Advances in Industrial Control

Péter Gáspár Zoltán Szabó József Bokor Balázs Németh

# Robust Control Design for Active Driver Assistance Systems

A Linear-Parameter-Varying Approach





## **Advances in Industrial Control**

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A Linear-Parameter-Varying Approach



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### Series Editors' Foreword

The series *Advances in Industrial Control* aims to report and encourage technology transfer in control engineering. The rapid development of control technology has an impact on all areas of the control discipline. New theory, new controllers, actuators, sensors, new industrial processes, computer methods, new applications, new design philosophies, new challenges. Much of this development work resides in industrial reports, feasibility study papers, and the reports of advanced collaborative projects. The series offers an opportunity for researchers to present an extended exposition of such new work in all aspects of industrial control for wider and rapid dissemination.

Road transportation has experienced significant control research and development over the last few decades. The introduction into vehicles and traffic-flow systems of reliable computing and information technologies along with robust sensor devices has produced a considerable change in the driving experience. Now prototype driverless vehicles are even appearing in the transport system. Creating a top-down view this control research in road transportation provides a useful framework for understanding the ongoing developments.

There are four major aspects to control research in road transport:

- i. the classes of vehicles;
- ii. the road transport infrastructure;
- iii. the environmental conditions; and
- iv. the issues arising from "human-in-the-loop" control.

These aspects then give rise to various interactions depending on the vehicle/traffic/environment situation being investigated. The "classes of vehicles" include: motor cycles, automobiles, light goods vehicles, heavy goods vehicles, and specialized vehicles (fire engines, and refuse-collection vehicles, for example). Each of these vehicle classes will have different travel objectives and quite different dynamics. However as more and more autonomy is introduced into vehicle control, the inclusion of the "human-in-the-loop" adds an addition level of complexity to

vehicle control. Road transport "infrastructure" includes urban road networks, rural road networks and then freeways, autobahns, or motorways. The quality of road surfaces, the density of traffic, the amount of roadside instrumentation, and the purpose of all these transport networks will differ considerably. Driving conditions as provided by the environment will depend on such factors as the weather, and the road topology. This type of overview can easily be transcribed into an interesting hierarchical diagram. Such an overview is useful to Series Editors as it enables them to place the many strands of road transport control research into a framework and allows them to identify new potential contributions for developing a well-balanced and up to date group of titles in the series.

One of the very few monographs in the *Advances in Industrial Control* series to deal with the characteristics of "human-in-the-loop" issues is the 1998 monograph *Modelling and Simulation of Human Behaviour in System Control* by Pietro C. Cacciabue (ISBN 978-3-540-76233-1, 1998). The practical issues of the balance between autonomy and human control intervention (from a driver, pilot, or operator) will undoubtedly receive more research input in future years across many different control application fields.

In the hierarchy of road transport, "infrastructure" is an important classifier with different types of network exhibiting different control requirements. The *Advances in Industrial Control* series has two monographs reporting developments in this growing field. The monograph *Feedback Control Theory for Dynamic Traffic Assignment* by Pushkin Kachroo and Kaan Özbay (ISBN 978-1-85233-059-0, 1998) is a seminal contribution (a second edition is currently in preparation) and the monograph *Hybrid Predictive Control for Dynamic Transport Problems* by Alfredo Núñez, Doris Sáez and Cristián E. Cortés (ISBN 978-1-4471-4350-5, 2012) reports some recent research on bus transport in urban road networks.

As evidenced by the frequent sessions at the IEEE control conferences, the exploitation of advanced control ideas for the automobile class of vehicles has received far more research input and the *Advances in Industrial Control* monograph series has several contributions:

- Dry Clutch Control for Automotive Applications by Pietro J. Dolcini, Carlos Canudas de Wit and Hubert Béchart (ISBN 978-1-84996-067-0, 2010);
- Active Braking Control Systems Design for Vehicles by Sergio M. Savaresi and Mara Tanelli (ISBN 978-1-84996-349-7, 2010); and
- *Optimal Control of Hybrid Vehicles* by Bram de Jager, Thijs van Keulen and John Kessels (ISBN 978-1-4471-5988-9, 2015).

In this same group of topics falls this monograph *Robust Control Design for Active Driver Assistance Systems: An LPV Approach* by Péter Gáspár, Zoltán Szabó, József Bokor and Balázs Németh. This particular monograph not only reports on control designs for driver-assist systems but is virtually a tutorial and case-study work on how to use the linear-parameter-variable method. This makes the volume doubly welcome in the *Advances in Industrial Control* monograph series being both applications- and technique-oriented. The readership for this monograph will not only encompass the specialist engineer in automotive engineering but will undoubtedly include the broader control engineering community.

Michael J. Grimble Michael A. Johnson Industrial Control Centre University of Strathclyde Glasgow Scotland, UK

## Contents

1	Intr	oductio	n	1
Part	<b>I</b> ]	Modelir	ng and Control of LPV Systems	
2	Mod	leling o	f LPV Systems	11
	2.1	LPV N	Model Structures	13
	2.2	Linear	ization Through LPV Modeling	15
		2.2.1	Jacobian Linearization	16
		2.2.2	Off-Equilibrium Linearization	18
		2.2.3	Fuzzy Linearization	19
		2.2.4	qLPV Linearization	19
		2.2.5	Non-uniqueness of the LPV Models	21
	2.3	Linear	ization by LFT Techniques	25
	2.4	Perform	mance-Driven LPV Modeling	27
	2.5	5 LPV Modeling of Two Subsystems		
		2.5.1	Modeling of the Vertical Dynamics	32
		2.5.2	Nonlinear Components of the Vertical Dynamics	36
		2.5.3	LPV Modeling of the Yaw–Roll Dynamics	41
	2.6	Grey-Box Identification and Parameter Estimation		
		2.6.1	Observer-Based Identification	48
		2.6.2	Adaptive Observer-Based Approach	49
	2.7	7 Parameter Estimation: Case Studies		50
		2.7.1	Identification of a Suspension System	50
		2.7.2	Identification of the Yaw–Roll System	55
		2.7.3	Fault Estimation in LPV Systems	65
3	Rob	ust Cor	ntrol of LPV Systems.	71
	3.1	The M	Iodeling of Performances.	71
	3.2	The Modeling of Uncertain Components		75
	3.3	Contro	Design Based on LPV Methods	76
		3.3.1	Formulation of a Nonlinear Controller	77
		3.3.2	Control Design Based on SLF Methods	78

