LEARNING MADE EASY

2nd Edition

Algebra I for dummies

A Wiley Brand

Common operations you'll encounter in algebra

Lessons on solving linear and quadratic equations

How to apply algebra in your everyday life

Mary Jane Sterling

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Algebra I For Dummies®, 2nd Edition

Published by
John Wiley & Sons, Inc.
111 River St.
Hoboken, NJ 07030-5774
www.wiley.com

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Library of Congress Control Number: 2010920659

ISBN 978-1-119-29357-6 (pbk); ISBN 978-1-119-29759-8 (ebk); ISBN 978-1-119-29756-7 (ebk)

Algebra I For Dummies, 2nd Edition (9781119293576) was previously published as *Algebra I For Dummies,* 2nd Edition (9780470559642). While this version features a new Dummies cover and design, the content is the same as the prior release and should not be considered a new or updated product.

Manufactured in the United States of America

10 9 8 7 6 5 4 3 2 1

Contents at a Glance

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Introduction

Let me introduce you to algebra. This introduction is somewhat like what would happen if I were to introduce you to my friend Donna. I'd say, "This is Donna. Let me tell you something about her." After giving a few well-chosen tidbits of information about Donna, I'd let you ask more questions or fill in more details. In this book, you find some well-chosen topics and information, and I try to fill in details as I go along.

As you read this introduction, you're probably in one of two situations:

- You've taken the plunge and bought the book.
- You're checking things out before committing to the purchase.

In either case, you'd probably like to have some good, concrete reasons why you should go to the trouble of reading and finding out about algebra.

One of the most commonly asked questions in a mathematics classroom is, "What will I ever use this for?" Some teachers can give a good, convincing answer. Others hem and haw and stare at the floor. My favorite answer is, "Algebra gives you *power.*" Algebra gives you the *power* to move on to bigger and better things in mathematics. Algebra gives you the *power* of knowing that you know something that your neighbor doesn't know. Algebra gives you the *power* to be able to help someone else with an algebra task or to explain to your child these logical mathematical processes.

Algebra is a system of symbols and rules that is universally understood, no matter what the spoken language. Algebra provides a clear, methodical process that can be followed from beginning to end. It's an organizational tool that is most useful when followed with the appropriate rules. What *power!* Some people like algebra because it can be a form of puzzle-solving. You solve a puzzle by finding the value of a variable. You may prefer Sudoku or Ken Ken or crosswords, but it wouldn't hurt to give algebra a chance, too.

About This Book

This book isn't like a mystery novel; you don't have to read it from beginning to end. In fact, you can peek at how it ends and not spoil the rest of the story.

I divide the book into some general topics — from the beginning nuts and bolts to the important tool of factoring to equations and applications. So you can dip into the book wherever you want, to find the information you need.

Throughout the book, I use many examples, each a bit different from the others, and each showing a different *twist* to the topic. The examples have explanations to aid your understanding. (What good is knowing the answer if you don't know how to get the right answer yourself?)

The vocabulary I use is mathematically correct *and* understandable. So whether you're listening to your teacher or talking to someone else about algebra, you'll be speaking the same language.

Along with the *how*, I show you the *why.* Sometimes remembering a process is easier if you understand why it works and don't just try to memorize a meaningless list of steps.

Conventions Used in This Book

I don't use many conventions in this book, but you should be aware of the following:

- When I introduce a new term, I put that term in *italics* and define it nearby (often in parentheses).
- I express numbers or numerals either with the actual symbol, such as 8, or the written-out word: *eight.* Operations, such as +, are either shown as this symbol or written as *plus.* The choice of expression all depends on the situation — and on making it perfectly clear for you.

What You're Not to Read

The *sidebars* (those little gray boxes) are interesting but not essential to your understanding of the text. If you're short on time, you can skip the sidebars. Of course, if you read them, I think you'll be entertained.

You can also skip anything marked by a Technical Stuff icon (see "Icons Used in This Book," for more information).

Foolish Assumptions

I don't assume that you're as crazy about math as I am — and you may be even *more* excited about it than I am! I do assume, though, that you have a mission here — to brush up on your skills, improve your mind, or just have some fun. I also assume that you have some experience with algebra — full exposure for a year or so, maybe a class you took a long time ago, or even just some preliminary concepts.

