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Yuri B. Zudin

Theory of Periodic Conjugate Heat Transfer

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To my beloved wife Tatiana, who has always been my source of inspiration, to my children Maxim and Natalya, and to my grandchildren Alexey and Darya

Preface

The material presented in this book crowns my long-term activity in the field of conjugate periodic heat transfer. Its first stage had passed under the scientific supervision of my teacher Professor Labuntsov (1929−1992), starting by publication in 1977 of our first article and finishing in 1984 by publishing our book in Russian: Labuntsov D.A., Zudin Y.B., "Processes of heat transfer with periodic intensity." This stage was marked by the defense in 1980 of my Candidate Thesis: Zudin Y.B., "Analysis of heat transfer processes with periodic intensity." The subsequent period of interpreting the already gained results and accumulation of new knowledge had taken seven years. In 1991 I started working on a new series of publications on this subject, which culminated in this book, the first edition of which appeared in 2007, and the second one (refreshed and extended), in 2011. This stage was also marked with my habilitation (Zudin Y.B., "Approximate theory of heat transfer processes with periodic intensity," 1996), as well as with fruitful scientific collaboration with my respected German colleagues: Prof. U. Grigull, Prof. F. Mayinger, Prof. J. Straub and Prof. T. Sattelmayer (TU München), Prof. W. Roetzel (Uni BW Hamburg), Prof. J. Mitrovic and Prof. D. Gorenflo (Uni Paderborn), Prof. K. Stephan, Prof. M. Groll, and Prof. B. Weigand (Uni Stuttgart).

The objective of the present monograph is to give an exhaustive answer to the question of how thermophysical and geometrical parameters of a body govern the heat transfer characteristics under conditions of thermohydraulic pulsations. An applied objective of this book is to develop a universal method for the calculation of the average heat transfer coefficient for the periodic conjugate processes of heat transfer.

As a rule, real "steady" processes of heat transfer can be looked upon as steady ones only on the average. In the actual fact, periodic, quasiperiodic, and various random fluctuations of parameters (velocities, pressure, temperatures, momentum and energy fluxes, vapor content, interface boundaries, etc.) around their average values always exist in any type of fluid flow, except for purely laminar flows. Owing to the conjugate nature of the interface "fluid flow-streamlined body," both the fluctuation and the average values of temperatures and heat fluxes on the heat transfer surface generally depend on thermophysical and geometrical characteristics of a heat transferring wall.

This suggests the principle question about the possible effect of the material and the thickness of the wall on the key parameter of convective heat transfer, namely, the heat transfer coefficient. Such an effect was earlier manifested in experimental investigations of heat transfer at nucleate boiling, dropwise condensation, and in some other cases. In these studies, the heat transfer coefficients, as defined as the ratio of the average heat flux on the surface and the average temperature difference "wall–fluid", could differ markedly for various materials of the wall (and also for different thicknesses of walls).

In 1977, a concept of a true heat transfer coefficient was first proposed in the work of Labuntsov and Zudin. According to this concept, the actual values of the heat transfer coefficient (for each point of the heat transferring surface and at each moment of time) are determined solely by the hydrodynamic characteristics of the fluid flow; as a result, they are independent of the parameters of a body. Fluctuations of parameters occurring in the fluid flow will result in the respective fluctuations of the true heat transfer coefficient, which is also independent of the material and thickness of the wall. This being so, from the solution of the heat conduction equation with a boundary condition of third kind, it is possible to find the temperature field in the body (and, hence, on the heat transfer surface), and as a result, to calculate the required experimental heat transfer coefficient as the ratio of the average heat flux to the average temperature difference. This value (as determined in traditional heat transfer experiments and employed in applied calculations) should in general case depend on the conjugation parameters.

The study of the relations between the heat transfer coefficients averaged by different methods (the true and experimental ones) laid the basis for the first edition of the present book, in which the following fundamental result was obtained: the average experimental value of the heat transfer coefficient is always smaller than the average true value of this parameter.

The first edition of this book (2007) involved seven chapters. The second edition (2011) was augmented with two new chapters. The third edition, incorporating (without any changes) the content of the second editions, contains three new chapters (8, 9, and 10). Below we give a brief summary of the contents of this book.

Chapter 1 gives a qualitative description of the method for investigations of periodic conjugate convective–conductive problems "fluid flow–streamlined body." An analysis of physical processes representing heat transfer phenomena with periodic fluctuations is also given.

In Chap. 2, a boundary problem for the two-dimensional unsteady heat conduction equation with a periodic boundary condition of third kind is examined. To characterize the thermal effects of a solid body on the average heat transfer, a concept of the factor of conjugation was introduced. The quantitative effect of the conjugation in the problem was shown to be rather significant.

Chapter 3 puts forward the construction of a general solution for the boundary value problem for the equation of heat conduction with a periodic boundary condition of third kind. Analytic solutions were obtained for the characteristic laws of variation of the true heat transfer coefficient, namely, the harmonic, inverse harmonic, stepwise and delta-like ones.