	3.4	3.3.3       Polytopic Approach.         3.3.4       An LFT-Based Design         Control Design Based on PDLF Methods.	78 81 84
		3.4.1 The Analysis of LPV Systems	84
		$\mathscr{L}_2$ -Norm Performance	86 91
			71
Part	t II	Vertical and Longitudinal Control	
4	Susp	pension Systems in Vertical Dynamics	95
	4.1	Modeling of Performances in the Vertical Dynamics	96
		4.1.1 Performance Specifications	96
		4.1.2 Weighting Functions in the Control Design	96
	4.2	Modeling of Vertical Dynamics by Using Uncertainties	99
		4.2.1 Parameter Uncertainties	99
		4.2.2 Weighting Functions	101
	4.3	Active Suspension Design Based on $\mathscr{H}_{\infty}$ Control	102
	4.4	Active Suspension Design Based on LPV Control	107
	4.5	Design of a Hierarchical Controller for an Active	
		Suspension System	111
		4.5.1 Modeling of the Actuator Dynamics	112
		4.5.2 Tracking Control Based on Backstepping Design	114
		4.5.3 Simulation Examples.	117
5	Anti	-roll Bars for Rollover Prevention	119
	5.1	Modelling of Performances in the Yaw–Roll Dynamics	121
		5.1.1 Rollover Threshold	121
		5.1.2 Design of Weighting Functions	123
	5.2	LPV Control Methods for Rollover Prevention Systems	127
	5.3	Design of a Fault-Tolerant Rollover Prevention System	130
6	Ada	ptive Cruise Control in Longitudinal Dynamics	135
	6.1	Adaptive Cruise Control.	135
	6.2	Model-Based Robust Control Design	137
		6.2.1 Modeling Longitudinal Dynamics	137
		6.2.2 Robust Control Strategy	138
		6.2.3 Modeling Actuator Dynamics	139
		6.2.4 Design of Feedback Controller	140
	6.3	Speed Design Based on Multiobjective Optimization	142
		6.3.1 Motivation of the Speed Design	142
		6.3.2 Design of Speed Profile	143
		6.3.3 Principles of the Optimization of the Look-Ahead	
		Control	144

8

9

10

7.3.3

7.3.4

7.3.5

7.4.1

7.4.2

7.4.3

7.4.4

7.4

8.1 8.2

8.3

9.1

9.2

9.3

9.4

9.4.1

9.4.2

6.4	Optimization of the Vehicle Cruise Control	146
	6.4.1 Handling the Preceding Vehicle in the Speed Design	148
	6.4.2 Motion of the Follower Vehicle in the Speed Design	148
	6.4.3 A Decision Method of the Lane Change	151
6.5	Implementation of the Method in the Driving/Braking	
	Systems	152
	6.5.1 SIL Implementation of the Controller	154
	6.5.2 Simulation Examples.	155
-		
Part III	Lateral and Integrated Control	
Part III 7 Desi	Lateral and Integrated Control         gn of Integrated Vehicle Control	161
<b>Part III</b> <b>7 Desi</b> 7.1	Lateral and Integrated Control         gn of Integrated Vehicle Control         Motivation of the Integrated Vehicle Control	161 161
<b>Part III</b> <b>7 Desi</b> 7.1 7.2	Lateral and Integrated Control         gn of Integrated Vehicle Control         Motivation of the Integrated Vehicle Control         LPV-Based Concept of the Integrated Control	161 161 164
<b>Part III</b> <b>7 Desi</b> 7.1 7.2 7.3	Lateral and Integrated Control         gn of Integrated Vehicle Control         Motivation of the Integrated Vehicle Control         LPV-Based Concept of the Integrated Control         Design of the Local and Reconfigurable Control Systems	161 161 164 166
<b>7 Desi</b> 7.1 7.2 7.3	Lateral and Integrated Control         gn of Integrated Vehicle Control         Motivation of the Integrated Vehicle Control         LPV-Based Concept of the Integrated Control         Design of the Local and Reconfigurable Control Systems         7.3.1         Design of the Brake System	161 161 164 166 168

Design of the Suspension System .....

Actuator Selection Procedure

Fault Information in the Decentralized Control

Modeling of Trajectory Tracking .....

Weighting Functions in the Control Design .....

Design of the Integrated Control.....

Simulation Results

Control Design of Trajectory Tracking .....

Control of the Variable-Geometry Suspension .....

Control Design of In-Wheel Motors.....

Driver Models in the Control Systems.....

10.1 Driver Model for Control Design Purposes.

10.2 Control-Oriented Model for Lateral Dynamics .....

10.3 Interconnection of the Driver-Vehicle System .....

10.1.1 Control-Oriented Driver Model .....

Robust Control of the Variable-Geometry Suspension

Lateral Dynamics of the Vehicle Model .....

Modeling of a Variable-Geometry Suspension System .....

Design of In-Wheel Motor Vehicle Control .....

High-Level Control Design of the LPV Controller .....

Control Implementation

Simulation Results

In-Wheel Motor Fault .....

Steering System Fault .....

