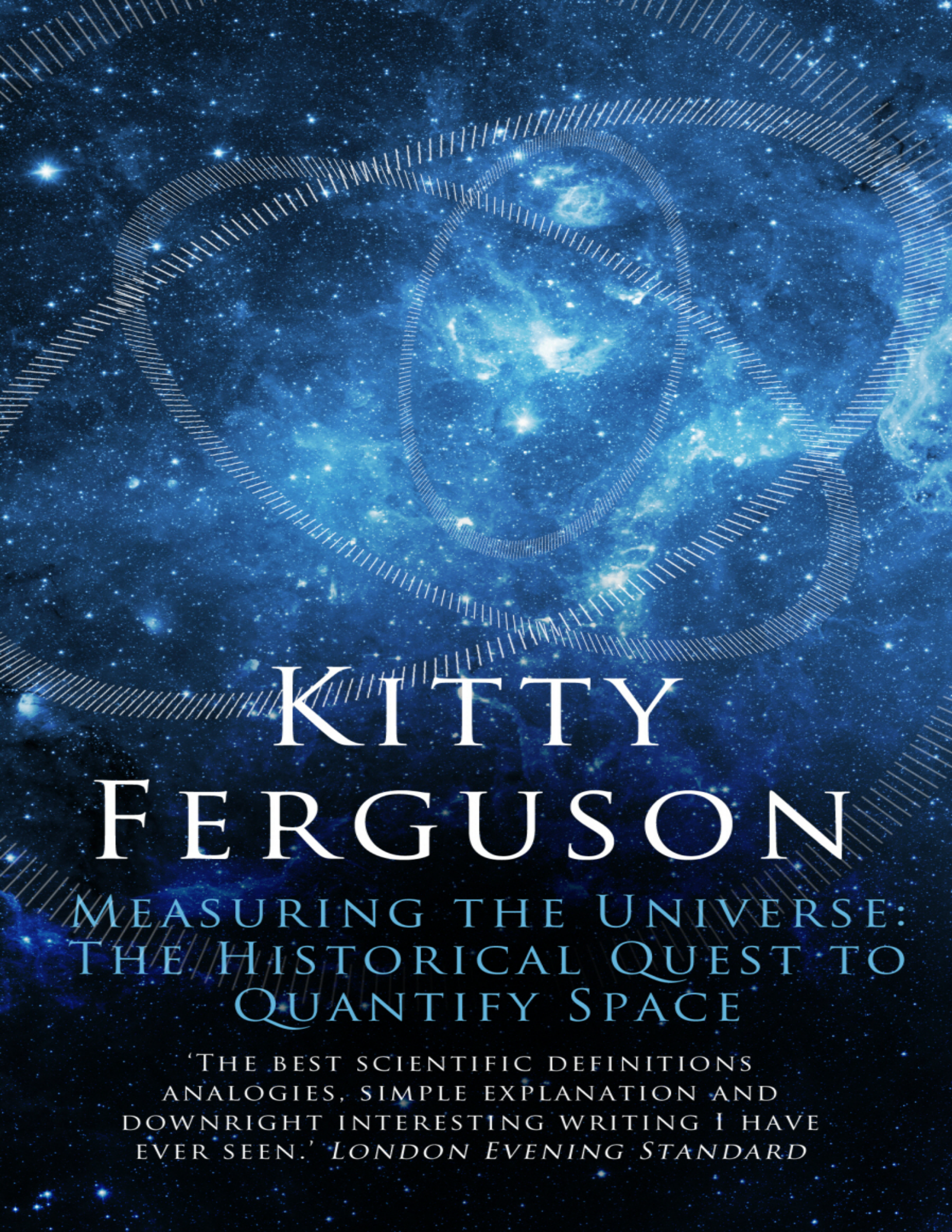




KITTY FERGUSON

MEASURING THE UNIVERSE:
THE HISTORICAL QUEST TO
QUANTIFY SPACE

'THE BEST SCIENTIFIC DEFINITIONS
ANALOGIES, SIMPLE EXPLANATION AND
DOWNRIGHT INTERESTING WRITING I HAVE
EVER SEEN.' *LONDON EVENING STANDARD*

About the Book

Suppose you and I still wondered whether all of the pinpoints of light in the night sky are the same distance from us. Suppose none of our contemporaries could tell us whether the Sun orbits the Earth, or vice versa, or even how large the Earth is. Suppose no one had guessed there are mathematical laws underlying the motions of the heavens.

