Springer Theses

Recognizing Outstanding Ph.D. Research

Adam Ross Solomon

Cosmology Beyond Einstein

Springer

Springer Theses

Recognizing Outstanding Ph.D. Research

Aims and Scope

The series “Springer Theses” brings together a selection of the very best Ph.D. theses from around the world and across the physical sciences. Nominated and endorsed by two recognized specialists, each published volume has been selected for its scientific excellence and the high impact of its contents for the pertinent field of research. For greater accessibility to non-specialists, the published versions include an extended introduction, as well as a foreword by the student’s supervisor explaining the special relevance of the work for the field. As a whole, the series will provide a valuable resource both for newcomers to the research fields described, and for other scientists seeking detailed background information on special questions. Finally, it provides an accredited documentation of the valuable contributions made by today’s younger generation of scientists.

Theses are accepted into the series by invited nomination only and must fulfill all of the following criteria

- They must be written in good English.
- The topic should fall within the confines of Chemistry, Physics, Earth Sciences, Engineering and related interdisciplinary fields such as Materials, Nanoscience, Chemical Engineering, Complex Systems and Biophysics.
- The work reported in the thesis must represent a significant scientific advance.
- If the thesis includes previously published material, permission to reproduce this must be gained from the respective copyright holder.
- They must have been examined and passed during the 12 months prior to nomination.
- Each thesis should include a foreword by the supervisor outlining the significance of its content.
- The theses should have a clearly defined structure including an introduction accessible to scientists not expert in that particular field.

More information about this series at http://www.springer.com/series/8790

Adam Ross Solomon

Cosmology Beyond Einstein

Doctoral Thesis accepted by
the University of Cambridge, UK

Springer

Author
Dr. Adam Ross Solomon
Center for Particle Cosmology
University of Pennsylvania
Philadelphia
USA

Supervisor
Prof. John D. Barrow
Department of Applied Mathematics
and Theoretical Physics
The Centre for Mathematical Sciences
University of Cambridge
Cambridge
UK

ISSN 2190-5053 ISSN 2190-5061 (electronic)
Springer Theses
ISBN 978-3-319-46620-0 ISBN 978-3-319-46621-7 (eBook)
DOI 10.1007/978-3-319-46621-7

Library of Congress Control Number: 2016953643

Printed on acid-free paper

This Springer imprint is published by Springer Nature
The registered company is Springer International Publishing AG
The registered company address is: Gewerbestrasse 11, 6330 Cham, Switzerland

The effort to understand the universe is one
of the very few things which lifts
human life a little above the level of farce
and gives it some of the grace of tragedy.

Steven Weinberg, *The First Three Minutes*

This thesis is dedicated to my parents,
Scott and Edna Solomon,
without whose love and support
I could never have made it this far.

Supervisor's Foreword

Einstein's general theory of relativity is one of the most impressive human achievements. It superseded Newton's great work of 1687 to provide us with a new theory of gravitation that extended Newton's theory to domains where velocities could approach that of light and gravitational forces were correspondingly strong. In the limit that speeds are slow and gravity is weak, Einstein's theory is well approximated by Newton's. This is where modern physics departs from the Kuhnian story of scientific 'revolutions'. Old theories are not simply replaced by new ones. Rather, they become limiting cases of the new theory which will hold good in extreme situations where the old one cannot remain consistent. Yet, Einstein's theory went further than merely extending the domain of applicability of our theory of gravity. For the first time it provided a collection of differential equations whose solutions, all of them, describe entire universes. For the first time, cosmology became a science. Physicists could try to solve Einstein's equations in simple cases where there was lots of symmetry to find possible descriptions of our entire astronomical universe. These solutions could then be tested against the astronomical evidence and the subject began to resemble other experimental sciences. Although you cannot experiment on the universe—we only have one universe on display—you can predict correlations that should be observed between different properties of the same mathematical universe and look to see if they exist. In this way, cosmology has become a major scientific enterprise. It makes use of a host of new technologies to create light detectors of previously unimagined sensitivity right across the electromagnetic spectrum and has even begun to see direct evidence of gravitational waves. It has joined forces with elementary particle physicists to share insights and constraints on the behaviour of matter at the highest possible energies. And it has fully exploited the massive increase in computational ability that allows us to simulate the behaviour of large and complicated agglomerations of matter to follow the processes that have led to the formation of galaxies in the universe.

