

Morgan J. Hurley
Editor-in-Chief

SFPE Handbook of Fire Protection Engineering

Fifth Edition



 Springer

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Volume I

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Editor-in-Chief

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This edition of the SFPE Handbook of Fire Protection Engineering is dedicated to Philip DiNenno, who was the editor in chief for the first four editions of this handbook. In the mid-1980s, Phil DiNenno, Jack Watts, Doug Walton, Craig Beyler, and Dick Custer had an idea to create a collection of calculation methods for fire protection engineering. From this idea emerged the SFPE Handbook of Fire Protection Engineering, which was first published in 1988.

*No other single event had as significant an impact on establishing the profession of fire protection engineering as the publication of this handbook. As Vyto Babrauskas said: “The field [of fire protection engineering] has made very gratifying progress in these last four decades. . . . The most remarkable positive achievement I think has been the SFPE Handbook, published first in 1988. . . . [W]ith the publication of the first edition of the SFPE Handbook in 1988, all of a sudden we could properly describe this as a science-based profession.” [Babrauskas, V. “Some Neglected Areas in Fire Safety Engineering,” *Fire Science and Technology* Vol. 32 No. 1 (2013) pp. 35–48.]*

When they began creating the first edition, Phil and his colleagues had no model other than handbooks used in other professions. Phil contributed the leadership, vision, and motivation necessary to develop the handbook, and he did so using entirely volunteer resources. This would be an incredible accomplishment for anyone. Phil did it before he turned 35.



Foreword

This edition marks a passing of the torch for the *SFPE Handbook of Fire Protection Engineering*. All of the editors of the prior editions except for two (Jack Watts and John Hall) have retired, and a new editorial team has taken their place. Additionally, Springer has assumed the role of publisher beginning with this edition.

For the first four editions, the *SFPE Handbook of Fire Protection Engineering* was published by the National Fire Protection Association. The Society of Fire Protection Engineers owes a debt of gratitude to NFPA. Without their encouragement and confidence, this handbook might never have existed.

With a new editorial team emerge many changes. The chapters relevant to human behavior in fire have been significantly refocused and augmented. The fundamental engineering chapters have been revised to provide a better foundation for the chapters that follow. Many new chapters related to fire protection system selection and design have been added. The chapters associated with fire resistance design have been modified to reflect advances over the last decade. And, this edition includes several new chapters pertinent to industrial fire protection.

The editors owe a debt of gratitude to those whom they follow. Continuing a successful endeavor is much easier than launching it.

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Metrication

The editors of the *SFPE Handbook of Fire Protection Engineering* have worked toward the expanded use of SI units for this fifth edition. In some instances, however, US customary units have been retained. For example, when equations, correlations, or design methodologies have input variables or constants that have been developed from data originally in US customary units, those units are retained. This is also the case for certain tables, charts, and nomographs. Where equations employing US customary units are used in worked examples, the results are presented as SI units as well.

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Bart Merci

Fluid Properties

In this section, a number of fluid properties are defined. An implicit assumption in the classical fluid mechanics is the ‘continuum hypothesis’, implying that we treat fluids as continuous media, not as an ensemble of individual molecules [1]. This is justified in ‘normal’ circumstances. This way, the fluid and flow quantities are continuous and local quantities to be interpreted as averages over a volume V^* which is very small (but still very large when compared to distances between molecules). This assumption allows to define local fluid and flow properties (e.g. velocity vectors). The continuum hypothesis is adopted here.

A fluid can be a liquid or a gas (vapour).

Density

The mass density is the amount of fluid mass inside a volume:

$$\rho = \frac{m}{V}. \quad (1.1)$$

Its unit is kg/m^3 .

In a variable density flow, the density can vary in space and time and the local density at a

certain time is defined as in Equation 1.1, taking the local limit for a small volume.

In an incompressible flow, the density does not vary. In general, liquids can be considered ‘incompressible’. In gases, the density can vary due to variations in pressure or temperature (see below: ideal gas law).

The reciprocal of density is the ‘specific volume’ (m^3/kg).

Viscosity

Fluids can flow. The viscosity is the fluid property that indicates how easily molecules can move with respect to each other. Fluid particles with different velocity have the tendency to evolve to the same common velocity, through exchange of momentum. In other words, fluid layers with different velocities exert a shear stress τ onto each other. Most technically relevant fluids are ‘Newtonian’: the shear stress increases linearly with the strain rate (or velocity gradient):

$$\tau = \mu \frac{dv}{dy}. \quad (1.2)$$

The unit of τ is Pa (= N/m^2).

The proportionality factor, relating the velocity gradient to the shear stress, is the dynamic viscosity μ (unit: Pa.s).

In gases, μ typically increases with temperature, whereas in liquids it decreases with increasing temperature.

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Sometimes, the kinematic viscosity is used:

$$\nu = \frac{\mu}{\rho}. \quad (1.3)$$

Its unit is m^2/s .

The shear stress, Equation 1.2, causes friction losses in case of flow. The higher the viscosity, the larger the flow losses become for the same velocity gradient. In other words, the resistance of the fluid against (imposed) flow increases with increasing viscosity.

The viscosity of a fluid is never zero. The important implication is that, whenever there is a solid boundary, this boundary always exerts an influence on the flow field (e.g. causing the development of a boundary layer).

Specific Heat

The specific heat or thermal capacity, c , is the amount of energy required to cause a temperature rise of 1 K (or 1 °C) in 1 kg of the fluid. Its unit is $\text{J}/(\text{kg}\cdot\text{K})$.

In gases the value of the specific heat depends on the circumstances under which the energy is supplied. If the pressure is kept constant, the notation is c_p . If the volume is kept constant, the notation is c_v . The difference between the two values is called the gas constant R (also in $\text{J}/(\text{kg}\cdot\text{K})$):

$$c_p = c_v + R. \quad (1.4)$$

For liquids and solids, $c_p \approx c_v$.