How would - how did - anyone begin to discover these numbers and these relationships without leaving the Earth? What made anyone even think it was possible to find out "how far," without going there? In *Measuring the Universe* we join our ancestors and contemporary scientists as they tease this information out of a sky full of stars. Some of the questions have turned out to be loaded, and a great deal besides mathematics and astronomy has gone into answering them. Politics, religion, philosophy and personal ambition: all have played roles in this drama.

There are poignant personal stories, of people like Copernicus, Kepler, Newton, Herschel, and Hubble. Today scientists are attempting to determine the distance to objects near the borders of the observable universe, far beyond anything that can be seen with the naked eye in the night sky, and to measure time back to its origin. The numbers are too enormous to comprehend.

Nevertheless, generations of curious people have figured them out, one resourceful step at a time. Progress has

owed as much to raw ingenuity as to technology, and frontier inventiveness is still not out of date.

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MEASURING
The **UNIVERSE**

**THE HISTORICAL QUEST
TO QUANTIFY SPACE**

KITTY FERGUSON

To my brother, David, who made himself ill as a child and
caused a family crisis, worrying about the size of the
universe

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PROLOGUE

Tilting at Windmills 1951



WHEN I WAS nine years old, my father suggested one morning that he and my brother and I go out and measure the height of the windmill on my grandparents' farm. My brother and I agreed that was a fine idea.

How would we do it? Climb the windmill, of course . . . at least my father would. My brother and I wouldn't be allowed to try anything so dangerous as that. When my father reached the top, there would still be the problem of how to measure the height. We didn't have a measuring tape that long. Would he take a yardstick and mark off the yards on the windmill as he climbed? Maybe he would drop the end of a long rope from up there and cut it off, and we would stand clear while the cut-off piece fell, and then we would measure it. That must be the plan, for he'd said my brother and I would help him.

My brother suggested that my father wouldn't need to climb the windmill at all. We could throw something over the top, just clearing it. Yes, I interrupted, attach a rope to the thing we threw, a rope with inches and feet marked on it, and then pull back on it gently so it would catch on the top of the windmill, and see what the measurement was to the ground! No, no, said my brother, who was two years younger than I but already very mathematically minded, we would measure the curve the object followed through the air. Good thinking, said my father, but, practically speaking,

more difficult than the original problem of measuring the height of the windmill.

I asked whether we might walk away from the windmill and measure how much smaller it looked as we got further away. More good thinking, said my father, but there was a better way.

He gave us a hint. He'd thought it was a wise idea to wait for a sunny day . . . and no one would have to climb the windmill or take a walk or risk wrecking the windmill with a bad throw . . . and the only tools we'd need would be a yardstick and our eyes and brains and a pencil and paper to do some calculations. And although at this latitude it would be possible to measure the windmill precisely at noon, it would be easier at another time of day.

Neither my brother nor I was clever enough to see where this was leading until my father said, 'The windmill does more than just pump water, you know. It casts a shadow, and so does a yardstick,' and then we began to understand how the trick could be done. We would stand the yardstick upright and measure its shadow. Then we would measure the windmill's shadow. If a shadow *this* long went with a three-foot stick, then a shadow *that* long went with a windmill of thus-and-so height. My brother and I didn't know how to make the comparison. My father taught us how and then pointed out that there was actually a more primitive way to find the answer. Wait for the time of day when the three-foot yardstick cast a three-foot shadow. At that moment the length of the windmill's shadow would be the same as the height of the windmill. We decided to use our newly acquired mathematics first, and then we checked our answer by sitting out in the Texas sun, watching the shadow of the yardstick creep along the ground.

That's how we measured the windmill, while above our heads the giant structure thrummed and creaked with the watery, metallic sounds windmills in central Texas made in those days, doing its work, turning and pumping, adjusting

its angle to catch a stronger breeze, not paying any attention to the mental exertions of three little people below who had captured its shadow.

I was elated. It seemed we had outwitted the windmill without so much as touching it, and now we knew a wondrous secret: Not the height of the windmill, but how to find it out. None of us thought to ask: why are we doing this? There was no need whatsoever for any of us three to know the height of a windmill that wasn't even our own.

