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Kathleen Clark

Jost Bürgi's Aritmetische und Geometrische Progreß Tabulen (1620)

Edition and Commentary

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Preface

The primary aim for writing this book was simple: to provide an edition and English translation of Jost Bürgi's *Aritmetische vnd Geometrische Progreß Tabulen/sambt gründlichem unterricht/wie solche nützlich in allerley Rechnungen zu gebrauchen/ vnd verstanden werden sol*[1] (1620). To clarify, when I refer to the *Aritmetische und Geometrische Progreß Tabulen* (the abbreviated title will be used hereafter), I mean the manuscript that contains both Bürgi's tables, which were printed with title page, and 23 pages of handwritten text (a 2-page foreword and 21 pages of "instruction" for how to use the tables). There are precious few copies of the *Aritmetische und Geometrische Progreß Tabulen* (and even fewer that contain the handwritten foreword and "instruction"), and the copy that was used to write this book is held in the Department of Special Collections of the Library of the Karl-Franzens-University Graz, in Graz, Austria.

This book is organized into the following chapters. Chapter 1 contains biographical and contextual content to familiarize readers for whom Bürgi is relatively unknown. Several biographies of Bürgi exist, which range from quite brief (e.g., the entry by Nový that appears in the *Dictionary of Scientific Biography*) to book length (e.g., Staudacher's recent book (in its second edition, with a third edition planned), in German and published in 2014). In Chapter 1, I provide enough detail about Bürgi's life and mathematical contributions in order to introduce the reader to a broader story than is typically provided in survey of history of mathematics textbooks. Thus, a secondary aim of this book is to offer readers the opportunity to examine Bürgi's role in the development of what John Wallis identified as one of "two developments that had greatly eased the labour of calculation" (Wallis 1685, pp. 22–23)[2] and to highlight an accurate telling of Bürgi's mathematical prowess that has not previously appeared in English.

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[1] *Arithmetic and Geometric Progression Tables/together with detailed instruction/how to use these in all sorts of useful calculations/and how they should be understood.*

[2] Wallis identified the two developments as the introduction of decimal fractions by Simon Stevin in 1585 and the invention of logarithms by John Napier in 1614 (Stedall 2008, p. 34).

By way of "full disclosure"—and with his permission—I have heavily drawn upon Fritz Staudacher's lovely book, *Jost Bürgi, Kepler und der Kaiser* (2014), for the purpose of providing a fluid timeline of Bürgi's life. Also, I chose to rely more on Staudacher's text than that of Ludwig Oechslin (*Jost Bürgi*, 2001; also only in German), since Oechslin concentrated more on Bürgi's mechanics, astronomy, and horology.

Chapter 2 provides brief descriptions for the known copies of the *Aritmetische und Geometrische Progreß Tabulen*, e.g., those that are printed (tables only) and those that include the "Kurzer Bericht" (printed tables and handwritten instructions), as well as a detailed description of the copy that is the focus of this book and which is located in the Department of Special Collections of the Library of the Karl-Franzens-University Graz, in Graz, Austria.

Chapter 3 begins with an orientation to the chapter and a few comments for reading the transcription and translation. Then, the complete facsimile of Bürgi's *Aritmetische und Geometrische Progreß Tabulen* (i.e., its title page and the text of the foreword and instruction for use of the tables) is given.[3] This facsimile is also available for download from www.springer.com/us/book/9781493931606. Next, I provide a corresponding transcription, as it was written, in order to preserve the original text (including errors and idiosyncrasies), as well as Bürgi's tone and style. Alongside this transcription, I also include a transcription of the Gdańsk (Poland)[4] manuscript, which is the copy used by Hermann Gieswald in his 1856 edition, so that readers may conveniently and closely examine the subtle and not-so-subtle differences between the two manuscripts. Finally, the translation and commentary is divided into seven subsections, according to the purpose of the text and the type of examples discussed. Heinz Theo Lutstorf published a similar work in 2005 (in German, with no accompanying English translation), in which he analyzed the copy of Bürgi's *Aritmetische und Geometrische Progreß Tabulen* that is held in Gdańsk, Poland. When appropriate, I have included references to Lutstorf's commentary to emphasize important points.

Chapter 4 summarizes my perspective on two questions that have been asked numerous times: Who is the copyist of the Graz manuscript of the *Aritmetische und Geometrische Progreß Tabulen*? And, what is the relationship between the Graz and Gdańsk manuscripts?

Although I have received much assistance from very competent writers, mathematics historians, and scholars while working on this project, I am not a traditionally trained historian. Consequently, if you have found your way to this book, I ask that you read it with the two stated aims in mind, as opposed to imposing a critical edition structure on what follows. Finally, I hope that this book provides an important addition to the known scholarship on Jost Bürgi.

Tallahassee, FL Kathleen Clark

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[3] However, the facsimile of the 58 pages of tables is given in Appendix C and can be downloaded from www.springer.com/us/book/9781493931606.

[4] Formerly Danzig, Prussia/Germany.