Einstein's theory of general relativity is a spectacular success and agrees with all the observational evidence to extraordinary accuracy. It would be fair to say that the agreement between theory and observation is so precise that in certain situations, like the binary pulsar's dynamics, it provides us with the surest and most accurate knowledge that human beings have of anything in their experience. So, why do we want to go 'beyond Einstein' in the words of Adam Solomon's thesis title? There are two main reasons. The first is that Einstein's theory has its limits of reliable applicability, just as Newton's theory does. When the density of matter gets too high, as it does near the apparent beginning of an expanding universe and near the centre of black holes, we expect quantum mechanics to modify the character of gravity in a way that will be described by some new theory of quantum gravity. Perhaps this future theory will modify general relativity so as to remove the 'singularities' that presently signal the beginning and of time at the beginning of the universe and the end of time in the inexorable contraction at the centres of black holes? The second reason to look beyond Einstein is of more recent origin. Just 17 years ago, astronomers first discovered that the expansion of the universe smoothly changed gear from deceleration to acceleration about 4 billion years ago. Why this occurred is a big mystery. Three different lines of astronomical evidence find the cause to be a ubiquitous form of energy in the universe—dubbed 'dark energy'—that is gravitationally repulsive. Physicists knew that quantum vacuum energy can have this repulsive effect because of its negative pressure, but no one expected the effects to be dramatically manifested so late in the universe's expansion history. About 70 % of the mass-energy in the universe seems to be in the form of this dark component. What is this dark energy? Is it just a new type of matter field that we have not identified and logged into the energy budget of the universe? This is one line of inquiry that cosmologists explore. The other is to investigate whether there are extensions of Einstein's theory of gravity which introduce new gravitational effects that act to accelerate the expansion of the universe when it is billions of years old and gravity is weak. These new behaviours of gravity need to be well circumscribed. They must not produce new adverse effects locally and in parts of cosmology where observations concur with Einstein's predictions to high accuracy. Adam Solomon's thesis explores a wide class of extensions to Einstein's theory to see whether they can potentially explain the observed acceleration of the universe and account for the existence of galaxies. These extensions cover theories which include a graviton with a non-zero mass and others, like bigravity, where there are two underlying spacetime metrics instead of one. These theories are mathematically more complicated than Einstein's and contain undesirable possibilities that need to be understood and excluded. Adam's thesis contains an elegant and systematic study of these theories, connecting abstract mathematical studies to astronomical predictions and observational tests of the theories. This analysis discovers new ways to solve the equations describing the growth of inhomogeneities and a facility with the observational data and statistical analysis needed to put them to the test. Adam combines a very wide range of mathematical skills and astrophysical understanding to advance our understanding of what a new theory of gravity that solves the dark energy problem is allowed to look like. The result is a valuable comprehensive study that will lead us a step closer towards the solution of the dark energy problem.

Cambridge, UK
August 2016

Prof. John D. Barrow

Abstract

The accelerating expansion of the Universe poses a major challenge to our understanding of fundamental physics. One promising avenue is to modify general relativity and obtain a new description of the gravitational force. Because gravitation dominates the other forces mostly on large scales, cosmological probes provide an ideal testing ground for theories of gravity. In this thesis, we describe two complementary approaches to the problem of testing gravity using cosmology.

In the first part, we discuss the cosmological solutions of massive gravity and its generalisation to a bimetric theory. These theories describe a graviton with a small mass, and can potentially explain the late-time acceleration in a technically natural way. I describe these self-accelerating solutions and investigate the cosmological perturbations in depth, beginning with an investigation of their linear stability, followed by the construction of a method for solving these perturbations in the quasistatic limit. This allows the predictions of stable bimetric models to be compared to observations of structure formation. Next, I discuss prospects for theories in which matter "doubly couples" to both metrics, and examine the cosmological expansion history in both massive gravity and bigravity with a specific double coupling which is ghost-free at low energies.

In the second and final part, we study the consequences of Lorentz violation during inflation. We consider Einstein-aether theory, in which a vector field spontaneously breaks Lorentz symmetry and couples nonminimally to the metric, and allow the vector to couple in a general way to a scalar field. Specialising to inflation, we discuss the slow-roll solutions in background and at the perturbative level. The system exhibits a severe instability which places constraints on such a vector–scalar coupling to be at least five orders of magnitude stronger than suggested by other bounds. As a result, the contribution of Lorentz violation to the inflationary dynamics can only affect the cosmic microwave background by an unobservably small amount.

