Applied and Numerical Harmonic Analysis

$$\hat{f}(\gamma) = \int f(x) e^{-2\pi i x \gamma}\, dx$$

Elena Prestini

The Evolution of Applied Harmonic Analysis

Models of the Real World

Second Edition

Forewords by Ronald Bracewell and Dušan Zrnić

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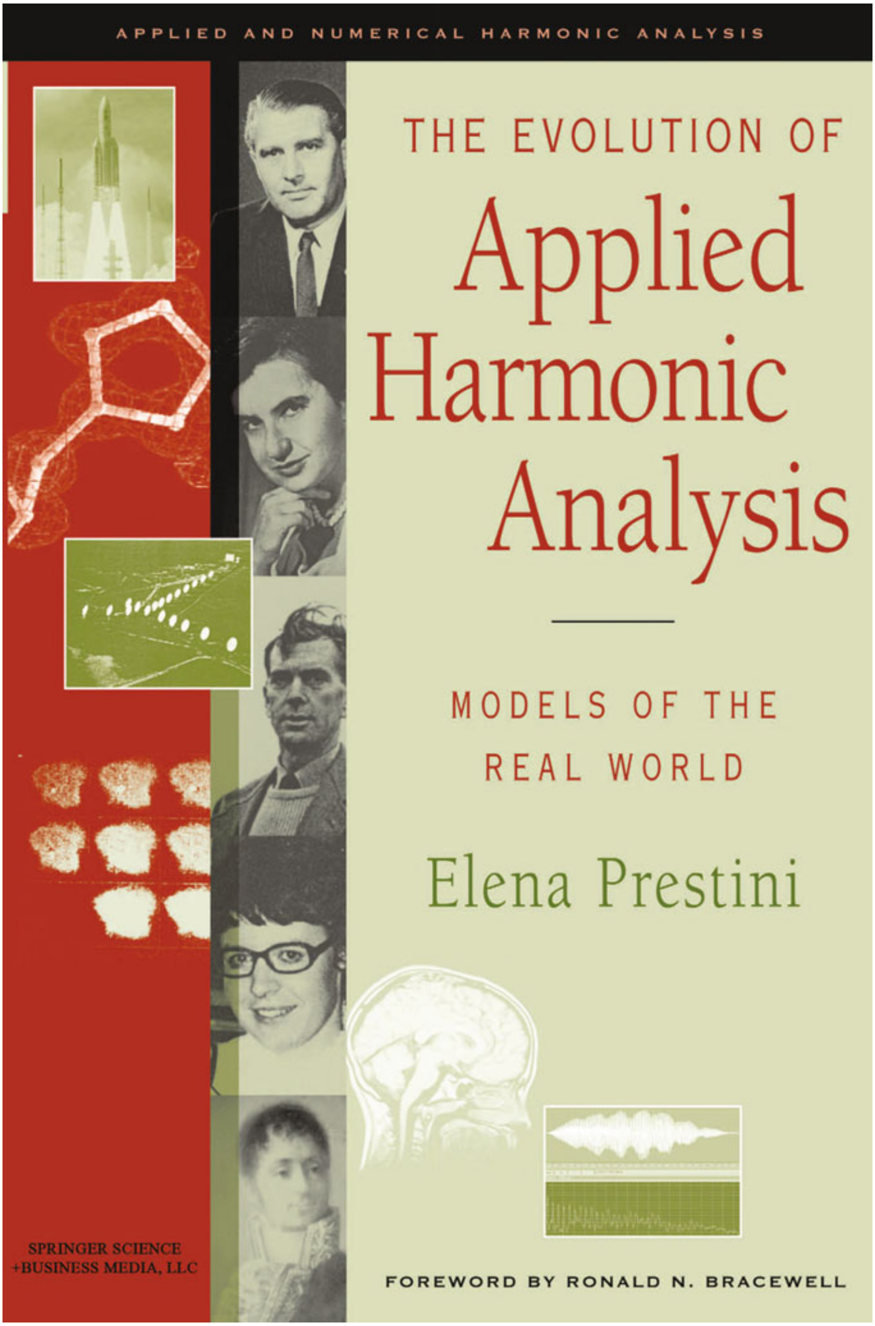

Cover of the first edition

Foreword to the Second Edition

The major contribution to this second edition is the addition of chapter ten which emphasizes applications to atmospheric science including measurements and probing instruments. As in the previous chapters the common thread is the Fourier method. The pinnacle of this chapter is that the most definite quantitative achievement in turbulence theory advanced by Kolmogorov is rooted in the statistical Fourier decomposition. The physical explanations and mathematics have enough details to be self-sufficient. The chapter will be of particular interest to atmospheric scientists and engineers.

Readers with experience in remote sensing will appreciate the discussion about the speed of sound in air and its wave properties of propagation, refraction, and reflection. The birth of atmospheric acoustics is attributed to men of peace (clergyman, and scientists) utilizing the works of men of war; we learn from the past that the firing of guns was used to make the first definitive measurements of the speed of sound in the atmosphere. To this day warfare technology provides unexpected benefits to society. Professor Prestini tells us that the field of radar meteorology owes its spectacular successes to the radar developments during World War II and thereafter. This can't help but leave us wondering how much good would come to humanity if its engagement in military compared to civilian sciences were reciprocal to what it is today. And what would ensue if we could eliminate conflicts altogether? The freed resources could tackle the numerous problems facing humanity including the enormous one of global warming. Speaking of which, the chapter lucidly articulates the evolution of understanding the physical principles that drive variations in climate. Milankovitch theory and its prediction of the three principal cycles of maximum insolation is a vivid example of the simplicity and predictive power rooted in Fourier analysis. Discussion of the discovery and subsequent effect of greenhouse gasses reminds us that too much of a perceived good (i.e., warming by CO_2 to prevent the earth from being "held fast in the iron grip of frost") can produce a bad outcome. The scientific method evoked so many times by the author points to the threat global warming poses to society.

