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Yingmin Jia
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*Editors*

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Volume I

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# Contents

# New Decoupling Conditions for Arbitrary Systems Based on Transcale Coupling to the Time-Derivative Order

**Mingxing Li and Yingmin Jia**

**Abstract** This paper proposes new coupling concepts: transcale coupling to the time-derivative order to give new decoupling conditions for a general system described by $(A, B, C, D)$ quadruples. Based on these new coupling concepts, novel conditions for the diagonal/diagonal block decoupling and triangular/triangular block decoupling are obtained in the time domain.

**Keywords** Linear system · Transcale coupling · Time domain · Unifying conditions

## 1 Introduction

The decoupling problem is extensively investigated over several decades and arises in four class decoupling results which are diagonal/diagonal block and triangular/triangular block. The diagonal decoupling problem (DDP) was first initiated in [1], a necessary and sufficient solvability condition was presented by [2] in state domain, and a numerically verifiable necessary and sufficient solvability condition was given in [3] recently. The triangular decoupling problem (TDP) was first formulated by [4]. Numerically reliable methods were developed by Chu [5–7] in time domain recently. The diagonal block decoupling (DBDP) was presented firstly by Sato in [8] and the triangular block decoupling problem (TBDP) was presented in [9] by a transfer matrix approach and a stable coprime factorization method in [10].

Above mentioned and other decoupling methods such as [11–13] are all very valid and valuable for solving the decoupling problem, separately. However, the solvability decoupling conditions are quite different. It leads us to study the problem that whether there is a unifying condition or index to describe the four different decou-

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pling results. This problem is answered partly in [14] by a frequency domain method. We will give a whole answer in time domain.

The paper is organized as follows: Sect. 2 provides some preliminary results. Section 3 establishes necessary and sufficient conditions of the four class decoupling results. Section 4 gives two examples to show the effectiveness of our results. Finally, Sect. 5 presents conclusions.

## 2 Problem Statement and Preliminary Results

Consider a proper LTI system described by

$$\begin{cases} \dot{x} = Ax + Bu \\ y = Cx + Du \end{cases} \tag{1}$$

where $x \in R^n, u \in R^m$ and $y \in R^p$ are the state, input, and output. For $v \in R^{m_v}$, permutation matrix $P_{y_I}$, and full rank $G$, substituting the state feedback law

$$u = Fx + Gv \tag{2}$$

into system (1) while let a new output $y_I = P_{y_I} y$, we obtain a closed-loop system which is

$$\begin{cases} \dot{x} = (A + BF)x + BGv \\ y_I = (C_I + D_I F)x + D_I Gv \end{cases} \tag{3}$$

where $C_I = P_{y_I} C$ and $D_I = P_{y_I} D$. The transfer function of system (1) while its output is taken as $y_I$ is $T(s) = C_I(sI - A)^{-1}B + D_I$. Relative to $(p_1, \dots, p_k)$, we partition $T(s)$, $C_I$ and $D_I$ into

$$T(s) = \begin{bmatrix} T_1(s) \\ \vdots \\ T_k(s) \end{bmatrix}, C_I = \begin{bmatrix} C_1 \\ \vdots \\ C_k \end{bmatrix}, D_I = \begin{bmatrix} D_1 \\ \vdots \\ D_k \end{bmatrix}, y_I = \begin{bmatrix} y_{p,1} \\ \vdots \\ y_{p,k} \end{bmatrix}. \tag{4}$$

where $C_i \in R^{p_i \times n}, D_i \in R^{p_i \times m}, p_i \in Z^+$, and $\sum_{i=1}^{k} p_i = p$.

**Definition 1** The static state feedback TBDP is solvable if there are $F, G$ and permutation matrix $P_{I_y}$ such that the transfer function of system (3) shown as

$$\hat{T}(s) \triangleq (C_I + D_I F)(sI - A - BF)^{-1}BG + D_I G \tag{5}$$

has the following triangular block form relative to partitions (4) while $G \in R^{m \times m_v}$ is full rank

$$\hat{T}(s) = \begin{bmatrix} \hat{T}_{11}(s) & 0 & \dots & 0 \\ \hat{T}_{21}(s) & \hat{T}_{22}(s) & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ \hat{T}_{k1}(s) & \hat{T}_{k2}(s) & \dots & \hat{T}_{kk}(s) \end{bmatrix} \tag{6}$$

where $\hat{T}_{ii}(s) \in R^{p_i \times m_i}, \hat{T}_{ii}(s) \neq 0, i = 1, \dots, k$. Furthermore, if $A + BF$ is stable, then the static state feedback TBDP with stability is solvable.

*Remark 1* We just give the TBDP definition since the DDP, DBDP and TDP are special cases of the TBDP. Notice that, if $p_i = 1, \hat{T}_{i,j} = 0, i \neq j$ then the DDP is obtained, if there is $p_i > 1, \hat{T}_{i,j} = 0, i \neq j$ then the DBDP is obtained, and if $p_i = 1$ and there are $\hat{T}_{i,j} \neq 0, i \neq j$ then the TDP is obtained.

