

Dominique Raynaud

Studies on Binocular Vision

Optics, Vision and Perspective from the
Thirteenth to the Seventeenth Centuries

Studies on Binocular Vision

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Preface

The aim of this book is to elucidate the question of the interrelationship between optics, vision and perspective before the Classical Age. In the Middle Ages and the Renaissance, the concept of *Perspectiva*—the Latin word for optics—encompassed many areas of enquiry that had been viewed since antiquity as interconnected, but which afterwards were separated: optics was incorporated into the field of physics (i.e., physical and geometrical optics), vision came to be regarded as the sum of various psycho-physiological mechanisms involved in the way the eye operates (i.e., physiological optics and psychology of vision) and the word ‘perspective’ was reserved for the mathematical representation of the external world (i.e., linear perspective).

However, this division, which emerged as a result of the spread of the sciences in classical Europe, turns out to be an anachronism if we confront certain facts from the immediately preceding periods. It is thus essential to take into account the way medieval scholars posed the problem—which included all facets of the Latin word *perspectiva*—when exploring the events of this period. What we now recognize as a ‘nexus’ between optics and perspective was at the time in fact seen as a single science. I submit that the earliest developments in linear perspective cannot be elucidated without reinserting them into the web of ideas that originally constituted *perspectiva*.

The central focus of this book is the theory of binocular vision, which has been virtually ignored in the field of perspective studies. This theory generated one of the most puzzling alternatives to linear perspective in the history of representation—two-point perspective which could be regarded as a ‘heterodox system’ inasmuch as linear perspective is taken to be the norm. However, linear perspective was not at all the standard until the late sixteenth century (Cinquecento). Before then many other systems were used, such that one would be justified in asking whether it would not be better to admit that different, parallel systems of perspective existed as late as the Renaissance. Since the norm was still to come, it was common to find painters and architects testing new methods that lay at the margins of linear perspective. As a result, there is no way to demonstrate that painters and architects as a whole were applying the rules of perspective from Brunelleschi’s time onward. Up until the end of the Cinquecento the word ‘perspective’ referred to a series of free and

uncoordinated systems, with debates being conducted in scholarly and artistic circles on the merits of each.¹

In Chap. 1 we will seek to define more clearly the similarities and differences between perspective and *perspectiva*, i.e., medieval optics. One of the main differences was the gradual trend to decouple linear perspective from medieval optics, the course of which included an entire chapter on the formation of binocular images.

Errors—Chap. 2 investigates the emergence of perspective as a geometric science and seeks to separate what is fact from what is fiction regarding the birth of perspective in Quattrocento Italy. Events that were codified into what may be regarded as the mythology of perspective are discussed, including Brunelleschi's untraceable *tavoletta*, Alberti's *costruzione legittima*, and the perspective in Masaccio's fresco of the *Holy Trinity* in the Church of Santa Maria Novella in Florence. This chapter will show how access to knowledge could change practices; it establishes, for instance, that the solutions found by draftsmen to the problem of how to draw the perspective view of a circle varied, depending on their degree of familiarity with optics and geometry. Chapter 3 provides a classification of the types of errors that may arise in perspective constructions, deepening our understanding of the problem by presenting several examples of works that depart from the rules of perspective. Chapter 4 scrutinizes a blatant example of mistaken judgment regarding the correctness of one specific case of perspective—the interpretation by Erwin Panofsky of Masaccio's *Trinity*. Although celebrated as a milestone in the history of perspective, this fresco is not a correct example of central perspective due to the many errors—both random and systematic—that can be found in its geometric construction. These results undermine the commonly held idea that linear perspective became the unspoken rule in Brunelleschi's time, with all other alternatives being gradually abandoned. Linear perspective was neither clearly defined nor followed as a general rule in these early stages, and there was not yet a sufficient consensus to limit alternative representational systems.

Theory—Chap. 5 outlines the theory of binocular vision presented by Ibn al-Haytham in *Kitāb al-manāẓir* and discusses the innovations and limitations of this medieval Arab scholar's work in the light of modern physiological optics. Chapter 6 seeks to retrace the impact of Ibn al-Haytham's theory on Latin medieval optics. There is evidence that the study of key sections of *Kitāb al-manāẓir* and the commentaries written by European scholars ensured the wide dissemination of his theory of binocular vision. Chapter 7 focuses on certain contemporary documents

¹The present book includes revised content from several papers, mostly in French, published in academic journals. Chap. 1: *Nel Segno di Masaccio*, ed. F. Camerota, Firenze, 2001, pp. 11–13. Chap. 2: *Les Espaces de l'homme*, eds. A. Berthoz and R. Recht, Paris, 2005, pp. 333–354. Chap. 3: *L'Hypothèse d'Oxford*, Paris, pp. 62–85. Chap. 4: *Nuncius* 17 (2003): 331–344. Chap. 5: *Arabic Sciences and Philosophy* 13 (2003): 79–99. Chap. 8: *Oriens/Occidens* 5 (2004): 93–131. Chap. 9: *Sciences et Techniques en Perspective* 2-1 (1998): 3–23. Chap. 10: *Zeitschrift für Kunstgeschichte* 67/4 (2004): 449–460. Chap. 11: *Physis* 45 (2008): 29–55. Appendix A: *L'Œuvre et l'artiste à l'épreuve de la perspective*, eds. M. Dalai Emiliani et al., Rome, 2006, pp. 411–430. The other parts of the book are new.

that explicitly condemned the practice of ‘two-point perspective.’ These texts, which were written by members of the earliest Italian academies and of the *Académie Royale de Peinture* in France, inform us that the theory and practice of monocular vision continued to encounter strong resistance during the Renaissance and well into the classical period.

Sifting the Hypotheses—Applying standard techniques of error analysis, Chap. 8 and Appendix 1 address the methodological issue of how to eliminate or reduce the errors that may be introduced during the *ex post* reconstruction of a perspective view. An in-depth analysis is presented of *The Saint Enthroned*, a fresco by Giusto de’ Menabuoi that illustrates the use of two-point perspective. The same methodology is then applied to 30 works produced in Italy between the Duecento and the Cinquecento in which the use of two-point perspective has been identified. The error analysis is supplemented by a reconstruction of the geometric plans and elevations in these paintings, working backward from the perspective views. This analysis based on a large number of works allows us to eliminate a series of alternative forms of representation, and the sifting of the different representational systems proves that binocular vision might have provided the foundations for the construction of these medieval and Renaissance perspectives.

However, the hypothesis that early works of perspective were constructed on the basis of binocular vision can be accepted only if all the competing assumptions are successfully rebutted. We therefore carried out an evaluation, one by one, of the various theses that currently dominate discussions of the history of perspective. In Chap. 9 we demonstrate the inconsistency on both logical and empirical grounds of the Hauck–Panofsky conjecture regarding ‘curvilinear perspective.’ Similarly in Chap. 10 we disprove the White–Carter conjecture regarding ‘synthetic perspective’ by pointing out a mathematical property that renders this system unlikely. Chapter 11 examines Andrés de Mesa Gisbert’s conjecture that medieval perspective was the result of an arithmetic method of construction, a solution that, while elegant, poses some serious difficulties.

