Theory and Applications of Transport in Porous Media

Hui-Hai Liu

Fluid Flow in the Subsurface

History, Generalization and Applications of Physical Laws

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Preface

This research monograph presents a systematic endeavor to generalize fundamental physical laws related to subsurface fluid flow that are important for a number of contemporary applications, such as recovery of subsurface energy resources, geological disposal of high-level nuclear wastes, CO_2 geological sequestration, and groundwater contamination in the vadose zone. The history of discovering these laws is also briefly presented within the context of discussing the ranges in which they are valid. This relatively new endeavor should be of interest to engineers, researchers, and students in the areas of reservoir engineering, hydrogeology, soil physics, and rock mechanics.

Darcy's law is the fundamental law of subsurface fluid flow. For low-permeability porous media, however, Darcy's law does not always hold because of the strong fluid–solid interaction. While this issue has been investigated by a number of researchers, Chap. 1 presents a new phenomenological relationship between water flux and hydraulic gradient, or a generalized Darcy's law. The traditional form of Darcy's law and two other generalizations for low-permeability media, proposed by other researchers, are shown to be special cases of our generalized Darcy's law. The implications and applications of this "non-Darcian" flow behavior are also discussed.

Edgar Buckingham independently discovered the relationship between water flux and hydraulic gradient for unsaturated (or multiphase) flow in porous media that has often been called Darcy's law in the literature. In this book, I call this relationship the Darcy-Buckingham law because in my view Buckingham's contribution to the theory of subsurface multiphase flow has been historically underestimated; multiphase flow is more complex than the single-phase flow that Darcy's law was developed for. The Darcy-Buckingham law, however, is only valid under the condition of local equilibrium. This condition does not hold in many cases, although the Darcy-Buckingham law has been used there because of the lack of alternatives. In Chap. 2, I introduce an optimality principle that unsaturated water flow patterns are self-organized in such a way that overall water flow resistance is minimal. Based on this principle, a generalized version of the Darcy-Buckingham law is derived in which unsaturated hydraulic conductivity is not only a function of water saturation (or capillary pressure) assumed in the Darcy-Buckingham law, but also a function of water flux. The consistency between our theoretical results and observations is demonstrated.

It is well known that subsurface fluid flow is coupled with mechanical deformation of subsurface media where fluid flow occurs; in many cases, this coupling can play a dominant role. Hooke's law is the most fundamental law governing elastic deformation of solids. Natural rocks, however, have unique features compared with other solids, one of which is the existence of small-scale deformation heterogeneities (such as microcracks). Acknowledgement of these unique features is important, because they justify that rock mechanics exists as a stand-alone discipline, rather than an adjunct of general solid mechanics. This also explains why elastic mechanical deformation of a natural rock does not follow exactly the traditional Hooke's law; the related mechanical properties, unlike those assumed by Hooke's law, are not constant under certain conditions. To better consider the impact of natural heterogeneity, Chap. 3 introduces the two-part Hooke model that was developed by dividing a natural rock into hard and soft parts. Remarkable consistency between the model and observations from different sources, for both rock matrix and fractures, has been achieved. The usefulness of the model in dealing with engineering problems is also demonstrated. Note that although a number of researchers have touched on the same issue by establishing empirical relations between stress and rock mechanical properties, the two-part Hooke model takes a much bolder and more systematic approach and is also more effective for practical applications.

Non-equilibrium thermodynamics (which is closely related to optimality principles) is the foundation for dealing with highly nonlinear problems. This branch of science, however, has not been well established yet. For example, contradictory optimality principles exist in the literature. As an applied scientist and engineer with a primary interest in applying basic scientific principles to my research areas, I initially tried to avoid, but eventually got into the study of the nonequilibrium thermodynamics because it is the true starting point to investigate subsurface multiphase flow and other related processes. Chapter 4 presents a new thermodynamics hypothesis that tries to answer under what conditions the optimality principles should apply and which of the simultaneously occurring physical processes, if not all, is subject to the optimization. This hypothesis seems to be able to reconcile different optimality principles proposed in several different areas.

The generalization of well-known fundamental physical laws is indeed an ambitious and a highly risky endeavor. It, however, was not motivated by academic interest, but by the needs of practical applications. As a modeler who is fully aware of and enjoys the increasing powerfulness of available computational capabilities, I am more and more convinced that the lack of appropriate physical laws at scales of practical interest is the weakest link in improving our modeling capability (especially the capability for prediction). For example, no matter how powerful computers are, we simply cannot use them to predict the observed non-Darcian flow with the traditional Darcy's law. Much more work is needed to substantiate our physical foundations for accurately modeling subsurface processes.