Conduction Coefficient

The conduction coefficient expresses how easily heat flows inside a material. Its value indicates the heat flux per unit area (W/m^2) related to a spatial temperature gradient (K/m):

$$\vec{q} = -k\nabla T = -\lambda\nabla T. \quad (1.5)$$

This is Fourier's law. The minus sign indicates that the heat flux is always from high temperature to low temperature.

The unit of the conduction coefficient (k or λ) is $\text{W}/(\text{m}\cdot\text{K})$.

The conduction coefficient, specific heat and density can be combined to obtain the thermal diffusivity:

$$\alpha = \frac{k}{\rho c} \quad (1.6)$$

The unit of α is m^2/s .

Diffusion Coefficient

In a mixture of fluids (see below), one species can diffuse in the mixture due to concentration gradients of that species in the mixture. It is common practice to apply Fick's law for many flows:

$$\vec{J}_k = -\rho D_k \nabla Y. \quad (1.7)$$

The diffusion coefficient D thus provides the relation between the diffusion flux J_k ($\text{kg}/(\text{m}^2\cdot\text{s})$) of species k and the spatial gradient of the local mass fraction Y_k (i.e. the amount of mass of species k per kg mixture) of that species. The minus sign expresses that the diffusion flux is always from higher concentration to lower concentration.

The unit of D is m^2/s .

Dimensionless Groups of Fluid Properties

By combining the fluid properties, dimensionless groups can be constructed. Indeed, the units of ν , α and D are the same (m^2/s). Physically, the interpretation is that ν tries to make the velocity field uniform inside a fluid

(through exchange of momentum), α tries to make the temperature field uniform (through heat exchange by conduction) and D tries to make the concentration field in a mixture homogeneous (through concentration gradient driven diffusion).

The resulting dimensionless groups read:

- The Prandtl number:

$$\text{Pr} = \frac{\nu}{\alpha} = \frac{\mu c_p}{\lambda} = \frac{\mu c_p}{k}. \quad (1.8)$$

- The Schmidt number:

$$\text{Sc} = \frac{\nu}{D}. \quad (1.9)$$

- The Lewis number:

$$\text{Le} = \frac{\alpha}{D}. \quad (1.10)$$

Clearly, these numbers are connected: $\text{Le} = \text{Sc} \cdot \text{Pr}^{-1}$.

It is important to note that the dimensionless numbers Equations 1.8, 1.9, and 1.10 are still fluid properties, not flow properties.

As long as no mixtures are considered, the Prandtl number is the most relevant dimensionless fluid property, when heat transfer is an issue.

State Properties

State properties describe the state of the fluid, not the material properties of the fluid.

Pressure

The pressure (p) can be defined as the normal force per unit area at a certain point. The unit is Pa. Pressure differences are the driving force for fluid flows.

Temperature

The unit of temperature (T) is Kelvin (K). The temperature must not be confused with heat (the unit of which is Joule, J).

Internal Energy

The local motion of molecules in a fluid is related to the internal energy (e or u , with unit J/kg). This is a measure for the thermal energy.

Enthalpy

The quantity (static) enthalpy (h , with unit J/kg) is related to the internal energy through addition of pressure, divided by mass density:

$$h = u + \frac{p}{\rho} = e + \frac{p}{\rho}. \quad (1.11)$$

Entropy

The entropy is a measure for the disorder in the fluid. It is related to the second law of thermodynamics. This quantity is typically not particularly relevant for fire related issues.

Equation of State

Liquids

In liquids, the density is essentially constant, relatively very weakly dependent on pressure and temperature. Yet, the general expression that provides the equation of state defines the relation between density, temperature and pressure:

$$\rho = f(p, T) \quad (1.12)$$

Gases: Ideal Gas Law

In gases, it is common practice to specify Equation 1.12 as the ‘ideal gas law’:

$$p = \rho RT \quad (1.13)$$

For fire related flows, this is justified. Most gases behave as air would do and air behaves as an ideal gas (with the exception of extremely low or

high pressure or temperature, but this is not relevant for real-life fire applications). The gas constant R (J/(kg.K)) has been introduced in Equation 1.4 and the temperature T is expressed in Kelvin (K).

Mixtures

In fire related flows, the fluid can be a mixture. Obvious examples are smoke or flames. A distinction must be made between chemical and physical issues. If toxicity is an issue, chemical aspects are important. As long as the flow itself is concerned, the physical behaviour of many gaseous mixtures resembles very much the behaviour of hot air. One reason is that the species most often encountered, have comparable diffusivities (with the important exception of hydrogen, which has a much higher diffusivity). Another reason is that typically by far mixtures in fire related flows consist mainly of air.

As a consequence, the simplification is made very commonly to treat a mixture of hot gases as hot air, applying the ideal gas law (Equation 1.13) with the gas constant for air and using the (temperature dependent) viscosity for hot air. Therefore, mixtures of gases do not receive much attention when fluid mechanics aspects are considered in case of fire.

Yet, a few definitions are introduced here. The mass fraction Y_i of species i is the ratio of the local amount of mass of species i to the local amount of mass of mixture. It is therefore a non-dimensional quantity. Conservation of mass leads to the statement that, everywhere in physical space, the sum of all mass fractions of all species equals unity: $\sum_{i=1}^N Y_i = 1$.

Using the notion of mass fractions, the fluid properties of mixtures can be determined from the fluid properties of their constituent species. E.g. the specific heat becomes $c = \sum_{i=1}^N Y_i c_i$.

Also state properties can be defined as such. E.g. static enthalpy becomes: $h = \sum_{i=1}^N Y_i h_i$.

Conservation Equations

Figure 1.1 visualises a streamline through a surface of a (control) volume. This concept will be used to develop the conservation equations in the integral formulation. A streamline is defined such that locally the velocity vector is tangent to the streamline. A collection of streamlines is called a stream tube.

