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Troels Andreasen · Henning Christiansen
Janusz Kacprzyk · Henrik Larsen
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Guy De Tré · Maria Amparo Vila
Adnan Yazici · Sławomir Zadrożny *Editors*

Flexible Query Answering Systems 2015

Proceedings of the 11th International Conference FQAS 2015, Cracow, Poland, October 26–28, 2015

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Foreword

This volume contains the papers presented at the Eleventh Flexible Query Answering Systems 2015 (FQAS-2015) held on October 26–28, 2015 in Cracow, Poland. The international conferences on Flexible Query Answering Systems (FQAS) are a series of premier conferences focusing on the key issue in the information society of providing easy, flexible, and intuitive access to information and knowledge to everybody, even people with a very limited computer literacy. In targeting this issue, the Conference draws on several research areas, such as information retrieval, database management, information filtering, knowledge representation, soft computing, management of multimedia information, and human-computer interaction. The Conference provides a unique opportunity for researchers, developers and practitioners to explore new ideas and approaches in a multidisciplinary forum. The previous FQAS conferences, which always attracted a large audience from all parts of the world, include: FQAS 2013 (Granada, Spain), FQAS 2011 (Ghent, Belgium), FQAS 2009 (Roskilde, Denmark), FQAS 2006 (Milano, Italy), FQAS 2004 (Lyon, France), FQAS 2002 (Copenhagen, Denmark), FQAS 2000 (Warsaw, Poland), FQAS 1998 (Roskilde, Denmark), FQAS 1996 (Roskilde, Denmark), FQAS 1994 (Roskilde, Denmark).

An important contribution of the Conference has also been the fact that has greatly facilitated, and often made possible, a deeper discussion on papers presented which as a rule has resulted in new collaborative works and a further progress in the areas.

The Workshop has been partially supported, financially and technically, by many organizations, notably: Systems Research Institute, Polish Academy of Sciences; Department IV of Engineering Sciences, Polish Academy of Sciences; Cracow Branch, Polish Academy of Sciences; Academia Europaea – The Hubert Curien Initiative Fund; Ghent University; Polish Association of Artificial Intelligence, and Polish Operational and Systems Research Society. Their support is acknowledged and highly appreciated.

We hope that the collection of main contributions presented at the Conference, completed with plenary talks by leading experts in the field, will provide a source of much needed information and inspiration on recent trends in the topics considered.

We wish to thank all the authors for their excellent contributions and their collaboration during the editing process of the volume. We are looking forward to the same fruitful collaboration during the next FQAS Conferences of this series that are planned for the years to come. Special thanks are due to the peer reviewers whose excellent and timely work has significantly contributed to the quality of the volume.

And last but not least, we wish to thank Dr. Tom Ditzinger, Dr. Leontina di Cecco and Mr. Holger Schaepe for their dedication and help to implement and finish this large publication project on time maintaining the highest publication standards.

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Part I
Preferences, Desires, Perception of Time, and Logical Foundations

Revising Desires – A Possibility Theory Viewpoint

Didier Dubois , Emiliano Lorini and Henri Prade

Abstract As extensively studied in the artificial intelligence literature, agents may have to revise their beliefs when they receive a new piece of information, for avoiding inconsistency in their epistemic states, since one cannot believe p and believe $\neg p$ at the same time. Similarly desiring p and $\neg p$ simultaneously does not sound reasonable, since this would amount to be pleased by anything. This motivates an approach for revising desires, a topic remained largely untouched. Desires do not behave as beliefs. While beliefs are closed under conjunction, one may argue that the disjunction of desires reflects the endorsement of each desire. In a possibility theory modeling setting, desires are expressed by a strong possibility set function, while beliefs are encoded by means of a necessity function. The paper outlines an original approach to the revision, the expansion, and the contraction of desires in the framework of possibility theory, and contrasts it with belief revision.

Introduction

Desires, goals, preferences, often used interchangeably (see, e.g., [23]), are all members of the family of *motivational* attitudes. This family is traditionally opposed to the family of *epistemic* attitudes including knowledge and belief. The distinction epistemic vs. motivational is in terms of the *direction of fit* of mental attitudes to the world. While epistemic attitudes aim at being true and their being true is their fitting the world, motivational attitudes aim at realization and their realization is the world fitting them [22, 25]. The philosopher J. Searle [26] calls "mind-to-world" the first kind of *direction of fit* and "world-to-mind" the second one.

While the word 'preferences' seems to have a generic meaning, *goals* are intended, they are like intentions, which is not the case for *desires*. Indeed a goal usually refers to a desire that has been selected by an agent in order to try to reach it. This distinction

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between desires and goals is supported by the possibilistic setting, where they are modeled differently. A goal p with a priority level α is translated by a constraint of the form $N(p) \geq \alpha$ where N is necessity measure. This fits with the idea that having $p \wedge q$ as a goal is the same as having p as a goal and having q as a goal. In that respect, see [19] for a possibilistic logic view of flexible querying in terms of prioritized goals.

As suggested in [6], and advocated in [9], a desire p is properly represented by a constraint of the form $\Delta(p) \geq \alpha$ which stands for "the agent desires p with strength at least α", where Δ is a strong possibility measure [13].

In the context of possibility theory, this concept of desire is also contrasted with the concept of belief that is properly represented by a constraint of the form $N(p) \geq \alpha$ which stands for "the agent believes p with strength at least α", where N is a necessity measure. However, the fact that beliefs and goals are represented by constraints of the same type does not mean at all that beliefs and goals are the same. Since they respectively represent what it is known about the current state of the world, and how the agent would like the state of the world becomes, it is crucial to keep beliefs and goals separated in two different possibilistic logic bases in decision under uncertainty problems; see [7].