As in many examples throughout the first nine chapters, details about the principal protagonists and their works are exposed and related to later developments and present understandings of the subjects. We are informed that Galileo was the first to recognize the wave nature of sound. Particularly striking is Leonardo da Vinci's brilliant conclusion that "the motion of the water, which resembles that of hair, which has two motions, of which one is caused by the weight of the hair, the other by the direction of the curls; thus the water has eddying motions." This partitioning of fluid motion into mean and turbulent flow predates Reynolds's idea by four centuries.

Sodars, radars, and lidars are the prime remote sensors that among other parameters can measure winds via the Doppler effect. Considerable space is devoted to these instruments and readers' understanding is enhanced by contrasting and/or comparing the nature of resulting measurements and the underlying properties of wind tracers.

The chapter provides well-articulated physical insights and explanations of atmospheric phenomena with sufficient mathematical exposure to be understandable but not overbearing. Particularly cute is the example of turbulence creation due to flow around a person and its capture followed by transport of odor, in this case perfume. The material is not meant to be a substitute for a working knowledge about the many subjects it touches upon, but informs about the essence and the diverse paths science takes in its quest for truth. Those with previous experience will be reminded about the complexities of atmospheric science and its interdisciplinary character. Those with less experience will be enriched with the newly acquired knowledge. To some, the chapter will serve as an excellent introduction into various modern aspects of atmospheric science. Reading this chapter was a pleasure to me and I hope you will enjoy it as much as I did.

Dr. Dušan Zrnić
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Foreword to the First Edition

Two hundred years ago Baron Jean Baptiste Joseph Fourier (1768–1830) championed a mathematical idea that was to have a profound and enduring influence far beyond what could be imagined at the time; understanding the significance of his work requires some historical perspective.

Arithmetic and trigonometry were already well polished. For example, the trigonometrical table published by Claudius Ptolemy (2nd c. A.D.) in the days of the emperors Hadrian and Antoninus was more precise and more finely tabulated than the four-figure table of sines used today in schools. Euclidean geometry was thoroughly familiar. Calculus, starting with Archimedes (287–212 B.C.), who had determined the volume of a sphere by the limiting process now familiar as integration, had been given its modern formulation by Gottfried Wilhelm Leibniz (1646–1716) and Isaac Newton (1642–1727). Calculus became a mighty tool of astronomy in explaining the movement of the Earth and planets and had an impact on philosophy by its success in showing that numerous puzzling features of the Universe were explicable by deduction from a few physical premises: the three laws of motion and the law of gravitation. But integral and differential calculus had not been brought to perfection; they suffered from limitations when confronted with entities that were deemed nonintegrable or nondifferentiable and from difficulties in handling certain notions involving infinity and infinitesimals.

Even today, in physics, as distinct from mathematics, one recognizes that the infinitely small has not been observed, nor has the infinitely large. Would it not therefore seem logical that finite entities, such as the integers, should suffice for physical theory?

Nevertheless, as is well known, theoretical physicists expertly wield the power of modern mathematical analysis, confidently canceling infinities and adding up infinitesimals. Modern analysis, their indispensable and versatile tool, sprang from Fourier's "Analytical Theory of Heat," published in 1822 (Fourier [1978]). It took half a century and works by several mathematicians from Dirichlet to Cantor to interpret Fourier's method of representing a function as a sum of infinitely many harmonics, laying the basis for modern analysis.

Fourier's idea was conceived while he was investigating the conduction of heat, in particular the heat flow resulting from a spatially dependent distribution of temperature along a metal bar. When the starting distribution is physically measurable, functions that are not integrable, not differentiable, not continuous, or not finite do not arise. But Fourier's theorem seemed to be capable of wider interpretation, and such proved to be the case as mathematicians, some of whom strongly criticized Fourier for failing to supply a rigorous proof, refined the very concept of a real function and lifted restriction after restriction on functions that could be analyzed by Fourier's method.

The idea of adding together a finite number of components of simple form to represent some previously observed phenomenon has a long history. It was used by Ptolemy when he accounted for the observed motion of the planets around the Earth. His system of deferents and epicycles was a geometrical device that added a periodic (annual) correction to the position of a point in circular orbit around the Earth to arrive at the longitude of a planet in the sky; the equant was another geometrical device that incorporated a trigonometrically calculable periodic correction for orbital eccentricity. Where further adjustment to improve agreement with celestial appearances was warranted, the addition of another epicycle would extend the trigonometric series to three terms. This procedure, in use for over a thousand years, was a precursor to the Fourier series which, though nominally infinite, are evaluated to a finite number of terms for practical purposes such as the annual publication of planetary ephemerides.