Acknowledgments

My interest in the history of logarithms dates back to my dissertation research that involved working with high school teachers on ways to teach students about logarithms and logarithmic functions using a historical perspective. I was greatly influenced by the writing of historians of mathematics dedicated to exploring the role of history of mathematics in teaching, particularly the work of the late John Fauvel. In his introduction to *Revisiting the History of Logarithms*, Fauvel (1995) quoted and shared the following:

> *My father was* l'ingegné *(the engineer), with his pockets always bulging with books and known to all the pork butchers because he checked with his logarithmic ruler the multiplication for the prosciutto purchase.* [Primo Levi, 12, p. 19]
>
> The subject of logarithms, like the notorious "asses' bridge" in Euclid (*Elements* I,5) for an earlier generation, seems to mark an intellectual rite of passage: before going over there is a sense of unfathomable mystery, even danger, ahead; afterwards there is still some wonder and perplexity at just what it is one has learned. Some stumble at the hurdle and feel forever excluded, like the lame boy of Hamelin; others press on and on and still do not come to the end of what is undeniably a paradigm of the rich complexity of mathematical concerns.
>
> All this remains true, even now that a traditional calculational justification for studying logarithms has passed into history…. (Fauvel 1995, p. 39)

This passage very much set the tone for my dissertation research, as I always believed "something more" could be cultivated (mathematically, culturally, historically) in the teaching of logarithms.

As part of my dissertation research, the classroom materials that I constructed and used with teachers (and, for subsequent use with their students) were informed by the work of John Napier, Henry Briggs, William Oughtred, and Leonhard Euler. Jost Bürgi was mentioned only briefly when I worked with the teachers, and this was primarily because of how the resources I used at the time treated his role in the development of the logarithmic relation. Even then, the brief references struck me as afterthoughts, as found in a short paragraph in Cajori (1915):

> The only possible rival of John Napier in the invention of logarithms was the Swiss Joost Bürgi or Justus Byrgius (1552–1632). … Bürgi published a crude table of logarithms six years after the appearance of Napier's *Descriptio*, but it seems that he conceived the idea

> and constructed that table as early, if not earlier, than Napier did his. However, he neglected to have the results published until after Napier's logarithms…were known and admired throughout Europe. (pp. 166–167)[1]

In 2009 I was awarded a research fellowship at the University of Canterbury (Christchurch, New Zealand) to conduct research in history of mathematics. The opportunity at Canterbury was the result of an effort by Clemency Montelle (and supported by the School of Mathematics and Statistics) to increase the production of research in history of mathematics. The first task of my fellowship was to respond to Clemency's request to describe possible connections for our research collaboration, and I immediately responded that I was keen to pursue what I felt was "the rest of the story" regarding the development of the logarithmic relation. Consequently, I felt Jost Bürgi's contribution would provide the missing piece to an incomplete story about the independent invention of logarithms.

I located the Graz copy of *Aritmetische und Geometrische Progreß Tabulen* (1620)[2] in January 2009, and with the assistance of Michaela Scheibl at the Department of Special Collections of the Library of the Karl-Franzens-University Graz, Clemency and I received a digital scan of the complete copy held there. Although we have presented papers and published articles about the parallel insights of John Napier and Bürgi in the early years of the seventeenth century, our initial research developed into something more from my perspective, and this book represents my desire to provide access to the life and one mathematical contribution of Jost Bürgi to non-German language readers.

This book would not have been possible without the encouragement and assistance of several individuals and institutions. I am indebted to Clemency Montelle for introducing me to many tools that made this scholarly "labor of love" a reality, not the least of which is having the confidence to live and work in multiple academic environments (mathematics, mathematics education, and mathematics history). I am fortunate to have been awarded the time and resources to live and work in Christchurch, New Zealand, and I will be forever grateful to the School of Mathematics and Statistics at the University of Canterbury.

I am also grateful to Michaela Scheibl at the Department of Special Collections of the Library of the Karl-Franzens-University Graz (Graz, Austria) for her assistance in providing me with the digital copies of the texts I needed for my research. She also assisted in reaching a publication agreement from her department and the library so that I and Birkhäuser Mathematics are able to provide others access to the manuscript that is the subject of this book.

I was fortunate that Fritz Staudacher contacted me in the autumn of 2013, inquiring about the insights we might discuss with each other concerning our shared interest in Jost Bürgi. His initial email led to an eventful trip to Zürich in March 2014

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[1] Almost 100 years later it is still often easier to find references to the development of logarithms that omit mention of Bürgi, including this example from Pesic (2010): "…long before John Napier, Stifel seems to have invented logarithms independently" (p. 506).

[2] The complete title is: *Aritmetische und Geometrische Progreß Tabulen/sambt gründlichem unterricht/wie solche nützlich in allerley Rechnungen zu gebrauchen/vnd verstanden werden sol.*

where I met Fritz, Jörg Waldvogel (Professor Emeritus, ETH-Zürich), and Christelle Wick (Toggenburger Museum, Lichtensteig, Switzerland). The assistance, encouragement, and discussions that I shared with each of these new friends (including Irene Waldvogel, Jörg's lovely wife) are what made the completion of this project possible, and I publicly offer them my sincerest thanks.

Ewa Lichnerowicz of the Library of the Polish Academy of Sciences (Gdańsk, Poland) provided much needed assistance at the end of my revision work, and I am very thankful for her kindness, patience, and ability to communicate with me in English.

The many hours, weeks, and years, as well as the financial commitment to see this book to fruition, were lovingly and consistently supported by my partner in life, Todd Clark. Without him, and the sacrifices he made, this book would not be possible.

Finally, I dedicate this book to my parents, John Edward McGarvey, who passed away suddenly at the age of 74 on 29 March 2014, and Mary Regina McGarvey, who lost her brave battle with cancer at the age of 72 on 3 October 2015. I miss you, Mom and Dad.