In Chap. 4, a universal algorithm of a general approximate solution of the problem is developed. On its basis, solutions are obtained for a series of problems with different laws of periodic fluctuations of the true heat transfer coefficient.

Chapter 5 deals with conjugate periodic heat transfer for involved cases of external heat supply: the heat transfer at a contact either with environment or with a second body. A generalized solution for the factor of conjugation for the bodies of the "standard form" is obtained. A problem of conjugate heat transfer for the case of bilateral periodic heat transfer is also investigated. The cases of asymmetric and non-periodic fluctuations of the true heat transfer coefficient are examined.

Chapter 6 includes some applied problems of the periodic conjugate heat transfer theory such as jet impingement onto a surface, dropwise condensation, and nucleate boiling.

Chapter 7 is concerned with effects of the thermophysical parameters and the channel wall thickness on the hydrodynamic instability of the so-called "density waves." The boundary of stability of fluid flow in a channel at supercritical pressures is found analytically. As an application, the problem of dealing efficient performance a thermal regulation system for superconducting magnets is considered.

In Chap. 8 the Landau problem on the evaporation front stability is generalized to the case of finite thickness of the evaporating liquid layer. The analysis of the influence of additional factors, the impermeability condition of solid wall and resulting pulsations of mass velocity, is carried out. Parametric calculations of the stability boundary are performed when changing the liquid film thickness and the relationship between phase densities in the framework of asymptotic Landau approach for the large Reynolds number. Approximate evaluation of the influence of liquid viscosity on the stability boundary has been done.

Chapter 9 deals with the hyperbolic heat conduction equation. An extension of the algorithm of computation of the factor of conjugation is given. The limiting case described by the telegraph equation is considered. The boundary between the Fourier and Cattaneo–Vernotte laws is found.

Chapter 10 is concerned with the derivation of the generalized Rayleigh equation that describes the dynamics of a gas bubble is given. Its solution has spherical and cylindrical asymptotics. A periodic quantum mechanical model is offered for the process of homogeneous bubble nucleation. The droplet size distribution in a turbulent flow is examined.

Chapter 11 examines the periodic slug flow in a two-phase media. One of the important parameter of periodic two-phase flows (the rise velocity of the Taylor bubbles in round pipes) is determined.

Chapter 12 develops an analytic method for calculation of heat exchange for a turbulent flow in a channel of fluid in a region of supercritical pressures. This method is capable of taking into account the effect of variability of thermophysical properties of a fluid on the heat transfer coefficient, as averaged over the period of turbulent pulsations.

In Appendix A, proofs are given of some properties of the two-dimensional unsteady equation of heat conduction with a periodic boundary condition of third kind. As a corollary, we find the limiting values of the factor of conjugation.

Appendix B examines the eigenfunctions of the solution to the two-dimensional unsteady equation of heat conduction, as obtained by the method of separation of variables.

In Appendix C, the problem of convergence of infinite continued fraction was considered. An extension of the proof of Khinchin's third theorem to the case where the terms in the fraction possess a negative sign was obtained.

In Appendix D, a proof of divergence of infinite series obtained in Chap. 3 for the particular solution of the heat conduction equation is given.

In Appendix E the approximate solutions from Chap. 4 are corrected for various laws of oscillation of the THTC (harmonic law, inverse harmonic law, and step law).

I am deeply grateful to Prof. Wilfried Roetzel (Helmut-Schmidt-Universität/Universität der Bundeswehr Hamburg), the meeting with whom in 1995 served as a starting point in planning the present book and in the formation of its ideology. During each subsequent stay in Germany I enjoyed fruitful discussions with Prof. Roetzel, which have substantially helped me in the preparation of the book.

I would like to deeply thank the Director of the ITLR, Series Editor Mathematical Engineering of Springer-Verlag, Prof. Dr.-Ing. habil. Bernhard Weigand for his strong support of my aspiration to successfully accomplish this work, as well as for his numerous valuable advices and fruitful discussions. My collaboration with Prof. Bernhard Weigand (Universität Stuttgart) started in 2005, who actively supported my idea to write a book and repeatedly invited me to visit the Institute of Aerospace Thermodynamics to perform joint research.

I am deeply indebted to Dr. Jan-Philip Schmidt, Editor of Springer-Verlag, for his keen interest in the publication of this book and his successful marketing of this book.

The publication of all three editions of this book would have been impossible without the long-term financial support of my activity in German universities (TU München, Uni Paderborn, Uni Stuttgart, HSU/Uni Bundeswehr Hamburg) from the German Academic Exchange Service (DAAD), which I very gratefully acknowledge. Being happy sevenfold (!) grantee of DAAD, I would like to express my sincere gratitude to the people who have made it possible: Dr. T. Prahl, Dr. G. Berghorn, Dr. P. Hiller, Dr. H. Finken, Dr. W. Trenn, and also to all other DAAD employees both in Bonn, and in Moscow.

Particular gratitude is due to my wife Tatiana, who always served me as an invaluable moral support in my lifelong scientific activity. I am greatly obliged to my beloved spouse for my academic degree of Prof. Dr.-Ing. habil. and also for the appearance of all three editions of my book.