#### xi

170

172

175

176

176

178

181

183

187

187

189

193

199

200

201

204

206

207

210

213

215

215

216

217

10.4 Performance Specifications of the Driver Assistance System	219	
10.4.1 Formulation of Performances	219	
10.4.2 Weighting Strategy of Performances	220	
10.5 Integrated Control Design of the Driver Assistance System	222	
10.5.1 Simulation Results	223	
10.5.2 Simulation Environment of the Driver Model	225	
Appendix A: Modeling of LPV Systems	231	
Appendix B: Robust Control of LPV Systems		
References	277	
Index	291	

## Abbreviations

DOF	Degrees of Freedom
FC	Full Control
FD	Finite Dimension
FDI	Fault Detection and Isolation
$H_{\infty}$	H-infinity
$H_{\infty}/\mu$	H-infinity with $\mu$
HIL	Hardware in-the-Loop
HJE	Hamilton-Jacobi Equations
НЛ	Hamilton-Jacobi Inequalities
LFT	Linear Fractional Transformation
LMI	Linear Matrix Inequality
LPV	Linear Parameter Varying
LS	Least Squares
LTI	Linear Time Invariant
LTV	Linear Time Varying
MIMO	Multi-Input Multi-Output
NLTI	Nonlinear Time Invariant
OE	Output Estimation
OF	Output Feedback
PDLF	Parameter-Dependent Lyapunov Functions
qLPV	Quasi LPV
RS	Robust Stability
RLS	Recursive LS
RP	Robust Performance
SF	State Feedback
SIL	Software in-the-Loop
SISO	Single-Input Single-Output
SLF	Single Lyapunov Functions
SSR	State Space Representation

## Chapter 1 Introduction

#### **Driver** Assistance Systems

Active driver assistance systems are able to assist the driver in enhancing passenger comfort, road holding, the efficiency and safety of transport, etc. At the same time the responsibility remains with the driver, since the driver is able to override the assistance. The demand for vehicle control methodologies that include the driver, the vehicle and the road arises at several research centers and automotive suppliers.

The book focuses on active driver assistance systems, which influence the dynamics of the vehicle. On the level of the individual vehicle components the control problem is formulated and solved by a unified modeling and design method provided by the linear parameter varying (LPV) framework. The requested global behavior is achieved by a judicious interplay between the individual components guaranteed by an integrated control mechanism. The integrated control problem is also formalized and solved in the LPV framework.

The main contributions of the book include

- application of the LPV paradigm in the modeling and control design methodology,
- application of the robust LPV design as a unified framework for setting control tasks related to active driver assistance,
- formulation and solution proposals for the integrated vehicle control problem,
- proposal for a reconfigurable and fault-tolerant control architecture.

#### **Design** Tools

Modeling and control of mechanical systems form an important class of nonlinear and linear systems, which have widespread application in science and industry. There are three approaches to describe the equation of motion for mechanical systems: Newtonian, Lagrangian, and Hamiltonian mechanics. Newtonian mechanics is used for simple mechanical systems because it is an intuitive and non-systematic method. By contrast, Lagrangian and Hamiltonian mechanics are used for complex multi-body mechanical systems because they are systematic approaches. A mechanical system

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is usually nonlinear in nature. Since the problem formulation of the output-feedback nonlinear control problem usually results in highly nonlinear partial differential equations and in a large number of theoretical and practical difficulties, it is difficult to solve in practice.

In modern control design, the approximation of nonlinear models with linear models is often based on a quasi LPV (qLPV) description. This approach is based on the possibility of rewriting the plant in a form in which nonlinear terms can be hidden by using suitably defined scheduling variables. For a successful analysis and design, it is crucial to obtain a model that captures the essential behaviors of the system under consideration. An advantage of qLPV models is that in the entire operational interval nonlinear systems can be defined while a well-developed linear system theory to analyze and design nonlinear control system can be used.

The purpose of modeling is control design, thus the model of the systems must be augmented with performance specifications and model uncertainties. Performance signals show the quantitative behavior of the controlled system, i.e., control systems are designed to maintain the system outputs at a desired value. In control systems usually a great number of predefined performance specifications must be formalized, e.g., passenger comfort, road holding, suspension deflection, tire load variation, energy consumption. The purpose of the control system is to guarantee the performance specifications. However, one of the properties might only be improved to the detriment of other properties, i.e., if one of the performance properties is enhanced at the same time another performance is usually degraded or hurt. For example, the performance demands of improving passenger comfort and road holding are in conflict. The conflict between different performance demands must be resolved in such a way that a balance between performances is achieved.

Uncertainties of the model are caused by neglected components, unknown or littleknown parameters. The uncertainties are modeled by both unmodeled dynamics and parametric uncertainties. In the vehicle model there are unknown parameters which vary in normal operation both in a short time period, e.g., mass, and in a long time period, abrasion. The estimation of the uncertain interval around its nominal value is important in the control design. If the uncertain interval is selected too large, the designed controller will be conservative. In this case the controller is designed in such a way that it will guarantee performances even in extreme conditions that do not occur. The unmodeled dynamics must be reduced by using a more appropriate estimation of the difference between the model and the actual plant. If parametric uncertainties of mechanical components are known, the uncertainties for unmodeled dynamics can also be reduced.

Weighting functions are applied to the performance signals to meet performance specifications and guarantee a trade-off between performances. The uncertainties are modeled by both unmodeled dynamics and parametric uncertainties. As a result of this construction, a linear fractional transformation (LFT) interconnection structure, which is the basis of control design, is achieved.

In model-based controller synthesis, a model describing the physical system is used to determine the controller such that the specifications on the closed-loop system are satisfied. However, the model used in the controller synthesis is just an approximation of the dynamics of the real physical system. In addition, there is always a presence of disturbances and measurement noises, which enter the system in an unpredictable way. The purpose of robust control methods is to design controllers with model uncertainties and disturbances, and at the same time they must satisfy the closed-loop system specifications.

Several control design methods have been proposed for linear or linearized models. In practice, the control design problem usually requires several control design methods, and the selection of the appropriate controller is carried out in the implementation phase. The robust control design methods which are usually applied fit in the so-called  $\mathcal{H}_{\infty}$  and the  $\mathcal{H}_{\infty}/\mu$  framework.

It is apparent that there is a great amount of analogy between classical adaptive schemes and the qLPV design philosophy. The parameters that are estimated during operational time and which are used to tune the actual controller in an adaptive scheme play the same role as the scheduling variables in the qLPV context. From this latter perspective the difference is in the acquisition of the scheduling variable, i.e., in the adaptive case the values of the scheduling variable are not directly available by measurement and need to be obtained by a specific estimation process based on the directly available data. This observation leads us to propose a unified view of both control design strategies cast in the qLPV design framework by extending the set of scheduling variables with parameters that might not be directly measured but estimated using a suitably designed procedure.