The generalization of the physical laws uses different approaches in this book, ranging from purely phenomenological ones to theoretical derivations based on newly introduced principles. But all generalized laws have the relatively simple mathematical form and a small number of parameters. This largely reflects my own research philosophy as an engineer: a good model or theory should be able to adequately capture the essence of physics and, at the same time, has a relatively simple form or structure. This happens to be consistent with the point of view of Nobel Laureate Richard Feynman, who in his celebrated book "*Characters of Physical Laws*", concluded that the mathematical forms of well-known physical laws are always simple, although they describe very complex phenomena.

I did initially have a reservation to publish a book dealing with the physical laws that are at the heart of several research areas. I would feel much more comfortable doing so after the related work becomes more mature. At the same time, I also feel the urgency to get the message out that we do need to revisit the fundamental laws that have generally been viewed as sacrosanct doctrines by many people. Thus, I view this book as a messenger or a starting point for this revisiting. I would also like to make it clear that the focus of this book is on the work mainly done by me and my collaborators. Although I try to briefly cover the related work of others as well, the citation of their work is by no means intended to be exhaustive.

Houston, TX, USA Hui-Hai Liu

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I have been blessed in my career by being exposed to projects of national and international importance and by having some world-class scientists/engineers as colleagues in both Lawrence Berkeley National Laboratory and Aramco Research Center. Many materials here result from dealing with technical issues of these projects and from collaborations with colleagues whose names can be found from the author lists of the cited papers or reports (in the reference sections) that I coauthored. I appreciate their company in my journey to revisiting the relevant physical laws that could have been very lonely otherwise. Especially, I like to thank some of my postdoc researchers and research students working with me when I was in Lawrence Berkeley National Laboratory; some contents of this book are directly connected to their work and/or stimulated by discussions with them. They are Drs. Feng Sheng, Lianchong Li, Yu Zhao, Jianping Zuo, Marco Bianchi, Mingyao Wei, Run Hu, Jiangtao Zheng, Delphine Roubinet, and Emma Engström. Dr. Run Hu also helped with preparing the initial version of this book. I have had a chance to work with many young and talent Chinese colleagues, because one of my responsibilities at Lawrence Berkeley National Laboratory was to help to establish connections between the Earth Sciences Division over there and Chinese research organizations in the relevant areas.

The draft version of this book was reviewed by Prof. Fred J. Molz from Clemson University (Chaps. 2 and 4), Prof. YuShu Wu from Colorado School of Mines

(Chaps. 1 and 5), and Dr. Yanhui Han from Aramco Research Center-Houston (Chap. 3). Mr. Jilin Zhang from Aramco Research Center-Houston reviewed the initial version of book and helped check its final version sent to the publisher. Their constructive comments considerably improved the readability of the book. Ms. Ebony N. Fondren from Aramco Research Center-Houston took care of the paperwork related to copyright issues for previously published illustrations used in this book. I also thank the management of the Aramco Research Center and Saudi Aramco, especially Drs. Ashraf M. Al-Tahini (Director of Aramco Research Centers) and Daniel T. Georgi (Lead of Reservoir Engineering Technology Team from Aramco Research Center—Houston), for supporting the publication of this book.

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Finally, I very much appreciate the support and encouragement from my wife Wei Huang and my son Neil Liu while writing this book. I initially thought that writing a book was something that I should do after retirement. The other consideration is that the related work may become more mature at that time. Wei and Neil were able to make me change my mind based on the argument that writing a book to summarize previous research work does not necessarily slow down the current research activities; instead it provides an opportunity to rethink some important issues. It turns out that they are right!

Contents

Chapter 1
Generalization of Darcy's Law: Non-Darcian Liquid Flow in Low-Permeability Media

Abstract Darcy's Law was discovered by Henry Darcy (1803–1858) based on experimental observations of steady-state water flow through sand columns. It states that water flux in saturated porous media is linearly proportional to hydraulic gradient. For low-permeability porous media, however, Darcy's law is not adequate because of the strong fluid–solid interaction that results in non-linear flux-gradient relationships. This chapter presents a new phenomenological relationship between water flux and hydraulic gradient, or a generalized Darcy's law. The traditional form of Darcy's law and two other generalizations for low-permeability media, proposed by other researchers, are shown to be special cases of the generalization. The consistency between the generalization and experimental observations from different sources is demonstrated. The generalized Darcy's law and its variations are used to attack several key technical issues facing the geoscience community, including the relative importance of diffusion in the excavation damaged zone for a shale repository of high-level nuclear waste, the accurate measurement of relative permeability for multiphase flow in a low-permeability porous medium, non-Darcian flow behavior during imbibition of fracturing fluids into a shale gas reservoir, and formation of the pressure seal in shale formations.