Even earlier, Babylonian planetary positions were predicted by summing periodic corrections that were triangular rather than sinusoidal (and therefore easier to calculate). Fourier's contribution called for successive sinusoids that were in a harmonic relationship; for example, synthesizing the mean annual temperature variation for a particular location would require the first sinusoid to have a period of one year, the second to have a period of half a year, the third one-third of a year, and so on. If only a few terms are to be summed this proposal has the same pragmatic ring as the practices of the Babylonians and Greeks. But Fourier provocatively implied that if the summing were continued forever, the final sum would agree exactly with the original data. This is certainly not obvious. In spite of that, another astronomical precedent was set, some years before Fourier, by Carl Friedrich Gauss (1777–1855), who computed a cometary orbit as a sum of sinusoids. Not only did he observe the harmonic relationship of the periods chosen, but he fixed the amplitudes and phases of the components by the numerical algorithm now known in computing circles as the *fast Fourier transform*. In 1965 J. W. Cooley and J. W. Tukey brought the fast algorithm, now universally referred to as the FFT, to the attention of a much wider audience than the few to whom it was already known.

Harmonic analysis is a mathematical tool, but Professor Prestini interprets it in its modern mature form in a variety of scientific and technological aspects so diverse that any connection among them would hardly be suspected. Certainly it is well known that radio and television broadcasting, and wireless telephony, are conducted in frequency bands that are allocated by government, while music produced by a piano is clearly composed of the approximately sinusoidal air vibrations launched by the strings when hit by the hammers. It is by appeal to the familiar vibrating string that Professor Prestini introduces the mathematical theory.

To understand the analysis of music, speech, television, and cell-phone transmission into sinusoids and, conversely, the synthesis of the corresponding time-varying signals from sinusoids is to possess the key to understanding how astronomical images, medical images, and images of protein molecules are formed. In these cases we are dealing with sinusoids in space instead of in time and with two (or three) dimensions instead of one. However, once the temporal analysis and synthesis are grasped, the transfer of the concepts to the spatial domain is facilitated. And in each such application this book explains how the traditional vocabulary of each field is related to the terminology of vibrating strings. We learn that what we called the resolution of a television image is analogous to the pitch of a musical sound: Pitch is connected with the number of vibrations per second, while spatial resolution has to do with the number of lines per centimeter. If you know some property of a beep of a certain pitch and temporal duration, you know a corresponding property of a patch of image of a certain resolution and spatial extent.

Learning the analogies between the various fields treated confers powerful knowledge, but each field also has its own distinctive and fascinating history of development which the author incorporates into each chapter.

The final result is an impressive overview of a range of modern technologies, an implicit lesson in the interplay between technology and one segment of mathematics, and a stimulus to thinking about possible futures of the fields discussed and about inventions in other fields that have not yet been made. Why exact mathematics, a product of the human mind, should have proved so useful in science and technology, remains a mystery. The book can be thoroughly recommended to any reader who is curious about the physical world and the intellectual underpinnings that have led to our expanding understanding of our physical environment and to our halting steps to control it; and everyone who uses instruments that are based on harmonic analysis will benefit from the clear verbal descriptions that are supplied.

September 2003

Ronald Bracewell
Stanford University

Preface to the Second Edition

A unifying view of many fields of science is a main feature of this book. This is done via Fourier analysis, a branch of mathematics that—with its dual stance of time and frequency (or space and frequency)—happens to permeate most of modern science.

Following this first edition thread—besides some change and an update of Section 3.12—a new chapter has been added. It deals with five intertwining topics: turbulence, sodar, radar, lidar, and climate. They are described through their historical developments, followed by some more technical considerations. Then modern results, also providing images for the figures, are discussed.

From the point of view of harmonic analysis, wind is an interesting topic. Among sections concerning remote sensing (Sections 10.6–10.18), a few are dedicated to wind in order to describe different methods to determine wind velocity and direction (Sections 10.11, 10.14, 10.15, 10.18). The important role played by harmonic analysis in the theory of turbulence is mentioned in Section 10.5 and shown in Section 10.9.

In a few instances this new chapter briefly outlines the life of scientists that, with their intuition and dedication, made fundamental advances.

In the last section that deals with Earth temperature and climate, the time scale changes. The values of astronomical parameters, such as Earth eccentricity and tilt, are reconstructed hundred of thousand years back and predicted hundred of thousand years hence. Moreover in two hundred years the temperature of the Earth, a problem dear to Fourier, has become a matter of worldwide attention. This problem, from which sprang the theory of harmonic analysis, opens the book and the problem of the evolution of the temperature of the Earth closes the book.

I hope readers will find this book useful and enjoyable.

Acknowledgments

When my editor contacted me to suggest a second edition, my reply was not really enthusiastic: the first edition has been so much work and took so long. Then a friend Alberto Mugnai, a scientist at the Institute of Atmospheric Sciences and Climate

(ISAC) of the National Council of Research (CNR) of Italy, mentioned a colleague of him, Giuseppe Mastrantonio, very much interested in harmonic analysis and went ahead to arrange a meeting. Mastrantonio is a historical scientist, having collaborated with Giorgio Fiocco to the realization of the first sodar in Italy. I talked to him on the matter, and my curiosity lit up again. I am grateful for his help during all stages of this work, for providing much literature on sodar, as well as many pictures. He also introduced me to his colleagues Luca Baldini, Gian Paolo Gobbi, Davide Dionisi, and Fernando Congeduti.