Contents

Figure 1 Frontispiece of Benjamin Bramer's *Bericht zu M. Jobsten Burgi seligen Geometrischen Triangular Instruments mit schönen Kupfferstücken hierzu geschnitten* (1648; image courtesy of Toggenburger Museum, Lichtensteig, Switzerland)

Chapter 1
A Brief Biography of Jost Bürgi (1552–1632)

Introduction

Several German- and French-language resources contain brief biographies of Jost Bürgi (e.g., Cantor 1900; Lutstorf 2005; Montucla 1758; Naux 1966; Wolf 1858). No substantial personal information on Jost Bürgi[1] exists in the English language, other than the short (just over one page) account by Nový (1970) in the *Dictionary of Scientific Biography*. We can, however, construct a decent timeline of Bürgi's life from German-language resources (see Appendix A), particularly when it is situated with respect to Bürgi's contemporaries who were engaged in or aided in the development of scientific work dependent upon the logarithmic relationship. Staudacher (2014) published (in German) a quite extensive account of Bürgi's life, which included content on his mathematical and scientific achievements and contributions, as well as accompanying obstacles, family relationships, and other personal attributes. Using translations of Staudacher's text, as well as more traditional sources of biographical information on Bürgi, the major aspects of Bürgi's professional life are highlighted in the brief biography presented here.

Lichtensteig and Surrounds: Bürgi's Early Life and Work (1552–1579)

Bürgi was born 28 February 1552, in Lichtensteig in the Toggenburg, a 70 km long alpine highland valley along the Thur River and southwest of Mount Säntis in the Canton of St. Gallen, Switzerland. Jost and his parents were Protestant, which was

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[1] Bürgi's given name is sometimes given as Joost, Jobst, or Justus (when used with the Latinized version of his surname, Byrgius).

K. Clark, *Jost Bürgi's Aritmetische und Geometrische Progreß Tabulen (1620)*,
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representative of the majority of Roman Catholic and Protestant families living in this small village of approximately 400 inhabitants (Figure 1.1). We do not know anything of substance about Bürgi's early learning (Waldvogel 2012, p. 3), except that he probably received an almost complete 6-year formal education that was typical of boys in Bürgi's time and until the beginning of the twentieth century (Staudacher 2014). In 1564, Bürgi finalized his formal education, but due to religious battles as a result of the Counter-Reformation in Switzerland, Lichtensteig was often left without a teacher. Consequently, Jost and his classmates may have lost 1 year of the 6-year formal education. Although the majority of people of the Toggenburg Valley supported and followed the Protestant teachings of Ulrich Zwingli (1483–1531), the citizens were almost always overruled by the duke-abbot of the St. Gallen monastery. According to Staudacher (2014), the lessons in public schools were composed of up to 50 % choral singing lessons, with the remainder in computing, reading, and writing. Bürgi did not know Latin (and certainly did not write or publish in Latin) and regarding his knowledge of scientific languages, Bürgi stated:

> Weil mir auß mangel der sprachen die thür zu den authoribus nit alzeitt offen gestanden, wie andern, hab jch etwas mehr, als etwa die glehrte vnd belesene meinen eigenen gedanckhen nachhengen vnd newe wege suechen müessen. (List and Bialas 1973, p. 7)[2]

After his early and brief education and beginning in 1565 (Staudacher 2014), Bürgi began training in various trades that later contributed to the craftsmanship necessary for instrument making by working with his father, who was a locksmith.[3] Bürgi possibly trained as a goldsmith between 1565 and 1567 with David Widiz (~1535–1596), when Widiz relocated to Lichtensteig from Augsburg (Staudacher, p. 52).

Bürgi most likely apprenticed with someone with experience in making technical instruments, such as clock- and watch-making. Faustmann (1997) and Naux (1966) noted that Bürgi possibly worked as a traveling apprentice in Straßburg, where he may have come in contact with the teachings of Conradus Dasypodius[4] (~1531 to ~1601). According to Sesiano (in the *Historical Dictionary of Switzerland*, 1986), Dasypodius was a mathematics professor at the Academy of Straßburg from 1562, where he also took care of Swiss fellows studying there. Dasypodius also continued the design and construction of the second version of the astronomical clock for the Straßburg Cathedral (built during 1570–1574), and Bürgi may have participated in the construction of this clock (Waldvogel 2014). Some experts still believe this hypothesis, put forth by Rudolf Wolf (1858), made sense at the time due to Bürgi's potential training trajectory.

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[2] *Because I did not know other languages, the doors to the well-known scientists were not always open for me. So, opposite to the well-educated scholars, I had to think a little bit more by myself and find my own ways.*

[3] In the sixteenth century, the professions of locksmithing and making clocks were closely connected.

[4] Dasypodius' German surname was "Rauchfuss." Rauchfuss followed the practice of his time and grecianized his name to "Dasypodius."

Figure 1.1 Part of a stained glass coat of arms that mentions the grandparents of Bürgi (photo courtesy of Toggenburger Museum, Lichtensteig, Switzerland)

The construction of the second version of Straßburg Cathedral's clock was carried out by the well-known clockmakers Isaac and Josias Habrecht (of the Canton of Thurgau, Switzerland). This version of the astronomical clock, which operated well into the eighteenth century, was well known for its complexity because of its numerous devices, including indicators for planets and eclipses, calendar dials, and the astrolabe. Wolf's speculation that Bürgi apprenticed under the Habrechts during the construction of the cathedral's clock in Straßburg has persisted for more than 150 years, but today it is denied by experts such as Roegel, Oechslin, and Oestermann. Waldvogel (2014) and Staudacher (2014) speculated that Bürgi might have acquired his skills in Schaffhausen, Switzerland, which is closer to Lichtensteig in eastern Switzerland and where the Habrecht family built clocks until at least 1572 before moving to Straßburg. The Habrechts designed and constructed the Bern, Solothurn, and Schaffhausen astronomical clocks, as well as clocks in many cities of southern Germany, including Heilbronn, Donaueschingen, Ulm, and Altdorf near Nürnberg (Staudacher 2014, pp. 55–56).