From [15], if $\text{rank}(D) = \bar{m}_0$ then there exist orthogonal matrix $T \in R^{m \times m}$, permutation matrix $P \in R^{p \times p}$, matrix $L$ with non-zero rows, invertible $D_t$ and $\bar{m}_1, \bar{m}_2 \in Z$, such that $\bar{m}_0 + \bar{m}_1 + \bar{m}_2 = p$ and

$$PDT = \begin{array}{c} \begin{array}{cc} \bar{m}_0 & m - \bar{m}_0 \end{array} \\ \begin{bmatrix} D_t & 0 \\ 0 & 0 \\ LD_t & 0 \end{bmatrix} \end{array} \begin{array}{l} \\ \}\bar{m}_0 \\ \}\bar{m}_1 \, . \\ \}\bar{m}_2 \end{array} \tag{7}$$

Taking $y_p = Py$ and partitioning $P, y_p, PC$ and $T^{-1}u$ into

$$P = \left[P_1^T, P_2^T, P_3^T\right]^T, y_p = \left[y_{p,1}^T, y_{p,2}^T, y_{p,3}^T\right]^T \tag{8}$$

$$PC = \left[C_{p,1}^T, C_{p,2}^T, C_{p,3}^T\right]^T, T^{-1}u = \left[u_{t,1}^T, u_{t,2}^T\right]^T \tag{9}$$

give

$$\begin{cases} y_{p,1} = C_{p,1}x + D_t u_{t,1} \\ y_{p,2} = C_{p,2}x \\ y_{p,3} = (C_{p,3} - LC_{p,1})x + Ly_{p,1}. \end{cases}$$

The coupling of $y_{p,1}, y_{p,2}$ and $y_{p,3}$ has three type, one is there are no coupling, another is $L \neq 0$ and $C_{p,3} - LC_{p,1} \neq 0$, the last one is $L \neq 0$ and $C_{p,3} - LC_{p,1} = 0$. Thus, we have following three new coupling conceptions

**Definition 2 Complete transcale coupling to the time-derivative order (CTCTDO)** If there are $y_i, y_{i_0}$ which satisfy $y_i(s) = l_0(s)y_{i_0}(s)$ for a rational fraction matrix $l_0(s)$, then we call this type coupling complete transcale coupling to the time-derivative order.

**Definition 3 Transcale coupling to the time-derivative order (TCTDO)** If there are $y_i, y_{i_0}$ which satisfy $y_i(s) = l_0(s)y_{i_0}(s) + g_i(s)u(s), y_{i_0}(s) = g_{i_0}(s)u(s)$ and Rank

$([g_{i_0}^T(s), g_i^T(s)]^T) > \text{Rank}(g_i(s))$ is physically realizable for rational fractions $l_0(s)$ and $g(s)$, then we call this type coupling transcale coupling to the time-derivative order.

**Definition 4 Coupling without transcale to the time-derivative order (CWTTDO)** If there are no $y_{i_0}$ such that $y_i(s) = l_0(s)y_{i_0}(s) + g(s)u(s)$ where $l_0(s)$ and $g(s)$ are physically realizable rational fractions, then we call this type coupling without transcale to the time-derivative order (CWTTDO).

Without loss of generality, let $G$ full column rank and $m_v \le \min\{m, p\}$. In [9], the necessary and sufficient conditions for the TBDP of system (1) with partitions (4) while $m_v = m$ are obtained which are the following lemma:

**Lemma 1** *The system (1) with transfer function $T(s)$, which is partitioned into Eq. (4) relative to $(p_1, \ldots, p_k)$, is triangularly block decouplable by a $m \times m$ proper admissible precompensator if and only if*

- $dim\left(\bigcap_{i=1}^{j-1} Ker(T_i(s))\right) > dim\left(\bigcap_{i=1}^{j} Ker(T_i(s))\right), j = 2, \ldots, k.$
- $dim(Ker(T_1(s))) < m.$

*Remark 2* If $rank(T(s)C(s)) = rank(T(s))$ then physical realizable $C(s)$ is an admissible precompensator of $T(s)$.

## 3 Necessary and Sufficient Conditions of the Decoupling

The necessary and sufficient conditions of the four class decoupling results are given in this section. To achieve this goal, we define

$$D_1^* = \left[(d_{1,1}^*)^T, \ldots, (d_{1,p}^*)^T\right]^T \tag{10}$$

$$d_{1,1}^* = \begin{cases} d_1, \text{ if } d_1 \neq 0 \\ c_1 A^{\rho_1} B, \text{ if } d_1 = 0 \quad \text{and} \quad c_1 A^{\rho_1} B \neq 0 \\ 0, \text{ if } d_1 = 0 \text{ and } c_1 A^i B = 0, i = 1, \ldots, n \end{cases} \tag{11}$$

$$d_{1,i}^* = \begin{cases} d_i, \text{ if } rank \begin{bmatrix} D_{1,i-1}^* \\ d_i \end{bmatrix} > rank(D_{1,i-1}^*) \\ c_i A^{\rho_i} B, \text{ if } rank \begin{bmatrix} D_{1,i-1}^* \\ c_i A^{\rho_i} B \end{bmatrix} > rank(D_{1,i-1}^*) \\ \qquad and\ rank \begin{bmatrix} D_{1,i-1}^* \\ d_i \end{bmatrix} = rank(D_{1,i-1}^*) \\ c_i A^{n-1} B, \text{ if } rank \begin{bmatrix} D_{1,i-1}^* \\ c_i A^j B \end{bmatrix} = rank(D_{1,i-1}^*) \\ \qquad \text{here } j = 1, \ldots, n \end{cases} \tag{12}$$

$$D^*_{1,i} = \left[(d^*_{1,1})^T, \ldots, (d^*_{1,i})^T\right]^T \tag{13}$$

$$\rho_i = \begin{cases} -1, \text{ if } d^*_{1,i} = d_i \\ \min\{j|d^*_{1,i} = c_i A^j B, j = 0, \ldots, n-1\}. \end{cases} \tag{14}$$

