Statistics and Probability in High School

Carmen Batanero and
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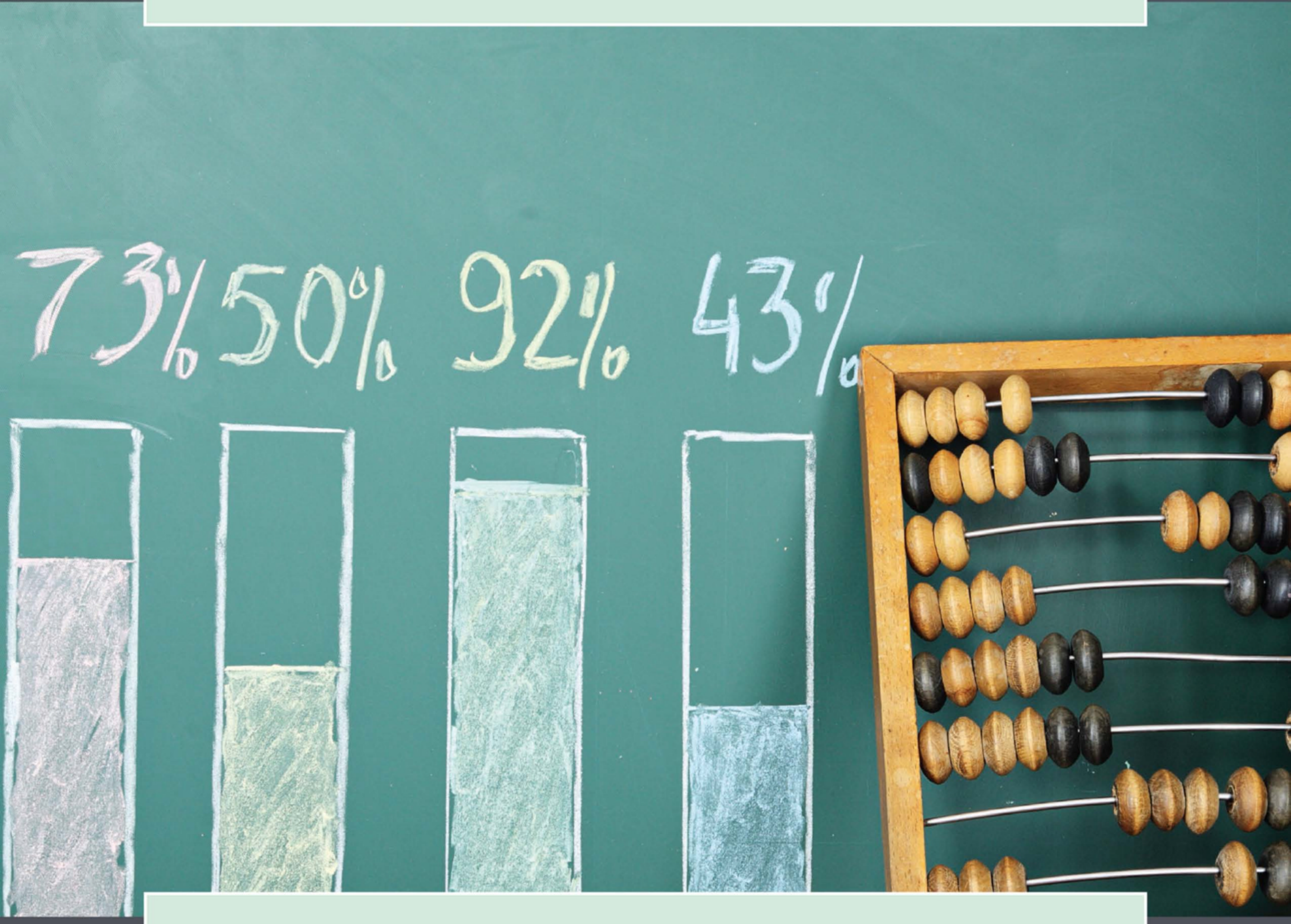

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PREFACE

Research in statistics and probability education has produced a variety of results that would be useful for both secondary and high-school mathematics teachers and the educators of these teachers. Although there are many good textbooks in different countries that describe statistical ideas with a formalisation level adequate for students, usually these textbooks are written in a sequential way so that the different concepts and procedures are introduced in turn, with insufficient connections between them and limited attention to students' underlying intuitions.

There are, of course, excellent exceptions such as the books produced by the Schools Council (1980) in Statistical Education Project in the 1980's; yet, even, these textbooks do not include a detailed summary of research related to the teaching of the concepts, which started to get shape only after the first International Conference on Teaching Statistics in Sheffield in 1982.

In the later stages of our careers and, after collaborating and corresponding for many years in different projects, we decided to write a book directed to reinforce the mathematical and didactical knowledge of high-school teachers in statistics and probability. At the same time, we wish to offer examples of potential activities useful to introduce the main statistics and probability concepts and enhance the underlying ideas at this school level.