If you went to junior high school or high school in the United States, you probably took an algebra class. If you're like me, you can distinctly remember your first (or only) algebra teacher. I can remember Miss McDonald saying, "This is an *n*." My whole secure world of numbers was suddenly turned upside down. I hope your first reaction was better than mine.

You may be delving into the world of algebra again to refresh those long-ago lessons. Is your kid coming home with assignments that are beyond your memory? Are you finally going to take that calculus class that you've been putting off? Never fear. Help is here!

How This Book Is Organized

Where do you find what you need quickly and easily? This book is divided into parts dealing with the most frequently discussed and studied concepts of basic algebra.

Part 1: Starting Off with the Basics

The "founding fathers" of algebra based their rules and conventions on the assumption that everyone would agree on some things first and adopt the process. In language, for example, we all agree that the English word for *good* means the same thing whenever it appears. The same goes for algebra. Everyone uses the same rules of addition, subtraction, multiplication, division, fractions, exponents, and so on. The algebra wouldn't work if the basic rules were different for different people. We wouldn't be able to communicate. This part reviews what all these things are that everyone has agreed on over the years.

The chapters in this part are where you find the basics of arithmetic, fractions, powers, and signed numbers. These tools are necessary to be able to deal with the algebraic material that comes later. The review of basics here puts a spin on the more frequently used algebra techniques. If you want, you can skip these chapters and just refer to them when you're working through the material later in the book.

In these first chapters, I introduce you to the world of letters and symbols. Studying the use of the symbols and numbers is like studying a new language. There's a vocabulary, some frequently used phrases, and some cultural applications. The language is the launching pad for further study.

Part 2: Figuring Out Factoring

Part 2 contains factoring and simplifying. Algebra has few processes more important than factoring. Factoring is a way of rewriting expressions to help make solving the problem easier. It's where expressions are changed from addition and subtraction to multiplication and division. The easiest way to solve many problems is to work with the wonderful *multiplication property of zero,* which basically says that to get a 0 you multiply by 0. Seems simple, and yet it's really grand.

Some factorings are simple — you just have to recognize a similarity. Other factorings are more complicated — not only do you have to recognize a pattern, but you have to know the rule to use. Don't worry — I fill you in on all the differences.

Part 3: Working Equations

The chapters in this part are where you get into the nitty-gritty of finding answers. Some methods for solving equations are elegant; others are down and dirty. I show you many types of equations and many methods for solving them.

Usually, I give you one method for solving each type of equation, but I present alternatives when doing so makes sense. This way, you can see that some methods are better than others. An underlying theme in all the equation-solving is to check your answers — more on that in this part.

Part 4: Applying Algebra

The whole point of doing algebra is in this part. There are everyday formulas and not-so-everyday formulas. There are familiar situations and situations that may be totally unfamiliar. I don't have space to show you every possible type of problem, but I give you enough practical uses, patterns, and skills to prepare you for many of the situations you encounter. I also give you some graphing basics in this part. A picture is truly worth a thousand words, or, in the case of mathematics, a graph is worth an infinite number of points.

Part 5: The Part of Tens

Here I give you ten important tips: how to avoid the most common algebraic pitfalls. You also find my choice for the ten most famous equations. (You may have other favorites, but these are my picks.)

Icons Used in This Book

The little drawings in the margin of the book are there to draw your attention to specific text. Here are the icons I use in this book:

MATH RULES

To make everything work out right, you have to follow the basic rules of algebra (or mathematics in general). You can't change or ignore them and arrive at the right answer. Whenever I give you an algebra rule, I mark it with this icon.

EXAMPLE

An explanation of an algebraic process is fine, but an example of how the process works is even better. When you see the Example icon, you'll find one or more problems using the topic at hand.

REMEMBER

Paragraphs marked with the Remember icon help clarify a symbol or process. I may discuss the topic in another section of the book, or I may just remind you of a basic algebra rule that I discuss earlier.