Darcy's law is the fundamental law for modeling subsurface fluid flow processes. This chapter briefly reviews Darcy's life and his law and presents a generalized Darcy's law for low-permeability media in which traditional Darcy's law does not hold for describing liquid flow. Applications of the generalization to several practical problems are also discussed. The discussions in this chapter are mainly based on materials from Liu et al. (2012, 2014), Liu and Birkholzer (2013) and Liu (2014).

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1.1 Henry Darcy and His Law for Subsurface Fluid Flow

The well-known Darcy's Law was discovered by Henry Darcy (1803–1858). It states that water flux in saturated porous media is linearly proportional to hydraulic gradient. Darcy's Law forms the quantitative basis of many science and engineering disciplines including hydrogeology, soil science, and reservoir engineering. Because of the unparalleled status of Darcy's law in the related areas, there are a number of excellent publications in the literature regarding Darcy's life and his law (e.g., Freeze 1994; Brown 2002; Simmons 2008). The materials presented in this section are based on these publications.

Henry Darcy (Fig. 1.1) was a distinguished engineer, scientist and citizen (Brown 2002). He was born on 10 June 1803 in Dijon, France, and entered L'Ecole Polytechnique, Paris, in 1821, where he started his science and engineering training. At that time Jean-Baptiste Joseph Fourier (1768–1830), who discovered Fourier Law for heat transfer, held a Chair position at L'Ecole Polytechnique. Thus, Simmons (2008) speculated that "it is therefore possible that Fourier taught Darcy his heat law and that the earliest seeds of Darcy's Law may have been planted at this point". Darcy was admitted to L'Ecole des Ponts et Chaussèes (School of Bridges and Roads), Paris, in 1823 when he was 20 years old. The school was closely associated with Le Corps Des Ponts et Chaussèes that had a mission to support the construction of infrastructure throughout France and was regarded an elite fraternity of engineers that had influential status in mid-nineteenth century in France (Freeze 1994; Simmons 2008). The school was created to train students to

Fig. 1.1 Henry Darcy (1803–1858) (https://en.wikipedia.org/wiki/Henry_Darcy)

be engineers in the Corps. Based on Darcy's class ranking in both L'Ecole Polytechnique and L'Ecole des Ponts et Chaussées, Brown (2002) noted that Darcy was a good, but not the best student in his classes.

In 1826, Darcy, at the age of 23, graduated from the School of Bridges and Roads and started a remarkable scientific and engineering career in the Corps. He spent almost all of his working life in his home city, Dijon. During 1827–1834, he performed feasibility studies of, and developed a plan for, the Dijon public water supply project. The plan was approved in 1835 and the project began in 1839 and was widely regarded one of the best European water supply systems at that time (Simmons 2008). Because of the success of the project, Darcy received a number of honors. He was awarded the Legion of Honor by King Louis Phillip in 1842. He also accepted a gold medal from the Municipal Council and a laurel wreath from the workmen when the project was completed in 1844, but he waived all the fees. As Philip (1995) put it, "Darcy, with great vision and skill, designed and built a pure water supply system for Dijon, in place of previous squalor and filth. Dijon became a model for the rest of Europe. Darcy selfless waived fees due to him from the town, corresponding to about $1.5 million today. Medals were struck recognizing his skill and a monument celebrates his great work".

Darcy's very success lay in his downfall (Philip 1995; Simmons 2008). During the last decade of his life (1848–1858), he suffered political persecution and his health deteriorated. Fortunately, he was able to focus on research activities very productively in this relatively short period of his life and made major scientific discoveries, including Darcy's Law. The period was called Darcy scientific legacy by Simmons (2008). As an engineer, Darcy's scientific contributions were clearly motivated by a deep desire to solve practical and relevant engineering problems that he had encountered. Darcy's other contributions than Darcy's law and the related historical backgrounds can be found in details in Simmons (2008).