Consequently, in this book we provide examples of teaching situations, while at the same time we review research on adolescents' stochastical[1] reasoning and literacy, with the aim to provide recommendations and orientations for teaching these topics within high-school mathematics. The expression "high school" relates to different educational levels depending on the country; in this book, we will consider students from ages 14 to 18 (grades 9–12 in the United States of America curriculum). The book is organised in five chapters:

In the first chapter, we present some principles we use to select the content analysed in the book and the approach to teach this content. These principles emerge from:

a. Our own teaching and research experience;

b. An analysis of stochastic high-school curricula in several countries (e.g., ACARA, 2010; NCTM, 2000; CCSSI, 2010, MEC, 2007);

c. The synthesis of available research (as summarised, for example, in Biehler, Ben-Zvi, Bakker, & Makar, 2013; Chernoff and Sriraman, 2014; Garfield & Ben-Zvi, 2008; Jones, 2005; Jones, Langrall, & Money, 2007; Shaughnessy, 1992, 2007; Shaughnessy, Garfield, & Greer, 1996);

[1] In some countries the term stochastics is used to highlight the mutual dependence between probabilistic and statistical knowledge and reasoning. Throughout the book we occasionally use stochastics for statistics and probability to express our view that these fields are tightly interconnected and should be taught together.

d. Our own conceptions of statistical and probabilistic literacy, thinking, and reasoning; and

e. Our extensive experience with strategies that may help support student development in stochastic literacy, thinking, and reasoning.

The first chapter sets out key educational principles. Each of the following chapters (Chapters 2–5) has a focus on a group of related fundamental stochastic ideas, while taking into account that high-school stochastics should be built on basic ideas that students have encountered at primary and middle-school levels. These chapters are organized according to a common structure, including an introduction, with a short analysis of the main stochastic ideas in the particular topic and its place in the curriculum; some initial and more advanced specific examples that may serve to involve learners actively as they progress in their development of the concepts, a summary of what is known about difficulties students encounter with the related concepts, a synthesis of the main learning goals in the chapter, and finally, some additional resources that may help teachers and students. When possible, we make connections between the different chapters and include some historical notes that shed light on ways of thinking about the concepts.

We have tried to give a balanced view on probability and statistics, with a focus on the interrelated nature of the concepts, integrating probabilistic ideas at a level suitable for high school teaching, including the step from descriptive statistics to statistical inference. Where ever we could do it, we have also tried to integrate mathematical concepts and contexts so that the mathematics developed becomes meaningful for the learners. May our exposition contribute to an increase in statistical and probabilistic literacy in our societies.

We hope the book will be both useful for practising teachers, as well as for researchers in statistics education and practitioners in teacher educators (teacher trainers). The different chapters contain original materials, but build upon our extended set of publications, part of which is listed in the references.

We thank our colleagues and students who have commented several drafts of the chapters. Among them we want to name especially two who accompanied us in our research work now for decades: Juan D. Godino and, particularly, Ramesh Kapadia who was also helpful for improving the English. Finally, we would like to express our deepest gratitude to our families and friends for their encouragement and support over the years when we were writing the book.

May, 2016
Carmen Batanero and Manfred Borovcnik

CHAPTER 1

EDUCATIONAL PRINCIPLES FOR STATISTICS AND PROBABILITY

In this chapter, we describe those principles, which reflect our view of how stochastics should be taught at high-school level. Firstly, we suggest the need to focus on the most relevant ideas for the education of students. Secondly, we analyse the complementary nature of statistical and mathematical reasoning on the one side and of statistical and probabilistic reasoning on the other side. We then examine the potential and limits of technology in statistics education and reflect on the different levels of formalisation that may be helpful to meet the wide variety of students' previous knowledge, abilities, and learning types. Moreover, we analyse the ideas of probabilistic and statistical literacy, reasoning and sense making. Finally, we demonstrate how investigations and experiments provide promising teaching strategies to help high-school students to develop these capabilities.

1.1. INTRODUCTION

We are surrounded by uncertainty that affects our lives in personal, biological, social, economic, and political settings. This fact suggests that we need to understand random phenomena to make adequate decisions when confronted with uncertainty. We meet arguments based on data in everyday life; to critically evaluate these arguments, we need to understand the way in which the data is produced and how the conclusions are obtained.

Three widely accepted reasons for including stochastics at the school level are the essential role of statistics and probability in critical reasoning, its instrumental role in other disciplines, and its key role for planning and decision making in many professions (Franklin et al., 2007; Gal, 2002, 2005; Wild & Pfannkuch, 1999). These reasons have been recognised for some time and, hence, the teaching of statistics at secondary school has a history of about 30 years in many countries, for example, in Australia, Austria, France, Israel, Germany, Spain, and in the US. A recent innovation is the extension of statistics teaching to lower grades so that it is now included throughout the curriculum starting from the first year of primary school to graduate courses and postgraduate training at universities.