Baldini is a radar expert; Gobbi, Dionisi and Congeduti are lidar experts. I like to thank them all for generously devoting their time to answer my questions and providing beautiful pictures as well. As for the lidar I was also in contact with Alcide Di Sarra of the Italian Ente Nazionale Energie Alternative (ENEA). And I like to thank Mariella Morbidoni, the head librarian of the Tor Vergata Area of Research (CNR-ARTOV), for her help and kindness.

Regarding turbulence, combustion, and aerospace issues I wish to thank Claudio Bruno at the time at United Technology Research Center (UTRC), in Connecticut.

Regarding climate I ought many thanks to André Berger of the Catholic University of Louvain in Belgium, who never failed to kindly answer my several e-mails. He also provided the image of a figure. I thank Eric A. Smith, Retired Professor of Atmospheric Science, and Christos Efthymiopoulos of the Athens Academy of Sciences for bibliographical information.

I thank Dušan Zrnić of the National Oceanic and Atmospheric Administration for suggesting improvements in the radar Sections 10.13–10.15. Also I thank him together with his colleague Richard Doviak, as well as Peyman Givi of the University of Pittsburgh, and James M. McDonough of the University of Kentucky for images that appear in the figures.

Finally I wish to thank Editors Allen Mann and Mitch Moulton for suggesting this second edition, and Ben Levitt with whom I worked in close contact during the latest stage of this publication.

Rome, Italy
June 2016

Elena Prestini

Preface to the First Edition

The suggestion to write a book on mathematics came from my Italian editor Ulrico Hoepli Jr. during a Christmas party years ago, about the time I returned to the University of Milano as an associate professor after being an instructor at Princeton University.

The suggestion appealed to me but I could not work on it right away. Years later, with a former colleague of mine Michele Sce, whom I wish to remember here, we proposed the contents and set a general plan. To my sorrow we could not go much further due to his sudden illness.

At that juncture the strongest input, which eventually shifted the topic of the book, came from my students. Over the years many, majoring in physics, chemistry, biology, and computer science, had attended my courses on mathematics. I wanted to write a book that would be easy for them to read and close to their scientific interests so that their studies would be better motivated.

I also wished to write a book for a broad audience. On more than a few occasions, after being introduced as a mathematician, I was told something like: "At school I liked math but never understood what it was used for." I wanted to meet this understandable curiosity and show how my field of interest is put to good use. Fortunately Fourier analysis has an extremely wide range of applications—reaching the everyday lives of everyone—in line with the goal Fourier assigned to mathematics: "the public good and the explanation of natural phenomena."

From that background the first book *Applicazioni dell'Analisi Armonica*, which appeared in 1996 in Italian, and the present book, double the size, were born. My Italian editor generously released his rights to this English version. I wish to thank him for that, for having accepted the change of topics, and for his original suggestion as well.

I also wish to thank John Benedetto, series editor in chief of Birkhäuser's Applied and Numerical Harmonic Analysis book series, for encouragement and support of this project.

An important contribution came from Ronald N. Bracewell of Stanford University, himself the author of a standard text on Fourier analysis and applications and whose range of interests covers a good part of the topics dealt with here.

He read my original book since, to my surprise, he knows Italian and kindly offered to assist me with English. Actually he did much more. I thank him for his steady support of the project, for his scientific advice, and for consultation on my English. The result is a presentation of much of modern science, from the billions of years of cosmic evolution to chemical reactions followed to the nanosecond, from the cosmic world populated by gigantic galaxies to the atomic world, and up to the ideal world of mathematics. Scientists, who are engaged in pushing ahead the frontier of knowledge, appear to share the feelings expressed in the poetic words of the ancient astronomer:

> When I search into the multitudinous revolving spirals of the stars my feet no longer rest on the earth, but, standing by Zeus himself, I take my fill of ambrosia, the food of gods. (Claudius Ptolemy, 2nd century A.D.)

Overview

The opening chapter of this book is devoted to Fourier's life, which was full of adventures and misadventures and equally filled with scientific and public work.

The second chapter introduces the mathematics that is common to the different applications. The historical introduction is followed by a presentation of basic concepts such as the Fourier series and Fourier transform, fundamental theorems such as the Dirichlet theorem and the inversion formula, and basic properties having to do with dilations and translations. The mathematical introduction given in Chapter 2 is as elementary as possible and illustrated by examples.

The enormous influence exerted upon science by Fourier's idea of decomposing a function into a sum of sinusoids or "waves"—influence that kept gathering power throughout the century that followed Fourier's work—is made apparent in the remaining seven chapters. They are devoted to different applications, each chosen for its significance and diversity. In all chapters the mathematical core is preceded by a historical account and followed by a description of recent advances.

Chapter 3, which begins with the history of the great electrical revolution—from Alessandro Volta to Alexander Bell and Guglielmo Marconi and up to satellite communications—has as a main topic signal processing, probably the best-known application of Fourier analysis. Filters, noise, quantization, sampling, and the sampling theorem are dealt with. The discrete Fourier transform (DFT) and the well-known algorithm for the fast Fourier transform (FFT) follow. The origin of this algorithm is traced back to Gauss in his successful attempt to compute the orbit of an asteroid. At this point the topic of space exploration appears as a natural link and ends the chapter. Here the historical part is dominated by the figures of the well-known German-born scientist Wernher von Braun and his Russian counterpart Sergei Korolev, while the technical part deals with aspects of propulsion and combustion.

In Chapter 4, after recalling how sound came to be understood and measured, the attention shifts to music, computer music, and its history. The analysis of musical sounds is dealt with, beginning with a single note and ending with the dynamic spectra of the performances of three tenors, which allows a quantitative and comparative analysis of their styles. Finally musical synthesis and some of the special effects it is used for are illustrated.