Parts of this thesis have been published in the following journal articles:

Following the tendency of modern research in theoretical physics, most of the material discussed in this dissertation is the result of research in a collaboration network. In particular, Chaps. 3–7 were based on work done in collaboration with Yashar Akrami, Luca Amendola, Jonas Enander, Tomi Koivisto, Frank Könnig, Edvard Mörtsell, and Mariele Motta, published in Refs. [1-5] while Chap. 8 is the result of work done in collaboration with John Barrow, published as Ref. [6]. I have made major contributions to the above, in terms of both results and writing.

References

1. Y. Akrami, T.S. Koivisto, and A.R. Solomon, *The nature of spacetime in bigravity: two metrics or none?*, *Gen.Rel.Grav.* **47** (2014) 1838, [arXiv:1404.0006].
2. A.R. Solomon, Y. Akrami, and T.S. Koivisto, *Linear growth of structure in massive bigravity*, *JCAP* **1410** (2014) 066, [arXiv:1404.4061].
3. F. Könnig, Y. Akrami, L. Amendola, M. Motta, and A.R. Solomon, *Stable and unstable cosmological models in bimetric massive gravity*, *Phys.Rev.* **D90** (2014) 124014, [arXiv:1407.4331].
4. J. Enander, A.R. Solomon, Y. Akrami, and E. Mörtsell, *Cosmic expansion histories in massive bigravity with symmetric matter coupling*, *JCAP* **1501** (2015) 006, [arXiv:1409.2860].
5. A.R. Solomon, J. Enander, Y. Akrami, T.S. Koivisto, F. Könnig, and E. Mörtsell, *Cosmological viability of massive gravity with generalized matter coupling*, arXiv:1409.8300.
6. A.R. Solomon and J.D. Barrow, *Inflationary Instabilities of Einstein-Aether Cosmology*, *Phys.Rev.* **D89** (2014) 024001, [arXiv:1309.4778].

Acknowledgements

First and foremost, it is my pleasure to thank my supervisor, John Barrow, for his consistent support throughout my Ph.D. I am also deeply indebted to the many collaborators with whom I embarked on the work contained in this thesis, including Jonas Enander, Frank Könnig, Edvard Mörtsell, and Mariele Motta, for their insights and enlightening discussions. I am extremely grateful to Luca Amendola and Tomi Koivisto for, in addition to their collaboration, their generous hospitality in Heidelberg and Stockholm, stays which have been memorable both for their productivity and for their great fun. My very special thanks, finally, go to Yashar Akrami, for being a constant collaborator and close friend.

The work discussed in this thesis has benefitted from conversations with many people, including Tessa Baker, Phil Bull, Claudia de Rham, Pedro Ferreira, Fawad Hassan, Macarena Lagos, Johannes Noller, Angnis Schmidt-May, Sergey Sibiryakov, and Andrew Tolley.

Within DAMTP I have had the privilege of engaging in one of the most active cosmological communities I have known. Particular thanks belong to Peter Adshead, Mustafa Amin, Neil Barnaby, Daniel Baumann, Camille Bonvin, Anne-Christine Davis, Eugene Lim, Raquel Ribeiro, Paul Shellard, and Yi Wang for fostering such a dynamic intellectual atmosphere.

DAMTP has also provided respite from work when times were tough. I am grateful to the many friends I have made in the department, who are too numerous to list, for their seemingly inexhaustible appetite for any event where coffee or beer could be found. Special thanks must go to the DAMTP Culture Club for their company practically any time a concert, opera, Shakespearean play, or other staging of the finer things graced Cambridge. It was my great fortune to come through the Ph.D. at the same time as Valentin Assassi and Laurence Perreault-Levasseur, who each are both brilliant cosmologists and great friends. I am extraordinarily grateful to have had an academic “big brother” in Jeremy Sakstein, who has been a tireless friend both scientifically and personally, always as willing to lend an ear, his knowledge, or career advice as to run off to Scotland in a motorhome to tour whisky distilleries.[1] Despite my technically not being in her group, Amanda Stagg has been a constant source of administrative and moral support, except when it came to switching offices. Finally, I am grateful to the many friends outside the department, from Wolfson College, Sidney Sussex College, and wherever else I somehow managed to run into such incredible people, who have made my years in Cambridge some of the most fun and exhilarating of my life.

Finally, all of my love and thanks go to my entire family. Once again there are far too many people to name, but being family I can assume every one of them knows how grateful I am to them personally. This section would not be complete, however, without special thanks to my aunt and uncle, (Unk) Dane and (Tía) Reyna Solomon, for their preternatural willingness to open their hearts and their home to me. And last but not least, words cannot begin to express my gratitude to my family at home, Mom, Dad, Arielle, Jess, Oscar, and even Bubba.