Statistical ideas originate from the natural sciences and demography and have transformed many fields of human activity over the past three centuries (Gigerenzer et al., 1989; Hacking, 1990). Today, statistics is pervasive; there is scarcely a political, scientific, or social issue without reference to statistical results. People encounter statistical information while shopping, watching TV, reading a newspaper, or surfing on the Internet. Furthermore, national statistics offices and international agencies such as the United Nations (UN) or the World Health Organisation (WHO) make their statistical studies available on the Internet.

Statistical methods are important not only in various disciplines in science but also for government and business systems, e.g., health records or retirement pension plans. It is essential to understand statistics and probability to critically evaluate how statistics are generated and to justify decisions, be they societal or personal (Hall, 2011). Accordingly, statistics educators, educational authorities, as well as statistical offices and societies call for a statistically literate society and support projects that help children and adults to acquire the competencies needed in the era of data information.

The aim of this chapter is to clarify what is meant by statistical literacy, statistical thinking, statistical reasoning, and sense making. These aspects of learning statistics have been widely discussed in the Statistical Reasoning, Thinking, and Literacy (SRTL) Research Forum, a series of conferences (srtl.fos.auckland.ac.nz/) starting in 1999 as well as in Ben-Zvi and Garfield (2004) and Garfield and Ben-Zvi (2008). We also suggest that students can acquire relevant competencies through statistical projects and investigations.

Teaching statistics and probability at high-school level is often embedded within mathematics. However, due to its peculiarities, statistics and probability require special attention on the part of teachers and curriculum designers in relation to the selection of content and the best way to make the statistical ideas accessible to the students. The goal of Chapter 1 is to present our overall perspective on the teaching of statistics and probability. This perspective is made more explicit in Chapters 2 to 5 that deal with the teaching of the main ideas of this subject at high-school level.

1.2. FUNDAMENTAL IDEAS IN STATISTICS AND PROBABILITY

Given that the time available for teaching is limited, it is important to select the key concepts that should be taught carefully. Several authors (e.g., Borovcnik & Kapadia 2014a; Borovcnik & Peard, 1996; Burrill & Biehler, 2011; Heitele, 1975) have investigated the history of statistics, the different epistemological approaches to probability and statistics, and the curricular recommendations in different countries, as well as the educational research. Based on their studies, they have proposed various lists of "fundamental" ideas. These ideas can be taught at different levels of formalisation depending on students' ages and previous knowledge (see, e.g., Borovcnik & Kapadia, 2011).

Our suggestions to teach the statistics content (Chapters 2 to 5) are organised around four main clusters of fundamental ideas. We consider these clusters as key foci around which activities can be organised by teachers in order to help students acquire the related key concepts. Each chapter analyses the essential content of one cluster using paradigmatic problems or situations that can be put forward to the students. When appropriate, we make connections across the content in the other chapters. In these chapters, we also inform teachers about the learning goals implicit in the activities, point out potential difficulties encountered by learners as described by research, and suggest promising teaching resources and situations that embed the ideas within instruction. A summary of the fundamental ideas included in each of these chapters follows.

1.2.1. Exploratory Data Analysis (Chapter 2)

Basically, statistics deals with collecting and analysing data, and making decisions based on data. The starting point in a statistical study is a real-world problem that leads to some statistical questions requiring data in order to be answered. To address the questions, such data may already exist; yet, often new data must be produced to provide sufficiently valid and reliable information to make a judgement or decision. Unlike mathematics, answers to statistical questions always involve an element of uncertainty. Furthermore, in contrast to mathematics, in statistics the context of the data is a critical component in the investigations.

For example, when studying the volume of a cylinder in mathematics, the same formula always applies no matter whether the cylinder is a juice can or part of a building. In statistics, however, the type of data and the context are essential for choosing the appropriate method of analysis and for interpreting the results. The challenge of statistical problem solving is the need to interpret the statistical concepts within a given context and choose the most suitable method from a variety of possible methods that may be applied for the problem. Although the computations in statistical procedures can often be completely outsourced to software, the decision about which procedures and techniques should be used and the interpretation of the results within the context of the data remain a big challenge when teaching statistics.

Today, there is a large amount of *data* accessible on the Internet on almost every topic that may interest students. This helps to facilitate working with real data, which can increase students' levels of motivation (see, e.g., Hall, 2011; Ridgway, 2015). Real data sets also help students investigate issues that are rarely mentioned in traditional problems in textbooks. For example, students can explore different ways of collecting data, design their own questionnaires or experiments, and gain a better understanding of different data types. When doing their own investigations, students can encounter additional problems in managing missing or incomplete data, come across data that are atypical (or even wrong due to a failure in the collection process). They have to refine the data set accordingly, or assess the reliability and validity of the investigated data.

