics that can be said to characterize the Information Age, neither freedom nor peace is one of them. The Information Age that promised infinite transparency, unlimited productivity, and true access to Knowledge (with a capital-K, but, quite distinct from “know-how”), requires a process of thinking, or imagination – the attribute that sets human beings apart.

Imagination is necessary for anyone wishing to make decisions based on science. Imagination always begins with visualization – actually, another term for simulation. Any decision devoid of a priori simulation is inherently aphenomenal. It turns out simulation itself has little value unless fundamental assumptions as well as the science (time function) are actual. While the principle of garbage in and garbage out is well known, it only leads to using accurate data, in essence covering the necessary condition for accurate modeling.

The sufficient condition, i.e., the correct time function, is little understood, let alone properly incorporated. This process of including continuous time function is emulation and is the principal theme of this book. The petroleum industry is known as the biggest user of computer models. Even though space research and weather prediction models are robust and often tagged as the “mother of all simulation”, the fact that a space probe device or a weather balloon can be launched – while a vehicle capable of moving around in a petroleum reservoir cannot – makes reservoir modeling more challenging than in any other discipline.

This challenge is two-fold. First, there is a lack of data and their proper scaling up. Second is the problem of assuring correct solutions to the mathematical models that represent the reservoir data. The petroleum industry has made tremendous progress in improving data acquisition and remote-sensing ability. However, in the absence of proper science, it is anecdotally

said that a weather model of Alaska can be used to simulate a petroleum reservoir in Texas. Of course, pragmatism tells us, we'll come across desired outcome every once in a while, but is that anything desirable in real science? This book brings back real science and solves reservoir equations with the entire history (called the 'memory' function) of the reservoir. The book demonstrates that *a priori* linearization is not justified for the realistic range of most petroleum parameters, even for single-phase flow. By solving non-linear equations, this book gives a range of solutions that can later be used to conduct scientific risk analysis.

This is a groundbreaking approach. The book answers practically all questions that emerged in the past. Anyone familiar with reservoir modeling would know how puzzling subjective and variable results – something commonly found in this field – can be. The book deciphers variability by accounting for known nonlinearities and proposing solutions with the possibility of generating results in cloud-point forms. The book takes the engineering approach, thereby minimizing unnecessary complexity of mathematical modeling. As a consequence, the book is readable and workable with applications that can cover far beyond reservoir modeling or even petroleum engineering.

# 1

## Introduction

### 1.1 Summary

It is well known that reservoir simulation studies are very subjective and vary from simulator to simulator. While SPE benchmarking has helped accept differences in predicting petroleum reservoir performance, there has been no scientific explanation behind the variability that has frustrated many policy makers and operations managers and puzzled scientists and engineers. In this book, a new approach is taken to add the Knowledge dimension to the problem. Some attempted to ‘correct’ this shortcoming by introducing ‘history matching’, often automatizing the process. This has the embedded assumption that ‘outcome justifies the process’ – the ultimate of the obsession with externals. In this book, reservoir simulation equations are shown to have embedded variability and multiple solutions that are in line with physics rather than spurious mathematical solutions. With this clear description, a fresh perspective in reservoir simulation is presented. Unlike the majority of reservoir simulation approaches available today, the ‘knowledge-based’ approach does not stop at questioning the fundamentals of reservoir simulation but offers solutions and demonstrates that proper reservoir simulation should be transparent and empower

decision makers rather than creating a black box. For the first time, the fluid memory factor is introduced with a functional form. The resulting governing equations become truly non-linear. A series of clearly superior mathematical and numerical techniques are presented that allow one to solve these equations without linearization. These mathematical solutions that provide a basis for systematic tracking of multiple solutions are emulation instead of simulation. The resulting solutions are cast in cloud points that form the basis for further analysis with advanced fuzzy logic, maximizing the accuracy of unique solution that is derived. The models are applied to difficult scenarios, such as in the presence of viscous fingering, and results compared with experimental data. It is demonstrated that the currently available simulators only address very limited range of solutions for a particular reservoir engineering problem. Examples are provided to show how the Knowledge-based approach extends the currently known solutions and provide one with an extremely useful predictive tool for risk assessment.