The topics in Chapter 5 are all interleaved with the long struggle to understand the nature of light, from Newton to Einstein. The Fourier transform in two dimensions, needed for the mathematical model of the so-called optical transforms, is introduced. Diffraction at infinity is the underlying physical phenomenon, which also lies at the heart both of radioastronomy, as illustrated in Chapter 9, and of x-ray crystallography, as illustrated in Chapter 6. Also relevant to x-ray crystallography is synchrotron radiation. With its unique features and the huge storage rings needed to produce it, it is the final topic of the chapter.

Chapter 6 is dedicated to x-ray crystallography, to its history from Nicolaus Steno to Max von Laue and Lawrence Bragg, and to the demanding task, so essential to the understanding of life, of determining the spatial atomic structure of human proteins, expected to number over a million. The first result, due to molecular biologist Max Perutz, dates from the 1960s. It could take work extending over a great deal of this century to see the end of it. The chapter would not be complete without mentioning the revolutionary work of biochemist James Watson and physicist Francis Crick on DNA, which was based on the experimental data of the crystallographer Rosalind Franklin.

Chapter 7 introduces the Radon transform, which is closely connected with the Fourier transform. Advanced in the early 1900s by the mathematician from whom it takes its name, it found its most important application in the CT scanners first built by the engineer Godfrey Hounsfield following the theoretical work by physicist Allan Cormack.

Chapter 8 begins with the history of nuclear magnetic resonance (NMR). This phenomenon, originally detected in the late 1920s by the physicist Isidore Rabin, aroused the interest of scientists from different fields, such as physicists Felix Bloch and Edward Purcell, chemists like Richard Ernst and Paul Lauterbur, and medical doctors such as Raymond Damadian. Their work lies at the foundations of the well-known magnetic resonance imaging (MRI). Also NMR spectroscopy, a technique that, like crystallography, is employed to determine the spatial atomic structure of proteins, is illustrated. The case of the prion protein, involved in the dreaded BSE, provides an example.

Chapter 9 takes us back to space. Introduced by a brief history of one of the oldest human activities, astronomical observations, this chapter focuses on radioastronomy. From its beginning in the 1930s, radioastronomy has revealed new celestial objects such as quasars and pulsars, led to the theory of the Big Bang, and following very recent measurements, pointed to an endless expansion of our universe.

Aims and Scope

In times like ours in which scientists tend to have a restricted and focused point of view, which is reflected in the scientific literature, I believe a book adopting a generalistic approach may have some value.

The book offers a brief introduction to several fields that are active and central in contemporary science together with the required Fourier analysis and landmarks in their fascinating historic development. Numerous illustrations make the reading easier, while the extensive bibliography may satisfy those in search of deeper knowledge, both scientific and historical.

Who might be interested in this book? To my mind, among the primary readers should be working scientists, particularly mathematicians and especially Fourier analysts wishing to know more about the applications of their own field of expertise, as well as scientists in one of the fields of application who want to know what workers in other fields do. In Ronald Bracewell's words: "To specialists already equipped with basic knowledge it could represent a way of efficiently building on their existing background, expanding their areas of expertise, and perhaps qualifying them for other jobs. One has to bear in mind that mathematicians are heavily outnumbered by people engaged in image engineering and in biomedicine. The rapid expansion of medical imaging will result in crystallographers, radioastronomers, and others being drawn from these areas in order to fill the immediate needs in medicine. Elena Prestini's book could facilitate the transfer into medical imaging of people whose primary exposure at graduate level was to one of the less populated specialities."

This book may also interest those concerned with history of science. I describe the ancient origins of every topic, move quickly to modern times—the primary focus—and finally reach the present.

The general reader might be interested in this work as well. I am thinking for instance of young people in the process of deciding the course of their studies. They might find useful information herein and derive inspiration from reading about the work of celebrated scientists. I have paid a lot of attention to the organization of the material and worked toward reaching clarity of exposition. This together with the selection of topics, I hope, will keep the readers interested until the very end.

While significant parts of every chapter are accessible to the general reader, some knowledge of calculus is required to understand the technical part. The book can be used as a complement in teaching at both the undergraduate and graduate levels, whenever the teacher feels comfortable with it. For instance I require my students majoring in chemistry—the corresponding level in the U.S. would be advanced undergraduate—to read it all (the original Italian version), and in my lectures I go over the technical parts of the topics relevant to chemistry. Even though it requires some extra effort from the students, I have found that it pays off for them to know some of the outstanding accomplishments in which the mathematical techniques that they are required to master are involved.

Acknowledgments

A book of such a wide scope cannot be reasonably written by a researcher working in isolation. It has been a pleasure of mine to discuss matters concerning the several fields presented here sometimes by e-mail but more often in personal meetings with the following scientists listed according to the order of the chapters:

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I wish to thank Ann Kostant, Elizabeth Loew, and Tom Grasso of Birkhäuser for the care put into editing my original text.

Finally my thanks go to the Mathematical Sciences Research Institute in Berkeley where I spent the month of July 2000, working hard while enjoying its warm hospitality and the beautiful landscape.