Representations of data play a major role in statistics as a variety of graphs can be used to display and extract information that may be hidden in the raw data. The process of changing the representation of data in order to find further information relevant to the initial problem is called *transnumeration* and is considered to be an important process in statistical reasoning (Wild & Pfannkuch, 1999).

Variation and distribution are two complementary concepts that play a vital role in statistics. Although variables and variation also appear in many areas of mathematics, mathematics focuses on functional (deterministic) variation while statistics deals with random variation. Hence, a goal of statistics education is to enable students to reason about data in context under conditions of uncertainty, and to discriminate between statistical reasoning and mathematical reasoning. Wild and

Pfannkuch (1999) suggest that the perception of random variation is an essential component of statistical thinking. Moreover, statistics provides methods to identify, quantify, explain, control, and reduce random variation.

Distribution is a term that is specific to statistics and probability; it is a collection of properties of a data set as a whole, not of a particular value in the data set. A distribution consists of all the different values in the data including the frequencies (or probabilities) associated with each possible value. Variation and distribution are linked to other fundamental statistical ideas such as centre (as modelled by mean, median, or mode), *spread* (as modelled by standard deviation or variance), and *shape* (for example, bi-modal, uniform, symmetric, or L-shaped). Measures of *centre* summarise the information about a distribution while measures of spread summarise the variability within the data. Each value of a variable shows some deviation from the centre. In the context of measuring an unknown quantity (signal), this deviation may be interpreted as an error in measurement (noise). Metaphorically, the distribution embodies an overall "model" for potential errors or noise, while the centre can be seen as the signal (Konold & Pollatsek, 2002).

1.2.2. Modelling Information by Probabilities (Chapter 3)

Descriptive statistics, probability theory, and statistical inference complement each other and view the same information from different angles. Descriptive statistics investigates the information from *one* sample (data set) and summarises it using single numbers and graphical displays. Statistical inference goes beyond the present data set and tries to generalise the findings, that is, to transfer them to a wider population to which the data set belongs. Probability takes the role of a mediator as it supplies a justification for a generalisation beyond the initial data.

The information extracted from the data by using descriptive statistics can only be generalised by inferential methods that are established by probability models. At the same time some probability concepts are more easily understood as a generalisation of descriptive statistics (e.g., the idea of expectation can be understood as a generalisation of the idea of mean). These links have influenced and shaped school curricula. Yet, the study of statistics is incomplete if the reference to probability is missing in our teaching. The fundamental purpose of probability is twofold: to *measure* or to *evaluate* uncertainty. The first idea considers probability as a property of physical objects like length, while the second notion points towards a qualitative procedure of subjectively attaching numerical values to uncertain phenomena. In the book, we take into account these two ideas and the different meanings of probability.

In recent years, there has been a shift in the way probability is taught at school level, from a classical Laplacean (or "axiomatic") approach (common until the 1980s) towards a frequentist conception of probability, that is, an experimental approach where probabilities are estimated from long-range relative frequencies (Batanero, Chernoff, Engel, Lee, & Sánchez, 2016). Simulations and experiments are used to support students in understanding the Law of Large Numbers and grasping the sophisticated interaction between the notion of *relative frequency* and the *frequentist conception of probability*. The *subjectivist view* of probability, which is widely used in applied statistics, has been developed hand in hand with the frequentist view so that the two complement each other (Hacking, 1975). Their interplay is relevant, especially for conditional probability. Bearing this in mind, we suggest a combination of both approaches in the teaching of probability.

The fundamental ideas of variable and distribution still apply for probabilistic modelling. However, probability distributions deal with the *potential* data of a "random experiment", which models how data will be generated, rather than with *actual* data that have been collected. A helpful idea is to think of repeated experiments that supply us with idealised frequencies. This metaphor helps us to transfer many concepts from descriptive statistics to probability and delivers a more concrete picture of what probability means.[1] A variety of representations of descriptive statistics can be transferred to probabilities. Furthermore, tree diagrams may be used to simplify the discussion of combined random experiments and the calculus of probabilities. All of these approaches are presented in Chapter 3.

1.2.3. Exploring and Modelling Association (Chapter 4)

In a statistical study, we often are interested in checking whether two (or more) variables are interrelated and whether some type of function may describe their interaction. Functions occupy a central place in mathematics and are used to describe connections between variables in many fields (such as economy or physics) in situations that have the following features. First, the function may be determined by general laws. Second, in practical experiments these functions may only be slightly blurred by small measurement errors so that the underlying functional relationship is still visible from graphs of the data. In physics, for example, the value of the independent variable *determines* a specific value of the dependent variable.