### 1.2 Opening Remarks

Petroleum is still the world's most important source of energy, and, with all of the global concerns over climate change, environmental standards, cheap gasoline, and other factors, petroleum itself has become a hotly debated topic. This book does not seek to cast aspersions, debate politics, or take any political stance. Rather, the purpose of this volume is to provide the working engineer or graduate student with a new, more accurate, and more efficient model for a very important aspect of petroleum engineering: reservoir simulations. The term, "knowledge-based," is used throughout as a term for our unique approach, which is different from past approaches and which we hope will be a very useful and eye-opening tool for engineers in the field. We do not intend to denigrate other methods, nor do we suggest by our term that other methods do not involve "knowledge." Rather, this is simply the term we use for our approach, and we hope that we have proven that it is more accurate and more efficient than approaches used in the past.

### 1.3 The Need for a Knowledge-Based Approach

In reservoir simulation, the principle of GIGO (Garbage in and garbage out) is well known (latest citation by Rose, 2000). This principle implies that the input data have to be accurate for the simulation results to be

acceptable. Petroleum industry has established itself as the pioneer of subsurface data collection (Islam *et al.*, 2010). Historically, no other discipline has taken so much care in making sure input data are as accurate as the latest technology would allow. The recent superflux of technologies dealing with subsurface mapping, real time monitoring, and high speed data transfer is an evidence of the fact that input data in reservoir simulation are not the weak link of reservoir modeling.

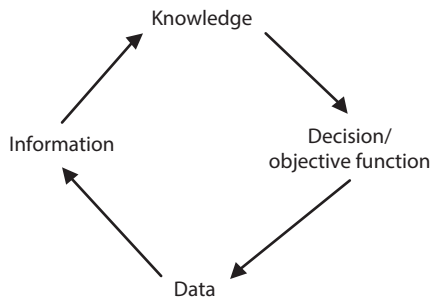
However, for a modeling process to be knowledge-based, it must fulfill two criteria, namely, the source has to be true (or real) and the subsequent processing has to be true (Islam *et al.*, 2012; 2015). The source is not a problem in the petroleum industry, as great deal of advances have been made on data collection techniques. The potential problem lies within the processing of data. For the process to be knowledge-based, the following logical steps have to be taken:

- Collection of data with constant improvement of the data acquisition technique. The data set to be collected is dictated by the objective function, which is an integral part of the decision making process. Decision making, however, should not take place without the abstraction process. The connection between objective function and data needs constant refinement. This area of research is one of the biggest strength of the petroleum industry, particularly in the information age.
- The gathered data should be transformed into Information so that they become useful. With today's technology, the amount of raw data is so huge, the need for a filter is more important than ever before. However, it is important to select a filter that doesn't skew data set toward a certain decision. Mathematically, these filters have to be non-linearized (Abou-Kassem *et al.*, 2006). While the concept of non-linear filtering is not new, the existence of non-linearized models is only beginning to be recognized (Islam, 2014).
- Information should be further processed into 'knowledge' that is free from preconceived ideas or a 'preferred decision'. Scientifically, this process must be free from information lobbying, environmental activism, and other forms of bias. Most current models include these factors as an integral part of the decision making process (Eisenack *et al.*, 2007), whereas a scientific knowledge model must be free from those interferences as they distort the abstraction process

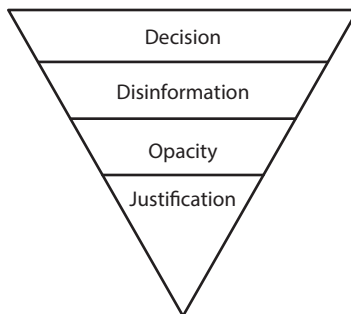
and inherently prejudice the decision making. Knowledge gathering essentially puts information into the big picture. For this picture to be distortion-free, it must be free from non-scientific maneuvering.