In 1570 or 1571, Bürgi most probably completed his professional trades training, and from about 1571 he worked as a clockmaker in various locations, possibly in Augsburg due to the many connections he held with people from there (e.g., Widiz), and later in Nürnberg. In 1576, Christoph Heiden (1526–1576), a famous mathematician and celestial-terrestrial globe inventor, died in Nürnberg, and Bürgi, who was in Nürnberg as well, finalized a celestial-terrestrial globe that was under construction in Heiden's workshop.[5] Heiden received orders directly from Emperor Maximilian II and also served as first president of Altdorf University in Nürnberg.

Also in 1576, Maximilian II died, and his son Rudolf II von Habsburg (1552–1612) was named successor and emperor of the Holy Roman Empire. Rudolf II was deeply interested in the arts and sciences, including alchemy. Since he was not as engaged in the political, ceremonial, and daily managerial duties of his position, he moved the seat of the Habsburg Empire from Vienna to Prague in 1583, to serve as better protection against the Ottoman Turks. In 1592 and upon the recommendation of Vice Chancellor Jacob Curtius (1554–1594), Rudolf II selected Nicolaus Reimers Baer, or Nicolaus Reimers Ursus[6] (1551–1600), as imperial mathematician. Then, in 1599 and after recommendation of his Imperial Physician Thaddäus Hagecius (1525–1601), he named Tycho Brahe (1546–1601) of Denmark as imperial astronomer to his court in Prague. Eventually, in 1601, Rudolf selected Johannes Kepler (1571–1630) as Brahe's successor, and, by following his own interest in goldsmithing and clockmaking, Rudolf selected Jost Bürgi as his imperial clockmaker in 1604.

However, before Bürgi worked in Nürnberg, close connections developed between Duke Wilhelm IV (1532–1592) and Georg Joachim Camerarius (1534–1598), as well as between Heiden, Camerarius, and Bürgi. In 1579, the duke invited Bürgi to court in Kassel to work as a clockmaker and also as a craftsman in his observatory (Staudacher 2014). To receive such an invitation from the duke would have meant that Bürgi was already established with most of the skills and knowledge to deserve such a prestigious appointment in the observatory in Kassel.

Connections in Kassel: 1579–1603

After arriving in Kassel in 1579, Bürgi was engaged in clock and instrument making, and later in astronomy and mathematics, as well. In 1580 he built his first Kassel celestial sphere, worked with astronomical instruments, and developed various metal sextants in brass, steel, and copper. In 1583, Bürgi invented his own type of proportional compass, and in 1584, he created the world's first clock precise to the second and which indicated seconds both visually and auditorily. As a prerequisite to this revolutionary observatory clock, Bürgi had to invent new methods and mechanical systems for smoothly and steadily distributing the initial forces of a weight or of a spring, which was realized by his inventions of the cross-beating escapement and of the rewound weight. Notably, both of these Bürgi inventions were in place 70 years before Huygens' and Newton's pendulum clocks and 120

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[5] This is a newly discovered fact taken from the inventory list of Emperor Rudolf II's Kunstkammer (i.e., a "collector's cabinet," which contains a collection of curiosities and treasures) in Prague (Staudacher 2014, p. 76).

[6] Several variations exist for Reimers' name, some of which include Reimarus Ursus, Raimarus Ursus, and Nicolaus Reymers Baer. In this chapter, I will use Reimers.

years before John Harrison's chronometer (Staudacher 2014). It is not surprising then that in a letter to Brahe in 1586, Wilhelm IV said: "…unsers Uhrmachers M. Just [Bürgi], *qui quasi indagine alter Archimedes* ist."[7]

Most importantly for the time period 1584/1585, Bürgi, Christoph Rothmann (1551–1600), and Wilhelm IV—all as astronomers in Kassel—began a new measurement program of the stars in order to obtain better data for navigation, astronomy, and astrology. Two years after beginning their work, the *Grand Hessiae Register of Stars* (in the original German: *Grosses Hessisches Sternverzeichnis*) was completed and included 383 newly measured stars (Staudacher 2014, p. 134).

In 1584, Paul Wittich (~1546–1586) arrived in Kassel and stayed several months, and during the same time period, Bürgi began a search for ways in which to improve methods and formulae for prosthaphaeresis.[8] As a result of his extraordinary mathematical and technical talent and from his experience in calculating and formulating gearings, Bürgi was well positioned to contribute to innovations necessary to improve upon astronomical calculations. And, in order to improve upon such work at the time, Bürgi would have needed to be knowledgeable of the notion of prosthaphaeresis and computation involving sines.

Prosthaphaeresis, a process that converts more complicated multiplication (or division) into simpler addition (subtraction), was probably well known to Islamic scientists from at least the eleventh or twelfth century. Prosthaphaeretic formulas, in modern trigonometric notation, include the identities

$$\cos(a+b)=\cos(a)\cos(b)-\sin(a)\sin(b)$$

and

$$\cos(a-b)=\cos(a)\cos(b)+\sin(a)\sin(b)$$

To observe the "product to sum" transformation, we first subtract the second formula from the first

$$\cos(a-b)-\cos(a+b)=2\sin(a)\sin(b);$$

and isolating the product term yields

$$\sin(a)\sin(b)=\frac{1}{2}\left[\cos(a-b)-\cos(a+b)\right].$$

Thus, when two angle measures are known, an easier calculation is made when subtracting the cosine of their sum from the cosine of their difference and then dividing the result by 2, as opposed to multiplying two sine values.