Conservation of Mass—Continuity Equation

Conservation of mass expresses the following principle:

The amount of mass that flows into a stationary volume per unit time, equals the outflow of mass per unit time out of that same volume plus the amount of mass accumulation per unit time in that same volume.

Mathematically, this is formulated as follows:

- The net outflow per unit time is given by a closed surface integral over the entire area of the manifold ∂V , enclosing the volume V :

$$\iint_{\partial V} \rho \vec{v} \cdot \vec{n} dA; \text{ in this expression, } \vec{v} \text{ is the local}$$

velocity vector at a certain position on ∂V , \vec{n} the local normal vector on the surface (i.e. the vector with length equal to 1, locally perpendicular to the surface and pointed outward) and dA the area of an infinitesimal element

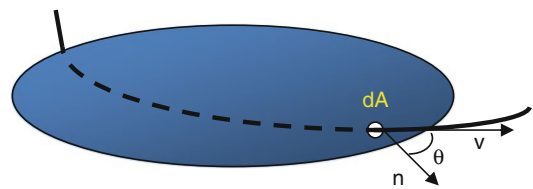


Fig. 1.1 Streamline through a surface. Notation: dA is the area of an infinitesimal part of the surface; n is the normal vector, with length equal to unity, perpendicular to dA and pointing 'outward' of the control volume, spanned by the surface; v is the local flow velocity vector at position dA ; θ is the angle between vectors n and v

on the surface; note that the inner product $\vec{v} \cdot \vec{n}$ > 0 for outflow, while $\vec{v} \cdot \vec{n} < 0$ for inflow;

- The accumulation of mass per unit time is obtained from a derivation with respect to time of the integral of the mass density over the entire volume: $\frac{\partial}{\partial t} \iiint_V \rho dV$.

The conservation of mass thus reads:

$$\frac{\partial}{\partial t} \iiint_V \rho dV + \iint_{\partial V} \rho \vec{v} \cdot \vec{n} dA = 0. \quad (1.14)$$

This equation is also called the *continuity equation*.

An important simplification is found in the case of *permanent* (or ‘*steady*’) *motion*. In that case, the time derivative disappears in Equation 1.14:

$$\iint_{\partial V} \rho \vec{v} \cdot \vec{n} dA = 0. \quad (1.15)$$

A further simplification concerns *incompressible fluids* (e.g. water in a pipe under normal conditions). In that case, density does not change, so that not only Equation 1.15 applies, but it further simplifies to read:

$$\iint_{\partial V} \vec{v} \cdot \vec{n} dA = 0. \quad (1.16)$$

A very simple illustration of Equation 1.15 is provided on the basis of Fig. 1.2. There is no flow through the solid boundaries (solid lines in

Fig. 1.2), so the only contributions to $\iint_{\partial V} \rho \vec{v} \cdot \vec{n} dA$

stem from surfaces 1 and 2. In surface 1, the velocity vector is pointing inward, while the normal vector is by definition pointing outward, so the contribution (under the simplified assumption of uniform flow through the cross-section) becomes: $-\rho_1 v_1 A_1$. On surface 2, the velocity and the normal vectors are pointing outward, leading to: $+\rho_2 v_2 A_2$. Equation 1.15 thus provides: $-\rho_1 v_1 A_1 + \rho_2 v_2 A_2 = 0 \rightarrow \rho_1 v_1 A_1 = \rho_2 v_2 A_2$. In case of incompressible flow (Equation 1.16) this further simplifies to: $v_1 A_1 = v_2 A_2$.

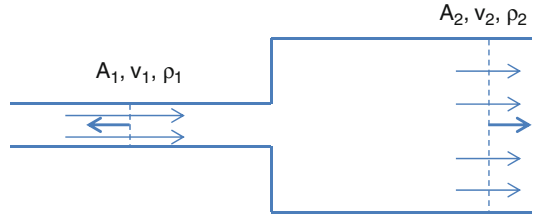


Fig. 1.2 Illustration of conservation of mass for steady flow (Equation 1.15) through a pipe expansion. *Dashed lines*: boundary of control volume. *Bold vectors*: normal vectors (unity length, perpendicular to surface and pointing outward). The other vectors indicate velocity vectors

The integral in Equation 1.14 in fact refers to the total net mass flow rate (kg/s) through a surface with area A :

$$\dot{m} = \iint_A \rho \vec{v} \cdot \vec{n} dA \quad (1.17)$$

If the mass density is not included, the total net volume flow rate (m^3/s) through a surface with area A is found:

$$\dot{V} = \iint_A \vec{v} \cdot \vec{n} dA. \quad (1.18)$$

Expression (1.14) can also be formulated in differential form, applying Green’s theorem:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \vec{v}) = 0. \quad (1.19)$$

The symbol ∇ is the divergence operator:

$$\begin{aligned} \nabla \cdot \vec{v} &= \left(\frac{\partial}{\partial x} \vec{1}_x + \frac{\partial}{\partial y} \vec{1}_y + \frac{\partial}{\partial z} \vec{1}_z \right) \cdot (v_x \vec{1}_x + v_y \vec{1}_y + v_z \vec{1}_z) \\ &= \frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z}. \end{aligned} \quad (1.20)$$

In Equation 1.20, $\vec{1}_x$ is the notation for the unity vector, i.e. a vector with length equal to unity, in the x -direction.

Expression (1.15), for steady flow, reads in differential form:

$$\nabla \cdot (\rho \vec{v}) = 0, \quad (1.21)$$

while expression (1.16), for incompressible fluids, becomes:

$$\nabla \cdot \vec{v} = 0. \quad (1.22)$$

This shows that the velocity field for any flow of an incompressible fluid is ‘divergence free’, or ‘solenoidal’.

Total Momentum

Now the integral formulation for the conservation of total momentum is discussed. Figure 1.1 again serves as the basic sketch.