TECHNICAL STUFF

The Technical Stuff icon indicates a definition or clarification for a step in a process, a technical term, or an expression. The material isn't absolutely necessary for your understanding of the topic, so you can skip it if you're in a hurry or just aren't interested in the nitty-gritty.

TIP

The Tip icon isn't life-or-death important, but it generally can help make your life easier — at least your life in algebra.

WARNING

The Warning icon alerts you to something that can be particularly tricky. Errors crop up frequently when working with the processes or topics next to this icon, so I call special attention to the situation so you won't fall into the trap.

Where to Go from Here

If you want to refresh your basic skills or boost your confidence, start with Part 1. If you're ready for some factoring practice and need to pinpoint which method to use with what, go to Part 2. Part 3 is for you if you're ready to solve equations; you can find just about any type you're ready to attack. Part 4 is where the good stuff is — applications — things to do with all those good solutions. The lists in Part 5 are usually what you'd look at after visiting one of the other parts, but why not start there? It's a fun place! When the first edition of this book came out, my mother started by reading all the sidebars. Why not?

Studying algebra can give you some logical exercises. As you get older, the more you exercise your brain cells, the more alert and "with it" you remain. "Use it or lose it" means a lot in terms of the brain. What a good place to use it, right here!

The best *why* for studying algebra is just that it's beautiful. Yes, you read that right. Algebra is poetry, deep meaning, and artistic expression. Just look, and you'll find it. Also, don't forget that it gives you *power.*

Welcome to algebra! Enjoy the adventure!

1 Starting Off with the Basics

IN THIS PART . . .

Could you just up and go on a trip to a foreign country on a moment's notice? If you're like most people, probably not. Traveling abroad takes preparation and planning: You need to get your passport renewed, apply for a visa, pack your bags with the appropriate clothing, and arrange for someone to take care of your pets. In order for the trip to turn out well and for everything to go smoothly, you need to prepare. You even make provisions in case your bags don't arrive with you! The same is true of algebra: It takes preparation for the algebraic experience to turn out to be a meaningful one. Careful preparation prevents problems along the way and helps solve problems that crop up in the process. In this part, you find the essentials you need to have a successful algebra adventure.

IN THIS CHAPTER

- **Giving names to the basic numbers**
- **Reading the signs — and interpreting the language**
- **Operating in a timely fashion**

Chapter 1

Assembling Your Tools

You've probably heard the word *algebra* on many occasions, and you knew that it had something to do with mathematics. Perhaps you remember that algebra has enough information to require taking two separate high school algebra classes — Algebra I and Algebra II. But what exactly *is* algebra? What is it *really* used for?

This book answers these questions and more, providing the straight scoop on some of the contributions to algebra's development, what it's good for, how algebra is used, and what tools you need to make it happen. In this chapter, you find some of the basics necessary to more easily find your way through the different topics in this book. I also point you toward these topics.

In a nutshell, *algebra* is a way of generalizing arithmetic. Through the use of *variables* (letters representing numbers) and formulas or equations involving those variables, you solve problems. The problems may be in terms of practical applications, or they may be puzzles for the pure pleasure of the solving. Algebra uses positive and negative numbers, integers, fractions, operations, and symbols to analyze the relationships between values. It's a systematic study of numbers and their relationship, and it uses specific rules.

Beginning with the Basics: Numbers

Where would mathematics and algebra be without numbers? A part of everyday life, numbers are the basic building blocks of algebra. Numbers give you a value to work with. Where would civilization be today if not for numbers? Without numbers to figure the distances, slants, heights, and directions, the pyramids would never have been built. Without numbers to figure out navigational points, the Vikings would never have left Scandinavia. Without numbers to examine distance in space, humankind could not have landed on the moon.

Even the simple tasks and the most common of circumstances require a knowledge of numbers. Suppose that you wanted to figure the amount of gasoline it takes to get from home to work and back each day. You need a number for the total miles between your home and business and another number for the total miles your car can run on a gallon of gasoline.

The different sets of numbers are important because what they look like and how they behave can set the scene for particular situations or help to solve particular problems. It's sometimes really convenient to declare, "I'm only going to look at whole-number answers," because whole numbers do not include fractions or negatives. You could easily end up with a fraction if you're working through a problem that involves a number of cars or people. Who wants half a car or, heaven forbid, a third of a person?