With $D^*_1$ of system (1), we have

**Lemma 2** *$rank(T(s)) = k_0$ if and only if $rank(D^*_1) = k_0$.*

The sufficiency is easily proved. The necessity is deduced by Corollary 1 in the following content, thus it is omitted here. Lemma 2 indicates that there are two groups positive integers $(\bar{i}_1, \ldots, \bar{i}_{k_0}), (\rho_{\bar{i}_1}, \ldots, \rho_{\bar{i}_{k_0}})$ and $l = 2, \ldots, k_0$ such that

$$\bar{i}_j < \bar{i}_{j+1}, j = 1, \ldots, k_0 - 1 \tag{15}$$

$$\bar{D}^*_{k_0} = \left[(d^*_{1,\bar{i}_1})^T, \ldots, (d^*_{1,\bar{i}_{k_0}})^T\right]^T \tag{16}$$

$$\bar{i}_1 = \min\{i|\text{rank}(D^*_{1,i}) = 1\} \tag{17}$$

$$\bar{i}_l = \min\{i|\text{rank}(D^*_{1,i}) > \text{rank}(D^*_{1,i-1}), i > \bar{i}\ \}. \tag{18}$$

and $\text{rank}(\bar{D}^*_{k_0}) = k_0$. Moreover, for $i = 1, \ldots, p$ define

$$\sigma_i = \begin{cases} -1, \text{ if } d_i \neq 0 \\ \min\{j|d_i = 0, c_i A^j B \neq 0, j = 0, \ldots, n-1\} \end{cases} \tag{19}$$

then $\rho_i \geq \sigma_i$. Moreover, by combining (13) and (14) with Lemma 1, we get

**Theorem 1** *For system (1) with partitions (4) relative to $(p_1, \ldots, p_k)$, the TBDP is solvable if and only if there exist $y_{\bar{i}_j}, j = 1, \ldots, k_0$ which is a vector component of $y_{p,i}, i = 1, \ldots, k$.*

*Proof* From (4), partitions of $T(s)$ can be rewritten as

$$T_i(s) = C_i(sI - A)^{-1}B + D_i, i = 1, \ldots, k \tag{20}$$

There are at least one $y_{\bar{i}_j}, j = 1, \ldots, k_0$ which is a vector component of $y_{p,i}$ for all $i = 1, \ldots, k$ implies

- $\dim\left(\bigcap_{i=1}^{j-1} \ker(T_i(s))\right) > \dim\left(\bigcap_{i=1}^{j} \ker(T_i(s))\right), j = 2, \ldots, k$
- $\dim(\ker(T_1(s))) < m$.

Thus, the sufficiency is derived by Lemma 1.

Assume there is $i_0 \in \{1, \ldots, k\}$ such that all components of $y_{p,i_0}$ are not $y_{\bar{i}_j}$ for any $j = 1, \ldots, k_0$. Since

$$y^{(n)}_{p,i_0} = C_{i_0} A^n x + C_{i_0} A^{n-1} Bu + \cdots + C_{i_0} Bu^{(n-1)} + D_{i_0} u^{(n)}$$

and applying Lemma 2 into $T_{i_0}(s)$ gives

$$\begin{aligned} T_{i_0}(s) &= \sum_{j=0}^{n-1} \frac{r_{j+1}}{s^{j+1}} C_{i_0} A^j B + D_{i_0} \\ &= \sum_{j=0}^{n} \frac{r_j}{s^j} \sum_{l=1}^{k_{1,0}} t_{j,l} d^*_{1,\bar{i}_l} \end{aligned} \tag{21}$$

where $k_{1,0} \in \{1, \ldots, k_0\}, r_0 = 1$, we have

$$\bigcap_{j=1}^{k_{1,0}} (\ker(d^*_{\bar{i}_j})) \subset \ker(T_{i_0}(s))$$

Furthermore, from Lemma 2 and above results, we get

$$\bigcap_{j=1}^{i_0-1} \ker(T_j(s)) \subset \bigcap_{j=1}^{k_{1,0}} (\ker(d^*_{\bar{i}_j})) \subset \ker(T_{i_0}(s)) \tag{22}$$

$$\bigcap_{j=1}^{i_0} \ker(T_j(s)) \equiv \bigcap_{j=1}^{i_0-1} \ker(T_j(s)) \tag{23}$$

then for system (1) partitioned into (4), the TBDP is solvable while (23) is established. This result conflicts with Lemma 1. Thus, the necessity is obtained and Theorem 1 is proved.

**Corollary 1** *For $i, i \notin \{\bar{i}_1, \ldots, \bar{i}_{k_0}\}$ and $i \in \{1, \ldots, p\}$, define*

$$j = \max\{l_1 | l_1 \in \{1, \ldots, k_0\}, \bar{i}_{l_1} < i\} \tag{24}$$

*then there are $h_l(s), l = 1, \ldots, j$ such that*

$$y_i = \sum_{l=1}^{j} h_l(s) y_{\bar{i}_l} \tag{25}$$

*for system (1) with new output $y_I$ and feedback law (2).*

*Proof* For $i$ with $i \notin \{i_1, \ldots, i_{k_0}\}, i \in \{1, \ldots, p\}$, $j$ defined by (24) exists. Using the Laurent expansion gets

$$t_i(s) = \sum_{l=0}^{n} \frac{r_l}{s^l} \sum_{\bar{l}=1}^{j} t_{l,\bar{l}} d^*_{1,\bar{i}_{\bar{l}}}$$

where $r_0 = 1$, $t_i(s)$ is the $i$th row vector of $T(s)$. Thus for any $i \notin \{\bar{i}_1, \dots, \bar{i}_{k_0}\}$, there exist rational $f_1(s), \dots, f_j(s)$ such that

$$t_i(s) = \sum_{\bar{l}=1}^{j} f_{\bar{l}}(s) d^*_{1,\bar{i}_{\bar{l}}} \tag{26}$$