One of the advantages of the proposed general qLPV framework, i.e., a robust adaptive control scheme using dynamic output feedback based on an LPV methodology, is that besides the introduction of the (parametric) model uncertainties in the design the LPV method also makes it possible to consider the unknown parameter variation rate, providing a framework to answer the long-standing question of whether or not the adaptation is limited fundamentally to slowly varying systems.

The solution to the LPV control synthesis problem is formulated as a parameter dependent linear matrix inequality (LMI) optimization problem, i.e., a convex problem for which efficient optimization techniques are available. This control structure is applicable whenever the value of the parameter is available in real-time. The resulting controller is time varying and smoothly scheduled by the values of the scheduling variables. Therefore qLPV models with LMIs, as the main design tool, seem to be the most efficient approach to achieve robust and non-conservative results. The LMI constraint set for qLPV problems is convex, however, it is usually not easily dealt with, since it represents an infinite number of conditions. One way to overcome this difficulty is to approximate the exact set by a tractable one. By choosing appropriate inner/outer approximations one may develop computable lower/upper bounds for certain performances, e.g., stability margins.

This basic setting for the controller synthesis can be varied depending on the problem at hand and the actual demands. The information on the change rate of the measured scheduling variables can also be introduced in the design.

A practically relevant control design task contains nonlinear components, e.g., the dynamics of the dampers and springs and nonlinear actuator dynamics. In order to handle the high complexity of the problem the design of a two-level controller is proposed in the integrated control framework. The required control force is computed by applying a high-level controller, which is designed using a LPV method. For the control design, the model is augmented with weighting functions specified by the performance demands and the uncertainty assumptions. The actuator generating the necessary control force is modeled as a nonlinear system for which a low-level forcetracking controller is designed. The proposed separation layers describe the intuitive structure of the different subsystems, i.e., the chassis and the actuators while keeping the complexity of the resulting control problems within reasonable bounds.

Each of the individual models is formulated in the LPV framework and contains the performance specifications and typical uncertainties. Thus, the primary models are augmented with the corresponding—preferably LPV—weights, which leads to the unified generalized plant structure, which is the starting point of the robust control design.

#### Integration of Vehicle Systems

Two different actuators might be able to influence the same vehicle dynamics. Thus, the role of the integrated vehicle control is to coordinate the local components and handle the interactions between them. Since the performance specifications of local controllers are often in conflict, they must also guarantee a balance or trade-off between them. This trade-off is formulated on the level of local controllers as a result of engineering knowledge. However, when an event occurs, the preferences, i.e., the trade-off levels, are subject to change.

The term configuration refers to a well-defined sensor and actuator set that is associated with a given functionality. Control reconfiguration is motivated by the following requirements: the achieved control performance in certain scenarios must be improved and increased reliability in the presence of sensor or actuator faults must be achieved. The term event is related to the occurrence of such a scenario. In a normal situation a baseline configuration is formed by a single local component, e.g., steering, otherwise it is composed of several local components that can cause the same functional behavior, e.g., steering and brake for generating yaw moment. The hierarchy of the configurations and corresponding scheduling variables ensure that the additional actuator(s) considered improve the stability properties of the given functionality.

The specification of the configuration sets and that of the corresponding reconfiguration policy are cornerstones of the proposed method and it may be a highly nontrivial task requiring considerable engineering knowledge. However, the analysis of the configurations, events, and possible reconfigurations is necessary for any reconfiguring control strategy.

The control solutions create a balance between driving (or road holding) and comfort and guarantee safety all the time. This balance often leads to compromises between vehicle functions, which may not be suitable for all the drivers. For example a driver who wants to minimize the length of the trajectory in the bend selects the curvature radius as small as possible, while the driver who requires comfort selects a larger curvature radius. At the same time, however, the selection of different curvature radiuses also corresponds to the possible speed selection, e.g., the larger radius allows the driver to select larger speed. The control solutions in practice are based on the drivers' behavior, which is learnt by the system during the journey. The driver input is not only a function of the planned trajectory but it also affects the dynamics of the vehicle.

Consequently, a driver model must be combined with the vehicle model in order that the driver behaviors and requirements are incorporated in the design of the control system. In the driver assistance system the interaction between the vehicle and the driver is taken into consideration.

The main chapter of this part offers a detailed presentation of the integrated control framework. An integrated control system is designed in such a way that the effects of a control system on other control functions are taken into consideration in the design process by selecting the various performance specifications. In order to impose performance requirements, a tight coupling among the elements of the integrated structure is needed, which is realized through a set of well-defined additional monitoring signals.

The plant can be considered as a core system which communicates with the environment through different peripheral components, while the controller is a program that executes a given task on the core system function of the available set of peripheries. In this respect, there is an analogy between a modern computer with its operating system and application programs and which is equipped with a set of peripheries. An important point here is that the actual peripheral subsystem plugged into this architecture can fulfill its intended task and the applications can use it quite flexibly without previous knowledge of the operation system about the internal details of the specific subsystem. The only constraint is that the information flow between these components should respect some well-defined protocols. This plug-and-play paradigm has been proven to be very fruitful in computer science and is considered to be a model that can be applied to the design of control systems as well.

In the context of control systems a plug-and-play control architecture provides the possibility to use sensors and actuators supplied by different vendors interchangeably on a system by guaranteeing a performance level and leaving the global controller intact. If a new control component is added, an old control is replaced by a new one, or an old component is removed, the structure of the system (or the control) changes. In these cases, the conventional control should be redesigned, which is expensive and takes a long time. In the integrated concept the control logic must be modified on the highest level.

Once the local controllers have been designed it is possible to perform an analysis step in the robust control on a global level to prove both global stability and performance. The presence of competing multiobjective criteria makes the applicability of the global approach difficult. It is a great challenge for research since the proof of global performance leads to a highly computation-intensive procedure. Although the analysis of global stability is an intensively researched area there are only few theoretical results. Moreover, although the analysis is fundamental in terms of distributed control, it is a fully open research field.