Algebra uses different sets of numbers, in different circumstances. I describe the different types of numbers here.

Really real numbers

Real numbers are just what the name implies. In contrast to imaginary numbers, they represent *real* values — no pretend or make-believe. Real numbers cover the gamut and can take on any form — fractions or whole numbers, decimal numbers that can go on forever and ever without end, positives and negatives. The variations on the theme are endless.

Counting on natural numbers

A *natural number* (also called a *counting number*) is a number that comes naturally. What numbers did you first use? Remember someone asking, "How old are you?" You proudly held up four fingers and said, "Four!" The natural numbers are the numbers starting with 1 and going up by ones: 1, 2, 3, 4, 5, 6, 7, and so on into infinity. You'll find lots of counting numbers in Chapter 6, where I discuss prime numbers and factorizations.

AHA ALGEBRA

Dating back to about 2000 B.C. with the Babylonians, algebra seems to have developed in slightly different ways in different cultures. The Babylonians were solving three-term quadratic equations, while the Egyptians were more concerned with linear equations. The Hindus made further advances in about the sixth century A.D. In the seventh century, Brahmagupta of India provided general solutions to quadratic equations and had interesting takes on 0. The Hindus regarded irrational numbers as actual numbers — although not everybody held to that belief.

The sophisticated communication technology that exists in the world now was not available then, but early civilizations still managed to exchange information over the centuries. In A.D. 825, al-Khowarizmi of Baghdad wrote the first algebra textbook. One of the first solutions to an algebra problem, however, is on an Egyptian papyrus that is about 3,500 years old. Known as the Rhind Mathematical Papyrus after the Scotsman who purchased the 1-foot-wide, 18-foot-long papyrus in Egypt in 1858, the artifact is preserved in the British Museum — with a piece of it in the Brooklyn Museum. Scholars determined that in 1650 B.C., the Egyptian scribe Ahmes copied some earlier mathematical works onto the Rhind Mathematical Papyrus.

One of the problems reads, "Aha, its whole, its seventh, it makes 19." The *aha* isn't an exclamation. The word *aha* designated the unknown. Can you solve this early Egyptian problem? It would be translated, using current algebra symbols, as: $x+\frac{x}{7}=19$. The unknown is represented by the *x*, and the solution is $x=16\frac{5}{8}$. It's not hard; it's just messy.

Wholly whole numbers

Whole numbers aren't a whole lot different from natural numbers. Whole numbers are just all the natural numbers plus a 0: 0, 1, 2, 3, 4, 5, and so on into infinity.

Whole numbers act like natural numbers and are used when whole amounts (no fractions) are required. Zero can also indicate none. Algebraic problems often require you to round the answer to the nearest whole number. This makes perfect sense when the problem involves people, cars, animals, houses, or anything that shouldn't be cut into pieces.

Integrating integers

Integers allow you to broaden your horizons a bit. Integers incorporate all the qualities of whole numbers and their opposites (called their *additive inverses*). *Integers* can be described as being positive and negative whole numbers: . . . −3, −2, −1, 0, 1, 2, 3, ...

Integers are popular in algebra. When you solve a long, complicated problem and come up with an integer, you can be joyous because your answer is probably right. After all, it's not a fraction! This doesn't mean that answers in algebra can't be fractions or decimals. It's just that most textbooks and reference books try to stick with nice answers to increase the comfort level and avoid confusion. This is my plan in this book, too. After all, who wants a messy answer, even though, in real life, that's more often the case. I use integers in Chapters 8 and 9, where you find out how to solve equations.

Being reasonable: Rational numbers

Rational numbers act rationally! What does that mean? In this case, acting rationally means that the decimal equivalent of the rational number behaves. The decimal ends somewhere, or it has a repeating pattern to it. That's what constitutes "behaving."

Some rational numbers have decimals that end such as: 3.4, 5.77623, −4.5. Other rational numbers have decimals that repeat the same pattern, such as $3.164164\overline{164}$, or $0.66666666\overline{6}$. The horizontal bar over the 164 and the 6 lets you know that these numbers repeat forever.