Furthermore, we define $\bar{T}_j(s)$ as

$$\bar{T}_j(s) = \left[ t^T_{\bar{i}_1}(s), \dots, t^T_{\bar{i}_j}(s) \right]. \tag{27}$$

It is easy to obtain that $\bar{T}_j(s)$ is full row rank and there is a rational matrix $\bar{T}_{j\perp}(s)$ such that $\bar{T}_{1,j}(s)$ invertible and

$$\bar{T}_{1,j}(s) = \left[ \bar{T}^T_j(s), \bar{T}^T_{j\perp}(s) \right]^T, \ \bar{T}_j(s) T^T_{j\perp}(s) \equiv 0.$$

Defining $h(s) = t_i(s)(\bar{T}^T_{1,j}(s)\bar{T}_{1,j}(s))^{-1}\bar{T}^T_j(s)$ gets

$$\begin{aligned} t_i(s) &= t_i(s)\bar{T}^T_{1,j}(s)\bar{T}_{1,j}(s)(\bar{T}^T_{1,j}(s)\bar{T}_{1,j}(s))^{-1} \\ &= t_i(s)(\bar{T}^T_{1,j}(s)\bar{T}_{1,j}(s))^{-1}\bar{T}^T_j(s)\bar{T}_j(s) \\ &= h(s)\bar{T}_j(s) \end{aligned} \tag{28}$$

From Eq. (28), we get Eq. (25) for system (1) with feedback law (2).

*Remark 3* From (28), $rank(T(s)) = rank(\bar{T}_{k_0}(s))$, thus the necessity of Lemma 2 is naturally concluded, Theorem 1 is established.

**Corollary 2** *The DBDP of system (1) with partitions (4) is solvable if and only if there are only coupling without transcale to the time-derivative order between* $y_{p,i}$ *and* $y_{p,j}$ *for* $i, j \in \{1, \dots, k\}$ *and* $j \neq i$*, and the DDP is solvable if and only if there are also have* $k_0 = p = m$*. TDP is solvable if there are no complete transcale coupling to the time-derivative order and* $k_0 = p = m$*.*

## 4 Numerical Example

Two more general examples are illustrated to show the effectiveness of our method. The first one is

$$A = \begin{bmatrix} -4.1476 & 1.4108 & 0.0633 \\ 0.2975 & -3.1244 & 0.0623 \\ -0.0429 & -0.1729 & -0.1325 \end{bmatrix}, \ B = \begin{bmatrix} 0.2491 & 0.0969 & -0.0112 \\ 0.2336 & 0.0335 & 0.0047 \\ 0.0624 & 0 & 0 \end{bmatrix}$$

$$C = \begin{bmatrix} 8.7379 & 0 & 0 \\ -3.3033 & 3.8052 & 0.0542 \\ 2.1940 & -2.5749 & -0.0295 \end{bmatrix}, \; D = \begin{bmatrix} 0 & 0 & 0 \\ 0.2383 & -0.2748 & 0.0224 \\ -0.1455 & 0.0580 & -0.2293 \end{bmatrix}$$

It is easy to obtain that $k_0 = 3$, $\rho_1 = \delta_1 = 0$ and $\rho_2 = \delta_2 = \rho_3 = \delta_3 = -1$, thus this system is diagonally decouplable. By our method, we get a diagonal decoupling controller as follows:

$$F = \begin{bmatrix} -14.7267 & -1.2687 & -0.2640 \\ -23.7392 & 12.1478 & -0.0292 \\ 12.9083 & -7.3517 & 0.0315 \end{bmatrix}, \; G = \begin{bmatrix} 2.1356 & -2.1400 & -0.4911 \\ 1.7782 & -5.4974 & -0.7719 \\ -0.9054 & -0.0326 & -4.2447 \end{bmatrix}$$

and the closed-loop system is stable and the transfer function is

$$T(s) = \begin{bmatrix} t_{11}(s) & 0 & 0 \\ 0 & t_{22}(s) & 0 \\ 0 & 0 & t_{33}(s) \end{bmatrix}.$$

Thus, the diagonal decoupling is achieved.

The second example is constructed to show our decoupling method further. Matrices $A, B, C, D$ of (1) are taken as

$$A = \begin{bmatrix} -1.3757 & 0.3261 & 0.4995 & -0.5721 & 0 \\ 0.2702 & -1.1435 & -0.8610 & 0.4206 & 0 \\ 0.8118 & -1.2524 & 0.3698 & -0.1262 & 0 \\ -0.5691 & 0.3981 & -0.0216 & -4.8505 & 0 \\ 0 & 0 & 0 & 0 & 1.5567 \end{bmatrix}$$

$$B = \begin{bmatrix} 13.1252 & -6.5409 & 4.0484 & 4.0484 & 0 \\ -0.4581 & -14.8132 & -4.3977 & -4.3977 & 0 \\ 20.8573 & 18.3359 & -3.7853 & -3.7853 & 0 \\ 1.5788 & -3.4030 & -1.3937 & -1.3937 & 0 \\ 0 & 0 & 0 & 0 & 2.3867 \end{bmatrix}$$

$$C = \begin{bmatrix} -0.0637 & -0.0519 & -0.0093 & 0.0041 & 0 \\ 0.0959 & -0.1180 & -0.0126 & -0.0323 & 0 \\ 0.0537 & -0.0397 & -0.0101 & 0.3089 & 0 \\ 0 & 0 & 0 & 0 & 4.6789 \end{bmatrix}$$

$$D = \begin{bmatrix} 1 & -1 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