All the competing assumptions having been disproved, I conclude that binocular vision and two-point perspective constituted a genuine alternative to linear perspective from the late Duecento onward. In this way a strong interdependence between optics and perspective is established that accords with the original meaning of the word *perspectiva* and opens up the possibility for a better understanding of how perspectives were constructed in the early modern period. I submit that binocular vision represents a key juncture point between the history of art and the history of science.²

²From this perspective, the binocular system makes a genuine difference with the foreshortening rule, which could have been derived from Euclid’s *Optica*, postulate 5, as well as from practical geometry, in particular the “*Turris altitudinem metiri*” section included in many treatises. See for instance Stephen K. Victor, *Practical Geometry in the High Middle Ages*, Philadelphia, 1979; Hubert L.L. Busard, “The ‘*Practica geometriae*’ of Dominicus de Clavasio,” *Archive for the History of Exact Sciences* 2 (1965): 520–575; and Cosimo Bartoli’s *Del modo di misurare*, Venezia, 1564.

The intent of this book is to explore the various explanations and past modes of rationalizing the phenomenon of vision that can be derived from the matrix of *Perspectiva*, thus contributing to the rewriting of an important chapter in the history of optics and perspective from an angle that takes into account the criticisms that have been brought to bear on linear perspective in the past, and that is more sensitive to the precarious balance that characterizes the early stages in any process of innovation.

I express gratitude to Lisa C. Chien, who translated several chapters from the French and diligently revised the whole text.

Saint-Martin
June 2015

Dominique Raynaud

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Chapter 1

Perspectiva Naturalis/Artificialis

Abstract Perspective, as a system of visual representation, draws its name from the medieval Latin term *perspectiva* which means ‘optics.’ We owe this linguistic connection to the fact that certain principles of perspective developed from theories of vision. Between the two sets of notions one can find relationships of both continuity and discontinuity. A study of textual parallels has established this continuity. However, there are clear distinctions between *perspectiva* and perspective. Apart from the close relationship between science and technique that characterized them both, medieval *perspectiva* was a tripartite science embracing *optica*, *catoptrica* and *dioptrica*, whereas perspective would focus exclusively on direct vision; *perspectiva* postulated the binocular vision whereas linear perspective would adopt the conditions of monocular vision. These were the two main bifurcations that led to the development of *perspectiva artificialis*.

The system of representation that we call today “perspective” derives its name from *perspectiva*, the term used in the Middle Ages to designate the science of optics (ὀπτική in Greek and *al-manāẓir* in Arab). This connection can be explained by the fact that certain principles useful to painters and architects are based on geometrical optics, beginning with the law that objects appear to diminish in size as a function of distance:

Alhacen: Perception of size is due only to a correlation of the base of the visual cone encompassing the size to the angle of the cone at the center of sight and to the length of the cone, which represents the magnitude of the distance of the visible object.¹

¹“Comprehensio magnitudinis non est nisi ex comparatione basis pyramidis radialis continentis magnitudinem ad angulum pyramidis qui est apud centrum visus et longitudinem pyramidis, que est remotio magnitudinis rei vise,” *Opticæ Thesaurus Alhazeni Arabi libri septem*, New York, 1972, p. 58; A. Mark Smith, *Alhacen’s Theory of Visual Perception*, Philadelphia, 2001, vol. I, p. 185.

Bacon: There can be no determination of the magnitude of an object in accordance with the size of the angle, but it is necessary that the angle be considered and the length of the pyramid.²

Pecham: Perception of the size [of an object] derives from perception of the radiant pyramid and comparison of the base to the length and to the size of the angle.³

These should not be viewed as isolated observations. A number of studies⁴ conducted over the past two decades have established the debt that *perspectiva artificialis* owes to *perspectiva naturalis*. This chapter will discuss some of the connections and divergences between these two sets of ideas.

1.1 Perspective in the Classification of the Sciences

The medieval classification of the sciences⁵ can help us to understand the links that existed between perspective, geometry and arithmetic. According to the classification by al-Fārābī, which was transmitted to the Latin world through the translations of Gerard of Cremona and Dominicus Gundissalinus,⁶ *perspectiva* is three-fold, consisting of *optica* (direct rays), *catoptrica* (reflected rays), and *dioptrica* (refracted rays). Pictorial perspective is tied only to *optica*. The relations between the sciences were understood through the Aristotelian concept of

²“Non potest esse certificatio magnitudinis rei secundum quantitatem anguli, sed oportet quod consideretur angulus et longitudo pyramidis,” *The ‘Opus majus’ of Roger Bacon*, ed. A.G. Little, reprint, Frankfurt am Main, 1964, pp. 115–116.

³“Comprehensionem quantitatis ex comprehensione procedere pyramidis radiose et basis comparatione ad quantitatem anguli et longitudinem distantie,” David C. Lindberg, *John Pecham and the Science of Optics*, Madison, 1970, p. 146.

⁴For example, Emma Simi Varanelli, “Dal Maestro d’Isacco a Giotto. Contributo alla storia della ‘perspectiva communis’ medievale,” *Arte medievale* 2. Ser. 3 (1989): 115–143; Luca Baggio, “Sperimentazioni prospettiche e ricerche scientifiche a Padova nel secondo Trecento,” *Il Santo*, 34 (1994): 173–232; Francesca Cecchini, “Artisti, commitenti e prospettiva in Italia alla fine del Duecento,” in *La prospettiva. Fondamenti teorici ed esperienze figurative dall’Antichità al mondo moderno*, ed. R. Sinisgalli, Fiesole, 1998, pp. 56–74; *Eadem*, “Ambiti di diffusione del sapere ottico nel Duecento,” in *L’Œuvre et l’artiste à l’épreuve de la perspective*, eds. M. Dalai Emiliani, M. Cojannot Le Blanc, P. Dubourg Glatigny, Rome, 2006, pp. 19–42.

⁵James A. Weisheipl, “Classification of the sciences in medieval thought,” *Mediaeval Studies* 27 (1965): 54–90; *Idem*, “The nature, scope, and classification of the sciences,” ed. D.C. Lindberg, *Science in the Middle Ages*, Chicago, 1978, pp. 461–482; Graziella Federici Vescovini, “L’inserimento della ‘perspectiva’ tra le arti del quadrivio,” *Actes du IVe Congrès international de Philosophie médiévale*, Montréal/Paris, 1969, pp. 969–974.

⁶Henri Hugonnard Roche, “La classification des sciences de Gundissalinus et l’influence d’Avicenne,” in *Études sur Avicenne*, eds. J. Jolivet and R. Rashed, Paris, 1984, pp. 41–63; Jean Jolivet, “Classification des sciences,” in *Histoire des sciences arabes*, ed. R. Rashed, Paris, 1997, 3, pp. 255–270.

subalternation.⁷ There is subalternation when a superior science (*scientia subalternans*) provides the *propter quid* of a fact presented by an inferior science (*scientia subalternata*). Ever since Aristotle's *Posterior Analytics*, optics has been subordinate to geometry, which has led either to its outright absorption into geometry, as in Boethius' *De Trinitate*, or to its insertion among the geometrical sciences, as in Dominicus de Clavasio's *Questiones super perspectiva*.⁸

Many classification systems made a clear distinction between the theoretical and the practical sciences, as in Isidorus of Seville's *Etymologiae* or the *Didascalicon* by Hugh of St Victor. In contrast, Arabic scholars saw a continuous gradation from the speculative sciences to the practical sciences.⁹ Along the lines of al-Fārābī, Dominicus Gundissalinus named seven mathematical sciences as having both theoretical and practical aspects, including optics (*de aspectibus*), statics (*de ponderibus*), and engineering (*de ingeniis*). Drawing on this same tradition, Roger Bacon devoted an entire chapter of *Communia mathematica* to "Geometria speculative et practica"¹⁰ and Fra' Luca Pacioli expounded on the "parte principale de tutta l'opera de Geometria, in tutti li modi theorica e pratica."¹¹ Such connections explain why perspective was so heavily dependent on the geometrical sciences and why, although a practical art, it benefitted from the contributions of speculative geometry and *perspectiva naturalis*.