In *all* cases, rational numbers can be written as fractions. Each rational number has a fraction that it's equal to. So one definition of a *rational number* is any number that can be written as a fraction, $\frac{p}{q}$, where p and q are integers (except q can't be 0). If a number can't be written as a fraction, then it isn't a rational number. Rational numbers appear in Chapter 13, where you see quadratic equations, and in Part 4, where the applications are presented.

Restraining irrational numbers

Irrational numbers are just what you may expect from their name — the opposite of rational numbers. An *irrational number* cannot be written as a fraction, and decimal values for irrationals never end and never have a nice pattern to them. Whew! Talk about irrational! For example, pi, with its never-ending decimal places, is irrational. Irrational numbers are often created when using the quadratic formula, as you see in Chapter 13.

Picking out primes and composites

A number is considered to be *prime* if it can be divided evenly only by 1 and by itself. The first prime numbers are: 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, and so on. The only prime number that's even is 2, the first prime number. Mathematicians have been studying prime numbers for centuries, and prime numbers have them stumped. No one has ever found a formula for producing all the primes. Mathematicians just assume that prime numbers go on forever.

A number is *composite* if it isn't prime — if it can be divided by at least one number other than 1 and itself. So the number 12 is composite because it's divisible by 1, 2, 3, 4, 6, and 12. Chapter 6 deals with primes, but you also see them in Chapters 8 and 10, where I show you how to factor primes out of expressions.

Speaking in Algebra

Algebra and symbols in algebra are like a foreign language. They all mean something and can be translated back and forth as needed. It's important to know the vocabulary in a foreign language; it's just as important in algebra.

- An *expression* is any combination of values and operations that can be used to show how things belong together and compare to one another. $2x^2 + 4x$ is an example of an expression. You see distributions over expressions in Chapter 7.
- A *term,* such as $4xy$, is a grouping together of one or more *factors* (variables and/or numbers). Multiplication is the only thing connecting the number with the variables. Addition and subtraction, on the other hand, separate terms from one another. For example, the expression $3xy + 5x - 6$ has three *terms.*
- An *equation* uses a sign to show a relationship — that two things are equal. By using an equation, tough problems can be reduced to easier problems and simpler answers. An example of an equation is $2x^2 + 4x = 7$. See the chapters in Part 3 for more information on equations.
- An *operation* is an action performed upon one or two numbers to produce a resulting number. Operations are addition, subtraction, multiplication, division, square roots, and so on. See Chapter 5 for more on operations.
- A *variable* is a letter representing some unknown; a variable always represents a number, but it *varies* until it's written in an equation or inequality. (An *inequality* is a comparison of two values. For more on inequalities, turn to Chapter 15.) Then the fate of the variable is set — it can be solved for, and its value becomes the solution of the equation. By convention, mathematicians usually assign letters at the end of the alphabet to be variables (such as x, y, and z).

- A *constant* is a value or number that never changes in an equation — it's constantly the same. Five is a constant because it is what it is. A variable can be a constant if it is assigned a definite value. Usually, a variable representing a constant is one of the first letters in the alphabet. In the equation $ax^2 + bx + c = 0$, a, b, and c are constants and the x is the variable. The value of x depends on what a, b, and c are assigned to be.
- An *exponent* is a small number written slightly above and to the right of a variable or number, such as the 2 in the expression 3^2. It's used to show repeated multiplication. An exponent is also called the *power* of the value. For more on exponents, see Chapter 4.

Taking Aim at Algebra Operations

In algebra today, a variable represents the unknown. (You can see more on variables in the "Speaking in Algebra" section earlier in this chapter.) Before the use of symbols caught on, problems were written out in long, wordy expressions. Actually, using letters, signs, and operations was a huge breakthrough. First, a few operations were used, and then algebra became fully symbolic. Nowadays, you may see some words alongside the operations to explain and help you understand, like having subtitles in a movie.

By doing what early mathematicians did — letting a variable represent a value, then throwing in some operations (addition, subtraction, multiplication, and division), and then using some specific rules that have been established over the years — you have a solid, organized system for simplifying, solving, comparing, or confirming an equation. That's what algebra is all about: That's what algebra's good for.