During the course of my Ph.D., I have been supported by the David Gledhill Research Studentship, Sidney Sussex College, University of Cambridge; and by the Isaac Newton Fund and Studentships, University of Cambridge.

[1] Slange var!

Contents

1 **Introduction** 1
1.1 Conventions 5
1.2 General Relativity 6
1.3 The Cosmological Standard Model 8
1.4 Linear Perturbations Around FLRW 12
1.5 Inflation 14
References 19

2 **Gravity Beyond General Relativity** 21
2.1 Massive Gravity and Bigravity 21
2.1.1 Building the Massive Graviton 22
2.1.2 Ghost-Free Massive Gravity 28
2.1.3 Cosmological Solutions in Massive Bigravity 34
2.2 Einstein-Aether Theory 41
2.2.1 Pure Aether Theory 42
2.2.2 Coupling to a Scalar Inflation 44
2.2.3 Einstein-Aether Cosmology 45
References 47

Part I A Massive Graviton

3 **Cosmological Stability of Massive Bigravity** 55
3.1 Linear Cosmological Perturbations 56
3.1.1 Linearised Field Equations 57
3.1.2 Counting the Degrees of Freedom 60
3.1.3 Gauge Choice and Reducing the Einstein Equations 61
3.2 Stability Analysis 62
3.3 Summary of Results 68
References 69

4 Linear Structure Growth in Massive Bigravity 71
4.1 Perturbations in the Subhorizon Limit 72
4.2 Structure Growth and Cosmological Observables 74
4.2.1 Modified Gravity Parameters 74
4.2.2 Numerical Solutions 76
4.3 Summary of Results 98
References 100

5 The Geometry of Doubly-Coupled Bigravity 103
5.1 The Lack of a Physical Metric 104
5.2 Light Propagation and the Problem of Observables 107
5.3 Point Particles and Non-Riemannian Geometry 109
5.4 Summary of Results 113
References 114

6 Cosmological Implications of Doubly-Coupled Massive Bigravity 117
6.1 Doubly-Coupled Bigravity 118
6.2 Cosmological Equations and Their Solutions 120
6.2.1 Algebraic Branch of the Bianchi Constraint 122
6.2.2 Dynamical Branch of the Bianchi Constraint 123
6.3 Comparison to Data: Minimal Models 125
6.4 Special Parameter Cases 128
6.4.1 Partially-Massless Gravity 128
6.4.2 Vacuum Energy and the Question of Self-Acceleration 129
6.4.3 Maximally-Symmetric Bigravity 131
6.5 Summary of Results 131
References 132

7 Cosmological Implications of Doubly-Coupled Massive Gravity 135
7.1 Cosmological Backgrounds 136
7.2 Do Dynamical Solutions Exist? 139
7.3 Einstein Frame Versus Jordan Frame 140
7.4 Massive Cosmologies with a Scalar Field 141
7.5 Adding a Perfect Fluid 143
7.6 Mixed Matter Couplings 147
7.7 Summary of Results 149
References 151

Part II Lorentz Violation

8 Lorentz Violation During Inflation 155
8.1 Stability Constraint in Flat Space 157
8.2 Cosmological Perturbation Theory 162
8.2.1 Perturbation Variables 162
8.2.2 Linearised Equations of Motion 163

8.3 Spin-1 Cosmological Perturbations 164
8.3.1 Slow-Roll Limit 167
8.3.2 Full Solution for the Vector Modes. 169
8.3.3 Tachyonic Instability 170
8.3.4 What Values Do We Expect for Λ? 173
8.4 Spin-0 Cosmological Perturbations: Instability and Observability 177
8.4.1 The Spin-0 Equations of Motion 178
8.4.2 The Instability Returns 179
8.4.3 The Small-Coupling Limit 180
8.4.4 The Large-Coupling Limit: The Φ Evolution Equation 181
8.4.5 The Large-Coupling Limit: CMB Observables 184
8.5 Case Study: Quadratic Potential 186
8.5.1 Slow-Roll Inflation: An Example 186
8.5.2 The Instability Explored 189
8.6 Summary of Results 192
References. 194

9 Discussion and Conclusions 197
9.1 Problems Addressed in This Thesis 197
9.2 Summary of Original Results 199
9.2.1 Massive Gravity and Bigravity 199
9.2.2 Lorentz-Violating Gravity 200
9.3 Outlook 201
References. 205

Appendix A: Deriving the Bimetric Perturbation Equations 207

Appendix B: Explicit Solutions for the Modified Gravity Parameters 213

Appendix C: Transformation Properties of the Doubly-Coupled Bimetric Action 215

Appendix D: Einstein-Aether Cosmological Perturbation Equations in Real Space 221

Curriculum Vitae 225

Chapter 1
Introduction

I am always surprised when a young man tells me he wants to work at cosmology;
I think of cosmology as something that happens to one, not something one can choose.