- Final decision making is knowledge-based, only if the abstraction from the above three steps has been followed without interference. Final decision is a matter of Yes or No (or True or False or 1 or 0) and this decision will be either knowledge-based or prejudice-based. Figure 1.1 shows the essence of the knowledge based decision making.

The process of aphenomenal or prejudice-based decision-making is illustrated by the inverted triangle, proceeding from the top down (Figure 1.2). The inverted representation stresses the inherent instability and unsustainability of the model. The source data from which a decision eventually emerges already incorporates their own justifications, which are then massaged by layers of opacity and disinformation.



**Figure 1.1** The knowledge model and the direction of abstraction.



**Figure 1.2** Aphenomenal decision-making.

The disinformation referred to here is what results when information is presented or recapitulated in the service of unstated or unacknowledged ulterior intentions (Zatzman and Islam, 2007a). The *methods* of this disinformation achieve their effect by presenting evidence or raw data selectively, without disclosing either the fact of such selection or the criteria guiding the selection. This process of selection obscures any distinctions between the data coming from nature or from any all-natural pathway, on the one hand, and data from unverified or untested observations on the other. In social science, such maneuvering has been well known, but the recognition of this aphenomenal (unreal) model is new in science and engineering (Shapiro *et al.*, 2007).

## 1.4 Summary of Chapters

Chapter 1 summarizes the main concept of the book. It introduces the knowledge-based approach as decision making tool that triggers the correct decision. This trigger, also called the criterion, is the most important outcome of the reservoir simulation. At the end, every decision hinges upon what criterion was used. If the criterion is not correct, the entire decision making process becomes aphenomenal, leading to prejudice. The entire tenet of the knowledge-based approach is to make sure the process is soundly based on truth and not perception with logic that is correct (phenomenal) throughout the cognition process.

Chapter 2 presents the background of reservoir simulation, as has been developed in last five decades. This chapter also presents the shortcomings and assumptions that do not have knowledge-base. It then outlines the need for new mathematical approach that eliminates most of the shortcomings and spurious assumptions of the conventional approach.

Chapter 3 presents the requirements in data input in reservoir simulation. It highlights various sources of errors in handling such data. It also presents guideline for preserving data integrity with recommendations for data processing that does not turnish the knowledge-based approach.

Chapter 4 presents the solutions to some of the most difficult problems in reservoir simulation. It gives examples of solutions without linearization and elucidates how the knowledge-based approach eliminates the possibility of coming across spurious solutions that are common in conventional approach. It highlights the advantage of solving governing equations without linearization and demarks the degree of errors committed through linearization, as done in the conventional approach.

Chapter 5 presents a complete formulation of black oil simulation for both isothermal and non-isothermal cases, using the engineering approach. It demonstrates the simplicity and clarity of the engineering approach.

Chapter 6 presents a complete formulation of compositional simulation, using the engineering approach. It shows how very complex and long governing equations are amenable to solutions without linearization using the knowledge-based approach.

Chapter 7 presents a comprehensive formulation of the material balance equation (MBE) using the memory concept. Solutions of the selected problems are also offered in order to demonstrate the need of recasting the governing equations using fluid memory. This chapter shows a significant error can be committed in terms of reserve calculation and reservoir behavior prediction if the comprehensive formulation is not used.

Chapter 8 presents formulations using memory functions. Such modeling approach is the essence of emulation of reservoir phenomena.

Chapter 9 uses the example of miscible displacement as an effort to model enhanced oil recovery (EOR). A new solution technique is presented and its superiority in handling the problem of viscous fingering is discussed.

Chapter 10 shows how the essence to emulation is to include the entire memory function of each variable concerned. The engineering approach is used to complete the formulation.