I dare to hope that the third edition of this book will be so favorably accepted by readers, as the first and second ones.

Stuttgart, Germany
October 2016

Yuri B. Zudin

Contents

Abbreviations

ATHTC	Averaged True Heat Transfer Coefficient
BC	Boundary Condition
EHTC	Experimental Heat Transfer Coefficient
FC	Factor of Conjugation
HN	Homogeneous Nucleation
HTC	Heat Transfer Coefficient
MRC	Method of Relative Correspondence
PTE	Parameter of the Thermal Effect
SCP	Supercritical Pressures
SRM	Surface Rejuvenation Model
TBC	Thermal Boundary Conditions
THTC	True Heat Transfer Coefficient

Symbols

A_k, A_k^*	Complex conjugate eigenvalues (–)
B_k, B_k^*	Complex conjugate eigenfunctions (–)
b	Amplitude of oscillations of the true heat transfer coefficient (–)
$C_f/2$	Friction factor (–)
c	Specific heat (J/(kg K))
d_0	Nozzle diameter (m)
F_k	Real parts of eigenfunctions (–)
h	True heat transfer coefficient (THTC) (W/(m^2 K))
$\langle h \rangle$	Averaged true heat transfer coefficient (ATHTC) (W/(m^2 K))
$\bar{h}$	Dimensionless averaged true heat transfer coefficient or Biot number (–)
h_m	Experimental heat transfer coefficient (EHTC) (W/(m^2 K))
h_0	Steady state heat transfer coefficient (W/(m^2 K))
$\bar{h}_0$	Dimensionless stationary heat transfer coefficient (–)
h_{fg}	Specific enthalpy of evaporation (J/(kg))
I_n	Imaginary parts of eigenfunctions (–)
Ja	Jacob number (–)
k	Thermal conductivity (W/(m K))
L	Distance between nucleate boiling sites (m)
m	Inverted Fourier number (–)
n_F	Number of boiling sites (1/m^2)
K	Ratio of thermal potentials of contacting media (–)
p	Pressure (Pa)
Pr	Prandtl number (–)
q	Heat flux density (W/(m^2))
$\langle q \rangle$	Averaged heat flux density (W/(m^2))
$\hat{q}$	Oscillating heat flux density (W/(m^2))
q_V	Volumetric heat source (W/(m^3))
R_n	Real parts of eigenvalues (–)
R_*	Critical radius of vapor nucleus (m)

St	Stanton number (–)
t	Dimensionless time (–)
T_s	Saturation temperature (K)
u	Velocity (m/s)
u_0	Free stream velocity (m/s)
u_*	Friction velocity (m/s)
U	Overall heat transfer coefficient (W/(m^2 K))
$\langle U \rangle$	Averaged true overall heat transfer coefficient (W/(m^2 K))
U_m	Experimental overall heat transfer coefficient (W/(m^2 K))
$\langle \bar{U} \rangle$	Dimensionless averaged true overall heat transfer coefficient (–)
E	Generalized factor of conjugation (–)
X	Spanwise coordinate (m)
x	Dimensionless spanwise coordinate (–)
Z	Coordinate along the surface of heat transfer (m)
Z_0	Spatial periods of oscillation (m)
z	Dimensionless coordinate along the heat transfer surface (–)

Greek Letter Symbols

α	Thermal diffusivity (m^2/s)
Γ	Shear stress (N/m^2)
δ	Wall thickness (flat plate) (m)
$\bar{\delta}$	Dimensionless wall thickness (flat plate) (–)
δ_f	Thickness of liquid film (m)
ε	Factor of Conjugation (FC) (–)
ϑ	Temperature (K)
$\langle \vartheta \rangle$	Averaged temperature (K)
$\hat{\vartheta}$	Oscillating temperature (K)
ϑ_0	Free stream temperature (K)
$\vartheta^\bullet$	Gradient of oscillating temperature or dimensionless heat flux density (K/m)
ϑ_Σ	Total temperature difference in the three-part system (K)
θ	Dimensionless oscillations temperature (–)
$\theta^\bullet$	Dimensionless gradient of the oscillation temperature (or dimensionless heat flux density) (–)
ξ	Generalized coordinate of a progressive wave (–)
ξ_ϑ	Phase shift between oscillation of true heat transfer coefficient and temperature (–)
ξ_q	Phase shift between oscillation of true heat transfer coefficient and heat flux (–)
μ	Dynamic viscosity (kg/m s)
ν	Kinematic viscosity (m^2/s)
ρ	Density (kg/m^3)
σ	Surface tension (N/m)

τ	Time (s)
τ_0	Time period of oscillation (s)
Φ_k	Imaginary parts of eigenfunctions (–)
χ	Parameter of thermal effect (PTE) (–)
ψ	Periodic part of the heat transfer coefficient (–)
ω	Frequency (1/s)

Subscripts

+	Active period of heat transfer
−	Passive period of heat transfer
f	Fluid
g	Gas
0	External surface of a body (at $X = 0$)
δ	Heat transfer surface (at $X = \delta$)
min	Minimal value
max	Maximal value
w	Another (second) body