Last but not least, it must be mentioned that the mathematical sciences were divided based on their subject matter—arithmetic was the science of discrete quantities (πλῆθος) while geometry was the science of continuous quantities (μέγεθος). This dichotomy remained in place from the time of Aristotle, Proclus,

⁷Aristotle rejects the mixing of genres during the course of a demonstration but admits the subordination of the sciences under certain conditions; Aristotle, *Posterior Analytics*, ed. H. Tredennick, Cambridge, 1966, I, IX, 66–69 and I, XIII, 88–90. He recognized, for example, that optics was subordinate to geometry, I, XIII, 88–90. Later, metaphysical considerations sometimes contributed to emancipate optics from pure mathematics. On *subalternation scientiae* in the Middle Ages, see Steven J. Livesey, "Science and theology in the fourteenth century: the subalternate sciences in Oxford commentaries on the sentences," *Synthese* 83 (1990): 273–292.

⁸"It is known that the mathematical sciences are five—namely arithmetic, geometry, music, astronomy and perspective—which differ, as was seen in the first conclusion/*Est sciendum quod quinque sunt scientiae mathematicae, scilicet arismetrica, geometria, musica, astrologia et perspectiva quae differunt secundum quod visum in prima conclusione*," Dominicus de Clavasio, *Quaestiones perspectivae*, Florence, BNCF, San Marco, Conv. Soppr. J X 19, qu aest. 1, ff. 44r-v; Graziella Federici Vescovini, *Studi sulla prospettiva medievale*, Turin, 1964, p. 210.

⁹The inclusion of the practical sciences in the overall classification of the sciences seems to have begun with the ancient Greeks. Pappus reports that Heron's disciples divided mechanics into two parts: (i) the theoretical, which included geometry, arithmetic, astronomy, and physics, and (ii) the manual, which included architecture (οικοδομική), ironworks (χαλκευτική), carpentry (τεκτονική), and painting (ζωγραφική), Pappi Alexandrini *Collectionis quae supersunt*, ed. F. Hultsch, Berlin, 1876–8, pp. 1022.3–1028.3 (VIII, praef. 1–3).

¹⁰Roger Bacon, *Communia mathematica Fratris Rogeri*, ed. R. Steele, Oxford, 1940 (I, 3, 2).

¹¹Luca Pacioli, *Summa de aritmetica, geometria, proportionione et proportionalita*, Venice, 1494, fol. 75r.

and Geminus¹² up to the Italian Renaissance treatises that identified devices *per numero* and *per linea*.¹³ In the light of these categories we can better understand why optics as a geometrical science guided the earliest experiments on perspective.

1.2 The Phases in the Development of Optics

If one examines the literature on the history of the classification of the sciences,¹⁴ one finds that the boundaries of optics were particularly labile and the place it occupied on the tree of scientific knowledge was subject to marked fluctuations. The only scientific classification systems that even mention optics before the advent of modern science were those of Aristotle in the *Nicomachean Ethics* (ca. 340 BC), al-Fārābī in his work *Kitāb iḥṣā' al-'ulūm* (*Opusculum de scientiis*, ca. 950), and the English friar Robert Kilwardby in *De ortu scientiarum* (ca. 1250).

If one compares the chronology of optical treatises to these milestones, one immediately notes that the introduction of optics into the classification of the sciences coincided with those periods in which research in this area was most prolific. This correlation should not surprise us for it is when new knowledge emerges that the need arises to assign it a place reflecting its importance. The first period of intense activity was seen in antiquity, with the work of Euclid (ca. 300 BC), Hero of Alexandria (ca. 70 AD), Damianus (ca. 100), Ptolemy (ca. 127), and Theon of Alexandria (before 405).¹⁵ The second period corresponded to the study of optics in the Arab world, which it would perhaps be more accurate to refer to as the science of optics in the Arabic language, given the significant contributions of Greek, Nestorian and Persian savants who expressed themselves in this language. The best known texts are those of al-Kindī (ca. 846), Ḥunayn ibn Ishāq (ca. 857), Qusṭā ibn

¹²Bernard Vitrac in Euclide, *Éléments*, vol. 2, pp. 19, 22.

¹³Luca Pacioli, *Divina proportione*, Venice, 1509; Pietro Cataneo, *L'Architettura*, Venice, 1567; Andrea Palladio, *I Quattro Libri di architettura*, Venice, Domenico dei Franceschi, 1570. On the devices *per numero* and *per linea*, see Samuel Gessner, *Les Mathématiques dans les écrits d'architecture italiens, 1545–1570*, Paris, 2006, pp. 109–144.

¹⁴James A. Weisheipl, “Classification of the sciences in medieval thought,” *Mediaeval Studies* 27 (1965): 54–90; Graziella Federici Vescovini, “L’inserimento della ‘perspectiva’ tra le arti del quadrivio,” *Arts libéraux et philosophie au Moyen Âge*, Paris/Montréal, 1969, pp. 969–974; Jean Jolivet, “Classification des sciences” in *Histoire des sciences arabes*, eds. Roshdi Rashed and Régis Morelon, Paris, 1997, vol. 3, pp. 255–270.

¹⁵*Euclidis opera omnia*, vol. VII: *Optica... Catoptrica cum scholiis antiquis*, ed. J.L. Heiberg, Leipzig, 1895; Wilfred R. Theisen, “Liber de visu: The Greco-Latin Tradition of Euclid’s Optics,” *Mediaeval Studies* 41 (1979): 44–105; *Heronis Alexandrini opera quae supersunt omnia*, vol. II. *Mechanica et Catoptrica*, eds. L. Nix and W. Schmidt, Stuttgart, 1900; *Damianos Schrift über Optik*, ed. R. Schöne, Berlin, 1897; Albert Lejeune, *L’Optique de Claude Ptolémée dans la version latine d’après l’arabe de l’émir Eugène de Sicile*, Leiden, 1989; *Euclidis opera omnia*, vol. VII: *Opticorum recensio Theonis*, ed. J.L. Heiberg, Leipzig, 1985.

