History of Mathematics Education

Kay Owens and Glen Lean with
Patricia Paraide and Charly Muke

History of Number

Evidence from Papua New Guinea and Oceania

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History of Number

Evidence from Papua New Guinea and Oceania

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Foreword

This book is remarkable in many ways. It contributes fresh data to discussions about the history of numbers in the South Sea regions. The data sets are huge, reflecting collections of number information over some 20 years. The book was not written by the chief researcher Glendon Lean, who sadly died in 1995. It was he who collected data on the many field trips he made while living in Papua New Guinea. He was fluent in the lingua franca and a couple of the PNG languages with some understanding of several others and thus was well able to interpret the rich data sources available to a scholar like him.

I was privileged to collaborate with him and finally to help him complete his PhD, which consisted of the thesis plus four equally huge appendices. These together document not just the routine number ideas that we take for granted in our western European culture but the many and varied ways in which number and number representations can be found in diverse cultures, still to this day. In addition, he used the historical and linguistic background data to illustrate the antiquity of when these seem likely to have developed. He also illustrates how variations and similarities occurred.

Dr Kay Owens is equally to be congratulated on creating a book which helps to clarify several of the current issues about the development of number ideas in a rich cultural context like that of PNG. With Patricia Paraide and the other researchers, Kay has illustrated the significance of this history of number to mathematics education today. It redresses the lack of previous knowledge of these ancient systems. It is a formidable resource for future scholars, and hopefully this book will inspire them to continue the analyses that Glendon Lean and Kay Owens have started. The ethnographic database that Dr Owens has created at the University of Goroka has helped not just the PNG scholars but scholars from everywhere to explore what this study has started. It would be a tragedy if it were unavailable to others or left like that. Glendon Lean's commitment to his colleagues and communities in PNG deserves much more.

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Preface

Papua New Guinea and Oceania (including West Papua, the other half of New Guinea) are unique in having the highest number of distinct languages in the smallest land mass in the world and also in having related languages spread across huge ocean spaces. This extraordinary ecology creates an incredible opportunity to explore the history of number. There are indeed 1 300 languages in this region. That means 1 300 cultures. The story that unfolds in this book reaches back to an era between 1 500 and 40 000 years ago, and yet the story is still active and part of the lives of the same genealogies of people today. This active story was very much the case during the period of first contact with Europeans which began sporadically in the late 1700s but was only significant after the 1880s. Places in this story have been among some of the last to be infiltrated by European colonialists, some as recently as 60 years ago. Some of the data drawn upon for this book were the first to be recorded for the language and culture at first contact by Europeans, but much of the data were less than 50 years old. Data from living cultures were mostly recorded in the 1970s and have been compared where possible with older records. Based on variations between existing languages that are linguistically and mathematically analysed together with evidence from other disciplines such as archaeology, the ancient story of number is told.

My research and experience in Papua New Guinea (PNG) has informed my writing of this story as it did an earlier Springer book *Visuospatial Reasoning: An Ecocultural Perspective on Space, Geometry and Measurement Education.* When that book was finished, Ken Clements, one of the editors of the new series on the history of mathematics education, emailed me. He asked if I would be willing to write a history of number from the perspective of Papua New Guinea and Oceania based on Glen Lean's work. This request came because he was well aware, unlike most historians of mathematics and mathematics education, of the incredible work of his friend and colleague Glendon Angove Lean, and he was aware that my interest in Glen's work and experience in Papua New Guinea provided a good basis for writing this book. It is a story that needs to be known by others.