Chapter 11 highlights the future needs of the knowledge-based approach. A new combined mass and energy balance formulation is presented. With the new formulation, various natural phenomena related to petroleum operations are modeled. It is shown that with this formulation one would be able to determine the true cause of global warming, which in turn would help develop sustainable petroleum technologies. Finally, this chapter shows how the criterion (trigger) is affected by the knowledge-based approach. This caps the argument that the knowledge-based approach is crucial for decision making.

Chapter 12 shows how to model unconventional reservoirs. Various techniques and new flow equations are presented in order to capture physical phenomena that are prevalent in such reservoirs.

Chapter 13 presents the general conclusions of the book.

Chapter 14 is the list of references.

Appendix-A presents the manual for the 3D, 3-phase reservoir simulation program. This program is attached in the form of CD with the book.



# 2

## Reservoir Simulation Background

The Information Age is synonymous with Knowledge. However, if proper science is not used, information alone cannot guarantee *transparency*. Transparency is a pre-requisite of Knowledge (with a capital-K).

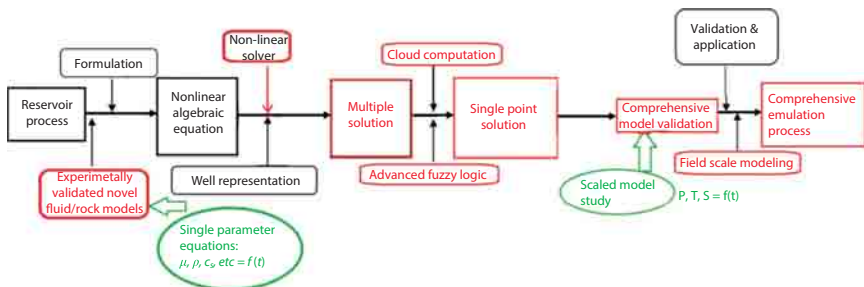
Proper science requires thinking or imagination with conscience, the very essence of humanity. Imagination is necessary for anyone wishing to make decisions based on science and always begins with visualization – actually, another term for simulation. There is a commonly-held belief that physical experimentation precedes scientific analysis, but the fact of the matter is that the simulation has to be worked out and visualized even before designing an experiment. This is why the petroleum industry puts so much emphasis on simulation studies. Similarly, the petroleum industry is known to be the biggest user of computer models. Unlike other large-scale simulations, such as space research and weather models, petroleum models do not have an option of verifying with real data. Because petroleum engineers do not have the luxury of launching a ‘reservoir shuttle’ or a ‘petroleum balloon’ to roam around the reservoir, the task of modeling is the most daunting. Indeed, from the advent of computer technology, the petroleum

industry pioneered the use of computer simulations in virtually all aspects of decision-making. From the golden era of petroleum industries, a very significant amount of research dollars have been spent to develop some of the most sophisticated mathematical models ever used. Even as the petroleum industry transits through its “middle age” in a business sense and the industry no longer carries the reputation of being the ‘most aggressive investor in research’, oil companies continue to spend liberally for reservoir simulation studies and even for developing new simulators.

## 2.1 Essence of Reservoir Simulation

Today, practically all aspects of reservoir engineering problems are solved with reservoir simulators, ranging from well testing to prediction of enhanced oil recovery. For every application, however, there is a custom-designed simulator. Even though, quite often, ‘comprehensive’, ‘All-purpose’, and other denominations are used to describe a company simulator, every simulation study is a unique process, starting from the reservoir description to the final analysis of results. Simulation is the art of combining physics, mathematics, reservoir engineering, and computer programming to develop a tool for predicting hydrocarbon reservoir performance under various operating strategies.