Lūqā (ca. 860), Aḥmad Ibn ‘Īsā (after 860), Ibn Sahl (ca. 985), and above all Ibn al-Haytham, known in the Latin world as Alhacen (d. after 1040).¹⁶

The third great period in the history of optics was that of the thirteenth century in Europe and the most significant contributions are associated with the names of Robert Grosseteste (ca. 1235), Roger Bacon (ca. 1266), Witelo (ca. 1277), John Pecham (ca. 1279), and their fourteenth-century epigones, including Egidius de Baisiu (ca. 1300), Dietrich de Freiberg (ca. 1304), Dominicus de Clavasio (before 1362) and Biagio Pelacani da Parma (ca. 1390).¹⁷

Issuing from this intense activity, *perspectiva* could already lay claim to being a synthesis of physical optics (treating such problems as the multiplication of species, the instantaneous versus the temporal propagation of light), geometric optics (the images reflected in mirrors, the source of the moon’s light, the theory of the rainbow), physiological optics (the anatomy of the eye, the phenomenon of the persistence of vision, the conflicting theories of intromission and extramission of visual rays), and psychological optics (used, for example, to explain optical illusions).¹⁸

1.3 The Similarities Between *Perspectiva* and Perspective

Before embarking on a discussion of the relationship between *perspectiva* and perspective, it should be pointed out that this relationship falls into the category of “a necessary but not sufficient condition.” Not sufficient because there were many determining factors in the emergence of perspective—not only the development of theories of vision, but also the support of medieval theologians for iconography and

¹⁶Elaheh Kheirandish, *The Arabic Version of Euclid’s Optics: Kitāb Uqlīdis fī ikhtilāf al-manāẓir*, New York, 1999; Roshdi Rashed, *Optique et mathématiques*, Aldershot, 1992; *idem*, *Géométrie et Dioptrique au Xe siècle: Ibn Sahl, al-Qūhī et Ibn al-Haytham*, Paris, 1993; *idem*, *Œuvres philosophiques et scientifiques d’al-Kindī: L’optique et la catoptrique*, Leiden, 1996; *idem*, *Geometry and Dioptrics in Classical Islam*, London, 2005; Abdelhamid I. Sabra, *The Optics of Ibn al-Haytham, Books I-III: On Direct Vision*, London, 1989; *idem*, *The Optics of Ibn al-Haytham, Books IV-V: On Reflection and Images Seen by Reflection*, Kuwait, 2002; A. Mark Smith, *Alhacen’s Theory of Visual Perception*, Philadelphia, 2001; *idem*, *Alhacen on the Principles of Reflection*, Philadelphia, 2006; *idem*, *Alhacen on Image-Formation and Distortion in Mirrors*, Philadelphia, 2008; *idem*, *Alhacen on Refraction*, Philadelphia, 2010.

¹⁷Ludwig Baur, “Die philosophischen Werke des Robert Grosseteste,” *Beiträge zur Geschichte der Philosophie des Mittelalters* 9 (1912): 1–778; David C. Lindberg, *Roger Bacon and the Origins of Perspectiva in the Middle Ages*, Oxford, 1996; Witelo, *Opticae Thesaurus*; David C. Lindberg, *John Pecham and the Science of Optics*, Madison, 1970; José-Luís Mancha, “Egidius of Baisiu’s theory of pinhole images,” *Archive for History of Exact Sciences*, 40 (1989): 1–35; Maria Rita Pagnoni-Sturlese, Rudolf Rehn and Loris Sturlese, *Dietrich von Freiberg. Opera Omnia, IV. Schriften zur Naturwissenschaft*, Hamburg, 1985; Graziella Federici Vescovini, “Les questions de ‘perspective’ de Dominicus de Clivaxo,” *Centaurus* 10 (1964): 236–246; Blaise de Parme, *Questiones super perspectiva communi*, eds. G. Federici Vescovini et al., Paris, 2009.

¹⁸Gérard Simon, *Le Regard, l’être et l’apparence dans l’optique de l’Antiquité*, Paris, 1988.

the use of imagery as a mnemonic technique,¹⁹ the desire for social mobility on the part of artisans (which motivated them to reduce the gulf between the mechanical arts and the liberal arts by demonstrating that their work was based on knowledge of the *quadrivium*),²⁰ etc.

The number of underlying factors can be greatly reduced if one focuses on the study of the textual parallels linking Renaissance treatises on perspective with the treatises on optics written in previous epochs. These parallels, which sometimes bordered on outright copying, can be found in the writings of Lorenzo Ghiberti,²¹ Leon Battista Alberti, Piero della Francesca, and Leonardo da Vinci. A passage typical of such undeclared borrowings can be found in the *Codex Atlanticus*, fol. 543r:

English translation: Light produces an impression in the eye that is directed toward it. This result is proved by an effect, for when the eye sees brilliant lights, it suffers and endures pain. Also, after a glance [at bright lights], images of intense brightness remain in the eye, and they cause a less illuminated place to appear dark until the traces of the brighter light have disappeared from the eye.

Leonardo: La luce operando nel uedere le chose contra se conuerse alquanto le spezie di quelli ritiene. Questa conclusione si pruoua per li effetti perche la uista in uedere luce alquanto teme. Ancora dopo lo sguardo rimangono nel locchio similitudine della chosa intensa e fanno parere tenebroso il luogo di minor luce per insino che dallochio sia spartito il uestigio de la impression de la magiore luce.²²

Pecham: Lucem operari in uisum supra se conuersum aliquid impressiue. Hec conclusio probatur per effectum, quoniam uisus in uidendo lucis fortes dolet et patitur. Lucis etiam intense simulacra in oculo remanent post aspectum, et locum minoris luminis faciunt apparere tenebrosus donec ab oculo euauerit uestigium luminis maioris.²³

This is the literal translation of a paragraph from John Pecham's *Perspectiva communis* on which, however, Leonardo does not elaborate. Research that I have devoted to these parallel texts allows me to formulate certain conclusions regarding the history of perspective.²⁴

¹⁹Alain Besançon, *L'Image interdite. Une histoire intellectuelle de l'iconoclasme*, Paris, 1994; Emma Simi Varanelli, "Arte della memotecnica e primato dell'immagine negli ordines studentes," *Bisancio e l'Occidente: arte, archeologia, storia*, Rome, 1996, pp. 505–525.

²⁰Robert E. Wolf, "La querelle des sept arts libéraux dans la Renaissance, la Contre-Renaissance et le Baroque," *Renaissance, Maniérisme, Baroque*, Paris, 1972, pp. 259–288.

²¹Klaus Bergdolt, *Der dritte Kommentar Lorenzo Ghibertis. Naturwissenschaften und Medizin in der Kunsttheorie der Frührenaissance*, Weinheim, 1998.

²²Leonardo da Vinci, *The Notebooks of Leonardo da Vinci*, ed. Jean Paul Richter, New York, Dover, 1970, vol. I, p. 24.

²³David C. Lindberg, *John Pecham and the Science of Optics*, p. 62.

²⁴For a detailed study of the textual parallels, see Raynaud, *L'Hypothèse d'Oxford*, Paris, 1998, pp. 163–209; *idem*, "L'ottica di al-Kindī e la sua eredità latina. Una valutazione critica," in *Lumen, Imago, Pictura*, Atti del convegno internazionale di studi (Rome, Bibliotheca Heriziana, 12–13 April 2010), eds. S. Ebert-Schifferer, P. Roccasecca and A. Thielemann, Rome (in press); *Idem*, "An unknown treatise on shadows referred to by Leonardo da Vinci," in *Perspective as Practice. An International Conference on the Circulation of Optical Knowledge in and Outside the Workshop*, eds. S. Dupré and J. Peiffer, Max Planck Institut für Wissenschaftsgeschichte (Berlin,