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[7] "*…our clockmaker Jost Bürgi,* who is almost on the way of another [a second] Archimedes" (Roegel 2010a, p. 5).

[8] *Prosthaphaeresis*, from the Greek *prosthesis* (addition) and *aphaeiresis* (subtraction).

There has been much speculation about Bürgi's contribution to the improvement of prosthaphaeresis, as well as his construction of a table of sines. For example, Thoren (1988) discussed Bürgi's role in the evolution and publication of the trigonometric formulas that reduce a more complicated operation (multiplication) into a simpler one (subtraction), as in the formula above. In his account, Thoren traced the first publication of the method of prosthaphaeresis to Reimers, who first mentioned Bürgi's calculations in 1588. Attributing this "first" to Reimers is questionable, according to Thoren, and he discussed the potential contribution of Tycho Brahe, Paul Wittich, and Jost Bürgi to the use, publication, and geometrical proof of prosthaphaeretic formulas (e.g., for computing $\sin(a)\sin(b)$). Moreover, Thoren stated that:

> Ursus…issued a disclaimer in 1597…. According to him, Wittich…brought the *method* to the astronomical observatory of the Landgrave [Landgraf] of Hessen-Cassel in 1584; but what he brought was only one prosthaphaeretic equation (for $\sin A \sin B$), and no *proof* for it! It had been the Landgrave's [Duke's] clock-maker, Joost Bürgi, Ursus said, who devised a geometrical proof for that identity. (Emphasis in the original, p. 33)

In approximately 1586 or 1587, Bürgi designed and constructed a three-dimensional planetarium (i.e., a planetary model) for Reimers, of his "Tychonian" world model (Staudacher 2014, p. 119). The Tychonian model of the universe was a hybrid model of Ptolemy's geocentric model, where the sun and planets orbit around the Earth, and of Copernicus' heliocentric world model, which places the sun at the center. The hybrid model had the support of the Jesuits and also had two inventors, Reimers and Tycho Brahe, each of whom fought hard for his own priority until the death of Reimers in 1600. The hybrid world model shows the Earth in the center, surrounded by the moon and the sun. The other planets revolve about the sun, and all together they revolve around the Earth. Bürgi then constructed a second version of the planetary model at the request of Wilhelm IV and which incorporated feedback from Rothmann. In 1587, Reimers translated Copernicus' *De revolutionibus orbium coelestium* into German for Bürgi. Despite Bürgi's lack of Latin ability, his friend Reimers—imperial mathematician to Emperor Rudolf II—also likened Bürgi's abilities to those of Euclid an Archimedes (Gaulke 2015).

Afterwards (from 1587 until 1591), Bürgi began new work on the measurement of celestial bodies in order to define better orbital paths of the sun, Earth, and moon. And, in December 1590 until 1597, "Bürgi…regularly determined the angular distances of the planets and the Moon from those of the fixed stars recorded in the [*Grand Hessiae Register of Stars*] catalogue of 1587" (Gaulke 2015, p. 45). He needed these data for computations and to design a mechanically working device of Copernicus' moon theory to be integrated in the equation clock (or solar and lunar anomalies clock) of 1591.[9] This small table clock showed the mean moon and sun positions, as well as the highly accurate relative positions of the sun, the moon, (including eclipses), and the fixed stars (astrolabium dial) through the creation of elliptic movements of epicyclical and differential-epicyclical gearings. To integrate

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[9] For a detailed discussion of this clock, see Gaulke (2015).

various paths, Bürgi selected the form of an elliptical movement, which is the same progression of the planets that Kepler discovered 15 years later. Thus, Bürgi's measurements and calculations would have required precision, and consequently, Bürgi needed methods for which he could carry out the computations. As an already skilled instrument maker, he needed mathematical tools to complete the work.

In 1588, Reimers published part of Bürgi's new mathematical methods in *Fundamentum Astronomicum*; however, Reimers published perhaps more than Bürgi would have actually agreed to—leading to a slightly strained relationship between the two men—and an unwritten or unspoken publication agreement of sorts was part of the problem. To prevent this undesirable outcome from happening again, Bürgi asked his friend and colleague Reimers to swear to keep quiet all of Bürgi's developments and innovations in future.[10] This misunderstanding (about what could and could not be published by Reimers) between Bürgi and Reimers in 1588 may have led to Bürgi being overly cautious about writing down his mathematical innovations and sharing them with others. For example, Bürgi's "Kunstweg" was a method that dealt with interpolation, and it was included in *Arithmetica Bürgii*, which was edited by Kepler in 1603.[11] Staudacher (2014), in following Ludwig Oechslin, is of the opinion that Bürgi had already prepared his *Aritmetische und Geometrische Progreß Tabulen* by this time, as he would have been able to create the tables and methods using his "Kunstweg," which included methods of interpolation.

German mathematics historian Menso Folkerts further supported this claim. Folkerts located a handwritten (allegedly by Bürgi himself) document titled *Fundamentum Astronomiae*—a document very similar to Reimers' *Fundamentum Astronomicum*—in the Biblioteka Uniwersytecka we Wrocławiu (Wroclaw University Library, Poland). The manuscript was personally given to Emperor Rudolf II as a gift 10 days after Bürgi's first audience with the emperor in June 1592.[12] The analysis and publication on the results of this Bürgi text on trigonometry, which includes algorithms for building sine tables and his "Kunstweg" method of interpolation, was published in 2015 (Folkerts, Launert, and Thom). The sine tables included in this document could be the same as shown to Brahe, which also took place in 1592.