The advantage of the integrated control is to provide reconfigurable and faulttolerant structures. If a performance degradation or fault occurs in the system and it has been detected, the role of the degraded controller may be substituted by another controller. The fault-tolerant local controllers also require components for monitoring fault information. Faults in the operation of an actuator can be usually detected by using a built-in self-diagnostic method. In this case, fault information is sent by the actuator itself to the supervisor. Any reconfiguration scheme relies on a suitable fault detection and isolation (FDI) component.

The basic objective of a fault detection methodology applied to dynamic systems is to provide techniques for the detection and isolation of failed components. Using a mathematical model of the system it is possible to exploit the principle of analytical redundancy, which allows to check discrepancies between the real behavior of the system and its idealized mathematical description or model. Model-based FDI relies on analytical redundancy to generate fault indicators, called residuals.

There are many analytical redundancy methods for linear and nonlinear systems available in the literature. While recent nonlinear approaches are useful for the analysis, and partly in the design of detection filters, they are largely incapable of solving synthesis problems because of the computational burden they usually pose to the implementation.

As a high-level approach, the FDI filter design problem can often be cast in the model matching framework. To achieve robustness in the presence of disturbances and uncertainty, multiobjective optimization-based FDI schemes can be proposed where an appropriately selected performance index must be chosen to enhance sensitivity to the faults and to simultaneously attenuate disturbances.

This is a typical worst-case filtering problem and the corresponding design criteria can be formulated as a convex optimization problem by using LMIs. The main problem here is that the sensitivity and robustness conditions are in conflict. In the linear time invariant (LTI) framework, it means that sensitivity to faults and insensitivity to unknown inputs cannot be achieved simultaneously at the same frequencies. Faults having similar frequency characteristics to those of disturbances might go undetected. While the design problem is nonconvex, in general, a scheme that can handle the problem by using LMI techniques is presented.

#### Structure of the Book

The book includes three parts and appendix. The first part focuses on the modeling and control of LPV systems. In Chap. 2, the construction of the LPV model of the physical system and the linearization methods are presented. Two examples are presented, i.e., the LPV modeling of the vertical dynamics and that of the yaw–roll dynamics. Since the parameters of the LPV models usually are not necessarily known, a graybox identification method is applied. In Chap. 3 the model is augmented with the performance specifications and uncertainties in order to form a control-oriented LPV model. Both constant Lyapunov function and parameter varying Lyapunov functions are applied for stability and  $\mathcal{L}_2$  performance. Finally, they are extended to LPV systems when the measured varying parameters do not exactly fit the real one. The theoretical part is extended by several important components in the Appendix.

In the second part of the book, the control methods for both vertical and longitudinal vehicle dynamics are presented. In Chap. 4 both the linear  $\mathscr{H}_{\infty}$  methods and LPV methods are applied for the control design. This chapter also presents the hierarchical structure of the control design. The high-level control focuses on the performance specifications and calculates a required control signal. The required signal is tracked by a low-level controller by setting the actuator dynamics. In Chap. 5, active anti-roll bars are applied for preventing the rolling over. It is combined with the active brake in order to improve the efficiency of the LPV control design method. Moreover, this combination guarantees the fault-tolerant operation of the control system. In Chap. 6, a classical control problem is presented, i.e., the adaptive cruise control in the longitudinal dynamics. In the robust control design both the driving and the braking systems are combined. The control algorithm is implemented in a SIL environment. An extension of the adaptive cruise control is the speed design in which several road and traffic conditions must be taken into consideration in order to reduce the control energy and keep the time requirement.

In the third part of the book, the control systems focus on the lateral dynamics. Since the control systems may affect the same vehicle dynamics, their operations must be integrated. An integrated control system is designed in such a way that the effects of a control system on other vehicle functions are taken into consideration in the design process by selecting the various performance specifications. The principles of the design methods are presented in Chap. 7. In this chapter, the operation of the integrated control is presented through trajectory tracking as a driver assistance system. In the integrated control three control components are applied simultaneously such as the brake, the steering and the suspension systems. Concerning the lateral vehicle dynamics the variable-geometry suspension system plays an important role. In Chap. 8, the modeling and control of the variable-geometry suspension system is presented. Moreover, the integration of the construction and the control design is also presented. In Chap. 9, the control design of the in-wheel motors for a trajectory tracking problem is presented. It leads also a hierarchical control, in which the retired longitudinal force and the yaw moment are calculated in the high-level while the torques of the in-wheel motors are designed in the low level. In Chap. 10, the drivers' behavior is analyzed. In the control design a simplified driver model is combined with the control-oriented vehicle model.

In the Appendix, further components of the modeling and robust control of LPV systems are included. The modeling part presents the basic terms of the analysis, the identifiability, the adaptive observers and the geometric approach of the FDI design. The robust control presents the structured uncertainty, the components of the nonlinear  $\mathscr{H}_{\infty}$  methods and the LFT-based qLPV design.

## Part I Modeling and Control of LPV Systems

## Chapter 2 Modeling of LPV Systems

#### Introduction

In general terms, control theory can be described as the study of how to design the process of influencing the behavior of a physical system to achieve a desired goal. An open-loop control is one in which the control input is not affected in any way by the actual (measured) outputs. If the system changes during the operational time then the control performance can be severely reduced. In a closed-loop system the control input is affected by the measured outputs, i.e., a feedback is being applied to that system. Very often a reference input is given, which is directly related to the desired value of system outputs, and the purpose of the controller will be to minimize the error between the actual system output and the desired (reference input) value.

There are two main features in the analysis of a control system: system modeling, which means expressing the physical system under examination in terms of a model (or models) which can be readily dealt with and understood, and the design stage, in which a suitable control strategy is both selected and implemented in order to achieve a desired system performance. Forming a mathematical model which represents the characteristics of a physical system is crucially important as far as the further analysis of that system is concerned.

Traditionally controllability and observability are the main issues in the analysis of a system before deciding the best control strategy to be applied, or whether it is possible to control or stabilize the system. Controllability is related to the possibility of forcing the system into a particular state by applying an appropriate control signal while observability is related to the possibility of reconstructing, through output measurements, the state of a system.

The model should not be over simple so that important properties of the system are not included, something that would lead to an incorrect analysis or an inadequate controller design. In some cases the nonlinear characteristics are so important that they must be dealt with directly, and this can be quite a complex procedure.