By directly computing, we get $\rho_1 = \rho_4 = -1$, $\rho_2 = 0$, $\rho_3 = 4$ and $rank(D^*_{1,k}) = 2$ where

$$D^*_{1,k} = \begin{bmatrix} d_1 \\ c_2 B \\ c_3 A^4 B \\ d_4 \end{bmatrix} = \begin{bmatrix} 1 & -1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 \\ 625 & -625 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

thus this system is transcale coupling. Furthermore, $k_0 = k = 3 < p$, thus this system is triangular block decouplable, $y_{p,1} = y_1$, $y^T_{p,2} = [y_2, y_3]$, $y_{p,3} = y_4$, $\bar{i}_1 = 1, \bar{i}_2 = 2, \bar{i}_3 = 4$, and

$$C = \begin{bmatrix} C_1 \\ C_2 \\ C_3 \end{bmatrix} = \begin{bmatrix} c_1 \\ \begin{bmatrix} c_2 \\ c_3 \end{bmatrix} \\ c_4 \end{bmatrix}, \; D = \begin{bmatrix} D_1 \\ D_2 \\ D_3 \end{bmatrix} = \begin{bmatrix} d_1 \\ \begin{bmatrix} d_2 \\ d_3 \end{bmatrix} \\ d_4 \end{bmatrix}$$

$$\bar{C} = \begin{bmatrix} c_1 \\ c_2 \\ c_4 \end{bmatrix}, \; \bar{D} = \begin{bmatrix} d_1 \\ d_2 \\ d_4 \end{bmatrix}, \; P_0 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}, \; P_{y_I} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & I_{2\times 2} & 0 \end{bmatrix}$$

then, we get

$$F = \begin{bmatrix} 0.0035 & 0.0753 & -0.0967 & 0.0190 & 0 \\ -0.0594 & 0.0308 & -0.1045 & -0.0023 & 0 \\ -0.1406 & 0.1580 & 0.1018 & 0.0481 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -3.1662 \end{bmatrix}$$

$$G = \begin{bmatrix} 2.7194 & -0.1090 & -3.4559 & 2 & 1 \\ 0 & 0 & -1 & 1 & 4 \\ 0.0081 & 0.0081 & 0.0993 & 0 & 0 \end{bmatrix}^T$$

The transfer function from $v$ to $P_{I_y} y$ has the following form:

$$T(s) = \begin{bmatrix} T_{11}(s) & 0 & 0 \\ T_{21}(s) & T_{22}(s) & 0 \\ T_{31}(s) & 0 & T_{33}(s) \\ T_{41}(s) & 0 & T_{43}(s) \end{bmatrix}$$

Thus $P_{y_I} y$ and $v$ is triangularly block decoupled and stable.

## 5 Conclusions

New coupling concepts, CTCTDO, TCTDO, and CWTTDO, are proposed to give new decoupling conditions for a general system described by $(A, B, C, D)$ quadruples. Based on these new coupling concepts, the new conditions for the diagonal/diagonal block decoupling and triangular/triangular block decoupling are obtained in the time domain. The numerical computable static feedback decoupling laws should be derived, robust decoupling and the combined problem with disturbance rejection should be pursued in more depth in the future.

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# Determination of the Vehicle Relocation Triggering Threshold in Electric Car-Sharing System

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**Abstract** The electric car-sharing system is an arising and promising urban transportation mode. Vehicle unbalance usually occurs in multi-station electric car-sharing systems. Threshold triggering method is the most practicable approach for vehicle relocation, while determination of thresholds is the key problem. This paper presents a method for the thresholds determination. First, a prototype of two-stage method is proposed to illustrate the function of upper and lower thresholds. Subsequently, an optimization-based model is derived to determine the thresholds under the objective of minimizing the out-of-service rate and number of moving vehicles. Order data of EVCard system in Shanghai, China was employed to test the method. The results indicate that the proposed calculative method lead to relative better service rate and less moving times.

**Keywords** Car sharing · Electric vehicle · Vehicle relocation · Threshold determination · EVCard

## 1 Introduction

Car-sharing system has been regarded as an emerging promising transportation mode, which contains a fleet of vehicles and several stations that allows members to access and return immediately and pay for their occupation hourly [1]. Technically, car-sharing system requires high quality of information accessibility and high efficiency of payment, while the popularity of mobile communication makes it possible for users to access information freely and instantly. Economically, con-

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temporary people are becoming more inclined to share cars instead of owning a car due to the low frequency of usage of a car and the low costs of electric vehicle (EV) [2]. Researches also suggest that car-sharing system has advantages on improving mobility, lowering emission, reducing traffic congestion, and saving parking space.

During the past decade, car-sharing mode has been widely practiced in Europe, North America, Japan, and Singapore [3–6]. Since the recent 5 years, as a result of a combination of the Internet revolution, the sharing economy boom, the awareness of resource and energy consumption, and the consciousness of carbon emission and pollution, car-sharing mode have obtained more and more attention as a new component in urban transportation structures. In China, the clear energy vehicle encouragement policy stimulates the electric vehicle sharing mode as a frontier to popularize the electric vehicle, which makes the electric car-sharing system as an arising market in urban transportation in China [7].