The use here of necessity measures and strong possibility measures for modeling the beliefs and the desires of agents, should not be confused with their use for modeling bipolar queries where we need to distinguish between what is compulsory to reach (since the opposite should be avoided), and what would be really satisfactory to get [14, 17]. Such a later bipolar view is left aside in the following, where one considers positive desires only (namely those that it would be really satisfactory to concretize). Modeling desires of endusers is a task ahead the expression of definite queries whose purpose is to check if intended desires are feasible and how.

"It makes no sense to want everything and its opposite at the same time" says the wisdom of mankind. Otherwise, one is led to indetermination. In other words, one cannot desire p and desire $\neg p$ at the same time. This parallels the fact that one cannot believe p and believe $\neg p$ at the same time, without being led to inconsistency. Revising beliefs copes with this constraint. Similarly, revising desires should cope with the previous constraint. But beliefs and desires obey different principles.

Beliefs, modeled by means of necessity measures, satisfy

$$N(p \wedge q) = \min(N(p), N(q))$$

i.e., believing p *and* q amounts to believing p and to believing q. Thus we have

$$\min(N(p), N(\neg p)) = N(\bot) = 0$$

where $\bot$ denotes contradiction. This expresses that one cannot believe in p and in $\neg p$ in the same time while remaining consistent. We also have

$$N(p \vee q) \geq \max(N(p), N(q))$$

which is nothing but the increasingness of N with respect to entailment (i.e., if $r \vDash s$ then $N(r) \leq N(s)$), and fits with the fact that one may believe $p \vee q$ without believing p or believing q more specifically.

Desires rather obey the principle

$$\Delta(p \vee q) = \min(\Delta(p), \Delta(q))$$

i.e., desiring p *or* q amounts to desiring p and to desiring q. Thus we have

$$\min(\Delta(p), \Delta(\neg p)) = \Delta(\top) = 0$$

where$\top$ denotes tautology. Moreover $\Delta(p \wedge q) \geq \max(\Delta(p), \Delta(q))$. This indicates that Δ is decreasing with respect to entailment (i.e., if $r \vDash s$ then $\Delta(r) \geq \Delta(s)$).

Since desires do not behave as beliefs – N increases while Δ decreases with respect to entailment – belief revision does not straightforwardly apply to desire revision. One needs a slightly different theory for desire revision.

After restating and explaining the modeling of desires in terms of Δ functions, and providing a refresher on belief revision in the setting of possibility theory, the paper introduces and discusses the revision of desires.

Modeling Desire Using Δ Function

We here assume that in order to determine how much a proposition p is desirable an agent takes into consideration the worst situation in which p is true. Let $\mathcal{L}(ATM)$ be the propositional language built out of the set of atomic formulas ATM and let Ω the set of all interpretations of this language, corresponding to the different possible states of the world that can be described by means of the language. Moreover, let $||p|| \subseteq \Omega$ denote the set of interpretations where the propositional formula p is true.

Let π and δ be two possibilistic functions with domain Ω and codomain a linearly ordered scale S with 1 and 0 as top and bottom elements. Functions π and δ capture, respectively, the degree of (epistemic) possibility and the degree of desirability of a given interpretation $\omega \in \Omega$. We assume that π and δ satisfy the following normality constraints: (i) there exists $\omega \in \Omega$ such that $\pi(\omega) = 1$ (i.e., at least one state of the world is fully possible), and (ii) there exists $\omega \in \Omega$ such that $\delta(\omega) = 0$ (i.e., at least one state of the world is not desired at all).

As suggested in [6] and advocated in [9], for a given formula p, we can interpret

$$\Delta(p) = \min_{\omega \in ||p||} \delta(\omega)$$

as the extent to which the agent desires p to be true. This can be contrasted with

$$N(p) = 1 - \max_{\omega \in ||\neg p||} \pi(\omega)$$

which estimates the extent to which the agent believes p to be true, all the more as $\neg p$ is found impossible. Indeed, the measure of necessity N is the dual of the possibility measure Π, namely $\Pi(p) = 1 - N(\neg p)$ (where $1 - (\cdot)$ denotes the order-reversing map of S). Let us justify the following two properties for desires:

$$\Delta(p \vee q) = \min(\Delta(p), \Delta(q)); \quad \Delta(p \wedge q) \geq \max(\Delta(p), \Delta(q)).$$

According to the first property, an agent desires p to be true with a given strength α and desires q to be true with a given strength β if and only if the agent desires p or q to be true with strength equal to $\min(\alpha, \beta)$. A similar intuition can be found in [6] about the min-decomposability of disjunctive desires, where however it is emphasized that it corresponds to a pessimistic view. Notice that in the case of epistemic states, this property would not make any sense because the plausibility of $p \vee q$ should be clearly *at least* equal to the maximum of the plausibilities of p and q. For the notion of desires, it seems intuitively satisfactory to have the opposite, namely the level of desire of $p \vee q$ should be *at most* equal to the minimum of the desire levels of p and q. Indeed, we only deal here with "*positive*"[1] desires (i.e., desires to reach something with a given strength).