Independent (explanatory) and dependent (response) variables occur in mathematics and statistics but the link between them is not as strong in the statistical context as it is in mathematics. Data on variables such as heights of fathers and sons might show a similar tendency but its pattern is less clear than in data on variables that follows a relationship in physics; yet, a description of the interrelationship might be useful. When we study the relation between smoking and the occurrence of lung cancer, we deal with qualitative variables. Moreover, while the percentage of people with lung cancer is greater among those who have previously been smokers – which indicates that it is better not to smoke – this association cannot be interpreted directly in causal terms as it could be induced by third factors (hidden variables) that may be operating in the background. One example is that problems in handling stress can lead people to smoke and make them more prone to lung cancer.

[1] In Chapter 3, we also see that in many situations probability models are applied to one-off decisions.

Three different problems can be studied when modelling statistical relations between variables:[2]

1. Are two variables interrelated? This leads to the concepts of association and correlation.
2. Is there a mathematical function that may be used to describe the relationship and is it possible to justify the criteria in selecting such a function? This is called the regression problem. At high-school level, teaching is usually restricted to linear functions to describe the relations between the variables as they are easier to interpret.
3. How well does a specific function describe the relationship between the variables? This leads to the problem of finding a suitable measure of the goodness of fit. For general functions, the coefficient of determination, which coincides with the square of the correlation coefficient for linear functions, is used.

Regression becomes more complex if the influence of several variables on one (response) variable is analysed. Therefore, the mathematical techniques needed to find suitable functions to model multivariate relationships between the target variable and several independent variables are usually outsourced to statistical packages. Yet, people who perform the analysis need to understand the basic ideas behind these methods. They also should understand that data becomes more reliable if a special design to control for third factors is followed. As the topic is so relevant, elementary parts of it are contained in school curricula with just two variables under scrutiny, while supported by technology both for the required calculations and for drawing specific graphs. The interpretation of the results can be sophisticated and is often a challenge for teaching.

1.2.4. Sampling and Inference (Chapter 5)

The fundamental idea of statistical inference is to generalise information from data sets to wider entities. It is related to *inductive logic*, which deals with principles of generalising empirical information (Hacking, 1965; Popper, 1959). Although mathematics derives true statements by rules of ordinary logic, in statistical inference the generalisations are constructed on the basis of hypotheses that include probabilities. In spite of being a very young field of study,[3] statistical inference has paved the way for our evidence-based society.

Fundamental ideas of inference are *population* and *sample* and their interrelationships. We might observe a process of data generation for some time (e.g., the weight of fabricated units in a factory) or a sample of data from a subset of a finite population (e.g., the weight of some eight-year-old children in a

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2 Chapter 4 is restricted to the descriptive study of the topic although all of these problems can be generalised with inferential methods.

3 Apart from rudimentary earlier traces, the methods were developed between 1920 and 1950. The successful axiomatisation of probability by Kolmogorov in 1933 increased the prestige of statistical methods, which are today applied in every area of human activity.

country). In both examples we might be interested in a number that describes the average weight of the population. In the first case the focus is on all future units produced, whereas in the second, the focus is all current eight-year-olds in the country.

An important idea is that of sample representativeness. If there are no biasing factors in how elements are selected, then the average weight of the sample data should be a reliable estimate of the average (expected) weight of the population. The statistical way to "guarantee" representativeness is to control the sampling process. More precisely, *random sampling* is a suitable method to obtain a representative sample, and it is possible to calculate the probability of obtaining a biased sample. The techniques for generalising the information from samples to the whole population are confidence intervals and tests of hypotheses.

Today, statistical inference has found its way into curricula all over the world with a variety of approaches that attempt to make the methods and the inherent notions more accessible to students. Specifically it is common to use simulation to facilitate parts of the computation and to visualise the sampling variability. More recently, resampling approaches have been used to simplify the probability models implicit in inference methods by focussing entirely on the data set that has to be analysed. All of these methods, as well as Bayes' rule to update information from empirical data, are presented in Chapter 5.

In conclusion, the most fundamental objective in probability and statistics is to offer models to understand and interpret real observations. A model does not completely represent reality; yet, it can be used for explorations that may lead to relevant results or insights about real data. A fundamental goal of teaching statistics is that students understand the *hypothetical character of probability and statistical models* and the possibility of applying these models in many different contexts.

1.3. COMPLEMENTARY VIEWS OF STATISTICS AND MATHEMATICS

Today, the teaching of statistics at university level is often separated from mathematics. For example, in countries like Spain or the US, distinct degrees or graduate programmes are offered in the training of mathematicians and statisticians. In other countries such as Austria or Germany, there are stronger connections between mathematics and statistics.