In 1856, Darcy published his most famous report on the construction of municipal water supply of Dijon, "Les Fontaines Publiques de la Ville de Dijon", or "The public fountains of the city of Dijon" in English. The report was about 680 pages long and contained 28 plates of figures. Part 2 of Note D of the report, with a subtitle of "Determination of the laws of water flow through sand", contains the results of his sand column experiments conducted with the set-up shown in Fig. 1.2. The motivation for the experiments was to investigate water flow through a sand filter. At that time, the water filtration method was a common practice to improve water clarity and, as a result, engineers were starting to think about the water flow behavior through filters (Simmons 2008). In the experiments, Darcy and his assistant applied the water on the top of the sand column (under the saturated condition), took measurements of discharge (flow) rate at the bottom, and at the same time monitored the water hydraulic heads at two points near the top and bottom (Fig. 1.2). Details of the experimental procedures and the observations are discussed in Freeze (1994), Brown (2002) and Simmons (2008). Using the obtained

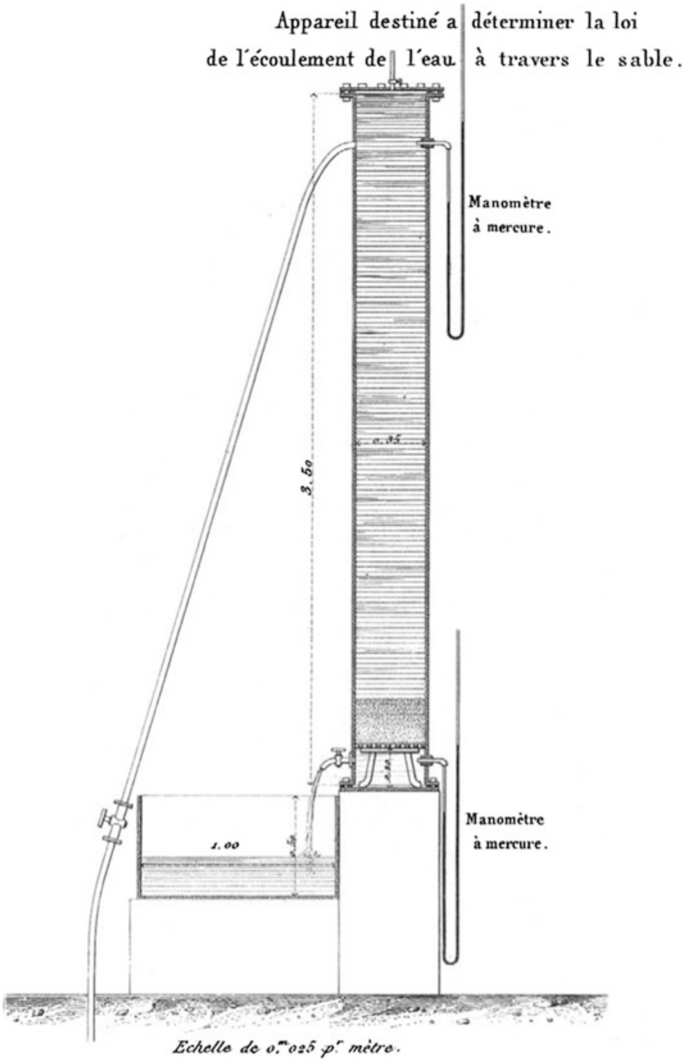

Fig. 1.2 Darcy sand column apparatus (Darcy 1856)

data, Darcy discovered an empirical relationship (Fig. 1.3), now called Darcy's Law, that the water flow rate is proportional to the cross-sectional area of the sand column and hydraulic gradient. This statement of Darcy's law is equivalent to the second sentence of this section.

Darcy passed away in 1858, two years after he published the fountain report. Although the Darcy's law was buried in the depth of the report, Simmons (2008) suggests that "Darcy understood that his discovery was new and significant". Supporting evidence includes the fact that Darcy dedicated almost half of the length

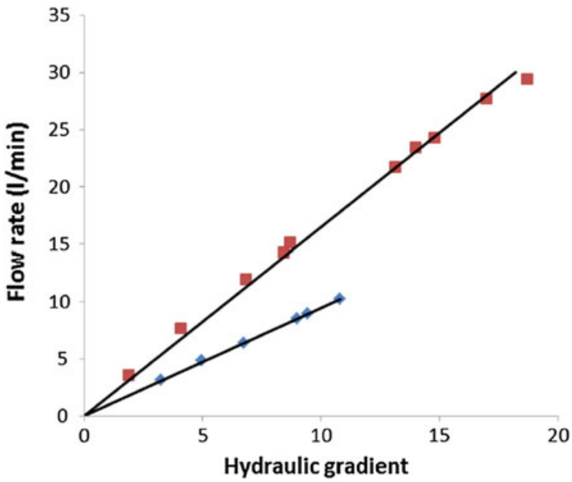

Fig. 1.3 Darcy's sand-column experimental results (the red and blue *squares*) (Darcy 1856). The *solid lines* are best linear fitting curves

of his preface to his report to a discussion on Darcy's Law. He also mentioned in the preface: "I have not seen the documents that are included in Note D collected in any special book. In particular, to my knowledge at least, no one has experimentally demonstrated the law of water flow through sand".