Sir William McCrea, *Presidential Address, Royal Astronomical Society*

One of the driving aims of modern cosmology is to turn the Universe into a laboratory. By studying cosmic history at both early and late times, we have access to a range of energy scales far exceeding that which we can probe on Earth. It falls to us only to construct the experimental tools for gathering data and the theoretical tools for connecting them to fundamental physics.

The most obvious application of this principle is to the study of gravitation. Gravity is by far the weakest of the fundamental forces, yet on sufficiently large distance scales it is essentially the only relevant player; we can understand the motion of the planets or the expansion of the Universe to impressive precision without knowing the details of the electromagnetic, strong, or weak nuclear forces.[1] As a result, we expect the history and fate of our Universe to be intimately intertwined with the correct description of gravity. For nearly a century, the consensus best theory has been Einstein's remarkably simple and elegant theory of general relativity [1, 2]. This consensus is not without reason: practically all experiments and observations have lent increasing support to this theory, from classical weak-field observations such as the precession of Mercury's perihelion and the bending of starlight around the Sun, to the loss of orbital energy to gravitational waves in binary pulsar systems, observations remarkable both for their precision and for their origin in the strongest gravitational fields we have ever tested [3].

[1]Modulo the fact that we need, as input, to know which matter gravitates, and that the quantum field theories describing these forces are essential to understanding precisely which matter we have.

A.R. Solomon, *Cosmology Beyond Einstein*, Springer Theses,
DOI 10.1007/978-3-319-46621-7_1

Nevertheless, there are reasons to anticipate new gravitational physics beyond general relativity. In the ultraviolet (UV), i.e., at short distances and high energies, it is well known that general relativity is nonrenormalisable and hence cannot be extended to a quantum theory [4]. It must be replaced at such scales by a *UV-complete* theory which possesses better quantum behaviour. The focus of this thesis is on the infrared (IR), i.e., long distances and low energies. While general relativity is a theoretically-consistent IR theory, the discovery in 1998 that the expansion of the Universe is accelerating presents a problem for gravitation at the longest distances [5, 6]. The simplest explanation mathematically for this acceleration is a cosmological constant, which is simply a number that we can introduce into general relativity without destroying any of its attractive classical features. However, from a quantum-mechanical point of view, the cosmological constant is highly unsatisfactory. The vacuum energy of matter is expected to gravitate, and it would mimic a cosmological constant; however, the value it would generate is as much as 10^{120} times larger than the value we infer from observations [7–9]. Therefore, the "bare" cosmological constant which appears as a free parameter in general relativity would need to somehow know about this vacuum energy, and cancel it out almost *but not quite* exactly. Such a miraculous cancellation has no known explanation. Alternatively, one could imagine that the vacuum energy is somehow either rendered smaller than we expect, or does not gravitate—and theories which achieve this behaviour are known [8, 9]—but we would then most likely need a separate mechanism to explain what drives the current small but nonzero acceleration.

For these reasons, it behoves us to consider the possibility that general relativity may not be the final description of gravity on large scales. To put the problem in historical context, we may consider the story of two planets: Uranus and Mercury. In the first half of the nineteenth century, astronomers had mapped out the orbit of Uranus, then the farthest-known planet, to heroic precision. They found anomalies in the observed orbit when compared to the predictions made by Newtonian gravity, then the best understanding of gravitation available. Newton's theory had not yet been tested at distances larger than the orbit of Uranus: it was, for all intents and purposes, the boundary of the known universe. A natural explanation was therefore that Newtonian gravity simply broke down at such unimaginably large distances, to be replaced by a different theory. In 1846, French astronomer Urbain Le Verrier put forth an alternative proposal: that there was a new planet beyond Uranus' orbit, whose gravitational influence led to the observed discrepancies. Le Verrier predicted the location of this hitherto-unseen planet, and within weeks the planet Neptune was unveiled.