Under the proviso that we deal with positive desires, the level of desire of $p \wedge q$ cannot be less than the maximum of the levels of desire of p and q. According to the second property, the joint occurrence of two desired events p and q is more desirable than the occurrence of one of the two events. This is the reason why in the right side of the equality we have the operator max. This latter property does not make any sense in the case of epistemic attitudes like beliefs, as the joint occurrence of two events p and q is epistemically less plausible than the occurrence of a single event. On the contrary it makes perfect sense for motivational attitudes likes desires, as suggested by the following example.

Example 1. Suppose Peter wishes to go to the cinema in the evening with strength α (i.e., $\Delta(goToCinema) = \alpha$) and, at the same time, he wishes to spend the evening with his girlfriend with strength β (i.e., $\Delta(stayWithGirlfriend) = \beta$). Then, according to the preceding property, Peter wishes to to go the cinema with his girlfriend with strength at least $\max(\alpha, \beta)$ (i.e., $\Delta(goToCinema \wedge stayWithGirlfriend) \geq \max(\alpha, \beta)$). This is a reasonable conclusion because the situation in which Peter achieves his two desires is (for Peter) at least as pleasant as the situation in which he achieves only one desire.

One might object that if it is generally the case that satisfying simultaneously two desires is at least as good as satisfying one of them, there may exist exceptional situations where it is not the case. Just imagine, in the above example, the case where Peter's

[1] The distinction between positive and negative desires is a classical one in psychology. Negative desires correspond to state of affairs the agent wants to avoid with a given strength, and then desires the opposite to be true. However, we do not develop this bipolar view [16] here.

girlfriend would be laughing aloud or crying all the time during movies, and so Peter would not like to go with her to the cinema. This is a situation of non monotonic desires that can be coped with in this setting; see [9]. It is a counterpart of non monotonic reasoning about beliefs where, while $N(p \vee q) \geq \max(N(p), N(q))$ expresses increasing monotonicity, this should be remedied by an appropriate prioritization as in the example "penguins (p) are birds (b), birds fly (f)", where one should block the consequences of $p \vDash b \Rightarrow N(\neg b \vee f) \leq N(\neg p \vee f)$); see [1] for details on non monotonic reasoning about beliefs in the possibilistic reasoning setting.

Besides, from the normality constraint of δ ($\exists \omega, \delta(\omega) = 0$, expressing that not everything is desired), we can deduce that if $\Delta(p) > 0$ then $\Delta(\neg p) = 0$. This means that if an agent desires p to be true – i.e., with some strength $\alpha > 0$ – then he does not desire at all p to be false. In other words, an agent's desires must be consistent. Note also that the operator Δ, which is monotonically decreasing, satisfies $\Delta(\bot) = 1$ by convention. There is no harm to desire $\bot$, which by nature is unreachable.

From Belief Revision to Desire Revision

It has been recognized early that the epistemic entrenchment relations underlying any well-behaved belief revision process obeying Gärdenfors' postulates [20] are qualitative necessity relations [10], thus establishing a link between belief revision and possibility theory [13]. In the possibility theory view of belief revision, the epistemic entrenchment is explicit and reflects a confidence-based priority ranking between pieces of information. This ranking is revised when a new piece of information is received. We restate the possibilistic expression of belief revision, before considering the revision of desires.

Belief Revision

Uncertain beliefs are represented in possibility theory by constraints of the form $N(p) \geq \alpha$, corresponding to possibilistic logic [15, 18] formulas (p, α), expressing that p is believed to be true, with a certainty level at least equal to $\alpha > 0$, where α belongs to a linearly ordered scale S with 1 and 0 as top and bottom elements. Necessity measures satisfy the characteristic axiom $N(p \wedge q) = \min(N(p), N(q))$ (one believes $p \wedge q$ at the extent to what the less believed propositions of p and q is believed). They are the dual of possibility measures Π such that $\Pi(p) = 1 - N(\neg p)$ ($1 - (\cdot)$ is the order-reversing map of S).

A set B of possibilistic logic formulas (p_i, α_i) (for $i = 1, \ldots, n$) is semantically associated to a possibility distribution

$$\pi_B(\omega) = \min_{i=1,\ldots,n} \max(||p_i||(\omega), 1 - \alpha_i),$$

where $||p_i||(\omega) = 1$ if $\omega \vDash p_i$ and $||p_i||(\omega) = 0$ otherwise [15]. π_B is the largest possibility distribution (minimum specificity principle) such that $N(p_i) \geq \alpha_i$ for $i = 1, \ldots, n$. The distribution π_B rank-orders the interpretations of the language induced by the p_i's according to their plausibility on the basis of the strength of the beliefs in B.

In qualitative possibility theory [13], conditioning is defined by means of equation

$$\Pi(p \wedge q) = \min(\Pi(q|p), \Pi(p)).$$

Applying the minimum specificity principle, we get the possibility distribution $\pi(\cdot|p)$ associated with the possibility measure $\Pi(\cdot|p)$:

$$\pi(\omega|p) = \left\{ \begin{array}{cc} 1 & \text{if } \pi(\omega) = \Pi(p) \text{ and } \omega \vDash p \\ \pi(\omega) & \text{if } \pi(\omega) < \Pi(p) \text{ and } \omega \vDash p \\ 0 & \text{if } \omega \vDash \neg p \end{array} \right\}.$$

Then the *revision* B_p^* of the belief base B revised by input p, is defined as:

$$\pi_{B_p^*}(\omega) = \pi_B(\omega|p).$$

It includes the *expansion* B_p^+ of B by p (where $core(\pi) = \{\omega \mid \pi(\omega) = 1\}$) as a particular case:

$$\pi_{B_p^+}(\omega) = \min(\pi(\omega), ||p||(\omega)) \text{ provided that } core(\pi) \cap ||p|| \neq \emptyset.$$

Besides, the *contraction* B_p^- of B by p is defined by [11, 12]:

$$\pi_{B_p^-}(\omega) = \left\{ \begin{array}{cc} 1 & \text{if } \pi(\omega) = \Pi(\neg p) \text{ and } \omega \vDash \neg p \\ \pi(\omega) & \text{otherwise} \end{array} \right\}.$$

In particular, if $\Pi(p) = \Pi(\neg p) = 1$, we have $\pi_{B_p^-}(\omega) = \pi(\omega)$.