Research in statistics and probability (stochastics) is promoted by the International Statistical Institute and other organisations with specific conferences (e.g., the World Statistics Congress) and journals (*Annals of the Institute of Statistical Mathematics* or *Computational Statistics*). Statistics education research has received wide input from areas different from mathematics, for example, psychology, mathematics education, and general education (see Batanero, Chernoff, Engel, Lee, & Sánchez, 2016; Jones, 2005; Jones, Langrall, & Mooney, 2007; Shaughnessy, 1992, 2007; Shaughnessy, Garfield, & Greer, 1996; Vere-Jones, 1995, for a detailed description). As discussed in Section 1.6.3, more recently the field of statistics education has grown to become a discipline in its own right.[4]

The specific character of statistics and probability is also reflected in the philosophical, ethical, procedural, and even political debates that are still ongoing within these areas and their applications, which does not often happen in mathematics.[5] Statistics and probability are closely related to other sciences such as demography, genetics, insurance, astronomy, and law, from which many statistical methods were developed. Furthermore, inferential statistics has formed the basis for a new scientific paradigm in social sciences, medicine, and other disciplines. These disciplines share the process of a scientific argument for *generalising empirical results* that leads beyond the subjectivity and the restrictions of a single experimental situation.

Subsequently, we describe the components of an empirical study, which starts with a contextual question (in Section 1.7.2). This question leads to an appropriate design with a corresponding statistical question as well as a plan on how to collect the data needed to address the question according to the chosen design. This careful planning is necessary in order to obtain useful information about the initial question and to keep the information free from confounding effects. The exploration and analysis of the data are followed by drawing some conclusion from the data (Wild & Pfannkuch, 1999). As we expose in Chapter 2, a crucial final step is the interpretation of the results in relation to the initial question given the context of the problem (see also the steps outlined in a statistical investigation in the GAISE project in Franklin et al., 2007).

The main interest in applying statistics in a real-world study concerns finding, describing, and confirming patterns in data that go beyond the information contained in the available data. Thus, statistics is often viewed as the science of data or as a tool for conducting quantitative investigations of phenomena, which requires a well-planned collection of observations (Gattuso & Ottaviani, 2011; Moore & Cobb, 2000). This feature of statistics explains why it is easy to establish connections between statistics and other school subjects and why it has sometimes been argued that statistics should be taught outside the mathematics classroom (Pereira-Mendoza, 1993).

Statistics is part of the mathematics curricula. As argued by Usiskin (2014), statistics involves a great deal of mathematical knowledge. Fitting statistics within mathematics teaching means that no additional time is needed for a separate subject in school, where time is limited. According to Usiskin, as well as to Scheaffer (2006), statistics and mathematics may support and complement one another in the school curriculum.

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[4] For example, in the US, there are graduate programmes in statistics education and doctoral degrees have been awarded in statistics education.

[5] An example is the controversy around the use of statistical tests (Batanero, 2000; Borovcnik, 1986a; Hacking, 1965).

An example described by Usiskin is the natural extension from the straight line that goes exactly through two points to linear regression where the interest is in finding a line that fits to more than two data points in an optimal way. Finding the line of best fit is part of mathematical modelling; however, statistical modelling does not stop there. Lines are compared to other functions that may also model the data and the interpretation of the fitted function depends on the context. Typical questions are:

Do other functions fit better? What does it mean if we describe the relation between the two variables under scrutiny by a line? Can we predict the value of the dependent variable outside the range of data?

According to Usiskin, statistics education can also benefit from the strong movement towards modelling developed since the late 1980s especially promoted by the ICTMA, the International Community of Teachers of Mathematical Modelling and Applications,[6] and introduced only more recently in statistics education through the Exploratory Data Analysis (EDA) movement.[7] In this modelling approach, the motivation for a particular concept emerges from the context; the concepts are developed interactively by contextual considerations and mathematical principles. The teacher should find an appropriate real situation in which the new concept makes sense for the students. Throughout the book we present such initial problems that help students understand the related concepts, followed by more complex situations when space permits.

Statistics at high school may sometimes be a separate course (e.g., the advanced placement course in the US) or included in other subjects and taught when it is needed to understand the current topics. We agree with Usiskin that regardless of the placement, all students should experience substantial school work in statistics. In this way, they become competent to appreciate and criticise empirically-based arguments around them and empowered to make adequate and informed decisions.

Probability versus statistics. Some authors consider that probability is more strongly linked to mathematics than to statistics. Throughout the book, we try to make clear that statistics and probability complement each other and should not be completely separated. The frequentist view of probability serves to connect probability with a wide range of applications and probability cannot be understood without a connection to relative frequencies. Moreover, modelling probability as the limit of relative frequencies provides a first basis for introducing statistical inference from a frequentist approach.