One may wonder why it was an "engineer" (Henry Darcy), not more theoretically oriented "scientists", who discovered such an important scientific (physical) law. It is a reasonable question to ask, because fluid mechanics and hydrodynamics theories had been well established at that time (Brown 2002). There might be many reasons for that. One speculation is that, when applying these theories, people often got stuck with details, such as the "messy" solid–water boundaries in a porous medium, and failed to see the key picture that pore spaces can be conceptually linked to the water flow in a capillary tube. Had they been able to, it would have been straightforward to deduce Darcy's law from the existing fluid mechanics theories, such as Poiseuille equation that gives pressure drop in a fluid flowing through a cylindrical pipe. This issue will be discussed further in Sect. 1.2. Thus, Darcy's success in discovering Darcy's law is largely because he was able to see the simplicity out of complexity. There are many famous quotes regarding simplicity and complexity in the literature. The quote that the author of this book likes the most is the following one from Steve Jobs, the late co-founder and CEO of Apple Inc. "That's been one of my mantras—focus and simplicity. Simple can be harder than complex; you have to work hard to get your thinking clean to make it simple." Although Jobs's statements are actually about technology invention and business, they can be equally applied to scientific discoveries as well.

While Darcy discovered Darcy's law using an empirical approach, he had the conceptual linkage in his mind and just used the test data to confirm his prediction (Simmons 2008). Darcy noted clearly in footnote 4 of Note D in his report, "I had already foreseen this curious result in my research on water flow in conduit pipes of very small diameters, …".

An interesting observation is that Darcy formally discovered his law at the age of 53. We are often told, especially by theoretical physicists, that one likely makes his or her most important scientific contributions at very young ages. Darcy's story suggests that it is not necessarily true at least for applied scientists or engineers who may have much longer career periods for significantly creative activities. This is really encouraging for many people in the engineering fields (including the author of this book) who are not very young now!

1.2 Relationship Between Water Flow Flux and Hydraulic Gradient in a Capillary Tube

The similarity of water flow in porous media to that in a capillary tube, first realized by Darcy (1856), is an important concept for studying flow and transport processes in porous media. Following this line, Dupuit (1857), according to Narasimhan (2005), further conceptualized the pore space in a porous medium as a collection of capillary tubes. This concept is critical for relating hydraulic properties to the pore size distribution for a porous medium and for extending Darcy's law to multiphase flow conditions (Chap. 2). A detailed derivation of a relationship between water flux and hydraulic gradient for a capillary tube with radius R (Fig. 1.4) is given

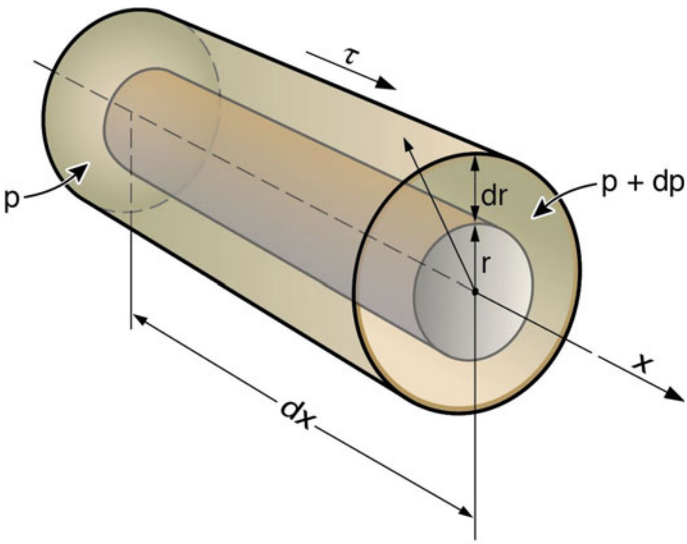

Fig. 1.4 A water element in a horizontal capillary tube with radius R (Liu et al. 2012) (reproduced by permission of Elsevier). The variable r is the radius of a water element within the capillary tube and ranges from zero to R

here; readers familiar with the subject may skip this section. For simplicity, we consider a horizontal capillary tube here, although this relationship could be easily extended to capillary tubes with other orientations.