Buoyed by his success, Le Verrier turned his sights to another planet whose orbit did not quite agree with Newtonian calculations: Mercury, the closest to the Sun. As is now famous, the perihelion of Mercury's orbit precessed at a slightly faster rate than was predicted. Le Verrier postulated another new planet, Vulcan, within Mercury's orbit. However, the hypothesised planet was never found, and in the early parts of the twentieth century, Einstein demonstrated that general relativity accounted precisely for the perihelion precession. In the case of Mercury, it was a modification to the laws of gravity, rather than a new planet, which provided the solution.

We find ourselves in a similar position today. Our best theory of gravity, general relativity, combined with the matter we believe is dominant, mostly cold dark matter, predict a decelerating expansion, yet we observe something different. One possibility is that there is new matter we have not accounted for, such as a light, slowly-rolling scalar field. However, we must also consider that the theory of gravity we are using is itself in need of a tune-up.

The project of modifying gravity leads immediately to two defining questions: what does a good theory of modified gravity look like, and how can we test such theories against general relativity? This thesis aims to address both questions, although any answers we find necessarily comprise only a small slice of a deep field of research.

Einstein's theory is a paragon of elegance. It is practically inevitable that this is lost when generalising to a larger theory. Indeed, it is not easy to even define elegance once we leave the cosy confines of Einstein gravity. Consider, as an example, two equivalent definitions of general relativity, each of which can be used to justify the claim that GR is the simplest possible theory of gravity. First we can say that general relativity is the theory whose Lagrangian,

$$\mathscr{L} = \sqrt{-g}R, \tag{1.1}$$

known as the *Einstein-Hilbert term*, is the simplest diffeomorphism-invariant Lagrangian that can be constructed out of the metric tensor and its derivatives.[2] Alternatively, we could look at general relativity as being the unique Lorentz-invariant theory of a massless spin-2 field, or *graviton* [4, 10–13].

These serve equally well to tell us why general relativity is so lovely, but they diverge once we move to more general theories. Consider, for example, modifying the Lagrangian (1.1) by promoting the Ricci scalar R to a general function $f(R)$,

$$\mathscr{L} = \sqrt{-g}f(R). \tag{1.2}$$

This is the defining feature of $f(R)$ gravity, a popular theory of modified gravity [14–16]. One can certainly make the argument that this is mathematically one of the simplest possible generalisations of general relativity. However, when considered in terms of its fundamental degrees of freedom, we find a theory in which a spin-0 or scalar field interacts in a highly nonminimal way with the graviton [17].

Alternatively, one can consider massive gravity, in which the massless graviton of general relativity is given a nonzero mass. While this has a simple interpretation in the particle picture, its mathematical construction is so nontrivial that over seven decades were required to finally find the right answer. The resulting action, given in Eq. (2.21), is certainly not something one would have thought to construct had it not been for the guiding particle picture.

There are additional, more practical concerns when building a new theory of gravity. General relativity agrees beautifully with tests of gravity terrestrially and in the solar system, and it is not difficult for modified gravity to break that agreement.

[2]For an introduction to the Lagrangian formulation of general relativity, see Ref. [2].

While this may be surprising if we are modifying general relativity with terms that should only be important at the largest distance scales, it is not difficult to see that this problem is fairly generic. Any extension of general relativity involves adding new degrees of freedom (even massive gravity has three extra degrees of freedom), and in the absence of a symmetry forbidding such couplings, these will generally couple to matter, leading to gravitational-strength fifth forces. Such extra forces are highly constrained by solar-system experiments. Almost all viable theories of modified gravity therefore possess *screening mechanisms*, in which the fifth force is large cosmologically but is made unobservably small in dense environments. The details of these screening mechanisms are beyond the scope of this thesis, and we refer the reader to the reviews [18, 19].

In parallel with these concerns, we must ask how to experimentally distinguish modified gravity from general relativity. One approach is to use precision tests in the laboratory [20–27]. Another is to study the effect of modified gravity on astrophysical objects such as stars and galaxies [28–31]. In this thesis we will be concerned with cosmological probes of modified gravity. Because screening mechanisms force these modifications to hide locally (with some exceptions), it is natural to look to cosmology, where the new physics is most relevant. Cosmological tests broadly fall into three categories: background, linear, and nonlinear. Background tests are typically geometrical in nature, and try to distinguish the expansion history of a new theory of gravity from the general relativistic prediction. Considering small perturbations around the background, we obtain predictions for structure formation at linear scales. Finally, on small scales where structure is sufficiently dense, nonlinear theory is required to make predictions, typically using N-body computer simulations.