Conservation of total momentum refers to the expression of Newton’s second law, applied to flows. The net change in momentum of a system per unit time in a certain sense and direction equals the net force on that system in that sense and direction.

Expressed for a stationary volume, this becomes:

The total force onto a stationary volume equals the sum of the net outflow of momentum per unit time out of that same volume plus the accumulation of momentum per unit time in that same volume.

The local amount of momentum per unit volume is $\rho \vec{v}$ (kg/(m²s)). Newton’s second law thus reads:

$$\frac{\partial}{\partial t} \iiint_{\partial V} \rho \vec{v} dV + \iint_{\partial V} \rho \vec{v} (\vec{v} \cdot \vec{n}) dA = \vec{F}_{tot}. \quad (1.23)$$

Note that Equation 1.19 is a vector equation, i.e. the equation is valid for each component/direction individually.

For a permanent (or ‘steady’) motion, expression (1.19) simplifies to:

$$\iint_{\partial V} \rho \vec{v} (\vec{v} \cdot \vec{n}) dA = \vec{F}_{tot}. \quad (1.24)$$

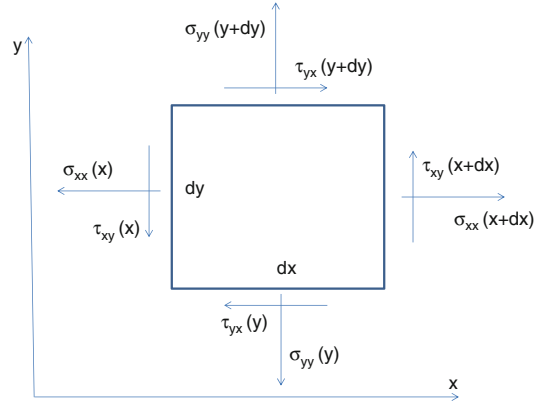


Fig. 1.3 Definition of normal stresses and shear stresses (2D)

The total force consists of:

- Surface forces:
 - Pressure (Pa);
 - Viscous stresses (Pa);
- Body forces:
 - Gravity (N);
 - Others (not relevant for fire related flows).

These forces are discussed now, in differential formulation:

$$\begin{cases} F_{tot,x} = \frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} + \frac{\partial \tau_{xz}}{\partial z} + \rho g_x \\ F_{tot,y} = \frac{\partial \tau_{yx}}{\partial x} + \frac{\partial \sigma_{yy}}{\partial y} + \frac{\partial \tau_{yz}}{\partial z} + \rho g_y \\ F_{tot,z} = \frac{\partial \tau_{zx}}{\partial x} + \frac{\partial \tau_{zy}}{\partial y} + \frac{\partial \sigma_{zz}}{\partial z} + \rho g_z \end{cases} \quad (1.25)$$

The final terms in Equation 1.25 refer to the gravity acceleration vector, multiplied with the local mass density. Figure 1.3 shows how the normal stresses and shear stresses are defined.

The shear stresses are found from Stokes’ law:

$$\begin{aligned} \tau_{xy} = \tau_{yx} &= \mu \left(\frac{\partial v_x}{\partial y} + \frac{\partial v_y}{\partial x} \right) \\ \tau_{xz} = \tau_{zx} &= \mu \left(\frac{\partial v_x}{\partial z} + \frac{\partial v_z}{\partial x} \right) \\ \tau_{yz} = \tau_{zy} &= \mu \left(\frac{\partial v_z}{\partial y} + \frac{\partial v_y}{\partial z} \right). \end{aligned} \quad (1.26)$$

The shear stresses are thus proportional to the dynamic viscosity and the local velocity gradients.

The normal stresses contain contributions from stresses due to fluid dilatation (for variable density flows only) and pressure:

$$\begin{aligned}\sigma_{xx} &= -p + \frac{2}{3}\mu \left(\frac{\partial v_x}{\partial x} - \frac{1}{3}\nabla \cdot \vec{v} \right) \\ \sigma_{yy} &= -p + \frac{2}{3}\mu \left(\frac{\partial v_y}{\partial y} - \frac{1}{3}\nabla \cdot \vec{v} \right) \\ \sigma_{zz} &= -p + \frac{2}{3}\mu \left(\frac{\partial v_z}{\partial z} - \frac{1}{3}\nabla \cdot \vec{v} \right).\end{aligned}\quad (1.27)$$

The above results in the *Navier–Stokes equations*:

$$\begin{cases} \frac{\partial}{\partial t}(\rho v_x) + \rho v_x \frac{\partial v_x}{\partial x} + \rho v_y \frac{\partial v_x}{\partial y} + \rho v_z \frac{\partial v_x}{\partial z} = -\frac{\partial p}{\partial x} + \frac{2}{3} \frac{\partial}{\partial x} \left(\mu \left(\frac{\partial v_x}{\partial x} - \frac{1}{3}\nabla \cdot \vec{v} \right) \right) + \frac{\partial \tau_{xy}}{\partial y} + \frac{\partial \tau_{xz}}{\partial z} + \rho g_x \\ \frac{\partial}{\partial t}(\rho v_y) + \rho v_x \frac{\partial v_y}{\partial x} + \rho v_y \frac{\partial v_y}{\partial y} + \rho v_z \frac{\partial v_y}{\partial z} = -\frac{\partial p}{\partial y} + \frac{2}{3} \frac{\partial}{\partial y} \left(\mu \left(\frac{\partial v_y}{\partial y} - \frac{1}{3}\nabla \cdot \vec{v} \right) \right) + \frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \tau_{yz}}{\partial z} + \rho g_y \\ \frac{\partial}{\partial t}(\rho v_z) + \rho v_x \frac{\partial v_z}{\partial x} + \rho v_y \frac{\partial v_z}{\partial y} + \rho v_z \frac{\partial v_z}{\partial z} = -\frac{\partial p}{\partial z} + \frac{2}{3} \frac{\partial}{\partial z} \left(\mu \left(\frac{\partial v_z}{\partial z} - \frac{1}{3}\nabla \cdot \vec{v} \right) \right) + \frac{\partial \tau_{xz}}{\partial x} + \frac{\partial \tau_{yz}}{\partial y} + \rho g_z \end{cases}\quad (1.28)$$

Note that the presence of the gravity force is essential in order to account for the Archimedes force. This is essential for buoyancy-driven forces, which is important in the context of fire.