Harper's and Levi's identities, which respectively relate expansion and contraction to revision, remain valid [11]. It can be checked that counterparts of Gärdenfors' postulates for expansion, contraction, and revision [20] hold in the possibilistic setting [11]. See [3] [4] for more thorough studies, and the syntactic counterpart in possibilistic logic of the above revision process.

In particular, the possibilistic base B_p^*, can be obtained syntactically as $\{(p_i, \alpha_i) \in B \text{ s.t. } \alpha_i > \lambda\} \cup \{(p, 1)\}$, where $\lambda = inc(B \cup \{(p, 1)\}) = 1 - \max_\omega \pi_{B\cup\{(p,1)\}}(\omega)$ is the degree of inconsistency [15] of $B \cup \{(p, 1)\}$. Let us illustrate the approach by a small example.

Example 2. Let $B = \{(p, \alpha), (p \vee q, \beta)\}$, with $\alpha < \beta$.
We have $\pi_B(pq) = \pi_B(p\neg q) = 1$; $\pi_B(\neg pq) = 1 - \alpha$; $\pi_B(\neg p\neg q) = 1 - \beta$.
Then, assume the input $\neg p$ is received.
It gives $\pi_{B^*_{\neg p}}(pq) = \pi_{B^*_{\neg p}}(p\neg q) = 0$; $\pi_{B^*_{\neg p}}(\neg pq) = 1$; $\pi_{B^*_{\neg p}}(\neg p\neg q) = 1 - \beta$
(since $\Pi_B(\neg p) = \max_{\omega\vDash\neg p} \pi_B(\omega) = 1 - \alpha$).
The syntactic counterpart is $B^*_{\neg p} = \{(\neg p, 1), (p \vee q, \beta)\}$
(where $inc(B \cup \{(\neg p, 1)\}) = \alpha$).

Desire Revision

As explained at the beginning of the paper, desires can be represented in terms of strong (or guaranteed) possibility measures (denoted by Δ). A desire for p is expressed by a constraint of the form $\Delta(p) \geq \alpha$. A desire p with strength α will be denoted $[p, \alpha]$. Strong possibility measures are governed by the characteristic property $\Delta(p \vee q) = \min(\Delta(p), \Delta(q))$. This implies that Δ is decreasing with respect to entailment; in particular $\Delta(\top) = 0$ and $\Delta(\bot) = 1$.

A set D of desires $[p_i, \alpha_i]$ (for $i = 1, \ldots, n$) is semantically associated to a possibility distribution

$$\delta_D(\omega) = \max_{i=1,\ldots,n} \min(||p_i||(\omega), \alpha_i).$$

δ_D is the smallest possibility distribution (maximum specificity principle) such that $\Delta(p_i) \geq \alpha_i$ for $i = 1, \ldots, n$. The distribution δ_D rank-orders the interpretations of the language induced by the p_i's according to their satisfaction level on the basis of the strength of the desires in D. Because we should have $\Delta(\top) = 0, \min_\omega \delta_D(\omega) = 0$ should hold. More generally,

$$una(D) = \min_\omega \delta_D(\omega)$$

may be viewed as a level of *unacceptability* of D. The larger $una(D)$, the more unacceptable the set of desires D.

The *contraction* of D by p amounts to no longer desire p at all after contraction:

$$\delta_{D_p^-}(\omega) = \left\{ \begin{array}{cc} 0 & \text{if } \delta(\omega) = \Delta(p) \text{ and } \omega \models p \\ \delta(\omega) & \text{otherwise} \end{array} \right\}.$$

In particular, we have $\delta_{D_p^-} = \delta(\omega)$ if $\Delta(p) = \Delta(\neg p) = 0$.

The *expansion* of a set of desires D by p amounts to perform the cumulate desire p with the desires in D, providing that the result is not the desire of everything to some extent (due to the postulate $\Delta(\top) = 0$). Thus, we have

$$\delta_{D_p^+}(\omega) = \max(\delta(\omega), ||p||(\omega))$$

under the proviso that $support(\delta) \cup ||p|| \neq \Omega$, where $support(\delta) = \{\omega | \delta(\omega) > 0\}$, Ω denoting the set of all possible interpretations.

The conditioning of a strong possibility measure Δ obeys the equation [2]:

$$\Delta(p \wedge q) = \max(\Delta(q|p), \Delta(p)). \quad (*)$$

Applying the *maximum* specificity principle, we get the smallest (i.e., corresponding to the least committed conditional desires) possibility distribution $\delta(\omega|p)$ obeying $(*)$:

$$\delta(\omega|p) = \left\{ \begin{array}{cc} 0 & \text{if } \delta(\omega) = \Delta(p) \text{ and } \omega \vDash p \\ \delta(\omega) & \text{if } \delta(\omega) > \Delta(p) \text{ and } \omega \vDash p \\ 1 & \text{if } \omega \vDash \neg p \end{array} \right\}.$$

As can be seen, what is no longer reachable is fully desirable by default ($\Delta(\neg p|p) = 1$), while what we have is no longer desired since $\Delta(p|p) = 0$, but still preserving what is strictly above $\Delta(p)$.