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Gain-scheduling is a technique widely used to control such systems in a variety of engineering applications. In the classical gain scheduling approach, having strong roots in flight control applications, the controller synthesis is based on local descriptions of the nonlinear system, that can most often be approximated by linear system properties. The gains of the gain-scheduled controllers are typically chosen using linear control design techniques and is a two step process. First, several operating points are selected to cover the range of system dynamics. At each of these points, the designer makes an LTI approximation to the plant and then, designs a linear compensator for each linearized plant. This process gives a set of linear feedback control laws that perform satisfactorily when the closed-loop system is operated near the respective operating points. A global nonlinear controller for the nonlinear system is then obtained by interpolating, or scheduling, the gains from the local operating point designs.

Since the synthesized controllers are guaranteed to satisfy specifications only locally, the designer typically cannot assess a priori the stability, robustness, and performance properties of gain-scheduled controller designs. While the local controller synthesis can be performed using the well established techniques of the linear system theory, it remains a non-trivial procedure to map the linear controllers such that non-local specifications of the closed loop system are kept.

The LPV paradigm provides a remedy to this problem, Shamma and Athans (1990), Shamma (1992). Initiated in Shamma and Athans (1991) LPV modeling techniques have gained a lot of interest, especially those related to vehicle and aerospace control, Becker and Packard (1994), Balas et al. (1997), Marcos and Balas (2001), Szászi et al. (2005). LPV systems have recently become popular as they provide a systematic means of computing gain-scheduled controllers. In this framework the system dynamics are written as a linear state-space model with the coefficient matrices functions of external scheduling variables. Assuming that these scheduling variables remain in some given range then analytical results can guarantee the level of closed loop performance and robustness. The parameters are not uncertain and can often be measured in real-time during system operation. However, it is generally assumed that the parameters vary slowly in comparison to the dynamics of the system. LPV based gain-scheduling approaches are replacing ad-hoc techniques and are becoming widely used in control design.

Many of the control system design techniques using LPV models can be cast or recast as convex problems that involve LMIs. Significant progress has been made recently in the use of LMI and  $\mathscr{H}_{\infty}$  optimization in gain-scheduled control. One such control design technique, described by Apkarian et al. (1995), is the Lyapunov function/quadratic  $\mathscr{H}_{\infty}$  approach wherein a single Lyapunov function is sought to bound the performance of the LPV system. Such a framework generally has a strong form of robust stability with respect to time-varying parameters. However, due to the continuous variation of scheduling parameters, such a synthesis approach is generally associated with a convex feasibility problem with infinite constraints imposed on the LMI formulation. This problem can be addressed by using affine LPV modeling that reduces the infinite constraints imposed on the LMI formation to a finite number.

Such a modeling approach has been used to solve design problems by Becker (1992), Sun and Postlethwaite (1998).

The above pure LPV model is not quite matched to the control problems where the scheduling variables are in fact system states (e.g., vehicle speed), rather than bounded external variables. An approach to this problem is to generate so-called quasi-LPV models, which are applicable when the scheduling variables are measured states, the dynamics are linear in the inputs and other states, and there exist inputs to regulate the scheduling variables to arbitrary equilibrium values.

These methods concentrate on robust performance, hence, robust stability of the controlled system. In this more general context such robust control problems—both analysis and synthesis—can be formulated using a generalized plant technique based on an LFT description of the uncertain LPV system, see, e.g., Iwasaki and Hara (1998), Iwasaki and Shibata (2001), Wu (2001). The controller synthesis leads to bilinear matrix inequalities (BMI) but often it is possible to reduce the problem to the solution of a finite set of LMIs, for details see, e.g., Scherer et al. (1997), Scherer (2001), Wu (2001).

#### 2.1 LPV Model Structures

The mathematical model of a dynamic evolution of a nonlinear, non-autonomous physical system is usually formulated as a state space representation in terms of the input  $u(t) \in \mathbb{R}^m$ , output  $y(t) \in \mathbb{R}^p$  and state signals  $x(t) \in \mathbb{R}^n$  related by a first-order differential equation:

$$\dot{x} = f(x, u, w), \tag{2.1}$$

$$y = h(x, u, w), \tag{2.2}$$

subject to the initial condition  $x(t_0) = x_0$ . Usually the model also describes the effect of the outer disturbances, which are modeled through the signal  $w(t) \in \mathbb{R}^d$ . In what follows for the sake of simplicity we concentrate on the undisturbed system, i.e., w will be suppressed from the model.

According to the LPV paradigm, parameter-dependent systems are linear systems, whose state-space descriptions are known functions of time-varying parameters. While the time variation of each of the parameters is not known in advance, it is assumed to be measurable in real time. Thus, in the LPV controller synthesis step the parameters are regarded as freely varying parameters taking arbitrary values in the region  $\Omega$  and, hence, the LPV description will differ from the nonlinear system. The larger this difference, the more conservatism is introduced in the LPV controller synthesis step. LPV descriptions of nonlinear systems are not unique: it is desirable to have an LPV description that in some sense is close to the nonlinear system for all parameter values.

Thus, the aim of the LPV modeling procedure is to find an LPV description of the nonlinear model on the form

$$\dot{x} = A(\rho)x + B(\rho)u \simeq f(x, u), \quad \rho \in \Omega$$
(2.3)

$$y = C(\rho)x + D(\rho)u \simeq h(x, u), \qquad (2.4)$$

where  $\rho$  is the, possibly state dependent, parameter vector varying within a region  $\Omega$ , such that the known relation  $\rho = \sigma(y, r)$  depends only on the measured signals y and exogenous signals r whose values are known in operational time.

This guarantees that the parameter values are available to the controller and that an explicit nonlinear feedback controller can be obtained from the designed LPV controller. In order to ensure that trajectories of the original nonlinear system are equal or at least closed to the trajectories of the LPV description, (2.3) should be as close to the nonlinear system as possible for all parameter values in the region  $\Omega$ .