In 1973, I went to live in Lae, PNG, and to work in the Mathematics Department at the PNG University of Technology beginning a life-long friendship with Glen Lean who sadly passed away in 1995, a week after a special ceremony to confer his PhD. For 20 years, Glen collected data on counting systems of Papua New Guinea and Oceania which culminated in his thesis being finished in 1993, but it remained unpublished until this book incorporated it. His academic mentor and executor, Alan Bishop, sent as much of Glen's collected materials to the University of Goroka (UoG) in PNG where the Glen Lean Ethnomathematics Centre (GLEC) was established. With my husband, we were able to catalogue Glen's photocopied papers, but unfortunately half of the material was lost through the climes and transport of PNG (some we were able to replace). The University of Goroka holds the most complete copy of his resources in one place. With funds from USA's National Science Foundation, with a small team of PNG lecturers and research assistants, we were able to work on Glen's collection of counting systems and put them into a database available from the GLEC website, but the database has not been readily available for some years. This database was developed from poor and incomplete electronic copies and a rare hard copy of his provincial summaries upon which he developed his thesis.

We prepared his thesis for the GLEC website together with copies of some journal articles and other materials that Glen had sourced. The references in this book support the arguments presented in the following chapters but also provide the data sources for around 100 languages given in the Appendices and selected as evidence for the arguments. The extent of the reference list indicates how much material Glen collated for 900 languages, that is, 75% of the region's languages. However, the data collected by Glen from students and others from 1968 to 1988 provide the really rich collection of living data that are evidentially telling the ancient story of their ancestors. I say "evidentially" as this book also presents the analytic, linguistic and archaeological evidence to support this statement.

Chapter 2 indicates the nature of Glen's data collection and reflects Glen's unique abilities that meant that he was able to collate the data from written records and thousands of questionnaires completed over 20 years by PNG and Oceania students and teachers. He searched maps, he asked questions, he visited remote places and with his linguistic, historical and mathematical skills, he was able to analyse and synthesise this data. His data from people also relied on his rapport with the many PNG people with whom he associated. Again Glen's rapport was an exceptional skill to which his many friends and colleagues can testify. My forays into collecting and checking data have indicated how extraordinary his work was and the nature of his decision-making in summarising the data (e.g. Owens, 2001). One of the main issues has been that of collecting data from people whose languages and to some extent cultures are rapidly changing. Another problem was in the multiple names given to any one language and the variations between villages who speak the same language or even the same dialect of the language.

To convert Glen's work into a book readable in APA style from the poor electronic copies of his thesis with its footnote format that referred by code to the tables in his huge appendices required considerable tedious effort and PNG knowledge. His work has been extended by more recent linguistic, anthropological and archaeological papers. While Glen did place his thesis into the cultural contexts of the languages and counting systems, this book goes further. It also gives credit to the work of Papua New Guineans and others who have pursued ethnomathematics as a significant area for PNG mathematics education. Patricia Paraide and Charly Muke are authors on chapters which specifically incorporate research on their own cultures. I would also like to acknowledge the team of linguists at the Australian National University; many SIL linguists and translators; Ralph Lawton and the team of Kilivila translators; Geoffery Saxe; my former UoG colleagues Wilfred Kaleva, Martin Imong, Rex Matang (deceased), Gairo Onagi and Samuel Kopamu who have shared their counting systems and ideas with me; my many PNG friends, colleagues and students who have accepted me into their families; and researchers Joseph Fisher, Peter Dwyer, Mark Donahue and Jadran Mimica whose works have significance for this book. Geoff Smith and Sue Holzknecht are long-term friends and were colleagues at the PNG University of Technology working in the fields of language and linguistics. They have shared much with me prior to my writing this book. I sincerely thank them.

This book portrays a history of number from an ancient time uninfluenced by events occurring in the Middle East region. It is a story that begins at an often much older time than developments in the Middle East and upon which the Indo-European centric histories of number are often based. Briefly, Lean argued that the types of counting systems within different phyla and protolanguages indicate the antiquity of the systems when combined with archaeological evidence. The system classifications also indicate how counting systems spread and changed. I have been fascinated by the alternative systems of counting portrayed in this story, and I trust the reader will also be enthralled. It is my belief that without this history being taught, there is a serious lack of richness in the history of mathematics and counting systems for any school student across the world.

Owens, K. (2001). The work of Glendon Lean on the counting systems of Papua New Guinea and Oceania. *Mathematics Education Research Journal, 13*(1), 47–71.