Figure 2.1 depicts the major steps involved in the development of a reservoir simulator (Odeh, 1982). In this figure, the *formulation* step outlines the basic assumptions inherent to the simulator, states these assumptions in precise mathematical terms, and applies them to a control volume in the reservoir. Newton’s approximation is used to render these control volume equations into a set of coupled, nonlinear partial differential equations (PDE’s) that describe fluid flow through porous media (Ertekin *et al.*, 2001). These PDE’s are then discretized, giving rise to a set of non-linear algebraic equations. Taylor series expansion is used to discretize



**Figure 2.1** Major steps involved in reservoir simulation with highlights of knowledge modeling.

the governing PDEs. Even though this procedure has been the standard in the petroleum industry for decades, only recently Abou-Kassem (2007) pointed out that there is no need to go through this process of expressing in PDE, followed by discretization. In fact, by setting up the algebraic equations directly, one can make the process simple and yet maintain accuracy (Mustafiz *et al.*, 2008). The PDEs derived during the *formulation* step, if solved analytically, would give reservoir pressure, fluid saturations, and well flow rates as continuous functions of space and time. Because of the highly nonlinear nature of a PDE, analytical techniques cannot be used and solutions must be obtained with numerical methods.

In contrast to analytical solutions, numerical solutions give the values of pressure and fluid saturations only at discrete points in the reservoir and at discrete times. *Discretization* is the process of converting the PDE into an algebraic equations. Several numerical methods can be used to discretize a PDEs. The most common approach in the oil industry today is the finite-difference method. To carry out discretization, a PDE is written for a given point in space at a given time level. The choice of time level (old time level, current time level, or the intermediate time level) leads to the explicit, implicit, or Crank-Nicolson formulation method. The discretization process results in a system of nonlinear algebraic equations. These equations generally cannot be solved with linear equation solvers and linearization of such equations becomes a necessary step before solutions can be obtained. *Well representation* is used to incorporate fluid production and injection into the nonlinear algebraic equations. *Linearization* involves approximating nonlinear terms in both space and time. Linearization results in a set of linear algebraic equations. Any one of several linear equation solvers can then be used to obtain the *solution*. The solution comprises of pressure and fluid saturation distributions in the reservoir and well flow rates. *Validation* of a reservoir simulator is the last step in developing a simulator, after which the simulator can be used for practical field applications. The validation step is necessary to make sure that no error was introduced in the various steps of development and in computer programming.

It is possible to bypass the step of formulating the PDE and directly express the fluid flow equation in the form of nonlinear algebraic equation as pointed out in Abou-Kassem *et al.* (2006). In fact, by setting up the algebraic equations directly, one can make the process simple and yet maintain accuracy. This approach is termed the “Engineering Approach” because it is closer to the engineer’s thinking and to the physical meaning of the terms in the flow equations. Both the engineering and mathematical approaches treat boundary conditions with the same accuracy if the mathematical approach uses second order approximations. The engineering approach is simple and yet general and rigorous.

There are three methods available for the discretization of any PDE: the Taylor series method, the integral method, and the variational method (Aziz and Settari, 1979). The first two methods result in the finite-difference method, whereas the third results in the variational method. The “Mathematical Approach” refers to the methods that obtain the nonlinear algebraic equations through deriving and discretizing the PDE’s. Developers of simulators relied heavily on mathematics in the mathematical approach to obtain the nonlinear algebraic equations or the finite-difference equations. A new approach that derives the finite-difference equations without going through the rigor of PDE’s and discretization and that uses fictitious wells to represent boundary conditions has been recently presented by Abou-Kassem (2007). This new approach is termed the “Engineering Approach” because it is closer to the engineer’s thinking and to the physical meaning of the terms in the flow equations. Both the engineering and mathematical approaches treat boundary conditions with the same accuracy if the mathematical approach uses second order approximations. The engineering approach is simple and yet general and rigorous. In addition, it results in the same finite-difference equations for any hydrocarbon recovery process. Because the engineering approach is independent of the mathematical approach, it reconfirms the use of central differencing in space discretization and highlights the assumptions involved in choosing a time level in the mathematical approach.