While the *revision* of a set of beliefs B by p exactly corresponds to the conditioning of π_B by p, this is no longer the case with respect to δ_D for the revision of a set of desires D by p. Indeed, while a belief input $(p, 1)$, i.e. $N(p) = 1$, really means that all the models of $\neg p$ should be impossible, i.e., $\Pi(\neg p) = \max_{\omega \vDash \neg p} \pi_B(\omega) = 0$, a desire input $[p, 1]$ means $\Delta(p) = \min_{\omega \vDash p} \pi_B(\omega) = 1$, which says that all the models of p are satisfactory after revision. Due to this change of focus from $\neg p$ to p, when moving from beliefs to desires, we state:

$$\delta_{D^*_p}(\omega) = \delta_D(\omega|\neg p) = \left\{ \begin{array}{cc} 0 & \text{if } \delta(\omega) = \Delta(\neg p) \text{ and } \omega \vDash \neg p \\ \delta(\omega) & \text{if } \delta(\omega) > \Delta(\neg p) \text{ and } \omega \vDash \neg p \\ 1 & \text{if } \omega \vDash p \end{array} \right\}$$

As easily seen, $\Delta_{D^*_p}(p) = 1$, which shows that the *success postulate* for desire revision is satisfied, in the sense that an agent desires p to be true after revising his desire base by p. The latter may be found too strong and weakened into $\Delta_{D^*_p}(p) > 0$. It can be defined by taking lesson of what is done in belief revision; see [3].

Let us illustrate the approach by some examples.

Example 3. Let $D = \{[p \wedge q, \alpha], [r, \beta]\}$, with $\alpha > \beta$.
We have $\delta_D(pqr) = \delta_D(pq\neg r) = \alpha$; $\delta_D(p\neg qr) = \delta_D(\neg pqr) = \delta_D(\neg p\neg qr) = \beta$; $\delta_D(p\neg q\neg r) = \delta_D(\neg pq\neg r) = \delta_D(\neg p\neg q\neg r) = 0$.
Clearly, $una(D) = 0$.
Now, assume we want to add desire $[\neg p, 1]$. Let us compute $\delta_{D^*_{\neg p}}$. We get:
$\delta_{D^*_{\neg p}}(pqr) = \delta_{D^*_{\neg p}}(pq\neg r) = \alpha$; $\delta_{D^*_{\neg p}}(p\neg qr) = \beta$; $\delta_{D^*_{\neg p}}(p\neg q\neg r) = 0$, which remain unchanged, while it gives
$\delta_{D^*_{\neg p}}(\neg pqr) = \delta_{D^*_{\neg p}}(\neg p\neg qr) = \delta_{D^*_{\neg p}}(\neg pq\neg r) = \delta_{D^*_{\neg p}}(\neg p\neg q\neg r) = 1$.
Observe that $una(D \cup \{[\neg p, 1]\}) = 0$,
which means that after addition of the new desire, the set of desires remains acceptable. In fact, we have just performed an expansion here.
As can be checked, we have $D^*_{\neg p} = D^+_{\neg p}$.
The syntactic counterpart is $D^*_{\neg p} = \{[p \wedge q, \alpha], [r, \beta], [\neg p, 1]\}$.

Let us now consider two other examples where the unacceptability level becomes positive.

Example 4. Let $D' = \{[p, \alpha], [r, \beta]\}$ with $\alpha > \beta$.
Then $\delta_{D'}(pr) = \delta_{D'}(p\neg r) = \alpha$; $\delta_{D'}(\neg pr) = \beta$; $\delta_{D'}(\neg p\neg r) = 0$, and $una(D') = 0$.
Now, let us add desire $[\neg p, 1]$.
We get $\delta_{D'^*_{\neg p}}(pr) = \delta_{D'^*_{\neg p}}(p\neg r) = 0$; $\delta_{D'^*_{\neg p}}(\neg pr) = \delta_{D'^*_{\neg p}}(\neg p\neg r) = 1$.
We have $una(D' \cup \{[\neg p, 1]\}) = \alpha$ and $D'^*_{\neg p} = \{[\neg p, 1]\}$.

Example 5. Now consider $D'' = \{[p, \beta], [r, \alpha]\}$ (always with $\alpha > \beta$), then $\delta_{D''}(pr) = \alpha$; $\delta_{D''}(p\neg r) = \beta$; $\delta_{D''}(\neg pr) = \alpha$; $\delta_{D''}(\neg p\neg r) = 0$, and $\delta_{D''^*_{\neg p}}(pr) = \alpha$; $\delta_{D''^*_{\neg p}}(p\neg r) = 0$; $\delta_{D''^*_{\neg p}}(\neg pr) = \delta_{D''^*_{\neg p}}(\neg p\neg r) = 1$.
Finally, $una(D'' \cup \{[\neg p, 1]\}) = \beta$, and $D''^*_{\neg p} = \{[r, \alpha], [\neg p, 1]\}$.

Generally speaking, it can be shown that only the desires strictly above the level of unacceptability are saved (the others are drown, as $[r, \beta]$ in D'):

$$D^*_p = \{[p_i, \alpha_i] \in D \text{ s.t. } \alpha_i > una(D \cup \{[p, 1]\}) \cup \{[p, 1]\}.$$

Conclusion

The paper has outlined a formal approach to the revision of desires. Much remains to be done: providing the postulates characterizing this type of revision, laying bare the counterparts of Harper's and Levi's identities for desires, studying iterated desire revision. Moreover, the success postulate is translated by $\Delta_{D^*_p}(p) = 1$, which may be found too strong; this may be weakened into $\Delta_{D^*_p}(p) > 0$, which corresponds to the idea of *natural* revision in the sense of Boutilier [5].