Dubbo, NSW, Australia Kay Owens

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This book is only possible because many, many Papua New Guineans and linguists shared their languages and aspects of their mathematical culture with others. I wish to thank them sincerely. I hope this book does justice to knowledge generated by the sharing of many, many communities. I wish to thank my very patient husband who has also proof read the whole book and Sandra Stewart, Melissa McNair and Laura White for their assistance with formatting the many tables at different stages of preparing this manuscript. I also acknowledge the encouragement of Ken Clements and Alan Bishop in their respective roles as series editor and Glen's supervisor and academic executor who are both pleased that Glen's work is being more widely recognised in the world as we believe it is ground--breaking. It has been an honour to extend his work in this book.

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Chapter 1
An Overview of the Studies, Papua New Guinea, Oceania, Languages and Migrations

Kay Owens and Glen Lean

Abstract This multidisciplinary study draws on archaeological information, linguistics, and an understanding of mathematics. A summary of the background archaeology for New Guinea and Oceania provides some evidence of the longevity of these cultures and the archaeological evidence for the spread of languages in New Guinea and Oceania. The diversity of language groups is a result of movement, colonisation, influence and innovation over time. The overview presented in this chapter permits the reader to link the pursuing discussion in a time and place. The chapter finishes with an overview of the book that sets out the diversity of counting system cycles, where they are established, and how they may have developed.

Keywords Lapita • migrations in the Pacific • Oceania • Papua New Guinea • prehistory • studies of counting systems • West Pacific languages

Introduction

Over the past 250 years the study of systems of numeration as they occur in natural languages has sometimes been of interest to mathematicians and others while at other times it has been of little interest. During the nineteenth century, we find that the published literature of the explorers, administrators and missionaries of the European colonising powers provides a rich source of information on the "exotic" languages and customs of the peoples of Africa, the Americas, the Far East, and Oceania. Indeed this material still serves today as a major source of data on non-European counting systems. The study of numerals and counting systems was initiated largely by scholars with an interest in linguistics and anthropology but during the twentieth century there was a marked drop in interest on the part of both linguists and anthropologists and the subject was relegated to one with only marginal status (Hurford, 1987). Numerals, it would seem, are strange animals which do not possess, for the linguist, the same intrinsic interest as other aspects of linguistic analysis, or, for the ethnographer, the same intellectual challenge as unravelling a complex kinship system or social structure. They are, perhaps, best left to the historians or philosophers of mathematics.

Philosophers of mathematics tend to take the view that the study of natural language numerals has little bearing on the understanding of the nature of number, an abstract concept independent of nominal linguistic vagaries. Those sections which deal specifically with number in standard histories of

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mathematics (Boyer & Merzbach, 1989; Dantzig, 1954; Struik, 1987), usually begin with the advent of written numerals perhaps for no other reason than that historians prefer to work from written records. Also, until relatively recently, there has been a marked Eurocentric bias in the Western accounts of the development of mathematical ideas, and even the contributions of India, China and Islam have been given scant attention (Joseph, 2000). In most cases, the occurrence of mathematical ideas as they existed in "primitive" tribes, is dealt with in a speculative or dismissive paragraph or two.

There are several points which should be made concerning this lack of interest in the numerals and counting systems which existed in societies outside the usual ambit of historians. First, those societies which do not possess a tradition of historical documentation are those in which the majority of the world's languages are spoken; their exclusion from the historical record means that the numerically greater part of the counting systems used by humankind is ignored. Second, those societies which are the focus of historical scholarship possess a relatively small range of counting system types. Generally speaking, their counting systems possess a base of 10, although several systems have features or irregularities which suggest vestiges of base 12 or base 20 systems: English and German being examples of the former, and French and Dutch being examples of the latter. The inference which may be drawn from this highly selective view is that humankind's response to the enumeration of its world is largely consistent across both time and cultures. Third, it may be useful to consider whether the absence of historical documentation for these societies means that we are unable to construct a *prehistory* of number. Given the information that has accumulated in the disciplines of anthropology and archaeology, an extensive database of natural language numerals and counting systems, together with the methods developed by comparative linguists, it may be theoretically possible to construct a tentative outline of the prehistoric development of the concept of number in human societies in the millennia preceding the advent of written numerals.