Deciphering the symbols

The basics of algebra involve symbols. Algebra uses symbols for quantities, operations, relations, or grouping. The symbols are shorthand and are much more efficient than writing out the words or meanings. But you need to know what the symbols represent, and the following list shares some of that info. The operations are covered thoroughly in Chapter 5.

- + means *add* or *find the sum, more than,* or *increased by;* the result of addition is the *sum.* It also is used to indicate a *positive number.*

- **–** means *subtract* or *minus* or *decreased by* or *less than;* the result is the *difference.* It's also used to indicate a *negative number.*
- × means *multiply* or *times.* The values being multiplied together are the *multipliers* or *factors;* the result is the *product.* Some other symbols meaning *multiply* can be grouping symbols: ( ), [ ], { }, ·, *. In algebra, the × symbol is used infrequently because it can be confused with the variable *x.* The dot is popular because it's easy to write. The grouping symbols are used when you need to contain many terms or a messy expression. By themselves, the grouping symbols don't mean to multiply, but if you put a value in front of a grouping symbol, it means to multiply.
- ÷ means *divide*. The number that's going into the *dividend* is the *divisor.* The result is the *quotient.* Other signs that indicate division are the fraction line and slash, /.
- $\sqrt{\ }$ means to take the *square root* of something — to find the number, which, multiplied by itself, gives you the number under the sign. (See Chapter 4 for more on square roots.)
- $|\ |$ means to find the *absolute value* of a number, which is the number itself or its distance from 0 on the number line. (For more on absolute value, turn to Chapter 2.)
- π is the Greek letter pi that refers to the irrational number: 3.14159… . It represents the relationship between the diameter and circumference of a circle.

Grouping

When a car manufacturer puts together a car, several different things have to be done first. The engine experts have to construct the engine with all its parts. The body of the car has to be mounted onto the chassis and secured, too. Other car specialists have to perform the tasks that they specialize in as well. When these tasks are all accomplished in order, then the car can be put together. The same thing is true in algebra. You have to do what's inside the *grouping* symbol before you can use the result in the rest of the equation.

Grouping symbols tell you that you have to deal with the *terms* inside the grouping symbols *before* you deal with the larger problem. If the problem contains grouped items, do what's inside a grouping symbol first, and then follow the order of operations. The grouping symbols are

- **Parentheses ( ):** Parentheses are the most commonly used symbols for grouping.

- **Brackets [ ] and braces { }:** Brackets and braces are also used frequently for grouping and have the same effect as parentheses. Using the different types of symbols helps when there's more than one grouping in a problem. It's easier to tell where a group starts and ends.
- **Radical $\sqrt{\ }$:** This is used for finding roots.
- **Fraction line (called the *vinculum*):** The fraction line also acts as a grouping symbol — everything above the line (in the *numerator*) is grouped together, and everything below the line (in the *denominator*) is grouped together.

Even though the order of operations and grouping-symbol rules are fairly straightforward, it's hard to describe, in words, all the situations that can come up in these problems. The examples in Chapters 5 and 7 should clear up any questions you may have.

Defining relationships

Algebra is all about relationships — not the he-loves-me-he-loves-me-not kind of relationship — but the relationships between numbers or among the terms of an equation. Although algebraic relationships can be just as complicated as romantic ones, you have a better chance of understanding an algebraic relationship. The symbols for the relationships are given here. The equations are found in Chapters 11 through 14, and inequalities are found in Chapter 15.

- $=$ means that the first value *is equal to* or the same as the value that follows.
- $\neq$ means that the first value *is not equal to* the value that follows.
- $\approx$ means that one value is *approximately the same* or *about the same* as the value that follows; this is used when rounding numbers.
- $\leq$ means that the first value is *less than or equal to* the value that follows.
- $<$ means that the first value is *less than* the value that follows.
- $\geq$ means that the first value is *greater than or equal to* the value that follows.
- $>$ means that the first value is *greater than* the value that follows.

Taking on algebraic tasks

Algebra involves symbols, such as variables and operation signs, which are the tools that you can use to make algebraic expressions more usable and readable. These things go hand in hand with simplifying, factoring, and solving problems,