Measuring is one of the more practical uses for mathematics, but our ability and desire to measure isn't always wrapped up with the need to know useful answers. Going with numbers where we can't go in person - whether that's to the top of a windmill or to the origin and borders of the universe - has been and still is one of humankind's favourite intellectual adventures. By the beginning of the twenty-first century it had outrun our practical requirements by billions of light years.

Compared with the adventure of finding them out, the actual measurements often seem dry as dust: The Sun is 149.5 million kilometres away (mean distance). The nearest star is 4.3 light years. The 'Local Group' of galaxies covers an area about 3 million light years in diameter. The distance to the edge of the observable universe is 13.7 billion light years. We shake our heads at how large these numbers are or admit their largeness makes them meaningless, remember them for a day or maybe long enough for a school exam . . . and then forget them. Science trivia.

Not trivial at all when you realize how hard-won these numbers are and what ingenuity it took and still takes to find them out. Can we even begin to imagine what it would be like if no one knew them? The night sky sparkles with pinpricks of light. Are these all the same distance from us? Suppose we didn't know. Suppose none of our

contemporaries knew whether the Sun orbits the Earth, or vice versa, or even how large the Earth is. Suppose no one had guessed there are mathematical laws underlying the motion of the heavens. How would – how did – anyone begin to discover these numbers and these relationships without leaving the Earth? What made anyone even think it was possible to find out ‘how far’? Without going there. Without climbing the windmill.

In the pages to come we’ll take many steps back, forget we know the measurements or how to make them, and join our ancestors as they tease this information out of a sky full of stars. The laboratory isn’t a neat, sterile room where carefully controlled experiments take place. Events in the heavens happen in their own good time and not before, and they are often not repeatable. We have learned to take what’s on offer and make the best of it.

Our human point of view is sorely limited. Until recently we had no ground on which to stand and take our measurements, no possible viewing platform, other than here on Earth. In the twentieth century we travelled to the Moon and looked back at our planet from space and sent probes out into the far reaches of our solar system. But by universal standards, by the standards of the distances we’ve learned to measure and still hope to measure, how pitifully close to home that is.

This book is a chronicle of how men and women over the course of two and a half millennia have built a ladder of measurement from our doorsteps to the borders of the known universe, and how the adventure has changed our ideas about the shape and nature of the universe and our place in it. It is not a history of all astronomy. There are fascinating discoveries, both in Western astronomy and in other cultures, that I have had to remind myself have no direct bearing on our knowledge of distances, size and shape. With regret, I have left them out, though the

temptation to embark on long digressions from the main theme of the book has been almost irresistible.

We shall however broaden our focus in another direction to examine the context in which the discoveries have taken place, for this is a story inextricably bound up with the rest of social, political and intellectual history. One of our tasks will be to look for reasons why a particular discovery or measurement happened when and where it did. What was it about that time and place, that society, that mindset or intellectual milieu, the available technology, the chain of previous discovery, the way some random occurrences fell out . . . perhaps most interesting of all, what was it about a specific individual that precipitated this advance in knowledge?

The story of our wanting to know ‘how far’ – to make ridiculously out-of-reach measurements – must surely have begun before the beginning of recorded history. The known story of our success began some 2,200 years ago in north Africa near the mouth of the Nile with the measurement of the circumference of our own planet, long before anyone was able to circumnavigate it. The Hellenistic librarian Eratosthenes didn’t need to know the circumference of the Earth. Nevertheless, he set about measuring it and he did it in a remarkably simple way. We now call Eratosthenes the father of the science of Earth measurement, ‘geodesy’. The word has the sound of ‘odyssey’ in it.

We learn from Eratosthenes what I learned from my father . . . and we shall see it demonstrated repeatedly in this book: what can’t be measured directly – what it is unthinkable that we should ever measure directly – *can* be measured in roundabout, inventive ways. In the first decade of the twenty-first century we determined the distance to the borders of the observable universe, far beyond any pinprick of light we see with the naked eye in the night sky, and measured time back to the origin of the universe. The numbers are indeed too enormous to make

sense to our little minds. Nevertheless our little minds have figured them out, one resourceful step at a time, each step building upon the last. It has been a history of astounding improvement in our technology, particularly in the twentieth century, but more than that, a history of raw ingenuity. It still is. Frontier inventiveness is not out of date.