Multi-station car-sharing systems in large scale usually allow users to pickup and return vehicles at different stations, which will bring convenience to costumers and improve the service quality, however, accompanied with unbalance of vehicles location [8]. Hence, vehicle relocation is quite necessary to satisfy users' demand, keep the vehicle supply in balance, make full use of facilities, and maximize the profits. With the unprecedented expansion of car-sharing services, especially EV-sharing services in China, the vehicle relocation theory and techniques become even more urgent. Taking the EVCard—an EV-sharing system operated by Shanghai automobile city—as an example, the system was established and tested in 2013–2014, and started to operate in 2015. The system owned 56 stations and 120 vehicles in April 2015, and more than 200 stations and 500 vehicles by December 2015. System unbalance problem appeared more and more serious and the vehicle relocation method is pivotal to support the system for sustainable operation and development.

Another fact is that car relocation is not as easy as bicycle or container relocation. Conventionally, bicycles or containers can be conveyed in large scale. However, car-sharing stations are generally scattered in intensive urban area, which disallow large freight truck to deliver [9]. Car towing and staff driving are two typical measures, where towing refers to the pulling of one car by another, and staff driving refers to relocating a car by a pair of staffs with a working car, but both of them have restricted relocation capability.

These facts have made the car relocation problem attractive to many researchers. According to the basic thinking, the vehicle relocation methods can be divided into three categories: demand prediction-based method, dynamic optimization method, and threshold method [10–12].

- Demand prediction-based method. It refers to acquiring the demand patterns and deploying vehicles as predicted demand. In practice, the demand patterns are not quite obvious and stable, and the users' demands are somehow stochastic, which makes demand prediction not reliable enough [13].

- Dynamic optimization method. These methods have been subsequently introduced to acknowledge the stochastic changing of demand, which are complicated on dynamic stochastic solving [14].
- Threshold method. It is a triggering mechanism to decide whether vehicles should move in or move out in a station, and is clearly available for operator to practice [15].

In practice of EVCard, we have found that the threshold method appropriate and adoptable, because of the very random demand and its request on simplex method. For the threshold method, paper [15] presents a three-phase method which employs station thresholds as the triggering values and regards threshold as a necessary step of their procedure. Paper [16–18] also involve the threshold as a condition of car relocation. However, to the best of our knowledge, these researches generally set the thresholds as known, or decide the thresholds empirically, and there are seldom researches that gives detail on how to determine the threshold of each station.

In this paper, the authors propose a method to determine the vehicle relocation triggering threshold, which is the key parameters in the mechanism to decide when the station should move in or move out vehicles (in Sect. 2). Section 3 presents the threshold determination method: first the state transition model is constructed to describe the dynamic process of the pickup and return phenomenon of a station, then based on the state variables transition, a vehicle moving number minimization model is developed to find out the optimum value of the thresholds. In Sect. 4, the order data and the station data of EVCard in April 2015 were adopted as a calculation example to demonstrate the process of the method.

## 2 Basic Thinking on Threshold Method

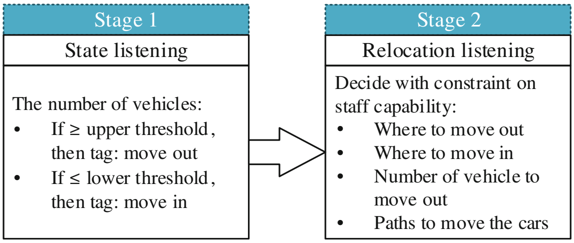
In this part, a basic two-stage prototype of threshold method is proposed to address the car relocation problem. In this prototype, we focus on the determination of the threshold, which is the main idea of this paper.

### *2.1 Stages of Threshold Relocation Method*

When should vehicles be moved into or out of a station? Typically, when there is no available car in the station (which means users cannot get a vehicle from the station or cannot access the service), vehicles should be located in; when there is no available parking space in the station (which means users cannot return a vehicle to the station or the service quality is bad), vehicles should be move out from the station.

So the number of vehicles in the station should be controlled by car relocation as much as possible to avoid the condition out of service. For a general station, the first

**Fig. 1** The two-stage threshold based relocation method




stage is to monitor the number of vehicles in the station and decide the occasion to move in or move out; the second stage is to match proper pairs of stations where one is to move out cars and the other is to move in cars, i.e., where the overflowing cars should be relocate to. This two-stage method is shown in Fig. 1.

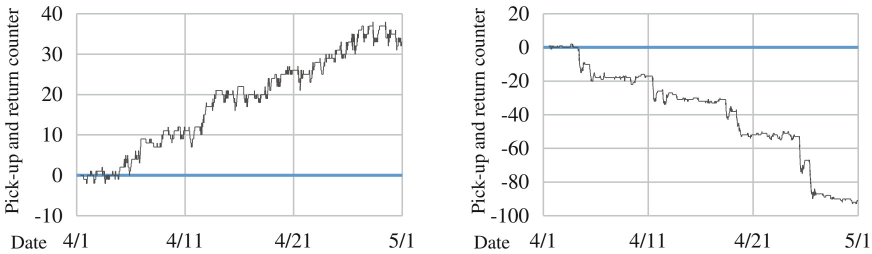
- **Stage 1**: state listening. The system listens to the change of the number of vehicles (the state variable) in each station. If the number of vehicles of a station accumulated above an upper threshold, tag the station as which need moving out cars (overflowing); if the number of vehicles of a station decreased below a lower threshold, tag the station as which need moving in cars (lacking).
- **Stage 2**: relocation listening. The system listens to the relocation tag of each station and decides the optimum solution of path to move vehicles from overflowing stations to lacking stations.

To this two-stage method, it should be noted that the two stages are relatively independent, i.e., the state listening decides when it should work and the relocation listening decide how it will work. In such case, relocating staff capability restricts the relocation processing in stage 2, but the stage 1 should tell whether each station should move cars in or out in advance, no matter whether the relocation would be successful. Another concern is that the upper and lower thresholds are pre-decided in order to make the monitoring mechanism work, so the method to determine the upper and lower thresholds of each station is necessary.