Rome, Italy Elena Prestini
September 2003

Contents

Chapter 1
Joseph Fourier: The Man and the Mathematician

1.1 An adventurous life

Orphaned when still a child, Joseph Fourier went through the French Revolution, the Napoleonic era, and the following Restoration as a high-ranking public servant. He learned on his own "how hard the way up and down another man's stairs is"[1] by experiencing directly how risky it is to please changing lords such as Robespierre, Napoleon, and King Louis XVIII. He started as a convinced Jacobin, maybe impulsive at the time, and ended up as a cautious liberal.

He was a distinguished administrator and diplomat, but more than that he was a scientist whose achievements were truly revolutionary in character. The most relevant aspect of his life, the one for which he is still remembered, is the contribution he made to the advancement of mathematics and physics. It is somewhat miraculous that his most important and creative work took place when he was prefect of Isère, an onerous administrative position he held with first class results. It suffices to mention the construction of the French part of a spectacular road through the Alps from Grenoble to Turin and the draining of twenty million acres of marshes around the village of Bourgoin midway between Lyon and Grenoble, a century-old proposal that nobody previously had been able even to start.

He ranks among the most important scientists of the nineteenth century for his studies on the propagation of heat, the consequences of which reach down to the present day. The question of terrestrial temperature was principally in his mind in establishing the mathematical theory of heat, and the paternity of the expression "greenhouse effect"— *effet de serre*—is attributed to Fourier. He was the first on record to hint at it, writing back in 1827: "The problem of global temperatures, one of the most important and difficult of the whole of natural philosophy, is composed by rather different elements that ought to be considered from a unique general

[1] "come è duro calle lo scendere e 'l salir per l'altrui scale" *The Divine Comedy*, Paradise, XVII, 59–60, by Dante Alighieri (1265–1321).

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point of view.” The novelty of his method, at least initially, perplexed outstanding mathematicians of his time, from Lagrange to Laplace and Poisson. The publication of Fourier’s work was consequently delayed as many as 15 years during which he tenaciously defended, explained, and extended it.

Moreover, it ought to be mentioned that Fourier is among the founders of modern Egyptology: He followed Napoleon in his Egyptian campaign, spent 3 years there, and organized an expedition to Upper Egypt that made important discoveries. They form the basis of a monumental work *Description of Egypt*—21 volumes—for which he supplied a historical introduction. Much under Fourier’s influence, Jacques Joseph Champollion-Figeac and his brother Jean François became important archeologists and Egyptologists.

1.2 The beginnings

Jean Baptiste Joseph Fourier was born on March 21, 1768 in the ancient and beautiful town of Auxerre, 150 kilometers south of Paris in a position dominating the river Yonne, with beautiful churches, above all the ancient fifth century abbey St. Germain and the Gothic cathedral of St. Étienne (Fig. 1.1). Joseph was the name of his father, a master taylor of Auxerre, who had been born to shopkeepers in a small town in Lorraine. Joseph might have been attracted to Auxerre by the rich and powerful

Figure 1.1 The Gothic cathedral of St. Étienne from the east bank of the river Yonne. (Photograph by Jasette Laliaux)

ecclesiastical establishment of the town, which had had its own bishop since Gallo-Roman times. He probably expected some special consideration in memory of his paternal great uncle Pierre Fourier, a leading figure of the Counter Reformation in Lorraine in the sixteenth and early seventeenth centuries. Joseph Fourier married twice. With his first wife he had three children and with the second one 12. Jean Joseph was the ninth of these. His mother Edmie died in 1777 and his father the following year.

Not yet ten years old, Jean Joseph was left an orphan. By all means the young Fourier could have then been considered “lost.” It did not go that way thanks to a certain Madame Moitton who recommended him to the bishop of Auxerre, thus sparing him a life of apprenticeship and servitude. At 12 he entered the local École Royale Militaire, run by the Benedictines.

The various military schools of the country, 11 in all, were placing special importance on the teaching of science and mathematics, due to the requirements of those pupils who were going to enter the specialist corps of artillery and engineers. They were periodically visited by a panel of inspectors, among them the *académicien* and mathematician Adrien Marie Legendre (1752–1833). Fourier, an exceptionally gifted student of happy nature and quick mind, soon showed a great passion for mathematics. He was only 13 when he got into the habit of collecting candle ends to use at night during the long hours he spent studying mathematics in some sort of store room. In so doing he succeeded in unintentionally scaring the deputy principal who, one night while making his rounds of the school, saw a light through the keyhole and rushed in, fearing a fire. Fourier’s overall excellence is confirmed by the prizes he won during 1782 and 1783: in rhetoric, mathematics, mechanics, and even singing. Then a year-long period of illness followed, perhaps due to the excessive intensity of his studying.

His main ambition at the time was a military career. At the age of seventeen, having completed his studies, he wished to enter the artillery or the engineers. In spite of the support of the inspectors of the school, the minister of war rejected his application. The fact that he was not a noble appears to have played a role.

As a second choice he decided to enter the Benedictines. To St. Benoît-sur-Loire, seat of an ancient and splendid basilica—hosting the body and relics of St. Benedict, which were transported there from Monte Cassino in the seventh century—Fourier arrived in 1787. He stayed two years preparing for his vows while teaching other novices. During this period of spreading riots and chaos, preparatory to the revolution, Fourier remained indifferent to political news and obtained results in the theory of equations. In 1789 he sent a paper to the Académie des Sciences in Paris. That was the year of the French Revolution and Fourier’s paper was one of the casualties.