This thesis is concerned with the construction of theoretically-sensible modified gravity theories and their cosmological tests at the level of the background expansion and linear perturbations. In the first part, we focus on massive gravity and its extension to a *bimetric* theory, or massive bigravity, containing two dynamical metrics interacting with each other. In particular, we derive the cosmological perturbation equations for the case where matter couples to one of the metrics, and study the stability of linear perturbations by deriving a system of two coupled second-order evolution equations describing all perturbation growth and examining their eigenfrequencies. Doing this, we obtain conditions for the linear cosmological stability of massive bigravity, and identify a particular bimetric model which is stable at all times. We next move on to the question of observability, constructing a general framework for calculating structure formation in the quasistatic, subhorizon régime, and then applying this to the stable model.

After this, we tackle the question of matter couplings in massive gravity and bigravity, investigating a pair of theories in which matter is coupled to both metrics. In the first, matter couples minimally to both metrics. We show that there is not a single effective metric describing the geometry that matter sees, and so there is a problem in defining observables. In the second theory, matter does couple to an effective metric. We first study it in the context of bigravity, deriving its cosmological background evolution equations, comparing some of the simplest models to data, and examining in depth some particularly interesting parameter choices. We next examine the cosmological implications of massive gravity with such a matter coupling. Massive gravity normally possesses a no-go theorem forbidding flat cosmological solutions, but coupling matter to both metrics has been shown to overcome this. We examine this theory in detail, finding several stumbling blocks to observationally testing the new massive cosmologies.

The remainder of this thesis examines the question of Lorentz violation in the gravitational sector. We focus on Einstein-aether theory, a vector-tensor model which spontaneously breaks Lorentz invariance. We study the coupling between the vector field, or "aether," and a scalar field driving a period of slow-roll inflation. We find that such a coupling can lead to instabilities which destroy homogeneity and isotropy during inflation. Demanding the absence of these instabilities places a constraint on the size of such a coupling so that it must be at least 5 orders of magnitude smaller than the previous best constraints.

The thesis is organised as follows. In the rest of this chapter, we present background material, discussing the essential ingredients of general relativity and modern cosmology which will be important to understanding what follows. In Chap. 2 we give a detailed description of the modified gravity theories discussed in this thesis, specifically massive gravity, massive bigravity, and Einstein-aether theory, focusing on their defining features and their cosmological solutions. In Chaps. 3 and 4 we examine the cosmological perturbation theory of massive bigravity with matter coupled to one of the metrics. In Chap. 3 we study the stability of perturbations, identifying a particular bimetric model which is stable at all times, while in Chap. 4 we turn to linear structure formation in the quasistatic limit and look for observational signatures of bigravity. In Chaps. 5–7 we examine generalisations of massive gravity and bigravity in which matter couples to both metrics. Chapter 5 focuses on the thorny problem of finding observables in one such theory. In Chap. 6 we examine the background cosmologies of a doubly-coupled bimetric theory, and do the same for massive gravity in Chap. 7. Finally, in Chap. 8 we study the consequences of coupling a slowly-rolling inflaton to a gravitational vector field, or aether, deriving the strongest bounds to date on such a coupling. We conclude in Chap. 9 with a summary of the problems we have addressed and the work discussed, as well as an outlook on the coming years for modified gravity.

1.1 Conventions

Throughout this thesis we will use a mostly-positive $(-+++)$ metric signature. We will denote the flat-space or Minkowski metric by $\eta_{\mu\nu}$. Greek indices $\mu, \nu, \ldots = (0, 1, 2, 3)$ represent spacetime indices, while Latin indices $i, j, \ldots = (1, 2, 3)$ are used for spatial indices. Latin indices starting from $a, b, c, \ldots$ are also used for field-space and local Lorentz indices. Partial derivatives are denoted by ∂ and covariant derivatives by ∇. Commas and semicolons in indices will occasionally be used to represent partial and covariant derivatives, respectively, i.e., $\phi_{,\mu} \equiv \partial_\mu \phi$ and $\phi_{;\mu} \equiv \nabla_\mu \phi$. Symmetrisation and antisymmetrisation are denoted by