1. This research shows first of all that there was a marked continuity between the study of optics and the study of perspective, thus greatly reducing the credibility of the classic thesis that a major rupture took place during the Renaissance in Italy. This was doubtless true on certain levels, but curiously enough the treatises on perspective seemed to form an exception to the rule.
2. Another finding is that the treatises on optics most often cited during the Renaissance were not those of antiquity but texts from the Arab and Latin Middle Ages. This appears to be quite strange given the fact that the Renaissance has been characterized by scholars as the period of the “rediscovery of the antique.”
3. Among the medieval authors, the ones most frequently cited belong to the tradition of the ‘Perspectivists,’ principally Alhacen and his Western successors. But here again is another source of surprise: Witelo, who had close connections with the papal court in Viterbo, is rarely quoted, and Biagio Pelacani da Parma hardly more often.
4. In *L'Hypothèse d'Oxford. Essai sur les origines de la perspective*, I proposed that these anomalies could be understood by introducing a socio-historic factor. *The texts on optics that the perspectivists of the Renaissance consulted were likely to have been the ones that were most accessible in terms of the number of manuscript copies in circulation.* The hierarchy between these texts can be reconstructed from their distribution: in libraries across Europe a total of 65 manuscripts by Bacon and 64 by Pecham can be counted, compared to 25 by Witelo and 16 by Biagio Pelacani da Parma.²⁵ Thus, the frequency with which authors borrowed from Bacon and Pecham could be due to the exceptional diffusion of their texts during the course of the thirteenth and fourteenth centuries.

(Footnote 24 continued)

12–13 October 2012), Berlin (in press); *Idem*, “Application de la méthode des traceurs à l’étude des sources textuelles de la perspective. Auteurs, traités, manuscrits,” in *Vision and Image-Making: Constructing the Visible and Seeing as Understanding*, Actes du colloque international, Centre d’Études Supérieures de la Renaissance et Le Studium CNRS, Orléans (Tours, 13–15 September 2013).

²⁵David C. Lindberg, *A Catalogue of Medieval and Renaissance Optical Manuscripts*, Toronto, 1975. With regard to the invention of perspective, links have also been drawn to the abacus, the cartographic projections of Ptolemy, the use of the astrolabe, or a combination of all of these sources, Birgitte Bøggild-Johanssen and Marianne Marcussen, “A critical survey of the theoretical and practical origins of the Renaissance linear perspective,” *Acta ad Archaeologiam et Artium Historiam Pertinentia* 8 (1981): 191–227. This knowledge probably contributed to the development of the perspective system, but in the Quattrocento their influence remained secondary to that of optics: (1) if perspective had been based on cartography, contemporaries would probably have spoken of “the cartography of painters” rather than “the perspective of painters”; (2) the identification of certain sources appears to be conjectural because they are not supported by a study of parallel texts (Raynaud, *L'Hypothèse d'Oxford*, pp. 165–167); (3) the notion of a “source” depends on one’s point of view. Simply because knowledge appears to us on logical grounds to be ‘pertinent’ to a subject does not necessarily mean that it would have been utilized.

5. A study of the holdings in Italian libraries sheds light on the context in which these borrowings unfolded.²⁶ For example, a comparative analysis of Florentine inventories before the middle of the Quattrocento shows that there were no treatises on optics in the Badia Fiorentina or the Medici library, but that they could be found in convent libraries. While the Dominicans of Santa Maria Novella had no manuscripts on *perspectiva*, the Augustinian order of the Basilica of Santo Spirito possessed one (*Perspectiva magistri Vitellonis*) and the Franciscans of Santa Croce no less than six (Robert Grosseteste, *De luce seu inchoatione formarum*; Bartholomew of England, *De proprietatibus rerum*; John Pecham, *Tractatus de perspectiva* and *Perspectiva communis*; Bartholomeus de Bononia, *De luce*; and Petrus Aureolus, *Scriptum in II Sententiarum*).
6. The large number of treatises on optics to be found in the libraries of the Franciscan convents during the Middle Ages can be explained by the conjunction of two factors: (1) a homophilic bias, that is, the preference of a religious community for authors belonging to the same order (thus, Dominican authors were over-represented in the libraries of Dominican convents, Franciscan authors in the collections of Franciscan libraries, and so on),²⁷ and (2) the strong commitment of Franciscans to the writing and copying of manuscript treatises on *perspectiva*. A tally beginning with the *Catalogue of Optical Manuscripts* shows that among 310 manuscripts from the thirteenth and fourteenth centuries preserved in European libraries, 92 % were redacted by clerics and of these 80 % were written by friars belonging to the mendicant orders. A total of 71 % (220 MSS) were the work of Franciscan friars, of which 66 % (205 MSS) were by just three authors—Grosseteste, Bacon and Pecham.²⁸

The interest of the Franciscans in the subject of optics, joined to the principle of homophily, properly explains the presence of Franciscan ‘best-sellers’ in Italian libraries. In *Optics and the Rise of Perspective* I used this data to show that the diffusion of optics was one of the pre-conditions for the development of linear perspective during the Renaissance.

The purpose of this book is different. It will test the hypothesis that there were close ties between optics and perspective, but from a different angle; that is, by asking whether long-abandoned medieval notions of optics may have left traces in the way perspective was envisaged in later epochs. From such traces—if they do exist—it should be possible to furnish proof of how close the relationship was between *perspectiva* and perspective. Since my aim here is more to lay out and conduct a scientific test than a discourse on culture, I will begin by reviewing the most salient differences between *perspectiva* and perspective.

²⁶Raynaud, *L'Hypothèse d'Oxford*, pp. 301–349.

²⁷See the statistical tables in Raynaud, *L'Hypothèse d'Oxford*, p. 329.

²⁸Raynaud, *Optics and the Rise of Perspective*, Oxford, 2014, chapter 3, especially pp. 64–65.

1.4 The Differences Between *Perspectiva* and Perspective

The existence of correspondences between *perspectiva* and perspective does not negate the possibility that differences exist between medieval optics and Renaissance perspective. In addition to the fact that their relationship was one of theory to practice, or of science to technology, two other bifurcations marked the passage from one to the other.

First of all, *perspectiva* as it was understood and taught during the Middle Ages was a tripartite science that comprised the study of direct rays (*optica*), reflected rays (*catoptrica*), and refracted rays (*dioptrica*).²⁹ By comparison, Renaissance treatises on perspective covered a much narrower field of investigation, ignoring for example the study of burning mirrors and such natural phenomena as the rainbow, the halo of the moon, and the apparent twinkling of the stars. An entire facet of *perspectiva* thus disappeared as scholars concentrated on direct vision.

Secondly, all medieval treatises on *perspectiva* speculated at length on the central conundrum of binocular vision—how do the separate images received by the two eyes come to be fused?³⁰ And yet modern summaries, as well as the sources of the period, continually underline the close ties that link the invention of perspective and the postulate of monocular vision. These presuppositions have been laid out by most historians of perspective. At the beginning of the twentieth century, Erwin Panofsky observed that in order to construct a perspective it is necessary to grant, “First, that we see with a single and immobile eye.”³¹ Thirty years later Gioseffi declared in his turn that *monocular vision* was the condition that guaranteed the integrity of the system of perspective.³² In his account of the history of perspective, Laurent expounded on this point: “The two eyes of binocular vision are reduced to a single one (monocular vision) called the eye and placed at the summit of the visual cone.”³³ The historical sources are no less prolix. The postulate of monocular vision figures prominently in Manetti’s *Vita* of Brunelleschi, in which the *tavoletta* of the baptistery of San Giovanni in Florence is described: “It is necessary that the painter postulate beforehand a single point from which his painting should be viewed/Il dipintore bisognia che presuponga un luogo solo

²⁹“Otherwise, vision is fundamentally triple, depending upon whether it is made of straight, refracted or reflected rays/Aliter vero triplicatur visio secundum quod fit recte, fracte et reflexe,” *The ‘Opus majus’ of Roger Bacon*, ed. Little, p. 162.