For laminar water flow in the capillary tube, the shearing stress is given by

$$\tau = -\mu \frac{du}{dr} \tag{1.1}$$

where τ is shearing stress, μ is water viscosity, u is water velocity along the longitudinal direction of the capillary tube, and r is the radial coordinate. For a water surface with radius r and length dx (Fig. 1.4), the total shearing force (F) acting on the cylindrical surface is equal to the shearing stress multiplied by the corresponding surface area, or

$$F = \tau(2\pi r)dx \tag{1.2}$$

Then, the net shearing force acting on a water element (shown in Fig. 1.4) with thickness dr within the capillary tube, dF, is given by differentiating Eq. 1.2 with respect to r, or $dF = 2\pi(dx)d(r\tau)$. For laminar flow, the inertial effect can be ignored because of the low water velocity. In this case, the water element should be subject to zero net force, with the shearing force being balanced by an opposing force (resulting from the pressure difference imposed on the water element) that is $(dP)(dr)(2\pi r)$. Therefore, we have

$$dF = 2\pi(dx)d(r\tau) = (dP)(dr)(2\pi r) \tag{1.3}$$

Dividing Eq. 1.3 by $dxdr$ and using Eq. 1.1, we obtain

$$r\frac{dP}{dx} = \frac{d(r\tau)}{dr} = \frac{-d(r\mu\frac{du}{dr})}{dr} \tag{1.4}$$

Note that the water pressure here is assumed to be uniform along the r direction. This is a common and reasonable assumption used for studying fluid flow in a capillary tube with a radius much smaller than its length. The above equation can be solved for velocity gradient, $\frac{du}{dr}$, for a constant pressure gradient, $\frac{dP}{dx}$. Thus we have

$$\frac{r^2}{2}\frac{dP}{dx} + C = -r\mu\frac{du}{dr} \tag{1.5}$$

where C is a constant that is equal to zero as a result of the following boundary condition (or the symmetry condition):

$$\left.\frac{du}{dr}\right|_{r=0} = 0 \tag{1.6}$$

Then Eq. 1.5 can be further simplified as

$$-\frac{du}{dr} = \left(\frac{dP}{dx}\right)\left(\frac{r}{2\mu}\right) \tag{1.7}$$

Based on the non-slip conditions on the solid surface of the capillary tube ($u = 0$ at $r = R$), we can obtain the solution to Eq. 1.7 as

$$u(r) = \frac{dP}{dx}\int_R^r \frac{r}{2\mu}dr = \frac{1}{4\mu}\frac{dP}{dx}\left[R^2 - r^2\right] \tag{1.8}$$

The above equation gives the velocity distribution along the radial direction within the capillary tube. The average water flux across the cross section of the tube is then determined by

$$q_x = \frac{\int_0^R u(r)(dr)(2\pi r)}{\pi R^2} = -\frac{1}{\mu}\frac{R^2}{8}\frac{dP}{dx} = -\frac{\rho g}{\mu}\frac{R^2}{8}\frac{d(P/\rho g)}{dx} \tag{1.9}$$

where ρ is water density and considered to be a constant and g is gravitational acceleration. The above equation is in fact equivalent to the well-known Poiseuille equation. It was derived for a horizontal capillary tube. For an arbitrarily orientated straight capillary tube, the similar derivation procedure will give rise to the following relation:

$$q_x = -\frac{\rho g}{\mu}\frac{R^2}{8}\frac{dH}{dx} \tag{1.10}$$

where H is called hydraulic head and given by

$$H = z + \frac{P}{\rho g} \tag{1.11}$$

where z is elevation of the cross-section's center of capillary tube and positive in the upward direction.

If one, like Henry Darcy, had foreseen the (conceptual) similarity between water flow in porous media and capillary tubes, he, based on the above results, could have immediately recognized two important points. The first is, of course, Darcy's Law that water flux or flow rate is proportional to hydraulic gradient. The second is that the proportionality, called hydraulic conductivity, consists of two parts related to fluid properties and medium properties, respectively. To the best of our knowledge, the second point seems to be first made by M. King Hubbert (1903–1989) in his master piece regarding groundwater flow (Hubbert 1940). Thus, Eq. 1.10 can be written as

$$q_x = -\frac{\rho g}{\mu} k \frac{dH}{dx} \tag{1.12}$$

where k is called permeability that is determined by geometry of pore space or flow paths, like the radius of capillary tube (R) in Eq. 1.10, and thus an intrinsic property of a porous medium.

Any conceptualization or model is an approximation of the reality. It must be confirmed by experimental results. Darcy's sand column experiments were the necessary step for discovering Darcy's law. Nevertheless, subsurface problems are generally very complex in terms of medium geometry and the involved physical processes. In this case, it would not be possible, or practically necessary, to consider all the details related to the problems of interest. The use of a simplified concept to capture the essence of physics in a real system and leaving the unconsidered complexity to parameters that need to be determined by test data seem to be an effective way to develop practically useful theories or models. In Eq. 1.12, the unconsidered complexity by using a capillary tube to conceptualize pore space of a porous medium is included in permeability k that generally needs to be experimentally determined. Readers will find the similar approaches used for developing theories or models in other chapters as well in this book.