In addition, many who study non-Western systems often consider them as ancient and of historical interest only. They are unaware that there are Indigenous communities today that can trace back their ancestry and may continue to use foundational practices for counting and mathematical practices today. Finally there are some that dismiss much of the old record of counting systems as irrelevant or simplistic and of need of being replaced by modern and universal systems of number. In so doing, they fail to realise the importance of identity in the learning of mathematics and its value in establishing mathematical identity. It is theorised that ecocultural identity is an important way forward in establishing a self-regulating and motivated learner who can think mathematically (Owens, 2014, 2015).

Fortunately, since the mid-1960s, there has been a renaissance of interest in documenting the mathematical ideas existing in non-Western societies. This has been partly due to the work of cross-cultural psychologists, for example Berry and Dasen (1974), Cole, Gay, Glick, and Sharp (1971), Dasen and de Ribaupierre (1987), Gay and Cole (1967), Saxe (1985, 1991, 2012; Saxe & Esmonde, 2005), and Pica, Izard and Dehaene and colleagues (e.g., Izard, Pica, Spelke, & Dehaene, 2011; Pica, Lemer, Izard, & Dehaene, 2004). All these researchers have been interested in situated cognition and the cultural context of learning and thinking, especially as it relates to the development of mathematical concepts. In particular, a literature of "ethnomathematics" has been established which has dealt, for example, with the mathematical ideas existing in African societies (Zaslavsky, 1973) and in North and Central American Indian societies (Ascher, 1994, 2002; Closs, 1986). The ethnomathematical approach has even been extended to include the study of identifiable cultural (and sub-cultural) groups within Western societies as well as non-Western societies, and is concerned, for example, with the mathematics used by gamblers (Pickles, 2013) or supermarket shoppers (D'Ambrosio, 2006; Lave, 1988). Paulus Gerdes (1999) and colleagues, in many publications, were able to link school mathematics to the clever cultural mathematics of many different cultural groups in many different practices, one of which was creating 3D objects from 2D surfaces or through different patterns and structures of weaving. More importantly Gerdes and his co-workers noted how new designs were being created and produced through apparent visual patterning. Adam (2010) indicated how advanced mathematical manipulation could be discussed with basket weavers who could discuss the relevance of certain changes. Eglash (1999) documented the use of fractals in African societies and subse-

quently with colleagues extended the link of ethnomathematics to technology (e.g., Eglash, 2010) while Rosa and Orey (2012) significantly illustrated the link between ethnomathematics and advanced mathematics through "ethnomodelling."

The renewed interest in counting systems of non-Western societies has made its mark in more recent histories of number and mathematics as may be found in works by Flegg (1984, 1989), Ifrah (1987), Van der Waerden and Flegg (1975a, 1975b) and Joseph (2000), but others such as Cooke (2011) recall the ancient systems with little update. Much of the material, however, on which these studies were based derives from the nineteenth century (Frobenius, 1900). Until the work of Pacific Resources in Education and Learning after 1990, there had not been an increase in the database commensurate with the increase of interest in the systems themselves. This is particularly true of the large geographical region which encompasses Melanesia, Polynesia and Micronesia and which has received little attention in the ethnomathematical literature and virtually none in the historical literature, even though this region accounts for over a quarter of the world's languages (Pawley, 1981). Goetzfridt's (2008) extensive bibliography and analysis of these references showed how significant the mathematics of these cultures were and that there was recorded material although much of this knowledge is oral (Goetzfridt, 2012). Lean's (1992) study, in part, attempted to redress the situation by, first, providing documentation of the natural language numeral systems used in the cultures of the New Guinea area and the Pacific, and, second, by investigating the implications of these for the history, or prehistory, of number. It remains the most extensive study of the region and a classic source for further exploration of the history of number or the number systems of the region. Some recent anthropological linguistic studies have critiqued earlier migration studies that were based on Lapita pottery findings across the Pacific. Lean had used the earlier theories to develop his thesis on migration of languages. For the most part we present Lean's argument and discuss recent research in Chapters 9 and 10 (Ross, 1988).