The subjectivist view of probability extends the applications of probability to decision making in one-off situations where the frequentist view does not apply. Additionally, Bayes' rule establishes strong links between these two perspectives on probability and allows combining subjective probabilities *and* statistical data to

6 The PPDAC cycle of Wild & Pfannkuch (1999) in Section 1.7.2 has close connections to the modelling cycle.

7 Although EDA also started in the 1980s, it was then mainly a movement towards methods that were attractive to teaching because of their simplicity.

update prior probability judgements and make them “more objective”. Finally, some probability ideas are needed to understand more informal approaches to inference when we use simulations or re-randomisation to generate empirical sampling distributions (see Chapter 5).

Thus, when teaching probability, one has to consider that probability is a theoretical concept – a virtual concept according to Spiegelhalter and Gage (2014) – and we speak about probability by using metaphors such as “propensity”, “degree of belief”, or “limit of frequencies”, which convey only parts of this abstract concept. Even though the relationship between probability and relative frequencies is fundamental for the comprehension of probability and statistical methods, this relationship is not always well understood as some students confuse frequency with probability.[8] Moreover, the different representations of measures of uncertainty (as absolute numbers[9] versus probabilities) may also involve different levels of difficulty in understanding probability models. In Chapter 3, we discuss other difficulties encountered by the students when interpreting small probabilities, which usually occur in the case of risks of adverse events such as a maximum credible accident of a nuclear power station or dying from lung cancer.

1.4. THE ROLE OF TECHNOLOGY

Technology has revolutionised the applications of statistics and likewise statistics education. With software such as *Fathom* (Finzer, 2007) or *Tinkerplots* (Konold & Miller, 2011), specially designed to support the learning of statistics and probability, with a spreadsheet, or even with Internet applets, data analysis is no longer the exclusive domain of statisticians (Biehler, 1997; Biehler, Ben-Zvi, Bakker, & Makar, 2013; Pratt, Davies, & Connor, 2011). As demonstrated throughout Chapters 2 to 5, software can facilitate computations and the production of graphical representations of data. Thus, students can use methods such as fitting a variety of models to a scatter plot (see Chapter 4) that were not accessible to them a few years ago.

With modern technology, students can represent abstract interrelationships and operations, interact with the setting, and see the changes in the representation or in the results when varying some data or parameters.[10] Technology provides the possibility of dynamic visualisations where the impact of crucial parameters on a graph can be traced. This technique may serve, for example, to explore the waiting time for the first success in Bernoulli experiments (as done in Chapter 3).

Due to facility and speed of computations, the size of data sets is no longer a limitation so that it becomes easier to use real data collected by the students or taken from the Internet (e.g., from CensusAtSchool, n.d., or from several statistical

[8] The expression “empirical probability” is unfortunate in this regard as a probability is always theoretical; only the frequencies are empirical.

[9] In the sense of Gigerenzer (1994).

[10] Another possibility not studied in the book is multivariate dynamic visualisation that can be implemented with tools like those available from Gapminder (Rosling, 2009).

offices). In Chapter 2, we suggest that students collect and analyse physical measures of themselves such as height and weight, arm span, shoe size, etc. However, teachers should be careful about issues such as students being able to understand the data within the context and to connect the data set to the investigated problem. Otherwise students may lose interest (Hall, 2011).

Another advantage of technology is that it can help build micro worlds where students can explore conjectures and establish ideas from analysing what happens. Some examples are given in Chapter 3, where students can explore simulations of experiments to understand essential properties of a frequentist concept of probability and perceive that the variability of relative frequencies becomes smaller in longer series of trials. Working with such environments may help students to realise and change their probability misconceptions that would persist within a formal approach to probability (see Jones, 2005; Jones, Langrall, & Mooney, 2007).

Furthermore, technology offers the opportunity to students to learn about modelling since it enables students to build their own models to describe data or to explore probability problems. For example, a table-oriented method allows calculating the posterior from prior probabilities and given data, which is a method vital for a Bayesian decision-oriented approach towards inference (see Chapter 5). Above all, technological support may enhance students' understanding of the complementary role of probability and statistics; for this reason, it is particularly useful in the introduction of the frequentist view of probability (Chapter 3) and in obtaining empirical sampling distributions with simulation or re-randomisation to explore inference from data to populations (Chapter 5).

The use of technology, however, may also have drawbacks. It may hide the mathematical difficulties of a concept using artefacts and change hands-on activities on physical objects (e.g., spinners) into virtual activities ("spinners" run by software). For example, drawing a scatter plot requires quite a few steps including attributing the variables to the coordinates, deciding the scale of the axes, and plotting the points. These steps build an operative understanding of the final plot. If technology automatically supplies these operations, students may use the default options (e.g., an inadequate scale) in an uncritical way.

Moreover, technology may bias educational efforts. On the one hand, teachers may use technology to overemphasise computation neglecting the understanding of the statistical concepts. On the other hand, the learning goals may be reduced to the learning of simulation techniques not paying attention to the concepts or the reasoning behind the method applied. Another drawback of the overuse of simulation is the reduction of probability to a mere frequentist concept. To avoid such problems, we put special emphasis on analysing the learning goals underlying the activities proposed for the work with the students in the classroom and, throughout the book, we use technology as a complement to rather than a substitute for statistical reasoning.