1.3 Generalized Darcy's Law for Water Flow in Low-Permeability Media

In his comments on Darcy's law, Freeze (1994) indicated that "it is fortuitous that God made life so simple, for if these relations (linear laws including Darcy's law) were nonlinear (if they had squared terms or cross products, for example), most of methods used by scientists to analyze the flow of water, heat, and electricity would be much messier." In the past two decades, significant progress has been made in understanding and applying nonlinearity sciences, including fractal geometry and chaotic systems. In fact, more and more scientists would agree on that God must be a nonlinearity expert who created so many beautiful things around us with nonlinearity, such as trees and flow-path patterns in river basins; most natural phenomena are actually associated with nonlinear processes, rather than linear processes. All the generalized physical laws, presented in this book, contain some nonlinear terms.

While significant attention has been historically given to flow processes in relatively high-permeability formations such as aquifers and conventional oil and gas reservoirs, fluid flow in low-permeability media (such as shale) becomes more and more important for a number of contemporary practical applications. For example, several countries have considered shale formations as potential host rocks for geological disposal of high-level radioactive nuclear wastes because shale has low permeability (on the order of 10–100 nano Darcy), low diffusion coefficient, high retention capacity for radionuclides, and capability to self-seal fractures (Tsang et al. 2012). Shale formations are also cap rocks for geological formations where supercritical CO_2 is stored for the purpose of CO_2 sequestration. Slow brine flow through the cap rock could be an important process that needs to be considered for managing pressure buildup owing to injection of CO_2 into storage formations below the cap rocks (Zhou et al. 2008). Nowadays, unconventional energy resources, including shale oil and shale gas, become an important part of recoverable hydrocarbon energy resources in the oil and gas industry. The recovery of these resources requires improved understanding of and modeling approaches for fluid flow within shale formations under different conditions.

The porous media used in Darcy's tests that led to the discovery of Darcy's law are sands, not shale or other low-permeability media. It has been well documented that Darcy's law is not adequate for the latter media. To provide some background for generalizing Darcy's law for the low-permeability media, this section first reviews the currently available relationships between water flux and hydraulic gradient and then develops an improved relationship that can capture water flow behavior in both high- and low-permeability media.

For one-dimensional flow systems, Darcy's law can be rewritten as

$$q_x = -Ki\mathrm{sgn}(\mathbf{i}_x) \tag{1.13}$$

where q_x is water flux with the magnitude of q, K is hydraulic conductivity and $\boldsymbol{i_x}$ is hydraulic gradient with a magnitude of i. The sgn($\boldsymbol{i_x}$) is the sign function of the hydraulic gradient and equals to 1, −1 and 0 for the positive, negative and zero gradients, respectively. The hydraulic conductivity is related to permeability, k, by

$$K = \frac{k\rho g}{\mu} \tag{1.14}$$

Non-Darcian flow is characterized by nonlinear relationship between water flux and hydraulic gradient (Liu et al. 2015). It is a result of strong solid–water interaction (e.g., Miller and Low 1963). The term "solid–liquid interaction" refers to combined effects of a variety of forces between molecules in solid and aqueous phases, including van der Waals forces. This interaction likely occurs in a thin fluid boundary layer close to solid surface where fluid properties are very different from those without the interaction. The thickness of the boundary layer is negligible in high-permeability media with relatively large pore sizes, but may be comparable to nanometer pore sizes in low-permeability media, such as shale formations. Thus, Darcy's law, based on the viscous laminar flow mechanism only, holds for high-permeability media, and is not adequate for fully describing liquid flow in low-permeability media.

The impact of solid–water interaction on water flow behavior has been demonstrated by a number of studies on nanoscale fluid flow based on molecular dynamics (MD) simulations (e.g., Chen et al. 2008; Ma et al. 2010; Farrow et al. 2011) and by experimental observations for water flow in microtubes (Xu et al.