The diversity of the languages and the rapid change in languages and cultural experiences and practices means that an awareness of the early contact data together with a recognition of developmental change are important for several reasons. First, they are important for transitions from cultural mathematics to school mathematics. It provides awareness that may enhance the value of retention of culture, promote identification and rediscovery of cultural knowledge, and most importantly use their strengths to initiate an understanding of school mathematics. Second, they are important for bilingual education in a Papua New Guinea (PNG) context where there is limited knowledge of how counting systems can be incorporated well into bilingual education or majority-English language of instruction (espoused by the current re-reform of education). Finally, they are important for a greater understanding of the history of number in the world and for an understanding of school mathematics in a global context, including any one child having a greater appreciation of the composite (group) units of the base 10 place value system.

Purpose of the Study

The original aims of this study were, first, to clarify the number and nature of the counting systems used in the foundational societies of Papua New Guinea and, second, to investigate whether the accumulated data had any implications for several theoretical questions which arise in the literature of the history and development of the concept of number in human societies. For example, were the structural/cyclic features of natural language numeral systems invented independently in various societies in different geographical locations, or were they diffused from a single source? Is there any evidence for a chronological developmental sequence of counting system types; that is to say, can we construct something approaching a prehistory of number?

Much of the existing data on numerals and counting systems derive, as noted above, from information collected during the nineteenth century in the foundational societies of Africa and the Americas. Published data are also available for a number of Polynesian societies of the Pacific region, however

the amount of data available for PNG appeared to be relatively small until Glen Lean began his work. Even though the population of PNG, being about 6.5 million, is a small proportion of the world's total, the linguistic situation is nevertheless very complex with a total of about 850 languages, or a sixth of the world's spoken languages within its borders (SIL, nd). To document the nature of the counting systems of even half the total number of these languages would make a significant addition to the database from which scholars have worked in the past. This was the significant 22-year study undertaken by Glendon Angove Lean (prior to his death in 1995) and this forms the basis of this book. We extend on this material and explore one question in more depth: how cultural contexts have impacted on counting and how that has influenced the development of number through different usages.

Tour of the Major Studies

Smith's Studies

We draw on several other studies of counting or number systems in Papua New Guinea that are forming significant parts of the argument for this book. These include Geoff Smith's (1984) study of Morobe's counting systems. This work informed Glen Lean and will be incorporated into the chapters of this book based on Lean's work. Smith made an exhaustive study of the Morobe counting systems. He collected his data by visiting most of the languages in situ in the Province especially if he was unable to source reliable older speakers in Lae, the capital. This meant long walks into the mountains on either side of the large Markham valley. Both Smith and Lean used a framework of cycle systems for describing the multitude of systems and both frameworks are elaborated further in Chapter 2. Lean based his database of counting systems in Morobe largely on Smith's work. A number of the Morobe languages are also spoken in neighbouring Provinces such as Madang and Eastern Highlands Province. Smith (1978) had earlier recorded the counting system of the Kiwai in Western Province and made a summary of counting in the highlands regions in terms of business encounters. In addition, he noted that in many areas the efficient formal counting of valuable objects on ceremonial occasions was necessary, and commented on how the people in the Highlands Region of PNG recorded the quantity of goods (Smith, 1981). He found complex cycles associated with the distribution of pigs, shells, and other valuables in the Western Highlands, Southern Highlands, and Enga Provinces. He noted that a variety of counting methods can be used for recording these transactions.