Besides, it is known that belief revision and nonmonotonic reasoning are two sides of the same coin [21]. This remains to be checked for nonmonotonic desires [9] and desires revision. Finally, we plan to extend the static modal logic of belief and desire we proposed in [8] by dynamic operators of belief revision and desire revision. This will provide a unified modal logic framework based on possibility theory dealing with both the static and the dynamic aspects of beliefs and desires, to be compared with the proposal made in [24].

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Discovering Consumers' Purchase Intentions Based on Mobile Search Behaviors

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Abstract Search activity is an essential part for gathering useful information and supporting decision making. With the exponential growth of mobile e-commerce, consumers often search for products and services that are closely relevant to the current context such as location and time. This paper studies the search behaviors of mobile consumers, which reflect their customized purchase intentions. In light of machine learning, a probabilistic generative model is proposed to discover underlying search patterns, i.e., *when to search*,*where to search* and *in what category*. Furthermore, the predicting power of the proposed model is validated on the dataset released by Alibaba, the biggest e-commerce platform in the world. Experimental results show the advantages of the proposed model over the classical content-based methods, and also illustrate the effectiveness of integrating contextual factors into modeling consumers search patterns.

Keywords Search patterns · Context-aware · Probabilistic model · Recommendation

1 Introduction

Nowadays, the Internet acts as a core information source for human worldwide, and many information gathering activities take place online [5, 10]. Search activities widely exist in daily life, from which irrelevant information is filtered and useful one is extracted to support decision making. For example, 'information retrieval' and 'database querying' are two common activities to extract relevant documents or other types of information. According to the CNNIC (China Internet Network Information Center) report, the number of search engine users in China had reached 522 million in 2014, with a growth rate of 6.7% compared to last year. In addition,

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mobile search engines also attract 429 million people and had an even higher growth rate of 17.6%. The search services have been extended to the combined presentation of pictures, applications, products and other types instead of just text and links. On one hand, search becomes an essential part for consumers to find useful information in their decision making processes. On the other hand, by keeping track of the search patterns of the consumers, online merchants can have a better understanding of the consumers' behaviors and intentions [5].

In mobile e-commerce, potential consumers search for product information before making purchasing decisions due to the overwhelming information [8, 12, 16]. Since search could reflect consumers' purchase intentions and affect their choices online [9], it is worthy of deep exploration and has attracted a lot of interest from both academia and practitioners. Moreover, as Bhatnagar & Ghose (2004) [3] indicated, consumers exhibit differences in their search patterns, i.e, time spent per search episode and search frequency, which are attributed to product categories and consumer characteristics.

In marketing practice, clickstream data are commonly used to quantify the customers' search behaviors [8, 9, 16]. Usually, the clickstream data provide information about the sequence of pages or the path viewed by the consumers as they navigate websites [18]. With the prevalence of smart mobile devices [6], the consumers' clickstream data have been enriched with various contextual information [25], such as geographical information, which poses significant new opportunities as well as challenges [13]. Some data mining techniques have been employed to extract consumers' context-aware preferences [12, 20]. However, these research efforts mostly focused on purchase records while ignoring the search activities. While purchasing indicates consumers' final preferences over different products in the same category, search is an essential reflection of their purchase intentions towards a specific category. Therefore, a more precise model is needed to capture each consumer's search behavior relating to the particular context.

In this paper, we aim to understand the mobile e-commerce consumers' potential purchase intentions by studying their search patterns. That is, because the examination and inspection of products/services come at the cost of the consumers' time and effort, search outcomes become informative about what the consumers want [12]. We start by analyzing the search history of each consumer and then examine whether there is a relationship between search activities and the contextual factors (i.e., time and location). Based on the assumption that search patterns are time and location dependent, a probabilistic generative process is proposed to model each consumer's search history, in which the latent context variable is introduced to capture the simultaneous influence from both time and location. By identifying the search patterns of the consumers, we can predict their click decisions in specific contexts and recommend the products/services with the maximum clicking probabilities of the consumers.

The remaining part of the paper is organized as follows. Section 2 reviews related research from three aspects: consumer information search, clickstream data, and context-aware preference. Section 3 presents the consumer search model and parameter estimation process. Section 4 demonstrates the experimental results on a real-world dataset and Section 5 concludes the paper.

2 Related Work

This section discusses the existing work that is related to this study, consisting of three aspects: consumer information search, clickstream data, and context-aware preference.

Consumer Information Search. Consumer information search is an important part of purchase decision making [4, 8, 19], which attracts continuous attention from researchers. By using page-to-page clickstream data, Moe (2003) [16] examined in-store navigational behavior in terms of the pattern and content of pages viewed, and classified consumers into four categories according to their underlying objectives, namely, direct buying, search and deliberation, hedonic browsing, and knowledge building. This helps understand the objectives of the consumers better, thereby providing some insights into purchasing behaviors. Huang et al. (2009) [8] investigated the differences in consumer search patterns between search goods and experience goods in the online context based on clickstream data. In the empirical examination, they found the type of information that the consumers seek, and the way they search and make choices, was different for the two types of goods. Further, these differences affected the amount of time spent per page of information, the number of pages searched, the likelihood of free riding, and the relative importance of interactive mechanisms. Branco et al. (2012) [4] discussed the optimal stopping strategy for consumer search. Specially, they provided a parsimonious model of consumer search for gradual information that captured the benefits and costs of search, resulting in the optimal stopping threshold where the marginal costs outweighed the benefits. Similarly, Kim et al. (2010) [12] introduced the optimal sequential search process into a model of choice and estimated consumer information search and online demand for durable goods based on dynamic programming framework.