## *2.2 Thinking on Threshold Determination*

Define the pickup and return counter as an accumulator which plus 1 if user returns one vehicle and minus 1 if user pickup one vehicle. If the initial number of the counter equals the number of vehicles in the station at the initial moment, the counter indicates the change of the number of vehicles in this station.

Figure 2 shows the pickup and return counter at two example stations in EVCard system. Figure 2a suggests that the returned cars are more than the pickup cars that would fill the station, which lead to the condition that user cannot return car to this station anymore if cars were not moved out. Figure 2b suggests that the pickup




a. Station A: Tongji University Jiading Campus

b. Station B: Jiatinghui Shopping Center

**Fig. 2** The pickup and return counter at two example stations

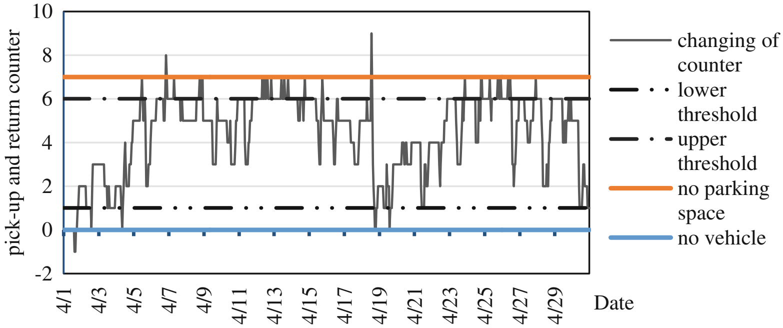



**Fig. 3** Thresholds to control the pickup and return counter changing within a proper range

demands are more than the returned cars, which make the station unavailable for following demands if cars were not moved in.

The aim of setting the upper and lower thresholds is to maintain the pickup and return counter within a proper range that can satisfy the pickup and return requests (as shown in Fig. 3).

Considering that the threshold determination is the precondition before the car relocation, and the staff capability only restricts the stage 2 as well as the thresholds in stage 1 only offer the if-or-not signal for stage 2, in threshold determination, suppose that all the move in or out operations can be satisfied in each station.

For each station, there are two basic principles to decide the thresholds

- Service unavailable rate—lower. This implies that the system should satisfy the users' demands as much as possible.
- Number of moving cars—lower. This implies that the operator wants to lower its costs on relocating cars.

## 3 Threshold Determination Method

Generally, the pickup or return requests happen stochastically with time, so that the pickup and return counter (or the number of vehicles in the station) would dynamically and randomly change with time (as illustrated in Figs. 2 and 3). The upper and lower thresholds are unknown but fixed decision variables. The solving method for the thresholds would be quite particular to conventional optimization method.

To solve the variables of thresholds, the first step is to establish the state transition equations for describing the feature of the change with time in a station. Subsequently, the optimization objectives can be inferred from the state transition equations, as well as the constraints.

### 3.1 Vehicle Counter State Transition Equations

Set the pickup and return counter (or the number of vehicles in the station) as the state variable, to monitor the change of the state of the station, and the state transition equation of the variable can be given as formula (1) and its supplementary formulae (2)–(6), where symbols are defined in Table 1.

$$X_t = X_{t-1} + r_t - d_t + \delta_t. \tag{1}$$

**Table 1** Definition of the symbols in the vehicle counter state transition equations

| Symbol | Definition |
|---|---|
| $t$ | The $t$th time segment, $t = 0, 1, \ldots, T$, and 0 is the initial time segment |
| $T$ | The number of total time valid segments |
| $X_t$ | The pickup and return counter in time $t$ |
| $r_t$ | The number of cars that are returned to the station and available in time $t$ |
| $d_t$ | The number of cars that are picked up at the station in time $t$ |
| $\delta_t$ | The number of cars actually moved in or out in time $t$ |
| $\delta_t^{'}$ | The number of cars that need to be moved in or out in time $t$ |
| $s_t$ | The tag on whether there are cars moved in or out in time $t$ |
| $u_t$ | The tag on whether the pickup or return requests are satisfied in time $t$ |
| $t_{in}$ | Average moving time on moving cars into the station |
| $t_{out}$ | Average responding time on moving cars out from the station |
| $N$ | The maximum number of the parking space in the station |
| $S_{\max}$ | Upper threshold to respond cars moving out |
| $S_{\min}$ | Lower threshold to respond cars moving in |

Define:

$$X_t' = X_{t-1} + r_t - d_t; \tag{2}$$

$$\delta_t' = \begin{cases} S_{\max} - X_t', & X_t' \geq S_{\max} \\ 0, & S_{\min} < X_t' < S_{\max} \\ S_{\min} - X_t', & X_t' \leq S_{\min} \end{cases}; \tag{3}$$

$$\delta_t = \begin{cases} \delta_{t-t_{out}}', & X_t' \geq S_{\max} \\ 0, & S_{\min} < X_t' < S_{\max} \\ \delta_{t-t_{in}}', & X_t' \leq S_{\min} \end{cases}; \tag{4}$$

$$s_t = \begin{cases} -1, & X_t' \geq S_{\max} \\ 0, & S_{\min} < X_t' < S_{\max} \\ 1, & X_t' \leq S_{\min} \end{cases}; \tag{5}$$

$$u_t = \begin{cases} -1, & X_t \geq N \\ 0, & 0 < X_t < N \\ 1, & X_t \leq 0 \end{cases}. \tag{6}$$

Formula (1) implies that the number of vehicles in the station in time $t$ equals the number of vehicles time $t - 1$ plus the number of cars that are returned to the station in time $t$ minus the number of cars that are picked up at the station in time $t$ then plus the number of vehicles that are actually moved into or out of the station. It should be noted that the number of cars that need to be moved in or out in time $t$ is judged by the state variable with the thresholds (given by formula (3)), but because of the responding delay for moving cars, the actual moved number of cars at time $t$ is given by formula (4). $s_t$ and $u_t$ are the detecting variables to summarize the number of moved cars and the service quality.