Lean's Study

Lean began the collection of data on PNG natural language numerals and counting systems early in 1968 and continued to collect these from both primary and secondary sources until 1988. The documentation of the accumulated data began in 1985 and a final revised version was completed at the beginning of 1991: this material can be found in the databases held on the Glen Lean Ethnomathematics Centre website (GLEC, 2008). In all, data were acquired on the numerals, counting or tally systems of 532 languages, or just over 70% of the total, then believed to be about 750. Generally speaking, the languages for which it was difficult to obtain data were those with 500 speakers or less; in the Madang Province alone some 93 languages fall into that category. In the ensuing years, SIL have determined many more languages identified by the people as distinct languages. In Lean's work much of this detail was lost when he collated the information from a number of different sources, especially those completed by students on his Counting System Questionnaire as he tried to select the best representation from his sources.

In contrast to the relatively homogeneous cultural and linguistic situation existing in Polynesia, the diversity of the cultures and languages existing in Melanesia has been commented on by many

scholars (e.g., Carrier, 1981; Kettenis, 1978; Lancy, 1983; Saxe, 1979, 1981a, 1981b, 1981c, 1982; Smith, 1978). This is particularly true of PNG where the diversity appears to be greatest and in this sense is untypical of its neighbours with the exception, perhaps, of Timor Leste and of West Papua (currently controlled by Indonesia), the complementary western half of the island of New Guinea. In order to see whether this diversity extended to counting systems and whether, therefore, the PNG data already gathered were untypical of the region as a whole, it was decided to extend the geographical scope of the original study to include the remainder of Melanesia, which is termed "Island Melanesia", and the islands of Polynesia and Micronesia.

This study, then, deals with the counting systems of Papua New Guinea and Oceania. Oceania includes West Papua, the Solomon Islands, Vanuatu, New Caledonia, Fiji, and Rotuma. In addition, Lean included all of Polynesia, both Triangle and Outlier, and those islands of Micronesia on which Oceanic Austronesian languages are spoken: effectively, that excludes the western part of Micronesia which contains the majority of the Marianas, where Chamorro is spoken, the Palau (Belau) Islands, and Yap. The islands to the west and south of West Papua, which include Timor, Halmahera, and the Kai and Aru Islands, have also been excluded although we do have some data from Timor Leste. The other major area excluded in the original analysis by Lean is Australia and although some reference will be made to Australian languages these are, nevertheless, untypical of the rest of Oceania whose languages are generally classified as either Austronesian (AN) or Non-Austronesian (NAN)–also called Papuan. These classifications are discussed further below. In Oceania, a further 450 languages or so are spoken, thus giving a total for the region under consideration, if we include PNG, of about 1 300 (see Figure 1.1 which indicates this region). Occasionally it has been useful to refer to New Guinea by which is meant the combined regions of West Papua and PNG. However, there are now many studies of number in Australian languages and these will be referred to in this book as comparisons and in terms of the importance of this historical study for mathematics education with Indigenous communities more widely.