The monastic orders, combining great wealth and a steadily shrinking number of inmates, had long been a strong temptation to a government continually on the verge of bankruptcy. During October 1789, by decree of the Constituent Assembly, it was forbidden to take any further religious vows. A few months later all religious orders were suppressed and subsequently all belongings confiscated.

Fourier himself, who in the meantime had manifested some uncertainty about the choice he had made, did not take his vows and returned to Auxerre to teach mathematics, rhetoric, history, and philosophy in his old school, now having as a second title that of collège national. A commissioner of the local directory who visited the college in October 1792 reported favorably on the health of the inmates and the liberal atmosphere. He deplored only the tendency to drive out Latin to make way for mathematics, so much in demand by the parents of the pupils. Fourier carried out his duties there with dedication and remarkable success that earned him a fine reputation.

1.3 The revolutionary

The town of Auxerre fortunately saw little or no bloodletting during the Revolution, but the local Société Populaire, associated with the revolutionary Jacobin party, was one of the most militant provincial clubs in the country. Fourier's involvement in politics did not occur officially until February 1793. The occasion seems to have been a speech before the local assembly following a decree of the Convention for the draft of 300,000 men. While the quotas of the various departments—the administrative regions into which France was divided in 1790—were fixed, it was left to each department through the vote of the citizens to decide how to meet its quota: by lot, by volunteering, or by other means. This hot issue was debated in Auxerre by a general assembly. Fourier intervened and proposed a plan that was then adopted. In a letter written later, in June 1795 from prison, he makes clear that he fully shared the ideals of the Revolution (Hérivel [975]): "As the natural ideas of equality developed it was possible to conceive the sublime hope of establishing among us a free government exempt from kings and priests, and to free from this double yoke the long usurped soil of Europe. I readily became enamored of this cause, in my opinion the greatest and most beautiful which any nation has ever undertaken."

In March 1793 he was invited to join the local Comité de Surveillance. It is not known whether Fourier wished the invitation or, on the contrary, would have preferred to decline it, after receiving it. Certainly, a refusal would not have been without risks, since by and large the town of Auxerre shared the Republican ideals and a refusal could have identified Fourier as an opponent of the patriot party. From what is known of his later involvement in politics, it is more likely that he eagerly embraced the chance to take part in the defense of the Republic, which was threatened by military reverses in Belgium and by the rebellion in Vendée.

Events were moving fast due to the mounting of internal opposition, which prompted the formation of the Tribunal Révolutionnaire, and the near famine conditions of the population. Already by September of the same year the innocuous committees of surveillance (for strangers and travelers) had been entrusted with universal surveillance and soon were to become an integral part of the Terror, having to proceed by the Law of Suspects of September 17, 1793 to arrest "those who by their conduct, relations or language spoken or written, have shown themselves partisan of tyranny or federalism and enemies of liberty." (During 1793–94 over 200,000 citizens were detained under this law and about 17,000 death sentences were handed down by the revolutionary tribunals and military commissions.) At this point Fourier, feeling "less suited than many others to execute this law," attempted to withdraw and submitted his written resignation. It was not well received. As Fourier relates (Hérivel [1975]):

> This move produced an opposite effect to what I had intended. In a reply sent to me I was reminded of a law which forbade any official to abandon his post and my resignation was rejected. At the same time other persons openly accused me of abandoning my colleagues at a moment when my help was about to become the most useful to them. I was reproached with the feebleness of my conduct and some even doubted the purity of my intentions.

Fourier was not a fanatic; he firmly believed in the ideals of the Revolution and to that he devoted his intelligence, eloquence, and zeal but always retained an independent judgment. This together with his juvenile impulsiveness led him into a dangerous situation. He was sent to the neighboring department of Loiret with the mission of collecting horses for the war effort. He completed that "with every possible success."

Unfortunately on his way through Orléans, in the course of his mission, he became involved in a local dispute. To Orléans, plagued by near starvation and declared in a "state of rebellion," a representative of the people was sent by the Convention. Immediately upon arrival he "purged" the administrative corps of the city, made numerous arrests, and threatened to bring in a movable guillotine like the one in Paris. Thereafter he turned against members of the local Société Populaire, members of his own party. It was at this point that Fourier, "behaving in conformity with the principles of Revolution" took the defense "perhaps imprudent but at least disinterested" of the heads of three local families. Fourier was immediately denounced to the Convention, his commission revoked, and himself declared "incapable of receiving such commissions [in the future]." This took place on October 29, thirteen days after the execution of the queen.

Fourier, in fear, returned to Auxerre where he would have faced the greatest possible danger if the Société Populaire and the Comité de Surveillance had not successfully intervened on behalf of their "young and learned compatriot." He remained a member of the local revolutionary party and kept on teaching in his old school, which was going to be closed down in August 1794. April of that year saw the execution of Danton and of his associates amid the mounting Terror.

Nothing is known of Fourier's feeling during this period except for what he wrote afterward, when he claimed he had spoken out in Auxerre against the excesses of the Revolution. By June 1794 he had become president of the Revolutionary Committee in Auxerre and, therefore, the foremost representative of the Terror in town. This high position is known from an entry in the local archives reporting his arrest: Fourier, always feeling injustice at the decree of the Convention that declared him unfit for "similar commissions" in the future, had gone in person to Robespierre in Paris to plead his case. Perhaps he made a bad impression on him. Certainly upon his return to Auxerre on July 4 he was arrested. Because of the high reputation he enjoyed in town, many interceded in his favor and he was released, only to be rearrested a few days later. Then an official delegation was sent to Saint Just in Paris to demand his release, but salvation came from a totally different route: on July 27 (9 Thermidor) the Convention ordered the arrest of Robespierre and Saint Just. They were to be promptly guillotined the next day without trial. The Terror was over. Fourier's life was saved and he regained his freedom.