## 3.2 *Thresholds Determination*

Optimization techniques are employed in the thresholds determination. The final result of this model is to output the upper and lower thresholds of a station, so the decision variables are $S_{\max}$ and $S_{\min}$. In the state transition equations, $S_{\max}$ and $S_{\min}$ are not explicit in equation but as the condition of the piecewise functions, which brings peculiarity to this model. $S_{\max}$ and $S_{\min}$ are implicated in detecting variables $s_t$ and $u_t$, which describe the performance of the model from two opposite direction.

Note that the principles to determine the thresholds include the moving costs and the service quality. If operator wanted to keep the relocation costs lower that may lead to more bad services, while if operator wanted to improve the service quality that may increase the relocation times. Giving the number of moved vehicle $\sigma$ as

$$\sigma = \sum_{t=0}^{T} s_t \delta_t, \tag{7}$$

where $\sigma \in [0, +\infty)$ and $\sigma$ is an integer. To make it compatible with the other objective, normalize $\sigma$ as

$$\sigma' = \sigma / \max_{\forall S_{\max}, \forall S_{\min}} \sigma. \tag{8}$$

To describe the service quality, give the ratio of out-of-service time to the total time $\rho$ as

$$\rho = \frac{1}{T} \sum_{t=0}^{T} |u_t|, \tag{9}$$

and $\rho \in [0, 1]$. To balance both sides of consideration, set weight $\omega_1$ and $\omega_2$. if $\omega_1 > \omega_2$, the model is more likely to take the costs in account, otherwise to consider service quality more.

$$\min y = \omega_1 \sigma' + \omega_2 \rho; \tag{10}$$

and for both weights there is

$$\omega_1 + \omega_2 = 1. \tag{11}$$

Constraints of this model are given as follows:
Subject to

(1) $S_{\max}$ and $S_{\min}$ are integers.
(2) Lower threshold should be lower than the upper threshold

$$S_{\min} < S_{\max}. \tag{12}$$

(3) Upper threshold should not be higher than the number of parking space

$$S_{\max} \leq N. \tag{13}$$

(4) Lower threshold should not be lower than 0

$$S_{\min} \geq 0. \tag{14}$$

(5) Give the initial condition of the pickup and return counter

$$X_0 = X. \tag{15}$$

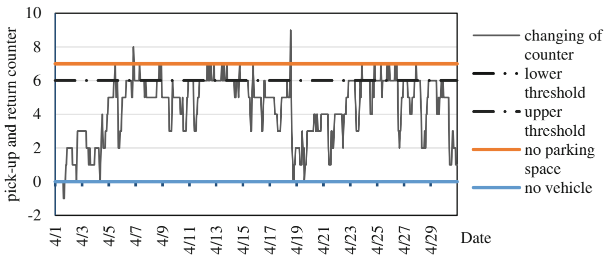
where $X$ is the initial number of vehicles in the station at the beginning of the study time segments. $X$ can be 0 if initial number of vehicles is unknown.

To solve this model, the major obstacle lies on the hidden variables in state transition equations. The decision variables $S_{\max}$ and $S_{\min}$ are implicit in the piecewise conditions and not explicit in optimization objectives. This feature makes the objective function nonlinear and hard to formularize. Since the decision variables $S_{\max}$ and $S_{\min}$ are integers and finite, and other variables are determinable through $S_{\max}$ and $S_{\min}$, enumeration would be feasible to determine the most proper solution.

## 4 Results and Discussions

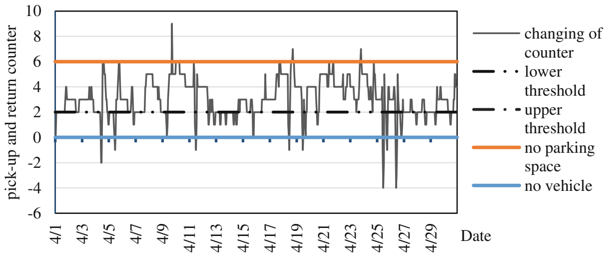
To demonstrate the threshold determination method, two typical stations given in Fig. 2—station A: Tongji University Jiading Campus and station B: Jiatinghui Shopping Center—are taken as examples. The orders of EVCard in April, 2015, including information about the pickup time, return time, pickup station, and return station are involved as the input data. Upper thresholds and lower thresholds of both stations are as the outputs. In this demonstration, we set $\omega_1 = \omega_2 = 0.5$ to take the relocation costs and service quality as the same importance. The state transition of the pickup and return counter graphs and the calculation results are shown in Fig. 4.

An interesting discovery from the results should be drawn that the station with more returning vehicles could get lower upper threshold and the station with more




| | |
|---|---|
| Parking space | 7 |
| Upper threshold | 6 |
| Lower threshold | 0 |
| Out-of-service rate | 7.36% |
| Number of moving | 34 |

a. Station A: Tongji University Jiading Campus




| | |
|---|---|
| Parking space | 6 |
| Upper threshold | 6 |
| Lower threshold | 2 |
| Out-of-service rate | 8.19% |
| Number of moving | 105 |

b. Station B: Jiatinghui Shopping Center

**Fig. 4** The pickup and return counter graphs and the calculation results