## 2.2 Assumptions Behind Various Modeling Approaches

Reservoir performance is traditionally predicted using three methods, namely, 1) Analogical; 2) Experimental, and 3) Mathematical. The analogical method consists of using mature reservoir properties that are similar to the target reservoir to predict the behavior of the reservoir. This method is especially useful when there is a limited available data. The data from the reservoir in the same geologic basin or province may be applied to predict the performance of the target reservoir. Experimental methods measure the reservoir characteristics in the laboratory models and scale these results to the entire hydrocarbons accumulation. The mathematical method applied basic conservation laws and constitutive equations to formulate the behavior of the flow inside the reservoir and the other characteristics in mathematical notations and formulations.

The two basic equations are the material balance equation or continuity equation and the equation of motion or momentum equation. These two equations are expressed for different phases of the flow in the reservoir and combine to obtain single equations for each phase of the flow. However, it is necessary to apply other equations or laws for modeling enhance oil recovery. As an example, the energy balance equation is necessary to analyze the reservoir behavior for the steam injection or in situ combustion reservoirs.

The mathematical model traditionally includes material balance equation, decline curve, statistical approaches and also analytical methods. The Darcy's law is almost used in all of available reservoir simulators to model the fluid motion. The numerical computations of the derived mathematical model are mostly based on the finite difference method. All these models and approaches are based on several assumption and approximations that may cause to produce erroneous results and predictions.

### 2.2.1 Material Balance Equation

The material balance equation is known to be the classical mathematical representation of the reservoir. According to this principle, the amount of material remaining in the reservoir after a production time interval is equal to the amount of material originally present in the reservoir minus the amount of material removed from the reservoir due to production plus the amount of material added to the reservoir due to injection.

This equation describes the fundamental physics of the production scheme of the reservoir. There are several assumptions in the material balance equation

- Rock and fluid properties do not change in space;
- Hydrodynamics of the fluid flow in the porous media is adequately described by Darcy's law;
- Fluid segregation is spontaneous and complete;
- Geometrical configuration of the reservoir is known and exact;
- PVT data obtained in the laboratory with the same gas-liberation process (flash vs. differential) are valid in the field;
- Sensitive to inaccuracies in measured reservoir pressure. The model breaks down when no appreciable decline occurs in reservoir pressure, as in pressure maintenance operations.

The advent of advanced well logging techniques, core-analysis methods, and reservoir characterization tools has eliminated (or at least created an opportunity to eliminate) the guesswork in volumetric methods. In absence of production history, volumetric methods offer a proper basis for the estimation of reservoir performance.

### 2.2.2 Decline Curve

The rate of oil production decline generally follows one of the following mathematical forms: exponential, hyperbolic and harmonic. The following assumptions apply to the decline curve analysis

- The past processes continue to occur in the future;
- Operation practices are assumed to remain same.

Figure 2.2 renders a typical portrayal of decline curve fitting. Note that all three declining curves fit closely during the first 2 years of production period, for which data are available. However, they produce quite different forecasts for later period of prediction. In old days, this was more difficult to discern because of the fact that a logarithmic curve was often used that skew the data even more. If any of the decline curve analysis is to be used for estimating reserves and subsequent performance prediction, the forecast needs reflect a “reasonable certainty” standard, which is almost certainly absent in new fields. This is why modern day use of the decline curve method is limited to generating multiple forecasts, with sensitivity data that create a boundary of forecast results (or cloud points), rather than exact numbers.

The usefulness of decline curve is limited under the most prevalent scenario of production curtail as well as very low productivity (or marginal

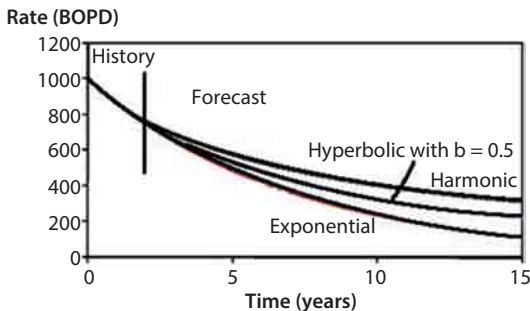


Figure 2.2 Decline curve for various forms.