Hence, an LPV model is defined as a linear model whose state-space matrices depend on a vector  $\rho$  of time-varying parameters of the form

$$\dot{x} = A(\rho)x + B(\rho)u, \qquad (2.5)$$

$$y = C(\rho)x + D(\rho)u, \qquad (2.6)$$

where it is often suppose that the parameter dependency has an explicit structure: namely either affine, polynomial, polytopic or an LFT dependency. Accordingly, if

$$S(\rho) = \sum_{i=0}^{n} \sum_{|\underline{j}|=i} \rho^{\underline{j}} S_{i,\underline{j}},$$
(2.7)

where  $\rho_1^{\underline{j}} = \rho_1^{j_1} \rho_2^{j_2} \cdots \rho_k^{j_k}$  with  $\rho_l$  are the components of the parameter vector  $\rho$ ,  $|\underline{j}| = \sum_{l=1}^k i_k$  and

$$S \sim \begin{pmatrix} A & B \\ C & D \end{pmatrix}, \quad S_{i,\underline{j}} \sim \begin{pmatrix} A_{i,\underline{j}} & B_{i,\underline{j}} \\ C_{i,\underline{j}} & D_{i,\underline{j}} \end{pmatrix},$$

then n = 1 corresponds to the affine models. Affine models are mostly involved in applications where geometric techniques are to be used.

For polytopic LPV models the system matrix  $S(\rho)$  varies within a fixed polytope of matrices: it is a convex combination  $S(\rho) \in \text{convex} \{ S_1, S_2, \dots, S_k \}$  of the system matrices (vertex systems), i.e.,

$$S(\rho) = \sum_{i=1}^{k} \rho_i S_i, \quad \rho_i \ge 0, \quad \sum_{i=1}^{k} \rho_i = 1.$$
 (2.8)



Since polytopic models are well suited for Lyapunov-based analysis and design, they are very popular model candidates in the LPV framework.

A more general representation is the LFT, see Fig. 2.1. LFT is a representation of a system using a feedback interconnection between two operators, a known causal system

$$M = \begin{pmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{pmatrix}$$

and a causal bounded system  $\Delta$  of proper dimension:

$$\mathscr{F}_L(M,\Delta) = M_{11} + M_{12}\Delta(I - \Delta M_{22})^{-1}M_{21}$$
(2.9)

$$\mathscr{F}_U(M,\Delta) = M_{22} + M_{21}\Delta(I - \Delta M_{11})^{-1}M_{12}$$
(2.10)

 $\Delta$  is typically norm-bounded,  $||\Delta||_{\infty} \leq 1$ , but otherwise unrestricted in form (structured/un-structured) or type (nonlinear/time-varying/constant). If some of the components in the  $\Delta$  operator are scheduling parameters an LPV system is obtained. This form is obtained by extracting a varying parameter from a system and placing it into a feedback loop, such that the remaining system, M, is time-invariant. Models with affine or polynomial parameter dependencies can be transformed exactly to a LFT. An important property of LFT systems is that their interconnection (e.g., sum, concatenation) and also the inversion, if it exists, always results in another LFT.

We emphasize that an LPV plant can be viewed either as an LTI plant subject to a time-varying parametric uncertainty  $\rho(t)$ , see, e.g., the LFT LPV structure or as a set of models of linear time-varying (LTV) plants, where each LTV system corresponds to a specific parameter trajectory. In the analysis and design process we chose the most convenient interpretation that fit the actual technique that we might use.

#### 2.2 Linearization Through LPV Modeling

Practically, concerning the structure of the models, prior to the design and analysis phase there is no significant difference between LPV models and those used for gain scheduling. All of them can be obtained by using different, application specific, methods. The direct linearization schemes applied to nonlinear systems can be roughly classified into the following types: linearization about an equilibrium, linearization about a parametrized state trajectory and global linearization. In the first case the system is represented as an LTI system locally around an equilibrium condition, while in the second approach the nonlinear system is to follow some prescribed trajectory around that it can be approximated by a family of parametrized linearizations. In the third case the original nonlinear system is approximated by a set of trajectories of a linear differential inclusion (LDI) which can represent it in the entire operation range. However, in this case there might be trajectories of the LPV model that are not actual trajectories of the original system. This might lead to a conservative analysis or design.

In what follows some of the most common techniques, e.g., classical, fuzzy and the off-equilibrium approaches, see, e.g., Leith and Leithead (2000), will be sketched.

#### 2.2.1 Jacobian Linearization

Often in industrial settings, a finite collection of linear models is used to describe the behavior of a system throughout an operating envelope. The linearized models describe the small signal behavior of the system at a specific operating point and the collection is parametrized by one or more physical variables whose values represent this specific point. If the state variables have physical meaning, then it makes sense to develop polynomial least squares fits of the state-space matrices to get a continuous parameterization of the operating envelope.

The classical approach, using Jacobian linearization of the nonlinear model about a manifold of constant equilibria, constant operating points or set-points, is called linearization-based scheduling. When a corresponding scheduling variable  $\rho$  is chosen appropriately to parameterize the set of linear models, a parameterized family of linearized models representing the original nonlinear model results.

Considering the nonlinear plant dynamics an equilibrium or constant operating point  $(x_e, u_e)$  is defined by the equilibrium condition  $f(x_e, u_e) = 0$ . Assuming *f* is continuously differentiable at the equilibrium point, the nonlinear model is approximated by

$$\delta \dot{x} = A \delta x + B \delta u \tag{2.11}$$

$$\delta y = C\delta x + D\delta u, \tag{2.12}$$

where

$$\delta u = u - \tilde{u}, \quad \delta y = y - \tilde{y}, \quad \delta x = x - \tilde{x},$$

and

$$A = \partial_x f(x_e, u_e), \quad B = \partial_u f(x_e, u_e), \quad C = \partial_x h(x_e, u_e), \quad D = \partial_u h(x_e, u_e).$$