With the benefit of hindsight we may be tempted to exclaim, 'Of course! Why . . . *I* could have thought of that! The windmill has a shadow. Of course!' for many of the methods we've devised to measure distances to out-of-reach places are simple enough for nearly everyone to understand – only a little more complicated than measuring my grandfather's windmill. But to figure these things out for the first time . . . how impossibly clever!

Kitty Ferguson,
November 2012

CHAPTER 1
A Sphere with a View
Third Century BC



The great mind, like the small, experiments with different alternatives, works out their consequences for some distance, and thereupon guesses (much like a chess player) that one move will generate richer possibilities than the rest . . . It still remains to ask how the great mind comes to guess better than another, and to make leaps that turn out to lead further and deeper than yours or mine. We do not know.

Jacob Bronowski

ASK WHO ERATOSTHENES of Cyrene was, and unless you are talking to someone who specializes in the minutiae of Hellenistic culture, you are unlikely to hear that he was a man who attempted to fix the dates of the major literary and political events from the conquest of Troy until his own time in the third century BC, that he composed a treatise about theatres and theatrical apparatus and the works of the best-known comic poets of the 'old comedy'; that he suggested a way of solving a problem that had tantalized mathematicians for two centuries - 'duplicating a cube'; that he let his voice be heard on the subject of moral philosophy and felt it essential to criticize those who were 'popularizing' philosophy, accusing them of 'dressing it up in the gaudy apparel of loose women'. It is true for Eratosthenes, as it is for many celebrated figures, that the strokes of genius for which he is revered were only a

minuscule part of a lifetime of achievement, and not necessarily the part he judged most important.

Nothing on the list above won Eratosthenes his place in the history books. Two additional accomplishments did: the invention of 'the sieve of Eratosthenes' - a method for sifting through all the numbers to find which are prime numbers; and his remarkably accurate measurement of the circumference of the Earth.

Dismiss any thought that before Columbus no one knew the Earth was round. Admittedly, the shape of the Earth probably wasn't of much daily practical interest to most people in the ancient world. However, long before even Eratosthenes, those few who were wondering about it at all were not seriously suggesting that the Earth was flat or, indeed, any shape but spherical. The Pythagoreans, a school of thinkers with particular genius for mathematics and music, had decided as early as the sixth and fifth centuries BC that the Earth is a sphere. Plato, still a century before Eratosthenes, pictured a cosmos made up of spheres within spheres, nested one within another, with a spherical Earth at the centre. Aristotle, only a little later than Plato, vigorously subscribed to the idea of a spherical Earth, and his defence proved convincing not only to the ancient world but also to the Middle Ages. The idea that scholars of the Middle Ages believed the world was flat is, in fact, a myth created in modern times.

Aristotle used a number of arguments. During an eclipse of the Moon, the shadow cast by the Earth on the Moon is always curved. When we on the Earth move from north to south or vice versa we notice what appears to be a change in the position of the stars in relation to ourselves. In Aristotle's words:

There is much change, I mean, in the stars which are overhead, and the stars seen are different, as one moves northward or southward. Indeed there are

some stars seen in Egypt and in the neighborhood of Cyprus which are not seen in the northerly regions; and stars which in the north are never beyond the range of observation, in those regions rise and set. All of which goes to show not only that the Earth is circular in shape, but also that it is a sphere of no great size: for otherwise the effect of so slight a change of place would not be so quickly apparent.

Aristotle speculated that the oceans of the extreme west and the extreme east of the known world might be 'one', and he reported with some sympathy the arguments of those who had noticed that elephants appeared in regions to the extreme east and the extreme west, and who thought therefore that those regions might be 'continuous'.

These reasons for belief in a spherical Earth came from observation, but Aristotle also argued on the basis of his philosophy. In that philosophy, five elements, earth, air, water, fire and aether, each have a natural place in the universe. The natural place for the element earth is at the centre of the universe, and for that reason earth (the element) has a natural tendency to move towards that centre, where it must inevitably arrange itself in a symmetrical fashion around the centre point, forming the sphere we call *the* Earth. Aristotle reported that mathematicians had estimated the Earth's circumference to be 400,000 stades; that is, about 39,000 miles or 63,000 kilometres (more than half again as large as the modern measurement). No record survives of the method they used to arrive at that number.