Clickstream Data. The widespread availability of Internet clickstream data has contributed greatly to marketing research [16], which allows both practitioners and academics to examine consumer search behaviors in a large-scale field setting. For example, Banerjee & Ghosh (2001) [2] clustered users based on a function of the longest common subsequence of their clickstreams, considering both the trajectory taken through a website and the time spent at each page. Montgomery et al. (2004) [18] used a dynamic multinomial probit model to extract information from consumers' navigation path, which is helpful in predicting their future movements. Kim et al. (2004) [11] used the clickstream data as implicit feedback to design a hybrid recommendation model, which resulted in better performance. Moe (2006) [17] proposed an empirical two-stage choice model based on clickstream data to capture observed choices for two stages: products viewed and products purchased. They found that the product attributes evaluated in Stage 1 differred from those evaluated in Stage 2. Overall, the clickstream data provided a great opportunity for researchers to dig into consumer search and purchase behaviors. Nevertheless, very little research has been conducted to describe the generative process of consumer search, especially the search behaviors in specific contexts.

Context-Aware Preference. Due to the exponential growth of mobile e-commerce, large volume of contextual information is available, which enables researchers to study the problem of personalized context-aware recommendation [13, 21, 24]. Shabib & Krogstie (2011) [21] proposed a step-by-step approach to assessing the context-aware preferences, which consists of four phases: product classification, interest matrix formation, clustering similar users and making recommendation. Zheng et al. (2010) [24] discovered useful knowledge from GPS trajectories based on users' partial location and activity annotations to provide targeted collaborative location and activity recommendations together. Specifically, they modeled the user-location-activity relations in a tensor and designed the algorithm based on regularized tensor and matrix decomposition. Liu et al. (2015) [13] considered users' check-in behavior in mobile devices to provide personalized recommendations of places, which integrated the effect of geographical factor and location based social network factor. Different from previous research, our study aims to formalize consumers' search behaviors as (*when, where, what*) patterns through a probabilistic generative model, which has better explanation ability and can be used to design appropriate recommendation strategies.

3 Consumer Search Model

3.1 Search Behavior Analysis

In the context of e-commerce, consumers commonly search for product information online before making purchase decisions [4]. Since more and more people access the Internet with their smart phone, the mobile search activities have distinctive features where contextual information can be captured by the search logs, including time and location [22, 25]. While consumers are often overwhelmed by excessive number of products in the platform, the specific category that products belong to is a good reflection of consumers' purchase intentions. In addition, the data of search log will be very sparse if each product is treated as an individual item, which may hardly be able to discover common search patterns. Therefore, we focus on the searched 'category' instead of 'product' in the following analysis. Predicting the category that consumers are most likely to click in a given timeslot and location is of great importance. If we can model a consumer's purchase intentions toward a specific category, we can recommend the appropriate products/services taking into account both his/her preferences and location information.

To analyze the possible factors that affect mobile consumers' search patterns, the relationships between the number of clicks and contextual factors will be explored. The clickstream data were released by Alibaba.com, which will be described in detail later. Figure 1 displays the click times in a range of 24 hours and Figure 2 shows the distribution of clicks under different geographic areas. Note that locations were clustered into different geographic areas without overlap.

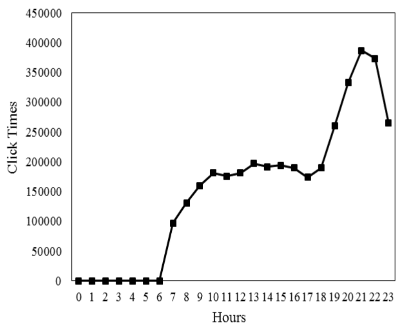

Fig. 1 Click times in different hours

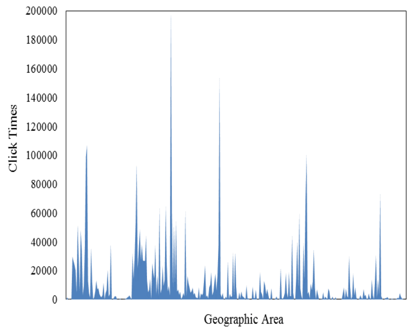

Fig. 2 Click times in different geographic areas

Here, the search events mostly occurred in the evening and reached the peak at 21:00. It can be seen that the consumers were more active in certain geographic areas, which illustrates the importance of location information. Thus, it is reasonable to assume that consumers search/click patterns are dependent on contextual factors including time and location.

Problem Definition: Suppose that we have consumers' clicking histories in mobile terminal, which reflect their search behaviors. For a consumer u, its clicking trace can be represented with a tuple (l, t, g) where a consumer u clicks category g at time t in location l. The problem is to model all the consumers' search histories and discover the patterns, in terms of *when to search, where to search*, and *in what category*. In this way, appropriate recommendation strategies can be designed so as to present the right product/service categories to right people at right time and right locations.

3.2 Generative Process

The advantage of probabilistic generative model is that it can mimic the process of a consumer's purchase behavior. Inspired by the classic topic model "Probabilistic Latent Semantic Analysis" (PLSA) [7, 15], the process of consumer search could be considered as an extension [23].