By considering an entire equilibrium family  $(x_e, u_e) \in \Omega_e$  yields to a linear parameter-dependent linearization family  $S(\rho_e)$ , locally describing the nonlinear model:

$$\delta \dot{x} = A(\rho_e)\delta x + B(\rho_e)\delta u$$
  
$$\delta y = C(\rho_e)\delta x + D(\rho_e)\delta u.$$

To obtain an LPV description for a nonlinear model, an interpolation of the stationary linearizations can be applied: e.g., by using a linear interpolation then system can be written as

$$\delta \dot{x}(t) = A(\rho)\delta x(t) + B(\rho)\delta u(t)$$

with

$$A(\rho) = \sum_{i=1}^{p} A_i \rho_i, \quad B(\rho) = \sum_{i=1}^{p} B_i \rho_i, \quad C(\rho) = \sum_{i=1}^{p} C_i \rho_i, \quad D(\rho) = \sum_{i=1}^{p} D_i \rho_i,$$
(2.13)

where

$$\sum_{i=1}^{k} \rho_{i} = 1, \ \rho_{i} \ge 0, \ \delta x = x - x_{e}(\rho), \ \delta u = u - u_{e}(\rho)$$

the points  $(x_e^i, u_e^i) \in \Omega_e$  being stationary. The procedure, however, may give an LPV system that does not include the original nonlinear system. But if the stationary points can be chosen such that

$$\{(\partial f_x(x, u), \partial f_u(x, u))\} \subset \operatorname{convex}\{(\partial f_x(x_e^i, u_e^i), \partial f_u(x_e^i, u_e^i))\}$$

then the LPV description (2.13) will include the nonlinear system.

A typical choice for  $\Omega_e$  is to take a specific trajectory  $(\tilde{x}, \tilde{u}, \tilde{y})$ , i.e., to perform the linearization of the nonlinear system (2.2) around a trajectory:

$$\delta \dot{x} = \partial_x f(\tilde{x}, \tilde{u}) \delta x + \partial_u f(\tilde{x}, \tilde{u}) \delta u,$$
  
$$\delta y = \partial_x h(\tilde{x}, \tilde{u}) \delta x + \partial_u h(\tilde{x}, \tilde{u}) \delta u,$$

where

$$\delta u = u - \tilde{u}, \quad \delta y = y - \tilde{y}, \quad \delta x = x - \tilde{x}.$$

By taking the parameters of the specific trajectory (flight envelop) as measurable scheduling variables, the desired LPV model will be of the form

$$\dot{\xi} = A(\rho)\xi + B(\rho)\delta u$$
  
$$\delta y = C(\rho)\xi + D(\rho)\delta u.$$

A parameterized family of linearized models resulting from linearization-based scheduling or a number of black-box point-designs are only locally valid. In case an LPV model is based on such a set of linearized models, the accuracy of the resulting linear parameter-dependent model with respect to the original nonlinear model or plant is unknown. Classical gain scheduling is mainly restricted to local controller synthesis in stationary points. Even though nonlinear systems can be linearized along a trajectory, no gain scheduling approaches available in the literature that extends the stability region using a family of linearizations along different trajectories.

#### 2.2.2 Off-Equilibrium Linearization

A disadvantage of classical linearization-based scheduling is the restriction to equilibrium-point modeling. Using the so-called velocity-based or off-equilibrium linearizations it is possible to enable linearization at every operating point: considering the nonlinear system

$$\dot{x} = f(x, u), \quad y = h(x, u),$$

the velocity linearization at a point  $(x_0, u_0)$  reads as

$$\begin{split} \dot{x} &= \zeta \\ \dot{\zeta} &= \partial f_x |_{(x_0, u_0)} \zeta + \partial f_u |_{(x_0, u_0)} \dot{u} \\ \dot{y} &= \partial h_x |_{(x_0, u_0)} \zeta + \partial h_u |_{(x_0, u_0)} \dot{u}. \end{split}$$

In this way there is a velocity-based linearization associated with every operating point of the original nonlinear system and the solutions may be pieced together. Thus, the resulting velocity-based linearization family, parameterized by  $\rho$ , globally approximates the trajectories of the nonlinear model to an arbitrary degree of accuracy. The velocity linearization is not limited to equilibrium points: as no restriction to equilibrium operating points is present, linear approximation of transient dynamics and operating points far from equilibrium operating points is also enabled.

Interpolation of linear controller based on velocity linearizations can be performed in a similar way to classical gain scheduling. However, since the velocity linearization is not an approximation in the same sense as a standard linearization scheme, it is easier to interpolate linearizations in a way such that the nonlinear system is included in the LPV description.

#### 2.2.3 Fuzzy Linearization

One approach to gain scheduling, and thus, to LPV modeling, uses ideas from fuzzy systems, see Takagi and Sugeno (1985) to describe the nonlinear system: the plant dynamics is formulated as a blended multiple model representation such as a Takagi-Sugeno model or local model network of the form

$$\dot{x} = \sum_{i} f_i(x, u) \mu_i(\phi),$$
$$y = \sum_{i} h_i(x, u) \mu_i(\phi)$$

where the function  $\phi(x, u)$  is the scheduling variable and the scalar blending weights  $\mu_i \ge 0$  often are normalized to  $\sum_i \mu_i = 1$ .

After a linearization and blending of the individual components the typical form of the LPV model will be of the form:

$$\begin{pmatrix} \dot{x} \\ y \end{pmatrix} = S(\rho) \begin{pmatrix} x \\ u \end{pmatrix}, \tag{2.14}$$

with

$$S(\rho) = S_0 + \sum_{i \in I} \rho_i S_i,$$
 (2.15)

where  $\rho_i$  will be the scheduling variables of the model.

#### 2.2.4 qLPV Linearization

Quasi-LPV scheduling tries to overcome the general shortcomings of classical linearization schemes regarding local validity of the resulting model: the idea is to transform the nonlinear model to an LPV form hiding the nonlinear terms by including them in the scheduling variable. Since this process involves a transformation rather than a linearization, the resulting LPV model exactly equals the original nonlinear model.

A qLPV model may arise by considering state transformations on a class of nonlinear systems of the form:

$$\dot{x}_1 = f_1(x_1) + A_{11}(x_1)x_1 + A_{12}(x_1)x_2 + B_1(x_1)u,$$
  
$$\dot{x}_2 = f_2(x_1) + A_{21}(x_1)x_1 + A_{22}(x_1)x_2 + B_2(x_1)u,$$
  
$$y = x_1.$$