Definition of Nondimensional Numbers and Groups

$\langle \bar{h} \rangle = \frac{\langle h \rangle Z_0}{k}$	Dimensionless averaged true heat transfer coefficient or Biot number
$\bar{h}_0 = \frac{h_0 Z_0}{k}$	Dimensionless stationary heat transfer coefficient
$\mathrm{E} = \frac{U_m}{\langle U \rangle}$	Generalized factor of conjugation
$Ja = \frac{\rho_f c_{pf} \vartheta}{\rho_g h_{fg}}$	Jacob number
$K = \sqrt{\frac{k\,c\,\rho}{k_f c_f \rho_f}}$	Ratio of thermal potentials of the contacting media
$m = \frac{Z_0^2}{\alpha \tau_0}$	Inverted Fourier number
$\Pr = \frac{\nu_f}{\alpha_f}$	Prandtl number
$St = \frac{q}{\rho_f c_f u_0 \vartheta_0}$	Stanton number
$\bar{U}_m = \frac{U_m Z_0}{k}$	Dimensionless averaged true overall heat transfer coefficient
$\bar{U}_m = \frac{U_m Z_0}{k}$	Dimensionless experimental overall heat transfer coefficient
$\bar{\delta} = \frac{\delta}{Z_0}$	Dimensionless wall thickness (flat plate)
$\varepsilon = \frac{h_m}{\langle h \rangle}$	Factor of conjugation

Chapter 1
Introduction

1.1 Heat Transfer Processes Containing Periodic Oscillations

1.1.1 Oscillation Internal Structure of Convective Heat Transfer Processes

Real stationary processes of heat transfer, as a rule, can be considered stationary only on the average. Actually (except for the purely laminar cases), flows are always subjected to various periodic, quasiperiodic, and other casual oscillations of velocities, pressure, temperatures, momentum and energy fluxes, vapor content, and interphase boundaries about their average values. Such oscillations can be smooth and periodic (wave flow of a liquid film or vapor, a flow of a fluctuating coolant over a body), sharp and periodic (hydrodynamics and heat transfer at slug flow of a two-phase media in a vertical pipe; nucleate and film boiling process), on can have complex stochastic character (turbulent flows). Oscillations of parameters have in some cases spatial nature, in others they are temporal, and generally one can say that the oscillations have mixed spatiotemporal character.

The theoretical base for studying instantly oscillations and at the same time stationary on the average heat transfer processes are the unsteady differential equations of momentum and energy transfer, which in case of two-phase systems can be notated for each of the phases separately and be supplemented by transmission conditions (transmission conditions). An exhaustive solution of the problem could be a comprehensive analysis with the purpose of a full description of any particular fluid flow and heat transfer pattern with all its detailed characteristics, including various fields of oscillations of its parameters.

However, at the time being such an approach cannot be realized in practice. The problem of modeling turbulent flows [1] can serve as a vivid example. As a rule at its theoretical analysis, Reynolds-averaged Navier–Stokes equations are considered, which describe time-averaged quantities of fluctuating parameters, or in other words

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turbulent fluxes of the momentum and energy. To provide a closed description of the process, these correlations by means of various semi-empirical hypotheses are interrelated with time-averaged fields of velocities and enthalpies. Such schematization results in the statement of a stationary problem with spatially variable coefficients of viscosity and thermal conductivity. Therefore, as boundary conditions here, it is possible to set only respective stationary conditions on the heat transfer surface of such a type as, for example, "constant temperature," "constant heat flux."

It is necessary to specially note, that the replacement of the full "instant"s model description with the time-averaged one inevitably results in a loss of information on the oscillations of fluid flow and heat transfer parameters (velocities, temperatures, heat fluxes, pressure, friction) on a boundary surface. Thus the theoretical basis for an analysis of the interrelation between the temperature oscillations in the flowing ambient medium and in the body is omitted from the consideration. And generally saying, the problem of an account for possible influence of thermophysical and geometrical parameters of a body on the heat transfer at such on approach becomes physically senseless. For this reason, such a "laminarized" form of the turbulent flow description is basically not capable of predicting and explaining the wall effects on the heat transfer characteristics, even if these effects are observed in practice. The problem becomes especially complicated at imposing external oscillations on the periodic turbulent structure that takes place, in particular, flows over aircraft and spacecraft. Unresolved problems of closing the Navier–Stokes equations in combination with difficulties of numerical modeling make a problem of detailed prediction of a temperature field in the flowing fluid very complicated. In some cases, differences between the predicted and measured local "heat transfer coefficient" (HTC) exceeds 100 %.

In this connection, the direction in the simulation of turbulent flows based on the use of the primary transient equations [2] represents significant interest. The present book represents results of numerical modeling of the turbulent flows in channels subjected to external fields of oscillations (due to vortical generators, etc.). It is shown that in this case an essentially anisotropic and three-dimensional flow pattern emerges strongly different from that described by the early theories of turbulence [1]. In the near-wall zone, secondary flows in the form of rotating "vortical streaks" are induced that interact with the main flow. As a result, oscillations of the thermal boundary layer thickness set on, leading to periodic enhancement or deterioration of heat transfer. Strong anisotropy of the fluid flow pattern results in the necessity of a radical revision of the existing theoretical methods of modeling the turbulent flows. So, for example, the turbulent Prandtl number being in early theories of turbulence [1] a constant of the order of unity (or, at the best, an indefinite scalar quantity), becomes a tensor.