Prior to Bürgi's first trip to Prague, he remained busy in Kassel, continuing to work on a system to measure planets, and he collects measurement data until 1597

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[10] Reimers must have kept his promise; he refused to divulge information about Bürgi's "Kunstweg" (meaning artful (or skillful) method), because he had promised Bürgi to keep all of his (Bürgi's) information confidential (Staudacher 2014, p. 181).

[11] This work came to be known as Bürgi's *Coss*. The *Coss* manuscript was never delivered to a printer for publishing; it was finally edited and published in 1973 by List and Bialas. In 1604, Kepler wrote a letter to Fabricius, stating that he now had an understanding of the "Kunstweg" after having edited the *Coss* manuscript (Staudacher 2014, p. 181). However, Kepler did not mention his *Coss* editing work for Bürgi and therefore did not compromise the secrecy agreement he held with Bürgi.

[12] In the forward for *Fundamentum Astronomiae*, Bürgi gives the date "Prag, am Tage Mariae Magdalenae, Anno Christi 1592" (Folkerts 2015, p. 109), which corresponds to 22 July 1592.

on more than 1000 planet positions.[13] Bürgi built a silver and gold planetary globe in 1591–1592, which is considered one of the most highly developed automated models ever built. It is this planetary globe that Rudolf II asked Bürgi (through Wilhelm IV) to bring to Prague and which Bürgi personally delivered to Rudolf II in 1592. The construction of the globe required precise astronomical values for planetary positions, which Bürgi was able to compute in his own work as an astronomer and also as a mathematics expert (Staudacher 2014, p. 147). Bürgi returned to Prague in 1596, most likely for the purpose of checking and servicing the planetary globe and observatory clocks. Bürgi also met and spoke with Rudolf II during this visit regarding distances to planets and other astronomical interests. They also spoke about Bürgis' work in trigonometry, including the trigonometry document (*Fundamentum Astronomiae*) that he left with the Emperor during his last audience with him in 1592.

In addition to Bürgi's extensive work on celestial measurements and the design and construction of intricate instruments, he also worked to finalize a table of sines, *Canon Sinuum*, during this time. The table was probably completed at the end of the sixteenth century (Roegel 2010a), with List and Bialas (1973) and Staudacher (2014) giving the year 1598. However, as with every other mathematical endeavor of Bürgi's, coupled with his fear of others publishing without his permission, Bürgi most likely carried a copy of the *Canon Sinuum* on his person and used the tables for his own and Kepler's purposes and calculations.[14] Bürgi's *Canon Sinuum* contained sines calculated to eight (8) places, at intervals of 2″ (2 s).

Also at this time (1597–1599), Bürgi was completing the manuscript for the previously mentioned mathematical work, *Arithmetica Bürgii* (Staudacher 2014, pp. 185–186). Bürgi certainly felt at a disadvantage due to his poor knowledge of languages and his need to work more intently to read and understand the solutions of mathematical authorities. Thus, he searched for someone to improve and edit his draft of his *Arithmetica*. Bürgi's relationship with Reimers made him a candidate as editor of the manuscript; however, Reimers was himself writing a new book on mathematics and algebra. Also at this time, Reimers, Brahe, and Kepler's paths were converging, and strained relations in Prague were due to the priority fight between Reimers and Brahe (regarding their model of the universe), in which Brahe already asked Kepler to write a study of the subject. Brahe would eventually hire Kepler as an assistant at the observatory in Prague to help with analyzing data on Mars, although Kepler held ill feelings toward Brahe's dealings with Reimers (particularly since Kepler had only favorable dealings with Reimers). Eventually, Reimers handed Bürgi's draft of the *Arithmetica* over to Kepler for editing.

Soon after, in August 1600, Reimers died of tuberculosis while awaiting trial in a case that Brahe brought against him for allegedly stealing Brahe's idea for a hybrid model of the universe. Brahe had the support of Rudolf II, and Brahe expected

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[13] The data was accessible to Kepler from 1603 until 1612, when both Kepler and Bürgi were in Prague.

[14] The *Canon Sinuum* was never published and most likely remains lost. However, it makes sense that if Bürgi kept it on his person, others would have seen it and stated that it did exist.

Reimers to be found guilty, the punishment for which would have entailed being "publicly beheaded, drawn, and quartered" (Staudacher 2014, p. 210).

Prague: 1603–1631

Upon arriving in Prague, Bürgi continued to produce specialized mathematical instruments and Kepler finalized his edited draft of Bürgi's *Coss*. Additionally, Bürgi's astronomical data, which had been recorded over a period of 12 years in Kassel, became available to his friend (and now Imperial Court Astronomer) Kepler in Prague from 1603 until 1612. Bürgi's strong need for secrecy (as agreed upon between Kepler, Bürgi, and Bürgi's brother-in-law, Benjamin Bramer (1588–1652)) was a major factor for his work and name as an astronomer to be all but forgotten and eliminated from any mention by Brahe's successors. However, as Staudacher (2014) claimed, without Bürgi it would have been difficult or nearly impossible for Kepler to define and to verify the small elliptical deviation of an only eight (8) arc minutes from a circular path in his calculation of planetary motion. Bürgi provided to Kepler not only the most precise instruments for time-second and angle-minute part measurements but also the mathematical methods necessary to accommodate this mass of spherical data.