Specifically, each search is composed of particular search time, location and category, which are correlated with each other. In order to discover the common search patterns that underly these tuples, we introduce a latent factor, i.e., *shared search context* c. Thus, each value of context c is assumed to be the mixture of time, location and category, representing some common patterns among all the consumers, hence consumers' preferences towards time, location, and category are simultaneously captured by the shared search context.

For example, in the case that u wants to choose a category, he/she may choose a context according to his/her personal preference distribution, which is commonly a multinomial distribution. After that, the selected context in turn 'generates' the specific search tuple (l, t, g) following the context's generative distribution. Thus, this model simulates the process of how u picks the category g at the specific time t and location l. To simplify the analysis, we assume that these specific tuples, including time, location and category, are conditionally independent given the latent context. Therefore, the generative process for each consumer's clicked category can be summarized as follows:

– For each consumer u:
  - For each search action (l, t, g) of consumer u:
    (1) Generate a shared search context $c \sim p(c|u)$, which is a multinomial distribution;
    (2) Generate a search location $l \sim p(l|c)$ conditional on the latent context;
    (3) Generate a search timeslot $t \sim p(t|c)$ conditional on the latent context;
    (4) Generate a search category $g \sim p(g|c)$ conditional on the latent context.

Note that $p(l|c)$, $p(t|c)$ and $p(g|c)$ are also assumed to be multinomial distributions. The above Bayesian generative process can be represented as a graphical model in which a directed graph is used to describe probability distributions (see Figure 3).

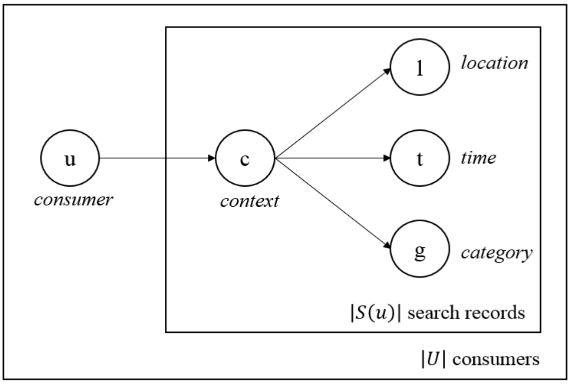

Fig. 3 A graphical representation of the probabilistic model

Thus, a consumer's search history is regarded as a sample of the following mixture model.

$$p(l, t, g|u) = \sum_c p(l, t, g|c) \cdot p(c|u) \tag{1}$$

Since l, t, g are assumed to be conditional independent with each other given the latent shared search context c, the above equation can be transformed to:

$$p(l, t, g|u) = \sum_c p(c|u) \cdot p(l|c) \cdot p(t|c) \cdot p(g|c) \tag{2}$$

The proposed model is a latent class statistical mixture model, which discovers: 1) a consumer's personal preference distribution over latent search context; 2) a category generative distribution for each latent context; 3) a consumer's preference distribution over locations/timeslots.

3.3 Parameter Estimation

To estimate parameters, the MLE (Maximum Likelihood Estimation) method is used to maximize the log likelihood of the collected search history for all consumers U that are generated by this model. The log likelihood function is given by:

$$\log p(U;\theta) = \sum_{u} \sum_{<l,t,g>\in S(u)} \{\log \sum_{c} p(c|u) \cdot p(l|c) \cdot p(t|c) \cdot p(g|c)\} \tag{3}$$

where $S(u)$ denotes the search history for consumer u; θ denotes all the parameters in the model including $p(c|u)$, $p(l|c)$, $p(t|c)$ and $p(g|c)$. Since it is difficult to directly optimize the above equation due to the log calculation being out of a summation, the EM (Expectation Maximization) algorithm is employed here to estimate these parameters.

In the **E-step**, the posterior distribution of hidden variable (i.e., context c) is computed, given the observed data and the current values of parameters according to Bayesian rule:

$$p(c|u,l,t,g) = \frac{p(c|u) \cdot p(l|c) \cdot p(t|c) \cdot p(g|c)}{\sum_{c'} p(c'|u) \cdot p(l|c') \cdot p(t|c') \cdot p(g|c')} \tag{4}$$

In the **M-step**, the new optimal values for parameters are obtained given the current settings of hidden variables calculated in E-step. By maximizing the log likelihood function, the parameters can be updated as follows:

$$p(c|u) = \frac{\sum_{<l,t,g>\in S(u)} p(c|u,l,t,g)}{\sum_{c'} \sum_{<l,t,g>\in S(u)} p(c'|u,l,t,g)} \tag{5}$$

$$p(l|c) = \frac{\sum_{u} \sum_{<l,t,g>\in S(u)} p(c|u,l,t,g)}{\sum_{l'} \sum_{u} \sum_{<l,t,g>\in S(u)} p(c|u,l',t,g)} \tag{6}$$

$$p(t|c) = \frac{\sum_{u} \sum_{<l,t,g>\in S(u)} p(c|u,l,t,g)}{\sum_{t'} \sum_{u} \sum_{<l,t,g>\in S(u)} p(c|u,l,t',g)} \tag{7}$$

$$p(g|c) = \frac{\sum_{u} \sum_{<l,t,g>\in S(u)} p(c|u,l,t,g)}{\sum_{g'} \sum_{u} \sum_{<l,t,g>\in S(u)} p(c|u,l,t,g')} \tag{8}$$