It is necessary to emphasize that all the mentioned difficulties are related to the nonconjugated problem when the role of a wall is reduced only to maintenance of a "boundary condition" (BC) on the surface between the flowing fluid and the solid wall.

1.1.2 Problem of Correct Averaging the Heat Transfer Coefficients

The basic applied task of the book is the investigation into the effects of a body (its thermophysical properties, linear dimensions, and geometrical configuration) on the traditional heat transfer coefficient (HTC), measured in experiments and used in engineering calculations. Processes of heat transfer are considered stationary on average and fluctuating instantly. A new method for investigating the conjugate problem "fluid flow—body" is presented. The method is based on a replacement of the complex mechanism of oscillations of parameters in the flowing coolant by a simplified model employing a varying "true heat transfer coefficient" specified on a heat transfer surface.

The essence of the developed method can be explained rather simply. Let us assume that we have perfect devices measuring the instant local values of temperature and heat fluxes at any point of the fluid and heated solid body. Then the hypothetical experiment will allow finding the fields of temperatures and heat fluxes and their oscillations in space and in time, as well as their average values and all other characteristics. In particular, it is possible to present the values of temperatures (exact saying, temperature heads or loads, i.e., the temperatures counted from a present reference level) and heat fluxes on a heat transfer surface in the following form:

$$\vartheta = \langle \vartheta \rangle + \hat{\vartheta} \tag{1.1}$$

$$q = \langle q \rangle + \hat{q} \tag{1.2}$$

i.e., to write them as the sum of the averaged values and their temporal oscillations. For the general case of spatiotemporal oscillations of characteristics of the process, the operation of averaging is understood here as a determination of an average with respect to time τ and along the heat transferring surface (with respect to the coordinate Z). The "true heat transfer coefficient" (THTC) is determined on the basis of Eqs. (1.1–1.2) according to Newton's law of heat transfer [3, 4]

$$h = \frac{q}{\vartheta} \tag{1.3}$$

This parameter can always be presented as a sum of an averaged part and a fluctuating additive

$$h = \langle h \rangle + \hat{h} \tag{1.4}$$

Averaged True Heat Transfer Coefficient

It follows from here that the correct averaging of the HTC is as follows:

$$\langle h \rangle = \left\langle \frac{q}{\vartheta} \right\rangle \tag{1.5}$$

Therefore we shall call parameter $\langle h \rangle$ an "averaged true heat transfer coefficient" (ATHTC). The problem consists in the fact that the parameter $\langle h \rangle$ cannot be directly used for applied calculations, since it contains initially the unknown information of oscillations $\hat{\vartheta}$, $\hat{q}$. This fact becomes evident if Eq. (1.5) is rewritten with the help of Eqs. (1.1–1.2)

$$\langle h \rangle = \left\langle \frac{\langle q \rangle + \hat{q}}{\langle \vartheta \rangle + \hat{\vartheta}} \right\rangle \tag{1.6}$$

Experimental Heat Transfer Coefficient

The purpose of the heat transfer experiment is the measurement of averaged values on averaged temperature $\langle \vartheta \rangle$ and a heat flux $\langle q \rangle$ on the surfaces of a body and determination of the traditional HTC

$$h_m = \frac{\langle q \rangle}{\langle \vartheta \rangle} \tag{1.7}$$

The parameter h_m is fundamental for carrying out engineering calculations, designing heat transfer equipment, composing thermal balances, etc. However it is necessary to point out that transition from the initial Newton's law of heat transfer (1.3) to the restricted Eq. (1.7) results in the loss of the information of the oscillations of the temperature $\hat{\vartheta}$ and the heat fluxes $\hat{q}$ on the wall.

Thus, it is logical to assume that the influence of the material and the wall thickness of the body taking part in the heat transfer process on HTC h_m uncovered in experiments is caused by non-invariance of the value of h_m with respect to the Newton's law of heat transfer. For this reason we shall refer further to the parameter h_m as to an "experimental heat transfer coefficient" (EHTC).

Distinction Between $\langle h \rangle$ and the h_m

Thus, we have two alternative procedures of averaging the HTC: true Eq. (1.5) and experimental Eq. (1.7). The physical reason of the distinction between $\langle h \rangle$ and the h_m can be clarified with the help of the following considerations:

- Local values $\langle \vartheta \rangle$ and $\langle q \rangle$ on a surface where heat transfer takes place are formed as a result of the thermal contact of the flowing fluid and the body.
- Under conditions of oscillations of the characteristics of the coolant, temperature oscillations will penetrate inside the body.
- Owing to the conjugate nature of the heat transfer in the considered system, both fluctuating $\hat{\vartheta}$, $\hat{q}$, and averaged $\langle \vartheta \rangle$, $\langle q \rangle$ parameters on the heat transfer surface depend on the thermophysical and geometrical characteristics of the body.