³⁰Alhacen, *Opticae Thesaurus*, pp. 76–87; Smith, *Alhacen’s Theory of Visual Perception*, vol. II, p. 562–582; *The ‘Opus majus’ of Roger Bacon*, pp. 92–99; Lindberg, *John Pecham and the Science of Optics*, pp. 116–118; Witelo, *Opticae Thesaurus... Item Vitellonis Thuringopoloni libri decem*, pp. 98–108.

³¹Erwin Panofsky, “Die Perspektive als symbolische Form,” *Vorträge der Bibliothek Warburg* 4 (1924/5): 258–331, *Perspective as Symbolic Form*, New York, 1991, p. 29.

³²Decio Gioseffi, *Perspectiva artificialis*, Trieste, 1957, p. 8.

³³Roger Laurent, *La Place de J.-H. Lambert (1728–1777) dans l’histoire de la perspective*, Paris, 1987, p. 37.

d'onde s'a a uedere la sua dipintura.”³⁴ The same condition is formulated in the commentary to *De visu* by Grazia de' Castellani: “And you put a single eye at point C where there is a small hole/E tu ponj un solo occhio al punto.c. doue è uno picholo bucho.”³⁵ As the vanishing point is the orthogonal projection of the eye onto the picture plane, the monocular postulate imposes the uniqueness of the vanishing point in a central linear perspective.

In contrast, the theory of monocular vision was much less developed in medieval optics and, it seems, was always seen in relation to the size of an object. This was illustrated by the classic experiment of the hand and the wall, which is cited in turn by Alhacen, Pecham and Alberti:

Alhacen: For instance, if an observer looks at a wall that lies at a moderate distance from the eye, and if he accurately determines the distance and size of that wall, and if he accurately determines the magnitude of its breadth, then, if the observer places his hand in front of one of his eyes between the center of sight and the wall *and closes the other eye*, he will find that his hand will cover a considerable portion of that wall.³⁶

Pecham: If a *one eyed man* looks at a large wall and, after certifying its size, places his hand before his eye, the hand will appear under an angle equal to or larger than that under which the wall is seen; nevertheless, the hand will appear to him smaller than the wall because it is less distant.³⁷

Alberti: I say that the part of the rod that lies between C and B goes as many times into the distance that is between B and D, i.e., between *your eye* and the foot of the rod, as many times as the height of the tower goes into the distance that is between your eye and the foot of the tower.³⁸

³⁴Antonio di Tuccio Manetti, *The Life of Brunelleschi*, by Antonio di Tuccio Manetti/Vita di Filippo di Ser Brunelleschi, eds. H. Saalman and C. Engass, University Park, 1970, p. 43.

³⁵Gino Arrighi, “Un estratto dal ‘De visu’ di M° Grazia de’ Castellani,” *Atti della Fondazione Giorgio Ronchi* 22 (1967): 44–58, p. 47; Filippo Camerota, “Misurare ‘per prospettiva’,” *La prospettiva. Fondamenti teorici ed esperienze figurative dall’Antichità al mondo moderno*, Fiesole, 1998, pp. 293–308.

³⁶“Verbi gratia, quod quando visus aspexerit parietem remotum a visu remotione mediocri, et certificaverit visus remotionem illius parietis et quantitatem eius, et certificaverit quantitatem latitudinis eius, deinde apposuerit aspiciens manum uni visui inter visum et parietem *et clauserit alterum oculum*, inveniet tunc quod manus eius cooperiet portionem magnam illius parietis,” Alhacen, *Opticae Thesaurus*, p. 52; Smith, *Alhacen’s Theory of Visual Perception*, vol. I, p. 171.

³⁷“Si *monoculus* aspiciat aliquem parietem magnum et quantitatem eius certificet deinde oculo suo manum anteponat, ipsa manus uidebitur sub eodem angulo uel sub maiori quam paries uisus est, nec tamen tanta ei apparebit quantus paries apparet quia minus distat,” Lindberg, *John Pecham and the Science of Optics*, p. 146.

³⁸“Dico che la parte del dardo quale sta fra C et B entra tante volte nella distanza quale sta fra B e D cioè fra l’occhio vostro e il piè del dardo, quante volte l’altezza della torre entra nella distanza quale è fra l’occhio vostro et il piè della torre,” Alberti, *Ex ludis rerum mathematicarum*, Cambridge, Mass., MS. Houghton Typ 422.2, fol. 1v.

Reducing the field of investigation to the study of direct vision (*optica*) and substituting the postulate of binocular vision for that of monocular vision would appear to be the two principal bifurcations—a consequence of the compartmentalization of the sciences—that set the seal on the continued evolution of *perspectiva artificialis*.

But there are just as many questions to be posed regarding the origins of the new theory of perspective, because the differences between *perspectiva* and perspective could have resulted from a lack of knowledge of the texts on optics, or a rejection of theoretical optics in favor of other sources such as the use of the astrolabe or practical geometry, or even the draconian selection from the textual sources of only those elements that were compatible with the development of linear perspective. The method used by Brunelleschi to depict the *tavoletta* of the baptistery could be viewed as part of a historical continuum, a logical consequence of the dearth of sources on monocular vision available during the Middle Ages. It could equally well be seen as an application of practical geometry to the measurement of inaccessible sizes,³⁹ thus favoring the discontinuity thesis. How might this issue be resolved?

The path that I will follow in this book differs from the one adopted earlier in *L'Hypothèse d'Oxford* and in *Optics and the Rise of Perspective*. If the origins of perspective are to be found *preponderantly* in medieval optics, then one should be able to identify some of its vestiges in the earliest experiments on perspective, which were conducted between the end of the Duecento and the second half of the Cinquecento. The uniformity of the procedure for creating perspective views was a consequence of its being taught as a regular part of the curriculum in the academies, beginning with the Accademia del Disegno (established in Florence in 1563) and the Accademia di San Luca (founded in Rome in 1577).⁴⁰ Before this time neither the concepts of perspective nor its methods were fixed and perspectivists, not being constrained to follow a definite set of rules, came up with a number of approaches that would all be regarded as “heterodox systems” once the rules for the representation of perspective were fixed and adopted. The period from the Duecento to

³⁹This exercise was included by many authors in their treatises on geometry, from Euclid to Johannes of Muris, and from Dominicus of Clavasio to Cosimo Bartoli; Euclid, *Liber de visu*, ed. W. Theisen, p. 72; Stephen K. Victor, *Practical Geometry in the High Middle Ages*, Philadelphia, 1979, p. 295; Hubert L.L. Busard, *Johannes de Muris. De Arte mensurandi*, Stuttgart, 1998, p. 145; *idem*, “The Practica Geometriae of Dominicus de Clavasio,” *Archive for the History of Exact Sciences* 2 (1965): 520–575, p. 539; Cosimo Bartoli, *Del modo di misurare le distantie, le superfitie, i corpi*, Venezia, 1564, fol. 19v, 24r.