In December 1604, Bürgi was officially named imperial clockmaker. There he maintained a clock- and watch-making workshop, with two employees, in the same building as Rudolf II's alchemy laboratory and artist Adriaen de Vries' atelier with metal casting equipment. Beginning in 1608, Bürgi owned a private house in the downtown area close to the Powder Tower, and with a monthly salary of 60 guilders, he was the third-highest paid employee of Rudolf II. For the next dozen years or so, Bürgi continued to develop instruments, clocks, and watches in his workshop and to support Kepler as an astronomical observer. Furthermore, others applied Bürgi's mathematical methods in their own work. For example, in the 1608 edition of *Trigonometria*, Bartholomaeus Pitiscus (1561–1613) published brief excerpts of Bürgi's new algebraic methods, including how to determine the direction and magnitude of eccentricity of the Earth's orbit and finding the sine of half-angle from the sine of an angle. In this edition of his *Trigonometria* (a book with examples from Bürgi), Pitiscus called Bürgi an "ingeniosissimus Mathematicus," or "ingenious mathematician" (Staudacher 2014, p. 187). One of the main reasons for the publication of Bürgi's mathematical examples in Pitiscus' books is the secrecy agreement between Bürgi and Kepler. That is, Kepler could publish Bürgi inventions in his own publications only after Bürgi had previously presented it himself in another publication. Therefore, it was necessary for Bürgi to hand over an example or excerpt for publication before a Kepler example was shown in *Astronomia Nova*.

A great deal has been written about when Bürgi began his work to construct the tables of the *Aritmetische und Geometrische Progreß Tabulen*, and a brief step back is in order. Nový (1970) speculated that Bürgi began computing his tables of logarithms as early as 1584. Grattan-Guinness placed Bürgi's computation of tables of logarithms as early as 1590 (1997, pp. 180–181). Many sources, however, quote Bürgi's brother-in-law, Benjamin Bramer, for a firsthand account of when Bürgi must have computed his tables of logarithms (actually, tables of antilogarithms). In his testimony, Bramer stated in a book published in 1630 that:

> [It] is on these principles that my dear brother and master Jost Bürgi, calculated, twenty years ago and more, a beautiful table of progressions, …, calculated to nine digits, [and] he did not print the [tables] until 1620 in Prague, so the invention of logarithms is not by Napier, but was made by Jost Bürgi long before." (translated from Montucla 1758, p. 10)

This passage has influenced some to place Bürgi's construction of tables as a result of his invention around the year 1610 (Roegel 2010a).

Refining the time frame for which Bürgi completed the construction of his tables of logarithms may be possible with Folkerts' forthcoming analysis of Bürgi's *Fundamentum Astronomiae* (which is dated to 1592). In particular, the first of the two books of the *Fundamentum Astronomiae* includes an explanation of the four basic arithmetic operations and root extraction using sexagesimal (base 60) numbers, a 12-page multiplication table (again, with sexagesimal numbers), a chapter dealing with prosthaphaeresis, and the calculation of the sine value for each angle, in increments of 1 min and to six places. The sheer amount of calculation work in the *Fundamentum Astronomiae*, coupled with the underlying similarity among the various calculation techniques required to construct tables of sines and to make the accurate calculations required to construct the astronomical models, could place Bürgi's construction of his tables of logarithms prior to 1592. That is, his method for simplifying all manners of calculations using logarithms (like those eventually needed in the *Fundamentum Astronomiae*) may have been the precursor to Bürgi's more complex mathematical texts.

Kepler, as his friend and colleague, urged Bürgi to print and disseminate his tables and instructions for their use as "an efficient method to carry out multiplications and divisions" (Waldvogel 2012, p. 13). Some time between 1600 and 1603 and in an effort to avoid a similar situation that Bürgi experienced with Reimers publishing his work without first establishing a proper agreement with Kepler, Bürgi arranged a secrecy agreement with him. Consequently, along with handing over of Bürgi's *Coss* draft to Kepler, Kepler and Bürgi swore to not betray each other and to keep the methods and innovations in mathematics of the other secret until he published them himself (Staudacher 2014).

Yet Kepler knew and worked with Bürgi's *Aritmetische und Geometrische Progreß Tabulen* while editing Bürgi's *Coss*, and from 1603 onward, Kepler worked in silence with both of Bürgi's innovative tables, the *Canon Sinuum* and the *Aritmetische und Geometrische Progreß Tabulen*, in order to calculate with a vast amount of observation data collected by Tycho Brahe. Then, in 1609 both Kepler and Bramer were convinced that Bürgi would bring both manuscripts to the printer. Unfortunately, Bürgi's first wife (Bramer's sister) died in 1609, and this, along with the growing trouble in Prague between Catholic League soldiers and of the people of Old Town Prague, made the eventual printing of Bürgi's manuscripts difficult. Bürgi would not start publication until 1620, and even then only the actual tables were printed as proofs and in small quantity and without the instructions necessary for their use. Whatever copies of the tables existed in 1620 were most likely lost during the Thirty Years' War. One battle—the Battle of the White Mountain—was fought just outside of Prague in November 1620 and 7000 men lost their lives there (González-Velasco 2011, p. 101).