- The ATHTC $\langle h \rangle$ directly follows from Newton's law of heat transfer (1.3) (which is valid also for the unsteady processes) and consequently it is determined by hydrodynamic conditions in the fluid flowing over the body.
- The EHTC h_m by definition does not contain the information on oscillations $\hat{\vartheta}$, $\hat{q}$, and consequently it is in the general case a function of parameters of the interface between fluid and solid wall.
- Aprioristic denying of dependence of the EHTC on material properties and wall thickness is wrong, though under certain conditions quantitative effects of this influence might be insignificant.

From the formal point of view, the aforementioned differences between the true (1.5) and experimental (1.7) laws of averaging of the actual HTC is reduced to a rearrangement of the procedures of division and averaging. This situation is illustrated evidently in Fig. 1.1.

Using the concepts introduced above, the essence of a suggested method can be explained rather simply. We shall assume that for the case under investigation the HTC h is known:$h = h(Z, \tau)$, where Z and τ are the coordinate along a surface where heat transfer takes place and the time, respectively. According to the internal structure of the considered processes this parameter should have periodic, quasiperiodic or generally fluctuating nature, varying about its average value $\langle h \rangle$: $h = \langle h \rangle + \hat{h}(Z, \tau)$. This information is basically sufficient for the definition of actual driving temperature difference $\vartheta(Z, \tau)$ heat fluxes $q(Z, \tau)$ in a massive of a heat transferring body, and, hence, on the heat transfer surface. Thus, the calculation is reduced to a solution of a boundary value problem of the unsteady heat conduction equation [5]

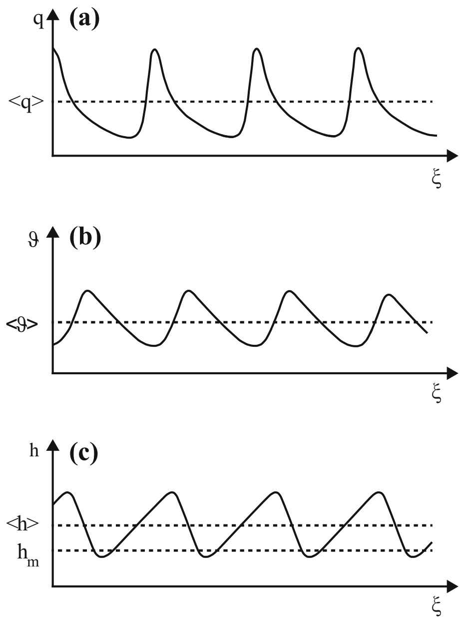

Fig. 1.1 True and experimental laws of the averaging of the heat transfer coefficient: **a** heat flux density on the heat transfer surface, **b** temperature difference "wall–ambience", **c** heat transfer coefficient

$$\frac{\partial \vartheta}{\partial \tau} = \alpha \left(\frac{\partial^2 \vartheta}{\partial X^2} + \frac{\partial^2 \vartheta}{\partial Z^2} \right) + \frac{q_V}{c\rho} \tag{1.8}$$

with the boundary condition (BC) of the third kind on the heat transfer surface

$$-k \frac{\partial \vartheta}{\partial X} = h\vartheta \tag{1.9}$$

and suitable BC on the external surfaces of the body.

It is essential for our analysis that up to the same extent in which the information about the function $h = h\,(Z,\, \tau)$ is trustworthy, the computed parameters $\vartheta\,(Z,\, \tau)$ and $q\,(Z,\, \tau)$ are determined also authentically. The basis for such a statement is the fundamental theorem of uniqueness of the solution of a boundary value problem for the heat conduction equation [5]. In other words, the temperature field ϑ and heat fluxes q found in the calculation should appear identical to the actual parameters, which could be in principle measured in a hypothetical experiment. Further basing on the known distributions ϑ and q, it is possible to determine corresponding average values $\langle \vartheta \rangle$ and $\langle q \rangle$, and finally (from Eq. (1.7)) the parameter h_m, which appears to be a function of the parameters of conjugation. It follows from the basic distinction of procedures of averaging of Eqs. (1.5) and (1.7) that an experimental value of the actual HTC is not equal to its averaged true value

$$h_m \neq \langle h \rangle \tag{1.10}$$

The analytical method schematically stated above, in which "from the hydrodynamic reasons" the following relation is stated:

$$h(z, \tau) = \langle h \rangle + \hat{h}(Z, \tau) \tag{1.11}$$

and further from the solution of the heat conduction equation in a body the parameter h_m is determined, outlines the basic essence of the approach developed in the present book. Different aspects of this method are discussed below in more detail.