⁴⁰Marica Marzinotto, “Filippo Gagliardi e la didattica della prospettiva nell'accademia di San Luca a Roma, tra XVII e XVIII secolo,” *L'Œuvre et l'artiste à l'épreuve de la perspective*, Rome, 2006, pp. 153–177.

the Cinquecento therefore offers an ideal field of investigation to explore whether the medieval principles of optics inspired systems of representation other than linear perspective.

We have characterized the passage from *perspectiva* to perspective in terms of two bifurcations: (i) the reduction of tripartite *perspectiva* (*optica*, *catoptrica*, *dioptrica*) to direct vision alone, and (ii) the adoption of the postulate of monocular vision. As *catoptrica* and *dioptrica* do not seem to have left any mark on the new system of perspective, the central axis of this book will consist in exploring whether the postulate of *binocular vision* could have inspired the many and varied systems of representation that were conceived beginning in the Duecento.

Part I

Errors

Chapter 2

Knowledge and Beliefs Regarding Linear Perspective

Abstract The aim of this chapter is to deconstruct the notion that linear perspective formed a stable system of representation beginning in the Quattrocento. Doubts must be raised because the history of perspective is in fact quite conjectural due to the many lacunae scattered along its path; one crucial example is the exact nature of the contributions of Brunelleschi, Alberti, and Masaccio. A second obstacle is the fact that a multiplicity of approaches were in use from the end of the Duecento to the Cinquecento, when the academies formally introduced the teaching of perspective techniques. Between these two time points perspectivists explored numerous systems of perspective, introducing errors and variations that can be explained by the uneven distribution of knowledge regarding the laws of optics and geometry.

The challenge facing the practitioner in representing space may be summed up as follows: how can one apprehend and capture the three-dimensionality of a solid in the two dimensions of a plane? Among the strategies commonly used, some consist in decomposing the object into a series of partial views—the horizontal plane, elevation, and profile—from which one can, with a little practice, mentally reconstruct the spatiality of the object. Other strategies instead attempt to provide a visual synthesis that is capable of immediately evoking the three-dimensionality of the solid. Parallel axonometric projections (isometric, dimetric, trimetric) and oblique projections (cavalier and military), both of which conserve the parallelism of an object's straight lines, fall into this category. Perspective itself rejects the property of parallelism for the principle of a gradual reduction in size, reproducing as closely as possible the conditions of natural vision: i.e., two straight lines that are not confined to the frontal plane converge toward a vanishing point. Linear perspective is just one of the systems that respects this principle (since it holds true for the curvilinear and synthetic perspectives as well), but it is the version that is generally considered in discussions of perspective *tout court*. I will conform to this usage by discussing only the case of linear perspective here.

The argument that can be advanced is that the characterization of perspective space as a unitary, coherent and stable representation is not sufficient because it fails to take into account the wide range of practices that are known to have existed.

Linear perspective constitutes an *open* rather than a closed *system*, one that reflected the mobilization over time of specific intellectual resources.

In the first part of this chapter, it will be shown that the work of Italian craftsmen at the beginning of the Quattrocento did not lead to a codified and homogeneous set of perspective practices (illustrating, in sociological terms, the effects of belief). In the second part it will be shown that the diversity of perspective conceptions in circulation can be explained differences in the optical-geometric resources available to the perspectivists (the effects of knowledge).

2.1 The Myth of Perspective

To begin, it will be useful to examine the supposedly stable nature of the perspective system. It is true that one finds, from Euclid¹ to Gibson,² unvarying expressions of the law of diminution in size as a function of distance. But the solidity of this principle has sometimes served as a pretext to impose the uniqueness of the perspective system and to reify it, particularly as far as the Renaissance is concerned, when in fact research on perspective often took the form of disparate and uncoordinated initiatives. Let us examine the contributions of Brunelleschi, Alberti and Masaccio, to whom have been attributed the invention, codification, and first major realization of the concept of perspective, respectively.

2.1.1 *Filippo Brunelleschi*

Filippo Brunelleschi (1377–1446) is usually credited with having realized the first rigorous work of perspective, in Florence around the year 1413. The documentation is scarce, but the artist apparently conducted an ingenious demonstration of the accuracy of his construction. He stood at a distance of three *braccia* (arm's lengths) from the main portal of the cathedral of Santa Maria del Fiore facing the Baptistery of San Giovanni, holding a mirror in one hand and a panel painting of the octagonal-shaped building in the other in such a way that he could observe, through a small hole pierced in the panel, the image of the painting reflected in the mirror. From his position he could see at the same time the image and the actual building, and thus judge the accuracy of his perspective drawing. The first difficulty regarding this experiment is that no material trace of it has survived. In particular, the

¹“Objects of equal size unequally distant appear unequal and the one lying nearer to the eye always appears larger/Aequales magnitudines inaequaliter expositae inaequales apparent et maior semper ea quae propius oculum adjacet,” *Optica*, ed. J.L. Heiberg, Leipzig, 1895.

²James J. Gibson, *The Ecological Approach to Visual Perception*, Boston, 1979.

tavoletta (panel painting) has been lost and there is no way of knowing what perspective method was used by the architect.

The only description that has come down to us is a second-hand account attributed to Antonio di Tuccio Manetti, who was born a decade after the experiment took place. What is more, his account does not grant Brunelleschi's display the status that is generally ascribed to it of an experiment in optics. In fact, Manetti never employs the word "experiment" although the term is amply attested to both in medieval Latin and in the Italian vernacular.³ He does couch his description in very concrete terms: "[Brunelleschi] put into practice" (*misse inatto*), "he displayed a panel" (*mostro una tavoletta*), "he made a painting" (*fecie una pittura*) ... but from this one cannot strictly speaking infer either an experiment of a public nature conducted before eyewitnesses, nor the existence of an experimental set-up of any kind. Hence, there is no concrete proof that Brunelleschi carried out a demonstration in the doorway of the cathedral of Florence. What Manetti's biographical account does offer is a fairly detailed description of his painting of the baptistery.

Let us identify the crucial points relating to perspective in this account, which are conditions A (the vantage point of the viewer), B (the scene depicted) and C (the size of the eyehole). These three conditions as described by Manetti in fact contradict one another. It is a simple matter to calculate the theoretical field of vision based on conditions A and B: the point of view chosen for the viewer ("some three *braccia* inside the central portal of Santa Maria del Fiore/*dentro alla porta del mezzo di Santa Maria del Fiore qualche braccia tre*") and the painted scene ("up to the arch and the corner of the sheep [market] ... up to the corner of the straw [market]/*insino all uolta e canto de Pecori ... insino al canto alla Paglia*") dictate a theoretical field of vision of 54°. The actual field of vision can be calculated from condition C: Manetti stated that the diameter of the eyehole at the end facing the observer was 5 mm ("a lentil bean/*una lenta*"), widening to 30 mm at its posterior end ("a ducat, or a bit more/*uno ducato o poco piu*"). For the eyehole to form a truncated cone ("it widened conically like a straw hat/*si rallargaua piramidalmente come fa uno capello di paglia*"), the minimal thickness of the panel must have been about 15 mm. In this case the actual field of vision based on the distance of the crystalline lens from the anterior end of the eyehole⁴ would have been between 13°

³One finds numerous references in the Latin and Italian translations of Ibn al-Haytham's *Kitāb al-manāẓir* (Alhacen's *De aspectibus/De li aspecti*). The terms that are attested to in Arabic, Latin and Italian are: *i'tibār* > *experientia-experimentatio* > *sperimento-sperimentatione*; *i'tabara* > *experimentare* > *sperimentare*; *mu'tabir* > *experimentator* > *sperimentatore*; cf. Abdelhamid I. Sabra, "The Astronomical Origin of Ibn al-Haytham's Concept of Experiment," *Actes du XIIIe Congrès International d'Histoire des Sciences*, Paris, 1971, tome IIIA, pp. 133–136.