Also note that pressure gradients (or pressure differences) are the driving force for flows, not the absolute pressure level.

Energy

Conservation of energy refers to the first law of thermodynamics:

The change (per unit time) of the total internal energy of a system equals the sum of the heat added (per unit time) to the system and the work (per unit time) exerted onto that system.

The total internal energy consists of:

- Static internal energy e (J/kg) or ρe (J/m³);
- Kinetic energy $\rho v^2/2$ (J/m³).

The mathematical formulation of the first law of thermodynamics for a stationary open system can be found in many textbooks (e.g. [2–9]). It reads:

$$\begin{aligned}\frac{\partial}{\partial t} \int_V \left(\rho e + \frac{1}{2} \rho v^2 \right) dV &= - \oint_{\partial V} \left(\rho e + \frac{1}{2} \rho v^2 \right) \vec{v} \cdot \vec{n} dS - \oint_{\partial V} p \vec{v} \cdot \vec{n} dS + \oint_{\partial V} \left(\vec{\tau} \cdot \vec{v} \right) \cdot \vec{n} dS \\ &\quad + \int_V \rho \vec{g} \cdot \vec{v} dV + \int_V \rho S_h dV - \oint_{\partial V} \vec{q} \cdot \vec{n} dS\end{aligned}\quad (1.29)$$

The terms on the right hand side are:

- First term: Net inflow of total internal energy into the control volume (‘convection’); the minus sign is necessary to comply with the

sign convention (see previous sections: the normal vector is pointing outward).

- Second term: Work of the flow against pressure. This is work from a force (pressure),

exerted onto the surface. The work by the pressure onto the flow is positive for inflow and negative for outflow, which explains the minus sign.

- Third term: Work by the viscous stresses. This is work from a force (viscous stresses, Equations 1.26 and 1.27, exerted onto the surface. With the sign conventions used (Fig. 1.3 and outward pointing normal vector), this is a term with a plus sign.
- Fourth term: Work by gravity. This is work by a volume force, exerted inside the volume. This work is positive for a downward flow,

so that with $\vec{g} = -g\vec{1}_y$ no minus sign is required in this term (if the y-direction is positive vertically upward).

- Fifth term: Volumetric source term of heat / internal energy (e.g. radiation). This term can be positive or negative.
- Final term: Net incoming flux of heat/internal energy (e.g. conduction). The flux with the flow cannot be added to this term (as it is already included in the convection term).

The energy equation can also be formulated, using enthalpy Equation 1.11:

$$\begin{aligned} \frac{\partial}{\partial t} \int_{CV} \left(\rho h + \frac{1}{2} \rho v^2 \right) dV &= \frac{\partial}{\partial t} \int_{CV} p dV - \oint_{\partial CV} \left(\rho h + \frac{1}{2} \rho v^2 \right) \vec{v} \cdot \vec{n} dS + \oint_{\partial CV} \left(\vec{\tau} \cdot \vec{v} \right) \cdot \vec{n} dS \\ &+ \int_{CV} \rho \vec{g} \cdot \vec{v} dV + \int_{CV} \rho S_h dV - \oint_{\partial CV} \vec{q} \cdot \vec{n} dS. \end{aligned} \quad (1.30)$$

In differential formulation, this reads:

$$\frac{\partial}{\partial t} \left(\rho h + \frac{1}{2} \rho v^2 \right) + \nabla \cdot \left(\rho \left(h + \frac{1}{2} v^2 \right) \vec{v} \right) = \frac{\partial p}{\partial t} + \nabla \cdot \left(\vec{\tau} \cdot \vec{v} \right) + \rho S_h + \rho \sum_{i=1}^N Y_i \vec{g} \cdot \vec{v}_i - \nabla \cdot \vec{q} \quad (1.31)$$

The (static) enthalpy is the mass-weighted sum of the enthalpies of species i :

$$h = \sum_{i=1}^N Y_i h_i. \quad (1.32)$$

The enthalpy h_i is the sum of a reference enthalpy (the chemical standard formation enthalpy of species i) and a ‘sensible’ (thermal) enthalpy [5–9]. For ideal gases this reads:

$$h_i(T) = h_{ref,i} + \int_{T_{ref}}^T c_{p,i}(T) dT, \quad (1.33)$$

with $c_{p,i}$ the specific heat of species i , defined above.

Note that in Equation 1.31, expressed in terms of enthalpy, the source term ρS_h contains e.g. radiation, but not a heat release rate due to

combustion. Combustion reactions transform chemically bound enthalpy into sensible enthalpy and as such cause a temperature rise, but the sum of sensible and chemical enthalpy does not change locally. If the energy equation is expressed in terms of temperature (or sensible enthalpy), a source term due to the combustion heat release rate does appear.

The final term in Equation 1.31 reads:

$$\nabla \cdot \vec{q} = -\nabla \cdot (\lambda \nabla T) - \nabla \cdot \left(\rho \sum_{i=1}^N h_i D_i \nabla Y_i \right) + D.E. \quad (1.34)$$

The abbreviation ‘D.E.’ stands for the ‘Dufour effect’, i.e. and additional enthalpy flux due to species concentration differences. This effect is

ignored in fire related flows. The first terms in Equation 1.34 refer to Fourier's law for heat conduction, Equation 1.5. The middle terms refer to an enthalpy flux due to diffusion, using Fick's law, Equation 1.7.