1.2 Physical Examples

For the practical realization of this method it is necessary for each investigated process to specify the parameter $h\,(Z,\, \tau)$ (i.e., THTC) periodically varying with respect to its average value. A difficulty thus consists in the fact that, generally speaking, a valid function outlining the change of the THTC (with all its details) is unknown for any real periodic process. Therefore, the specification of this parameter is possible only approximately. This freedom in choice of the THTC inevitably makes results of the analysis dependent on the accepted approximations and assumptions. Thus the approximate nature of the developed method consists namely in this aspect. From the mathematical point of view, all constructions, solutions, estimations, and conclusions are obtained quite strictly and precisely. Physical features of some characteristic processes of heat transfer with periodic oscillations are discussed below.

Slug Flow of a Two-Phase Medium

A schematic image of this type of flow frequently met in engineering applications is given in Fig. 1.2. Oscillations of the heat transfer intensity in each section of the channel are caused here by the periodic passage of a large steam bubble and a liquid volume. Instant picture of the HTC variation over the height of a pipe is shown in the same figure. The thickness of the liquid film δ_f formed on a wall during passage of a steam bubble, can be determined using known recommendations documented in [6, 7]. The THTC is practically equal to thermal conductivity of a liquid layer k_f/δ_f, where k_f is the heat conductivity of the liquid phase. During the passage of the liquid, the heat transfer intensity is determined by the relations for heat transfer to a turbulent flow. Thus the character of the variation of the THTC with respect to time and to the vertical coordinate can be considered periodic step function. The curve of $\delta_f\,(Z,\,\tau)$ here will move upwards with speed of movement of the steam bubbles along the wall of a pipe. For the considered case, it is essential that the function $h\,(Z,\,\tau)$ is determined by fluid flow peculiarities in the two-phase medium and consequently does not depend on the thermophysical properties and thickness of the wall.

Flow Over a Body in the Vicinity of the Stagnant Point

The schematization of this type of flow is shown in Fig. 1.3. It is easy to show that in the presence of the periodic oscillations of the velocity of a fluid about its average value, the heat transfer intensity will be also periodic in time. In other words, if the period of change in the fluid velocity is essentially larger than the time needed for the individual particles of a liquid to pass by zone where heat transfer is studied (in the vicinity of the frontal stagnation point K), the instant behavior of heat transfer

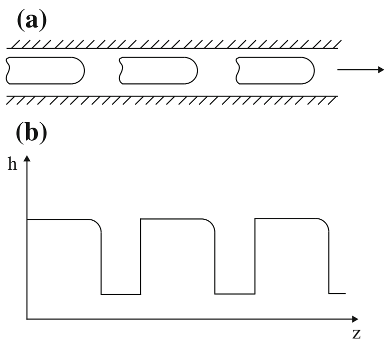

Fig. 1.2 Slug flow of a two-phase fluid: **a** schematic of the process, **b** variation of the THTC with the longitudinal coordinate

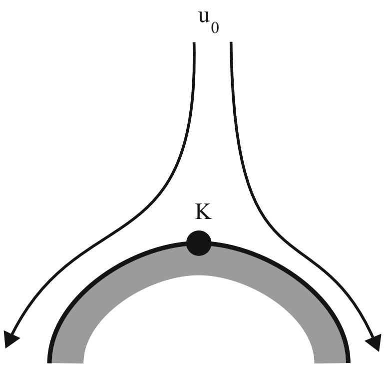

Fig. 1.3 Flow over a body in the vicinity of a critical point

can be considered quasi-stationary, with the function $h(\tau)$ being equal to the stationary dependence $h[u_0(\tau)]$.

In the considered case, the time variation of the heat transfer intensity follows from the hydrodynamic conditions of flow, and THTC remains actually constant for various materials of the surface.

Flow in a Laminar Boundary Layer

Let us consider stationary flow in a laminar boundary layer on which periodic velocity oscillations are imposed. From the same reasons, as in the example of the fluid flow over a body in the vicinity of a stagnation point considered above, the process of heat transfer here can be considered quasi-stationary: $h(\tau) = h[u_0(\tau)]$. For a case where the amplitude of the velocity oscillations is comparable to the velocity's average value, it is necessary to expect backward influence of the imposed oscillations on the average level of heat transfer. As known [4], a stationary HTC h_0 in a laminar boundary layer depends on the velocity as

$$h_0 = C\sqrt{u} \tag{1.12}$$

Here $C = 0.332\rho_f c_f / \mathrm{Pr}^{2/3}\sqrt{\nu_f/X}$, X is the distance from the initial stagnation point of a plate. Imposing of harmonic velocity oscillations on the stationary flow $u \to \langle u\rangle[1 + b\cos(2\pi\tau/\tau_0)]$ results in corresponding oscillations of the THTC —$h_0 \to h_0\left(1+\tilde{h}\right)$, so that Eq. (1.12) takes the following form:

$$h_0\left(1+\tilde{h}\right) = C\sqrt{\langle u\rangle[1 + b\cos(2\pi\tau/\tau_0)]} \tag{1.13}$$

Averaging Eq. (1.13) over the period of oscillations τ_0 gives

$$h = Cf(b)h_0 \tag{1.14}$$

Here $f(b)$ is a rather complex function of the oscillations amplitude, which weakly decreases with increasing b: $b = 0, f(b) = 1$; $b = 1, f(b) \approx 0.9$. Subtracting Eq. (1.14) of Eq. (1.13) term by term, one can find the fluctuating component of the