Table 1.1 Abbreviations used throughout this thesis

Abbreviation	Expression
BAO	Baryon-acoustic oscillations
CDM	Cold dark matter
CMB	Cosmic microwave background
FLRW	Friedmann-Lemaître-Robertson-Walker
GR	General relativity
SNe	Supernovae
VEV	Vacuum expectation value

$$S_{(\mu\nu)} \equiv \frac{1}{2}\left(S_{\mu\nu} + S_{\nu\mu}\right), \qquad A_{[\mu\nu]} \equiv \frac{1}{2}\left(A_{\mu\nu} - A_{\nu\mu}\right), \tag{1.3}$$

and similarly for higher-rank tensors. In lieu of the gravitational constant G we will frequently use the Planck mass, $M_{\rm Pl}^2 = 1/8\pi G$. Cosmic time is denoted by t and its Hubble rate is H, while we use τ for conformal time with the Hubble rate $\mathscr{H}$. For brevity we will sometimes use abbreviations for common terms, listed in Table 1.1.

1.2 General Relativity

This thesis deals with modified gravity. Consequently it behoves us to briefly overview the theory of gravity we will be modifying: Einstein's general relativity. The theory is defined by the Einstein-Hilbert action,

$$S_{\rm EH} = \frac{M_{\rm Pl}^2}{2}\int d^4x\sqrt{-g}R, \tag{1.4}$$

where $R = g^{\mu\nu}R_{\mu\nu}$ is the Ricci scalar, with $g_{\mu\nu}$ and $R_{\mu\nu}$ the metric tensor and Ricci tensor, respectively. Allowing for general matter, represented symbolically by fields Φ_i with Lagrangians $\mathscr{L}_m$ determined by particle physics, the total action of general relativity is

$$S = S_{\rm EH} + \int d^4x\sqrt{-g}\mathscr{L}_m\left(g, \Phi_i\right). \tag{1.5}$$

Varying the action S with respect to $g^{\mu\nu}$ we obtain the gravitational field equation, the Einstein equation,

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = 8\pi G T_{\mu\nu}, \tag{1.6}$$

where the stress-energy tensor of matter is defined by

$$T_{\mu\nu} \equiv -\frac{2}{\sqrt{-g}} \frac{\delta\left(\sqrt{-g}\mathscr{L}_m\right)}{g^{\mu\nu}}. \tag{1.7}$$

It is often convenient to define the Einstein tensor,

$$G_{\mu\nu} \equiv R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu}, \tag{1.8}$$

which is conserved as a consequence of the Bianchi identity,

$$\nabla_\mu G^\mu{}_\nu = 0. \tag{1.9}$$

Note that we are raising and lowering indices with the metric tensor, $g_{\mu\nu}$. The Bianchi identity is a geometric identity, i.e., it holds independently of the gravitational field equations. The stress-energy tensor is also conserved,

$$\nabla_\mu T^\mu{}_\nu = 0. \tag{1.10}$$

This is both required by particle physics and follows from the Einstein equation and the Bianchi identity, which is a good consistency check. A consequence of stress-energy conservation is that particles move on geodesics of the metric, $g_{\mu\nu}$,

$$\ddot{x}^\mu + \Gamma^\mu_{\alpha\beta}\dot{x}^\alpha \dot{x}^\beta = 0, \tag{1.11}$$

where $x^\mu(\lambda)$ is the position 4-vector of a test particle parametrised with respect to a parameter λ, an overdot denotes the derivative with respect to λ, and $\Gamma^\mu_{\alpha\beta} = \frac{1}{2}g^{\mu\nu}(g_{\alpha\nu,\beta} + g_{\beta\nu,\alpha} - g_{\alpha\beta,\nu})$ are the Christoffel symbols.

Einstein's equation relates the curvature of spacetime to the distribution of matter. Freely-falling particles then follow geodesics of the metric. The combination of the Einstein and geodesic equations leads to what we call the gravitational force. John Wheeler's description of gravity's nature is perhaps the most eloquent: "Spacetime tells matter how to move; matter tells spacetime how to curve" [32].

As discussed above, it seems clear to the eye that Eq. (1.4) is the simplest action one can construct for the gravitational sector, if one restricts oneself to scalar curvature invariants. Indeed, the simplicity of general relativity can be phrased in two equivalent ways. Lovelock's theorem states that Einstein's equation is the only gravitational field equation which is constructed solely from the metric, is no more than second order in derivatives,[3] is local, and is derived from an action [34]. Alternatively, as alluded to previously, the same field equations are the unique nonlinear equations of motion for

[3]The requirement that higher derivatives not appear in the equations of motion comes from demanding that the theory not run afoul of *Ostrogradsky's theorem*, which states that most higher-derivative theories are hopelessly unstable [33].