The general expression, Equation 1.31, can often be simplified. Many fire-induced flows are

low-Mach number flows (note: this is not true for explosions). The time derivative of pressure can often be ignored. Also the work done by gravity, by the viscous shear stresses and by the normal stresses becomes very small and the kinetic energy is negligible. Using Equations 1.8 and 1.9, the energy equation becomes:

$$\frac{\partial}{\partial t}(\rho h) + \nabla \cdot (\rho h \vec{v}) = \nabla \cdot \left(\frac{\mu}{\text{Pr}} \nabla h + \mu \sum_{i=1}^N \left(\frac{1}{Sc_i} - \frac{1}{\text{Pr}} \right) h_i \nabla Y_i \right) + \rho S_h \quad (1.35)$$

For unity Lewis number ($Le_i = 1$ for all i , Equation 1.10) fluids, this further simplifies to:

$$\frac{\partial}{\partial t}(\rho h) + \nabla \cdot (\rho h \vec{v}) = \nabla \cdot \left(\frac{\mu}{\text{Pr}} \nabla h \right) + \rho S_h. \quad (1.36)$$

Hydrostatics

Hydrostatics

From the general Navier–Stokes equations (1.28), the basic law for hydrostatics is immediately recovered. Indeed, setting all velocities in a certain environment equal to zero, the only terms remaining are:

$$\nabla p = \rho_{amb} \vec{g}. \quad (1.37)$$

Equation 1.37 is valid at any time (in the absence of motion). For the special case where $\vec{g} = -g \vec{1}_y$, with $g = 9.81 \text{ m/s}^2$, Equation 1.37 reads (in the y -direction):

$$\frac{dp}{dy} = -\rho_{amb} g \quad (1.38)$$

Note that Equation 1.37 in such circumstances also implies that pressure does not vary in the horizontal directions.

Equation 1.38 can be integrated:

$$p = p_{ref} - \rho_{amb} g (y - y_{ref}). \quad (1.39)$$

Buoyancy

The main relevance of the fundamental law of hydrostatics, Equation 1.37, lies in the fact that in many fire related flows, buoyancy plays a dominant role. This can be learnt from the Navier–Stokes equations, Equation 1.28, combining the forces due to pressure gradients and gravity. In the vertical direction (still with $\vec{g} = -g \vec{1}_y$), using Equation 1.38, the resulting force per unit area reads:

$$-\frac{dp}{dy} - \rho g = (\rho_{amb} - \rho)g. \quad (1.40)$$

In the process of getting to expression (1.40), the implicit assumption is made that pressure differences in the horizontal directions are small.

Equation 1.40 reveals that the driving force in situations where buoyancy dominates, stems from density differences, in the presence of a gravity field. This is known as Archimedes' law. Note that, since gravity acts in the vertical direction only, buoyancy forces by definition also act in the vertical direction only.

For small density differences, the approximation $\rho \approx \rho_{amb}$ is typically made in the Navier–Stokes equations, except that the difference $(\rho_{amb} - \rho)$ is accounted for in combination with gravity (Equation 1.40). This is called Boussinesq's approximation. In the context of small density differences, expression (1.40) can be developed further, using a Taylor series

expansion: $\rho = \rho(T, p) \Rightarrow \rho \approx \rho_{amb} + \left(\frac{\partial \rho}{\partial T}\right)_p (T - T_{amb}) + \left(\frac{\partial \rho}{\partial p}\right)_T (p - p_{amb})$. Typically the pressure correction is much smaller than the

temperature correction. Using the thermal volumetric expansion coefficient:

$$\beta = -\frac{1}{\rho} \left(\frac{\partial \rho}{\partial T}\right)_p, \quad (1.41)$$

the Archimedes force becomes:

$$(\rho_{amb} - \rho)g = \rho_{amb}\beta(T - T_{amb})g, \quad \text{if } \beta(T - T_{\infty}) \ll 1. \quad (1.42)$$

The basic expression is thus Equation 1.40, based on density differences, while Equation 1.42 is only valid for small enough temperature differences.

number, which is the ratio of inertial forces to viscous forces:

$$\text{Re} = \frac{\rho u L}{\mu} = \frac{u L}{\nu}. \quad (1.43)$$

Scaling Laws—Dimensionless Flow Numbers

In this section, starting from the governing equations, some scaling laws and non-dimensional flow numbers are introduced. The characteristic length scale is L , the characteristic velocity is u .

The viscous forces tend to damp the inherent instabilities in the non-linear convection terms in the Navier–Stokes equations, while these instabilities can evolve towards fully-developed turbulence for large enough Reynolds number. This is addressed in the next section.

When buoyancy is dominant, the proportionality $\frac{\rho u^2}{L} \sim \Delta \rho g$ leads to the Froude number, which is the ratio of inertial forces to the Archimedes force:

$$\text{Fr} = \frac{\rho u^2}{\Delta \rho g L}. \quad (1.44)$$

Dimensionless Flow Numbers

Examination of the terms in the Navier–Stokes equations, Equations 1.28 and 1.26, leads to the following proportionalities: $\frac{\rho u}{\tau} \sim \frac{\rho u^2}{L} \sim \frac{\Delta p}{L} \sim \Delta \rho g \sim \mu \frac{u}{L^2}$. Several non-dimensional flow numbers can be derived now, as follows. The importance of each of the numbers mentioned, depends on the importance of the corresponding terms in the Navier–Stokes equations. The convection term/inertia term is always important, as it characterizes the flow. Depending on the flow configuration, one or more terms are in competition with (or determine) the inertia term (or thus the flow). This is explained next.