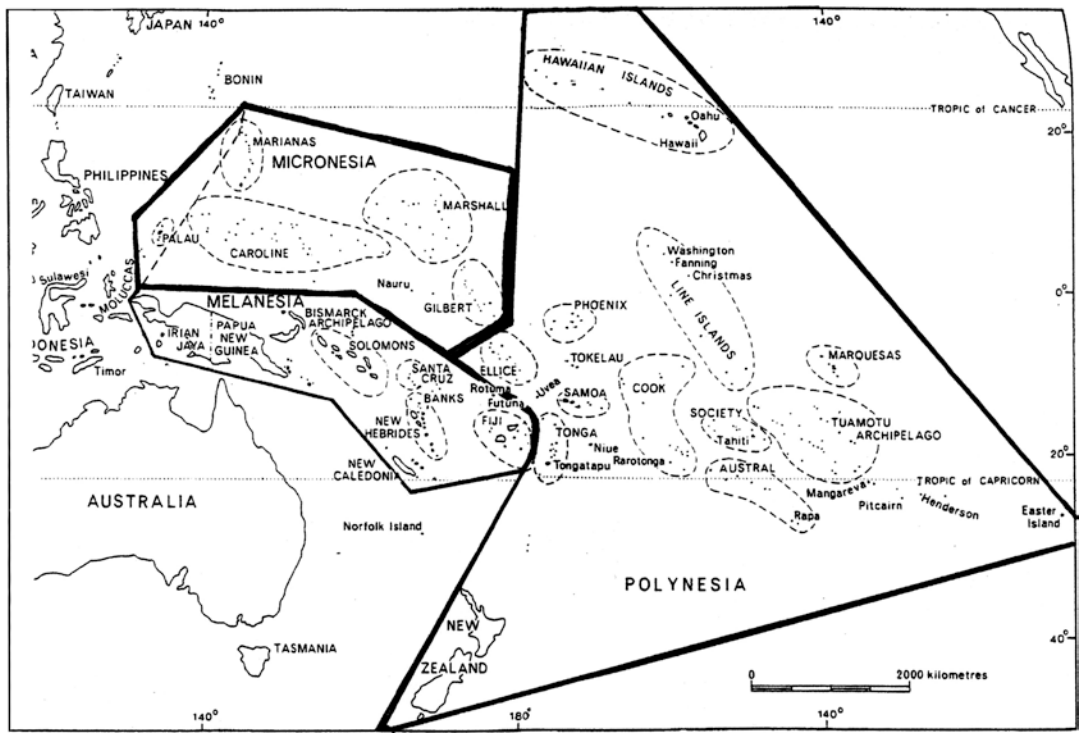

Figure 1.1. Melanesia, Micronesia, and Polynesia.

Muke's Study in Wahgi Valley, Jiwaka PNG

Charly Muke undertook a study of counting of his own language in Wahgi valley in the Jiwaka Province (formerly part of Western Highlands Province). Like Lean, Muke (2000) undertook a cultural study of counting based on the view that mathematics is not, as the Ancient Greeks and as mathematicians down the ages have suggested, as singular and definite but developed in a cultural context (Ernest, 1996, 2012). His ethnographic study of his own language addressed the following research questions:

- What are the Wahgi counting systems of both the past and present in terms of:
  - verbal communications: counting names
  - symbols: both written and on artefacts
  - practice: other practices of counting?
- What are the social relationships (e.g. conceptual, understanding, values, beliefs etc.) in the culture associated with counting practices?
- In what contexts are Wahgi counting systems found?
- How can this information be used to contribute to the learning of mathematics in the school system?

One strength of Muke's study is that he not only spoke the language dialect Yu Wooi with an understanding of the other dialects but also gathered data from 73 participants (30 teachers, 40 students and 3 villagers), 72 through questionnaire and one through lengthy discussions with his father, a villager. From this number of participants, 58 were speakers of Yu Wooi including the interviewee, living in nine tribes. The others spoke the Kumai, Yu Nimbang and North Wahgi dialects. As a result he was aware of the diversity within the counting system and its context.

Paraide's Study of Tolai Language and Cultural Practices

Patricia Paraide (2010) documented the number and measurement knowledge of her Tolai community (East New Britain Province) and their cultural activities including those related to business. One of her research questions was "How is Indigenous knowledge of number and measurement positioned and/or constructed, compared to Western number and measurement?" She particularly researched this question within the cultural context at the time of her study but cognisant of her cultural knowledge heritage. For the Tolai, number and measurement knowledge play an important part of their roles in community. Number and measurement knowledge is integrated into the people's everyday activities such as gardening, fishing, dancing, feasting, and building houses and canoes. Paraide also presented possible reasons for the diminishing status of some of the counting systems of particular items in this community. Moreover, she focused on the various power plays that occur between teachers, parents, and students which contribute to changes in attitudes and behaviour towards vernacular instruction and Indigenous knowledge.

For her case study, she used classroom and community observations, teachers, community members, parents' interviews, students' discussions, field notes, informal discussions, participation in the research participants' community activities, and her knowledge of her own Indigenous number and measurement, as sources of data which are commonly used in qualitative methods of research. Interviews with Elders were especially important. The students in this community speak