2007). These studies generally show that water properties and flow processes at that scale could be significantly different from those in less fine-grained materials. For example, the density of water near the solid surface of grain material is generally much higher than its bulk value. Also, the resistance to water flow at the nanoscale is dominated by friction between fluid and solid surfaces, and less so by internal friction between fluid layers, because of the small pore sizes (Chen et al. 2008). Several MD simulation results show that the flow rate of water through a nano-tube is a nonlinear function of shearing stress (equivalent to hydraulic gradient for steady-state flow) (e.g., Chen et al. 2008; Ma et al. 2010; Farrow et al. 2011), which is consistent with non-Darcian behavior observed from laboratory measurements (e.g., Miller and Low 1963). Most recently, the MD simulation results of He et al. (2015) offer further insights, although their study is about gas transport in nanometer pores, rather than liquid. They simulated gas transport between two parallel carbon plates and within a carbon nanotube. The distance between the two plates (4.4 nm) is the same as nanotube diameter. They found that average gas velocity for the carbon-plate case is linearly related to the forces imposed to the gas molecules along the flow direction, which is consistent with Darcy's law at the macroscopic scale. That relationship becomes nonlinear for the nanotube case, an indication of non-Darcian flow behavior. This is because the latter is subject to a stronger field of intermolecular forces; the nanotube case corresponds to a larger ratio of solid surface area to the associated fluid volume.

Xu et al. (2007) also experimentally investigated the relationship between flux of deionized water and hydraulic gradient in individual microtubes with diameters ranging from 2 to 30 μm. They demonstrated that water flow in microtubes with diameters larger than 16 μm is consistent with Darcy's law, but not for smaller diameters. In the latter cases, the solid–water interaction is relatively strong and consequently the relationship between water flux and hydraulic gradient becomes nonlinear.

A number of researchers have proposed parametric formulations for describing the non-linear relationship between water flux and hydraulic gradient. Hansbo (1960, 2001) reported that water flux in a low-permeability clay soil could not be described by Darcy's law, but is proportional to a power function of the hydraulic gradient when the gradient is less than a critical value, whereupon the relationship between water flux and gradient becomes linear for larger gradient values. Consequently, Hansbo's (1960, 2001) proposed the following relationship:

$$q_x = -K' i^N \mathrm{sgn}(\mathbf{i}_x) \quad \text{for} \quad i \le i_1 \tag{1.15-1}$$

$$q_x = -K' N i_1^{N-1} (i - I) \mathrm{sgn}(\mathbf{i}_x) \quad \text{for} \quad i \ge i_1 \tag{1.15-2}$$

$$i_1 = \frac{IN}{(N-1)} \tag{1.15-3}$$

The formulation of Hansbo (1960, 2001) includes three parameters K' (m/s), N (-) and I (m/m). Note that K' herein is not the hydraulic conductivity because

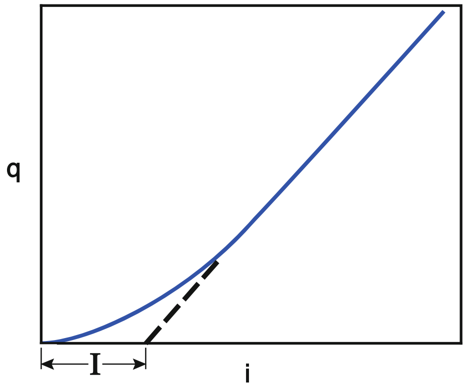

Fig. 1.5 Definition of threshold hydraulic gradient

Eq. 1.15-1 is not a linear function between water flux and hydraulic gradient i. Parameter I is called threshold gradient in this book and refers to the intersection between the i axis and the linear part of the relationship (Fig. 1.5). Hansbo (1960, 2001) explained the observed water-flow behavior by positing that a certain hydraulic gradient is required to overcome the maximum binding energy of mobile pore water. From their experiment results, Miller and Low (1963) also found the existence of a hydraulic gradient below which water is essentially immobile.

Hansbo (1960, 2001) demonstrated that Eq. 1.15 can fit related experimental observations and developed, based on Eq. 1.15, a theoretical approach to dealing with clay consolidation processes. As indicated by Swartzendruber (1961), however, Eq. 1.15 consists of two separate mathematical expressions and three related parameters and therefore is difficult to use in practice. To overcome this, Swartzendruber (1961), after analyzing several data sets for water flow in clay media, proposed a new version of the modified Darcy's law based on a relation for dq/di:

$$\frac{dq}{di} = K(1 - \mathrm{e}^{-i/I}) \tag{1.16}$$

For a large value of hydraulic gradient i, dq/di approaches a constant K that is hydraulic conductivity. Integrating Eq. 1.16 and using the condition that $q = 0$ at $i = 0$, Swartzendruber (1961) obtains

$$q = K[i - I(1 - \mathrm{e}^{-i/I})] \tag{1.17-1}$$

or

$$q_x = -K[i - I(1 - \mathrm{e}^{-i/I})]\mathrm{sgn}(\mathbf{i}_x) \tag{1.17-2}$$

There are two parameters K and I in Eq. 1.17. Compared with the common form of Darcy's law, it contains only one additional parameter (I). The equation of