When the viscous stresses prevail, the proportionality $\frac{\rho u^2}{L} \sim \mu \frac{u}{L^2}$ leads to the Reynolds

In the fire community, this is often simplified to:

$$\text{Fr} = \frac{u^2}{gL}. \quad (1.45)$$

Expression (1.44) resembles the underlying physics more than Equation 1.45. On the other hand, the difference between expressions (1.44) and (1.45) is no more than a numerical factor, depending on the densities at hand. Moreover, in many experiments it is much more straightforward to measure velocities than mass densities, so that it is easier to characterize the experimental set-up through formulation (1.45). This explains why the use of Equation 1.45 is popular in diagrams and correlations.

If large (imposed) pressure differences occur, sometimes the Euler number comes into play, through $\frac{\rho u^2}{L} \sim \frac{\Delta p}{L}$:

$$Eu = \frac{\Delta p}{\rho u^2}. \quad (1.46)$$

In fire related flows, this is often not relevant.

In buoyancy driven flows, applying Boussinesq's hypothesis, the driving force (Equation 1.42) can also be made dimensionless as:

$$Ra = \frac{L^3 g \beta \Delta T}{\alpha \nu}. \quad (1.47)$$

This is the Rayleigh number. Alternatively, the Grashof number can be used:

$$Gr = \frac{L^3 g \beta \Delta T}{\nu^2}. \quad (1.48)$$

The relation between the two is: $Ra = Gr.Pr$, with the Prandtl number as defined in Equation 1.8. The Grashof number can be interpreted as a ratio of buoyancy forces (with Boussinesq's approximation) to the viscous forces. This is relevant in boundary layers (see below).

Scaling

In this section, scaling is briefly discussed in the context of fluid mechanics. As such, only the momentum equation is considered, albeit that at the end of this section, some remarks are formulated on the fire heat release rate (using the energy equation) and the study of unsteady phenomena (using the mass conservation equation). As a consequence, no comments are formulated on e.g. convective heat transfer or conduction through solids, nor on radiation. For an extensive discussion on scaling, the reader is referred to [10, 11].

The main non-dimensional numbers in low-Mach number flows are the Reynolds number Equation 1.43 and the Froude number Equation 1.44 (or Equation 1.45). Firstly, it is mentioned that the only way to preserve both numbers when scaling (up or down) a flow in a

certain configuration, is through the use of different fluids. Indeed, assume that the fluid does not change (and that the densities do not change). Then preservation of Re reveals that: $Re_1 = Re_2 \Rightarrow \frac{u_1 L_1}{\nu} = \frac{u_2 L_2}{\nu} \Rightarrow u_2 = \frac{u_1 L_1}{L_2}$. Preservation of the Froude number (still with the assumption that densities do not change) leads to: $Fr_1 = Fr_2 \Rightarrow \frac{u_1^2}{g L_1} = \frac{u_2^2}{g L_2} \Rightarrow u_2 = u_1 \sqrt{\frac{L_2}{L_1}}$. Clearly, this is inconsistent with the requirement, stemming from the preservation of the Reynolds number. Both numbers can be preserved if, starting from the requirement for preservation of the Froude number, the fluid's viscosity is modified such that also the Reynolds number is preserved. This is not straightforward.

Fortunately, both the Reynolds number and the Froude number have the property that, as soon as they are large enough, their actual value becomes irrelevant. In other words, as soon as they are sufficiently high, the qualification 'high' is sufficient, not the exact number. This is due to turbulence, overwhelming molecular phenomena (see next section). This can also be understood intuitively. The Reynolds number is the ratio of inertia to viscous damping forces. Either the damping force is strong enough to overcome the inherent instabilities in the non-linear convection terms in the Navier–Stokes equations (laminar flow), almost strong enough (transitional flow) or not strong enough (turbulent flow). When turbulence is fully developed, the strength of the viscous stress becomes irrelevant, i.e. the true value of the Reynolds number becomes irrelevant. For the Froude number, it is most instructive to examine expression (1.44). The driving force for buoyancy is in the denominator. If density differences become small, buoyancy becomes irrelevant and the Froude number is high. As such, high values of the Froude number implies that buoyancy is not important and thus that the error is small when the Froude number is not preserved (as long as it stays sufficiently high).

Knowing this, it is instructive to examine the order or magnitude of Reynolds number and Froude number in fire related flows. Indeed, if one of the numbers can be expected to be high,

that number need not be preserved in scaling. Typical dimensions are in the order of 1 m: $L = O(m)$. Typical velocities are in the order of 1 m/s: $u = O(m/s)$. Densities are in the order of 1 kg/m^3 : $\rho = O(\text{kg/m}^3)$. The dynamic viscosity in gases is in the order of $10^{-6} \text{ Pa}\cdot\text{s}$: $\mu = O(10^{-6} \text{ Pa}\cdot\text{s})$. Using these numbers, the Reynolds number Equation 1.43 is: $\text{Re} = O\left(\frac{1 \cdot 1}{10^{-6}}\right) = O(10^6)$, while the Froude number Equation 1.44 is: $\text{Fr} = O\left(\frac{1 \cdot 1}{1 \cdot 10^{-6}}\right) = O(0.1)$. Obviously, these are rough order of magnitude analyses, but it is clear that in fire related flows, the choice will be made to preserve the Froude number, not the Reynolds number, when scaling is applied.

The energy equation also provides information regarding scaling laws. The simplified formulation (1.36) can be used for fire-related flows. Yet, temperatures are very important in fire related flows, so the energy equation should be interpreted in terms of sensible enthalpy, in which case the fire heat release rate (\dot{Q} , in W) comes into play. Knowing that, in terms of dimensions, (sensible) enthalpy differences can be re-written as the product of specific heat and temperature differences, Equation 1.36 leads to the following proportionalities: $\frac{\rho c_p \Delta T}{t} \sim \frac{\rho c_p \Delta T u}{L} \sim \frac{\dot{Q}}{L^3} \sim \frac{k \Delta T}{L^2}$.