THTC. In the case of negligibly small amplitude $b \to 0$, these oscillations will look like as a cosine function

$$h_0 = C(b/2)\cos(2\pi\,\tau/\tau_0) \tag{1.15}$$

In a limiting case of the maximal amplitude $b = 1$, it can be deduced from Eq. (1.13)

$$h_0 = C[\pi/2|\cos(\pi\,\tau/\tau_0)| - 1] \tag{1.16}$$

As it is obvious from Eq. (1.16), at transition from $b \to 0$ to $b = 1$ oscillations of the heat transfer intensity are strongly deformed: the period decreases twice, and the form sharpens and is pointed from top to bottom. On the other hand, the average heat transfer level changes thus only by ≅10 %: at maximal amplitude $(b = 1)$ the ATHTC equals to $h \approx 0.9\,h_0$. Thus, the strong change in the amplitude of oscillations leads only to minor change of the average heat transfer level.

Wave Flow of a Liquid Film

At film condensation of a vapor on a vertical surface and also at evaporation of liquid films flowing down, one can observe a wave flow of the film already at small values of the film Reynolds numbers [6, 7]. Under these conditions, the wavelength essentially exceeds the film thickness, and the phase speed of its propagation is of the same order as the average velocity of the liquid in the film. As the Reynolds numbers increase, the character of flow changes: a thin film of a liquid of approximately constant thickness is formed on the surface, on which discrete volumes of a liquid periodically roll down. At a wave mode of the film flow, the THTC is rather precisely described by the dependence $h(Z,\tau) = k_f/\delta_f(Z,\tau)$ specified for the first time by Kapitsa in his pioneer works [8, 9]. It follows from this dependence that at a harmonic film structure the THTC is characterized by an inverse harmonic function (Fig. 1.4). At a flow with a "rolling down" liquid, a description of the THTC can be constructed similar to the case of the slug flow of a two-phase medium considered above, i.e., also independently of the thermal influence of a solid body. At a wave mode of condensation of vapor of liquid metals (sodium, potassium), nonequilibrium molecular-kinetic effects in the vapor phase play a significant role, due to the process of capturing (condensation) of the molecules of vapor. Therefore for a calculation of the heat transfer for vapor condensation (as well as for liquid film evaporation) of a liquid metal, these effects should be taken into account together with the thermal resistance of the liquid film itself determined by the formula of Kapitsa.

Near-Wall Turbulent Flows

The structure of the hydrodynamic oscillations in the turbulent flows is very complex and includes a wide spectrum of oscillations with various scales and amplitudes. Along with the so-called stochastic noise, typical for casual processes in a flow, there exist also large-scale periodic oscillations caused by periodic entrainment of accelerated portions of a fluid from the core of the flow into the

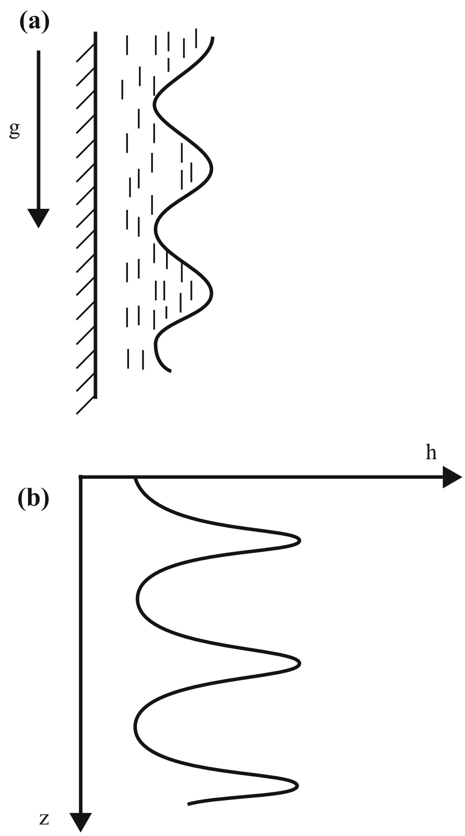

Fig. 1.4 Wave flow of a liquid film: **a** schematic of the process, **b** variation of the THTC with the longitudinal coordinate (*g* is the gravitational acceleration)

near-wall region. The average time intervals between these periodic entrainments, and also characteristics of oscillations of the wall friction have been determined in a number of experimental investigations (see, for example, [10, 11]). On the basis of the Reynolds analogy, it is possible to expect that the wall heat flux will also undergo similar oscillations. It is essential for our analysis that oscillations of parameters are connected with the movement of large turbulent vortical streaks and are consequently caused by the hydrodynamics of the flow. It is again obvious in the examined case that the THTC is independent of the material of a solid body.

1.3 Numerical Modeling of Conjugate Convective-Conductive Heat Transfer

The needs of modern engineering applications (in particular, aerospace engineering) dictate extremely strict requirements for thermal loaded surfaces and of critical conditions of the flow aerodynamics. In order to meet these requirements, it is