1.5. ADAPTING THE LEVELS OF FORMALISATION TO THE DIVERSITY OF STUDENTS

Various factors suggest the need to be able to teach the same topic at different levels of formalisation. Among them, we list the diversity of curricular guidelines around the world, the different educational requirements of similar high-school grades (e.g., technical versus social-science strands), as well as the differing abilities and competencies of the students. As the book is intended to serve a broad international audience, we have tried to implement the principle of adapting to a diversity of students throughout the text.

Let us consider, for example, statistical inference, a quite sophisticated topic, where starting with an informal approach has been recommended to reduce the technicalities when introducing the topic (e.g., Makar, Bakker, & Ben-Zvi, 2011; Rossman, 2008). This informal approach could be used as an introduction for the majority of students. However, in some countries (e.g., in New Zealand, Germany, or Spain), a more formal approach to inference (including an exposition of confidence intervals) is required in the last grade of high school for particular strands.

We therefore start the exposition of basic inference methods like tests of significance and confidence intervals in Chapter 5 with an informal approach where the sampling distribution is estimated via simulation. We then later suggest that students with experience in probability rules or the binomial distribution may use this previous knowledge to compute the exact sampling distribution. We add further activities related to statistical tests as decision making or use of Bayes' theorem to update prior information about a parameter for those students with more advanced experience in probability.

Other resources considering the same curricular content at different standards of formalisation can be found in the GAISE guidelines (Franklin et al., 2007). They provide examples of how different levels of presentation require and reflect an increasing sophistication in understanding and applying stochastic concepts.

1.6. STATISTICAL AND PROBABILISTIC LITERACY

Statistics is embedded in a methodology that serves to generate evidence from data. Statistical knowledge is vital in research and in public discussion where it is used to empower arguments pro or contra some issue. Without statistical knowledge it is difficult to discern misuse from proper use of data. Statistical knowledge involves thinking in models, being able to apply proper models in specific situations, considering the impact of assumptions, deriving and checking the results, and interpreting them in the context.

1.6.1. Statistical Literacy

The relevance of statistical reasoning and knowledge to functioning effectively in the information society led to the introduction of the term *statistical literacy:*

> The ability to understand and critically evaluate statistical results that permeate daily life, coupled with the ability to appreciate the contributions that statistical thinking can make in public and private, professional and personal decisions. (Wallman, 1993, p. 1)

Statistical literacy is emphasised in the GAISE report (Franklin et al., 2007) produced in collaboration between the American Statistical Association (ASA) and the National Council of Teachers of Mathematics (NCTM, 2000).

Literacy is defined as the ability to find, read, interpret, analyse, and evaluate written information, and to detect possible errors or biases within this information. Therefore, to be statistically literate, people need a basic understanding of statistics. This includes knowing what statistical terms and symbols mean; being able to read statistical graphs and other representations of data; understanding the basic logic of statistics; and understanding and critically evaluating statistical results that appear in everyday life. Statistical literacy should also enable people to question the thinking associated with a specific method, to understand certain methods and their limitations, or to ask crucial questions to experts and understand their answers. For example, to measure the success of some treatment as compared to alternative treatments, a doctor can use a variety of criteria such as life expectancy, 5-year survival rates, or the whole survival curve.

Several researchers have developed their own specific models of competencies to describe statistical and probability literacy. We describe some of these models subsequently.

1.6.2. Statistical Literacy Components

Watson (1997) described "statistical literacy" as a set of competencies that adults need to manage "life" in the information society, which include literacy, mathematical and statistical skills, as well as knowledge of context and motivation. She considered three levels of increasing complexity: a) a basic understanding of statistical terms and language; b) a more complete understanding of statistical language and the related concepts in the contexts where they are encountered; and c) a critical attitude and thinking ability to apply statistical ideas to analyse or debate statistical claims (see also Watson & Callingham, 2003).

Another widely accepted description of statistical literacy was proposed by Gal (2002) who identifies two interrelated components of the knowledge that an adult needs to become a competent "data consumer": a) the person's ability to interpret and critically evaluate statistical information, data-related arguments, or stochastic phenomena in diverse contexts; b) the related ability to discuss or communicate opinions on such information. These capabilities are based on statistical and mathematical knowledge, literacy skills, knowledge of the context, as well as specific attitudes such as a critical stance. Gal (2002, p. 47) suggests that the goal of statistical literate students is more easily reached when less emphasis is placed on the computational component of statistics teaching:

> Some schools […] teach statistics […] as part of mathematics, […] yet not in a way that necessarily emphasises the development of statistical literacy.