This reveals that:

$$\dot{Q} \sim u \rho c_p \Delta T L^2. \quad (1.49)$$

It is common practice to scale configurations such that the temperatures remain the same. This also implies that densities do not change (if the same fluid is applied). As has just been explained, the Froude number Equation 1.44 is preserved, so that the velocity scales as $\sim \sqrt{L}$. As a consequence, the fire heat release rate scales as:

$$\frac{\dot{Q}_1}{\dot{Q}_2} = \frac{\sqrt{L_1} L_1^2}{\sqrt{L_2} L_2^2} \Rightarrow \dot{Q} \sim L^{5/2}. \quad (1.50)$$

Finally, it is noteworthy that the conservation of mass, Equation 1.19, reveals that:

$$t \sim L/u. \quad (1.51)$$

Applying Froude scaling, the velocity scales as $\sim \sqrt{L}$, so that expression (1.51) reveals that the

temporal evolution of quantities (e.g. temperature) depends on the dimensions of the configuration as $t \sim \sqrt{L}$. This is relevant when unsteady phenomena are studied.

Turbulence

There are numerous text books on turbulence and turbulent flows, e.g. [12, 13]. Only some introductory comments are presented here.

Reynolds Number

In the previous section it has been mentioned that the Reynolds number Equation 1.43 is the ratio on inertia to viscous forces. It is well-known that the convection term in the Navier–Stokes equations (1.28) is inherently unstable and that the flow becomes turbulent when the viscous forces are not strong enough to damp the instabilities, i.e. when the Reynolds number becomes sufficiently high. Below a certain threshold number, the flow remains ‘laminar’. There is no sudden change from ‘laminar’ to ‘turbulent’: there is a ‘transition’ zone in between.

Care must be taken in the definition of this ‘critical’ Reynolds number, in the sense that the length scale must be defined. In flows over flat plates, it is common practice to use the distance from the leading edge and Re_c is in the order of 500.000. In pipe flows, it is common practice to use the pipe diameter as characteristic length scale and Re_c is in the order of 2.000.

It is important to stress that the Reynolds number is a flow property, not a fluid property.

Turbulence is typically defined on the basis of a number of properties [13]:

- Randomness: there are fluctuations in the flow;
- Three-dimensionality: even if the mean flow is 2D or axisymmetric, the vortices or ‘eddies’ are always three-dimensional;
- There is a wide range of length scales and time scales in the flow. The largest scales are determined by the configuration at hand,

while the smallest scales are determined by the Reynolds number. The smallest scales can easily be 10,000 times smaller than the largest scales.

- Turbulent mixing is very effective.
- There is a lot of diffusion and dissipation. Turbulence dies out quickly if not sustained by velocity gradients in the mean flow.
- There is vortex stretching, transferring energy from the mean flow to turbulent fluctuations.

It is instructive to briefly explain the randomness in the flow. Indeed, knowing that the Navier–Stokes equations (Equation 1.28) are deterministic, one may pose the question how it is possible that randomness occurs when applying deterministic boundary and initial conditions. The reason is that there are always small fluctuations, i.e. the boundary and initial conditions are never known with infinite precision. Due to the unstable convection terms in the Navier–Stokes equations, turbulent flows are extremely sensitive to details and this creates randomness in the instantaneous flow fields. This makes it impossible to make long-term predictions of instantaneous turbulent flow fields and explains why turbulent flows are tackled in simulation through statistical approaches (see below). Obviously, the mean flow can still be deterministic (see below).

Reynolds Averaging

As mentioned in the previous section, the fluctuations in a turbulent flow make a direct analysis through the Navier–Stokes equations (Equation 1.28) impossible. Therefore, a statistical approach is adopted. The primary interest is often the mean flow. To that purpose, the Navier–Stokes equations are averaged. The concept of Reynolds averaging is explained first.

Consider a turbulent flow. Measuring a velocity component (or e.g. a temperature) at a certain location will then yield a fluctuating signal, as explained. One can now determine the ‘average’ of that signal. The true definition of a Reynolds average [12, 13] is that many realizations of the

‘same’ turbulent flow are made, repetitive measurements of the quantity are made at the same location, and the average value of the measurements is determined. In a simplified manner, though, one can think of this procedure as a time averaging, where the averaging period Δt is sufficiently long, compared to the largest turbulent time scales, but sufficiently short compared to time scales associated with possible variations in the mean flow:

$$\bar{v}_x(t) = \frac{1}{\Delta t} \int_{t-\Delta t}^t v_x(t) dt; \bar{T}(t) = \frac{1}{\Delta t} \int_{t-\Delta t}^t T(t) dt. \quad (1.52)$$

It is clear that this is only possible if the turbulent time scales are short, compared to time scales in the mean flow. The ‘integral’ turbulent time scale is typically less than 1 s, so in many fire related flows this concept of Reynolds averaging is possible.

Using Equation 1.52, the instantaneous value can be expressed as the sum of the (Reynolds) averaged value and the instantaneous fluctuation around that value:

$$v_x(t) = \bar{v}_x(t) + v'_x(t); T(t) = \bar{T}(t) + T'(t). \quad (1.53)$$

Note that:

$$\begin{aligned} \overline{v'_x(t)} &= 0; \overline{T'(t)} = 0; \overline{\overline{v_x(t)}} \\ &= \overline{v_x(t)}; \overline{\overline{T(t)}} = \overline{T(t)}. \end{aligned} \quad (1.54)$$

Applying this averaging technique to the conservation equations (1.19), (1.28) and (1.36), the equations are obtained for the Reynolds-averaged quantities. They are very similar to the instantaneous equations, but some additional terms appear:

- Reynolds stresses in the momentum equations;
- Turbulent heat fluxes in the energy equation.

This is explained next. For the sake of ease, the energy equation is simplified here: it is expressed in terms of temperature and no chemical reactions, nor radiation, are considered. The