The subject of assigning a timeframe or year to Bürgi's construction of his tables of logarithms is often due to the question of priority with regard to the invention of logarithms. In 1614, John Napier (1550–1617) published his *Mirifici Logarithmorum Canonis Descriptio* (or the *Descriptio*), officially earning publication priority with regard to the invention of logarithms. However, for some, the priority issue is about more than the moment of publication. González-Velasco (2011) stated that "for the sake of fairness that the earliest discoverer of logarithms was Joost, or Jobst, Bürgi (1552–1632), a Swiss clockmaker, about 1588" (p. 100).

As was the case with Bürgi, Napier began working on his conception of logarithms some years before his first publication in 1614. Napier stated in his *Descriptio* that he worked some 20 years on the tables he presented within it, which would place the beginning of his work on logarithms in 1594. Interestingly and perhaps out of respect for his colleague and friend, Kepler did not show an official interest in Napier's logarithms since he had been urging Bürgi to publish the *Aritmetische und Geometrische Progreß Tabulen* for many years. In 1619, Kepler would have known that Bürgi's tables were being typeset for publication, and since they would soon be printed and distributed, Kepler no longer felt he was bound to secrecy. And his reaction was to not maintain allegiance to Bürgi but to align with Napier's (and, consequently, Briggs') tables of logarithms and, eventually, his own. In 1627 Kepler famously wrote in the foreword to *Tabulae Rudolphinae*: "Der zaudernde Geheimniskrämer liess sein Kind im Stich, anstatt es zum allgemeinen Nutzen grosszuziehen"[15] (Staudacher 2014, p. 206).

The discussion about assigning the title of inventor of logarithms to Bürgi or Napier is now over 400 years old. If we only consider publication date as the defining metric for priority, then Napier is the clear winner. Another dimension to the discussion, however, is to recognize that the parallel insights of both Napier and Bürgi occurred at approximately the same time. In the late sixteenth century and early seventeenth century, both Bürgi and Napier, in two different locations and engaged in very similar life's work (the need to perform a vast amount of difficult calculations, particularly with respect to astronomical computation applications), came to develop a mathematical method that enabled them to improve their own work and the work of others. Whereas Napier's original conception of the logarithmic relationship was dependent upon a kinematic argument (Appendix B), and which required complex calculations to construct his table of logarithms, Bürgi's original conception was algebraic in nature and much simpler in construction. It is unfortunate that because of Bürgi's need for secrecy to protect his innovations and methods until he believed them to be ready for publication and the events of the time (e.g., the worsening political conditions in Prague and the start of the Thirty Years' War),

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[15] "*The hesitant secretive [man] abandoned his child instead of raising it for the general benefit.*"

the *Aritmetische und Geometrische Progreß Tabulen* would not be published and enter into mainstream use as Napier's conception of logarithms did.

There are several resources that describe Napier's conception of the logarithmic relationship, as well as the method used to construct his tables, including Havil (2014), Katz (2009), and Roegel (2010b).

Return to Kassel: 1631–1632

In 1631, just before his death, Bürgi left Prague for the last time to return to Kassel. He died just 4 weeks shy of his 80th birthday on 31 January 1632, and without children of his own, his legacy died there as well. Although the grave no longer exists, a plaque was placed to commemorate his contributions:

> Auf diesem Friedhof liegt begraben
> der landgräflich- hessische und
> kaiserliche Uhrmacher sowie Mathematiker
> Jost Bürgi
> geb. 28.2.1552 in Lichtensteig, Schweiz
> gest. 31.1.1632 in Kassel.
> 1579–1604 und in späteren Jahren tätig in Kassel
> als genialer Konstrukteur von Messinstrumenten
> und Himmelsgloben, Erbauer der
> genauesten Uhren des 16. Jahrhunderts,
> Erfinder der Logarithmen. (Volk 2009)[16]

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[16] *On this cemetery lies buried/the Landgrave of Hessen and/the Emperor's watchmaker and mathematician/Jost Bürgi/born February 28th, 1552 in Lichtensteig, Switzerland/died January 31st, 1632 in Kassel/ingenious designer of measuring instruments/and celestial globes, builder of the/most precise clocks of the 16th century,/inventor of the logarithms.*

Chapter 2

Details of *Aritmetische und Geometrische Progreß Tabulen*: Printed Tables, Manuscripts, and Mathematical Details

Introduction

Two extant *Aritmetische und Geometrische Progreß Tabulen* manuscripts were considered for the commentary that appears in Chapter 3. In this chapter, the two copies (the Gdańsk (Gk) manuscript and the Graz (Gz) manuscript), as well as an example of a copy that contains only the title page and Bürgi's tables (e.g., the printed copy in München (Mn)), are briefly described. Then, further details of the Graz copy are given so as to inform the transcription, English translation, and commentary presented in Chapter 3.

Brief Descriptions of Extant Prints and Manuscripts

The München Print (Mn) of 1620

One copy of the *Aritmetische und Geometrische Progreß Tabulen*, comprising only the printed title page and 58 pages of tables, can be found in the Universitätsbibliothek of the Ludwig-Maximilians-Universität in München (Table 2.1). This was the first copy found by Rudolf Wolf in 1846 and was previously owned by Doppelmayer. As it does not contain any additional handwritten information (e.g., there is no accompanying written instruction manuscript), nobody understood it or could work with the tables. Furthermore, copies of this print containing only the title page and tables are also available online (http://daten.digitale-sammlungen.de/~db/0008/bsb00082065/images/). Analyses of the accuracy of Bürgi's tables are available (Roegel 2010a; Waldvogel 2012); however, there are print-quality discrepancies among the various copies used in the analyses.

K. Clark, *Jost Bürgi's Aritmetische und Geometrische Progreß Tabulen (1620)*,
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