⁴(A) "In order to paint it, it seems that he stationed himself some three *braccia* inside the central portal of Santa Maria del Fiore/E pare che sia stato a ritrarlo dentro alla porta del mezo di Santa Marie del Fiore qualche *braccia tre*..." (B) "In the foreground he painted that part of the piazza encompassed by the eye, that is to say, from the side facing the Misericordia up to the arch and corner of the sheep [market], and from the side with the column of the miracle of St. Zenobius up to the corner of the straw [market]/Figurandoui dinanzi quella parte della piazza che ricieue l'occhio cosi uerso lo lato dirinpetto alla Misericordia insino alla uolta e canto de Pecorj cosi da lo lato della

and 19° , i.e. only one-fourth to one-third of the expected theoretical value. When Brunelleschi's "experiment" was reproduced in situ in April 1995, it was found that conditions A, B, and C were in fact mutually exclusive. The field of vision carves out a square measuring 7–8 m on each side corresponding precisely to the door of the Baptistery. Since all the lines lie in the frontal plane containing the façade, this is not a perspective image.⁵ The results of a second experiment conducted in May 2001 as part of the 4th *ILabHS* were no more convincing as a demonstration of perspective.⁶ Despite the many positive analyses of this episode that continue to appear, all serious attempts to reconstruct Brunelleschi's experiment have failed and for one simple reason: it is *physically impossible* to reproduce the tableau based on the conditions described by Manetti.

If one adds to this the fact that the only work of perspective extant that can be attributed with any probability to Filippo Brunelleschi—an engraving on a silver plaque of *Christ Casting Out a Demon* (Louvre)—does not follow the rules of linear perspective,⁷ one is forced to conclude that Brunelleschi's contribution has been considerably overestimated. The doubts raised here do not concern his involvement in the development of perspective, which is incontestable, but the exact nature of this contribution, about which we know nothing. In truth only three pieces of evidence exist on the role played by the artist.

The first is a letter written by Domenico da Prato to Alessandro Rondinelli on 10 August 1413, in which Filippo Brunelleschi is described as "an ingenious man on perspective/*prespettiu ingegnoso uomo*," but this reference could simply attest to the fact that the architect took a general interest in the subject of optics (*perspectiva* in Latin); rigorously speaking it certainly does not allow a *terminus ante quem* to be fixed for the invention of perspective.

(Footnote 4 continued)

colonna del miracolo di Santo Zanobi insino al canto alla Paglia..., (C) "The hole was as tiny as a lentil bean on the painted side and it widened conically like a woman's straw hat to about the circumference of a ducat, or a bit more, on the reverse side/El quale buco era piccolo quanto una lenta da lo lato della dipintura et da rouescio si rallargaua piramidalmente come fa uno cappello di paglia da donna quanto sarebbe el tondo d'uno ducato o poco piu....," Antonio di Tuccio Manetti, *Vita di Filippo di Ser Brunelleschi*, eds. H. Saalman and C. Engass, University Park, 1970, p. 43ff. The first reassessment of this account was made by Martin Kemp, "Science, non-science and nonsense: The interpretation of Brunelleschi's perspective," *Art History* 1 (1978): 134–161.

⁵The field of vision is fixed by the distance between the centre of the crystalline lens and the anterior opening of the eyehole, that is, $a_0 \approx 15$ mm in the case of an exophthalmic eye and $a_1 \approx 22.2$ mm in the case of a normal eye. This information allows us to calculate $\alpha = \arctan(d/a)$: $13^\circ 05' < \alpha < 18^\circ 54'$; see Raynaud, *L'Hypothèse d'Oxford*, pp. 132–150.

⁶Filippo Camerota, "Brunelleschi's panels," *The 4th International Laboratory for the History of Science*, Florence, 25 May 2001 and personal communication; *Idem*, "L'esperienza di Brunelleschi," *Nel segno di Masaccio*, Florence, 2001, pp. 32–33: "Ma date le dimensioni, non consentiva di vedere tutto il dipinto, bensì solo una porzione piuttosto limitata della facciata del Battistero" [that is, nothing else than the door].

⁷Raynaud, *L'Hypothèse d'Oxford*, pp. 73–75.

Secondly, around 1461 Filarete wrote in his treatise on architecture, “I *believe* this is the way that Pippo di Ser Brunellesco found this perspective, which had not been used before,”⁸ a declaration that must be taken for what it is worth, as a statement of belief rather than an assertion of fact.

Finally, around 1480 Manetti asserted that: “[Brunelleschi] himself put into practice what painters today call perspective, because it is part of that science [i.e. optics],”⁹ but this claim was based on an inappropriate interpolation of the text, and he makes no mention of an “inaugural experiment” nor does he provide a method that would permit the reconstruction of his perspective.

None of these references can be regarded as unambiguous and beyond them, the rest remains conjecture. It is necessary therefore to retain a more nuanced picture of the contribution of Brunelleschi; his role in the development of perspective is in fact quite obscure.

2.1.1.1 Leon Battista Alberti

In *De pictura*, Leon Battista Alberti (1404–1472) sets out what is generally recognized to be the first codified procedure for the representation of perspective. Even today his method is often qualified as *costruzione legittima*, a term that gained wide currency thanks to Erwin Panofsky, who wrote: “Trecento pictures after the Lorenzetti became, so to speak, progressively more false, until around 1420, when *costruzione legittima* was (as we may well say) invented.”¹⁰ The expression is replete with meaning, because it implies the existence of a law for the representation of space that is universally true. As a consequence, it imposes the notion of a unified vision of perspective that formed at the beginning of the Quattrocento and still holds today. And yet any law, to be legitimate, must meet two conditions: it has to be based on a rational order, and it must be applied. Let us examine these two points.

With regard to the foundations of the rule of perspective, on re-reading *De pictura* it becomes clear that Alberti’s only intention in this text is to describe a series of *empirical* operations. He makes no attempt to justify these operations, either in terms of their correspondence to reality (perspective as the tracing of a visual experience) or their logical consistency (perspective as a system whose validity could be demonstrated).¹¹ The approach adopted by Alberti was strictly

⁸“Credo che Pippo di Ser Brunellesco trovasse questa prospettiva, la quale per altri tempi non s’era usata,” Antonio Averlino detto Il Filarete, *Trattato di architettura*, eds. A.M. Grassi and L. Finoli, Milano, 1972, p. 653.

⁹“Misse innatto luj propio quello che dipintorj oggi dicono prospettiva perche ella e una parte di quella scienza...,” Manetti, *Vita*, p. 43.

¹⁰Erwin Panofsky, “Die Perspektive als symbolische Form,” *Vorträge der Bibliothek Warburg* 4 (1924/5): 258–331, *Perspective as Symbolic*, New York, 1991, p. 62.

¹¹This question would not be raised in studies on perspective until much later. In 1585 Giovanni Battista Benedetti demonstrated that Alberti’s construction was correct; Judith V. Field, “Giovanni