1.4 At the École Normale

Fourier, back in Auxerre, was soon going to return to Paris for an event that would have a great impact on his life: the opening of the École Normale.

The Revolution had rid itself of the existing educational system but failed to replace it with anything new. That was not a good prospect for its civilian and military functions. To help repair the damage and remedy an acute shortage of elementary school teachers, by a decree dated October 30, 1794, the Convention set up a national college in Paris, the École Normale.[2] There were to be 1500 students chosen and financed by the districts of the Republic. Fourier was nominated by the neighboring district of St. Florentin, since Auxerre had already made its choice while he was in prison. He accepted after having requested and been granted authorization from "the constituted bodies of the commune of Auxerre." The view of one of his fellow students is interesting (Hérivel [1975]):

> When the pupils [of the École Normale] gathered together, France had only just emerged from beneath the axe of Robespierre. The agents of this tyranny were everywhere regarded with abhorrence: but the fear which they had inspired, joined to a fear of their return to power, retained for them some vestige of credit. They profited from this, by seizing the opportunity of quitting the scene of their vexatious act. Several had themselves named pupils of the École Normale. They carried there with the ignorance proper to them the hate, distrust and contempt which followed them everywhere. Beside them were men full of wisdom, talent and enlightenment, men whose names were celebrated in all Europe.

Fourier was likely to have few regrets at the prospect of leaving Auxerre where, as former president of the Revolutionary Committee, he was a marked man. Compared to the violent whirlpools of the Revolution, at the École Normale must have felt like paradise. The professors were chosen from among the foremost men in the country: Joseph Louis Lagrange (1736–1813), Pierre Simon Laplace (1749–1827) and Gaspard Monge (1746–1816) taught mathematics and for chemistry there was Claude Louis Berthollet (1748–1822). All of them were going to play a role in Fourier's life in the years to come.

The school opened with impressive dispatch on January 20, 1795, amid great enthusiasm. In a letter to J. A. R. Bonard, his former teacher of mathematics in Auxerre, Fourier conveyed a vivid description of the early sessions of the school (Hérivel [1975]):

> The École Normale holds its sessions at the Jardin des Plantes, in a middle-sized place of circular shape; the pupils who are very numerous are seated in rows on the tiers of a very

[2]The present day École Normale descends from the homonymous school established 16 years later during the Napoleonic era, dedicated to training professors of secondary and higher education.

> high amphitheater; there is no room for everyone and every day there are a fair number who find the door closed; if one is obliged to leave during the sessions, one cannot enter again. At the back of the room, within an enclosure separated by a railing, are seated several Parisian scientists and the professors. In front, on a slightly higher platform are three armchairs for the professors who are to speak and their assistants. Behind them, and on a second platform, are the representatives of the people in the uniform of deputies on detached service. The session opens at 11 o'clock when one of the deputies arrives; there is much applause at this moment and when the professor takes his place. The lessons are almost always interrupted and terminated by applause. The pupils keep their hats on, the professor who is speaking is uncovered; three quarters of an hour or an hour later, a second professor takes his place, then a third, and the usher announces that the session is ended.

Fourier, an experienced teacher himself, gives an account of the lecturing habits and idiosyncrasies of the professors. Lagrange, whom Fourier followed with eyes full of admiration, as deserved by "the first among European men of science," had a "rather poor reception" from the students. Incapable of preserving order, he showed his Italian origin by "a very pronounced accent" (he was born in Turin and spent his youth there). He could make some rather comic sentences, such as "There are still on this matter many important things to say, but I shall not say them." Nevertheless to Fourier "the hesitation and simplicity of a child," that sometimes could be seen in Lagrange, made only more apparent "the extraordinary man he is."

Of Laplace, "among the first rank men of science," Fourier writes that "he speaks with precision, but not without a certain difficulty" and that "the mathematical teaching he gives has nothing extraordinary about it and is very rapid." Berthollet, acknowledged as "the greatest chemist we have either in France or abroad," Fourier continues, "only speaks with extreme difficulty, hesitates and repeats himself ten times in one sentence" to conclude that "his course is only understood by those who study much or understand already." Monge instead "speaks with a loud voice" and "the science about which he lectures [descriptive geometry] is presented with infinite care and he expounds it with all possible clarity."

He reports on others including a certain Sicard, "well known as a teacher of deaf-mutes, full of enthusiasm and patience" but "mad." "His theory of grammar, which is brilliant in certain respects, is one of the craziest I know of" (Hérivel [1975]).

On May 1795, a few months after inauguration, the school was closed due to objective difficulties. In particular the seminars, which had been intended as the backbone of the system, were a failure because so few students had learned enough to be able to contribute. For the majority of the pupils the École Normale had been a waste of time, but for Fourier it was a turning point of his career: At those seminars he made his mark. Meanwhile disturbing rumors were coming from Auxerre.

1.5 From imprisonment to the École Polytechnique

The fall of Robespierre marked the beginning of the settling of scores with those associated with him. Many had used their power to commit all sorts of injustices and atrocities: It had been the "Terror." In Auxerre, Fourier's opponents wasted no time.