When Aristotle died in 322 BC at the age of 62, the military campaigns of his most highly achieving pupil, Alexander the Great, had just ended with the death of Alexander. Though there is a tendency to speak of 'the Greeks' and toss names like Eratosthenes into that file, the civilization and the culture we are dealing with after

Alexander was much larger both in territory and concept than what is implied in the word 'Greek'. Alexander's campaigns had carried Greek knowledge, language and culture throughout Asia Minor and Mesopotamia as far east as present-day Afghanistan and Pakistan, all the way to the Indus River, as well as to Palestine and Egypt. Vastly widened intellectual horizons were part of his extraordinary legacy. The culture of Greece and its colonies and the cultures of the conquered peoples began to mix and marvellously enrich one another. This was the dawn of the *Hellenistic* era, as opposed to the Hellenic era. That is, *Greekish*, as opposed to Greek.

At the time of Alexander's and Aristotle's deaths, within a year of one another, Athens was still the undisputed centre of the intellectual world. That pre-eminence was not to last. Alexander's generals divided his empire, and Ptolemy's portion was Egypt and Palestine. He made Alexandria, near the mouth of the Nile, his capital. This already prospering city began to grow in size and splendour, and Ptolemy and his successors, reputedly ruthless in their exploitation of the lands under their control, amassed a surplus of wealth, some of which they chose to spend on literature, the arts, mathematics and science. Scholars are divided as to which Ptolemy should get the credit (Ptolemy's successors were also named Ptolemy), but either the first or the second of them, and perhaps it took both, decided to extend the royal patronage to found a library and museum. This was not an institution of the sort we call 'museum' today. As that name suggests, it was a temple to the muses, which meant both a religious shrine and a centre of learning.

Meanwhile the old, justly famous schools across the sea in Athens – schools founded by Plato, Aristotle, Epicureus and the Stoics – were no longer producing vibrant new ideas to quite the extent they had once done, though they were still the places a young man of Eratosthenes's time

would have wished to go for his education. Alexandria began to rival and eventually supplanted Athens as the focal point of the intellectual world, and the museum and library there became *the* premier research institution. The library grew large, containing by one ancient estimate nearly 500,000 rolls. Eratosthenes was its director or librarian at the end of the third century BC, with a salary provided from the royal coffers.

Most of us have heard that one of the devastating tragedies in the history of humankind was the burning of the contents of the library at Alexandria. The story (now thought to be apocryphal) is that all those rolls were burned to heat the public baths for six months in the seventh century AD. Today there is a campaign underway to raise funds to rebuild the structure, but that effort seems rather pitiful and beside the point, in view of what can never be retrieved – the assembled knowledge of our ancestors in antiquity, hard-won over many centuries. We sense that something dreadful happened to us with the loss of all those rolls, whether it occurred quickly and calamitously in the seventh century or, more likely, gradually through neglect and the many political, military and religious turns of fortune that affected the city of Alexandria prior to that. Perhaps its loss was the symbol and symptom of a greater tragedy: the increasing lack of any widespread perception that such intellectual achievement was valuable. By the seventh century, there was probably little left to burn. It took centuries for humanity in the Western world to reach again an intellectual level on a par with the civilization that had produced that lost collection. But when Eratosthenes was librarian (235–195 BC) that was all in the future. He knew the Alexandria library in its heyday.

Scholars in the Hellenic and Hellenistic worlds would have been mystified by our present-day concept of ‘science’ as a distinct category of knowledge and pursuit of

knowledge. They had several different words for what we call 'science'. Some modern words have evolved from these terms, but the modern words don't have precisely the same meaning these had in ancient Athens and Alexandria. Some examples are: *peri physeos historia* (inquiry having to do with nature); *philosophia* (love of wisdom, philosophy); *theoria* (speculation); and *episteme* (knowledge). Hellenistic scholars thought of 'physics' as one of three branches of philosophy. The other branches were 'logic' and 'ethics'.

The financial support of the Ptolemys and their efforts to outbid all competitors when it came to collecting the masterpieces of Greek literature and encouraging distinguished scholars to flock to Alexandria were motivated by desire for prestige – to add to the lustre and apparent power of the dynasty. They were also far from displeased when research could be applied to problems connected with weaponry. However, a key difference between the ancient way of thinking and ours is that although Hellenic and Hellenistic scholars didn't ignore the possibility that their study might serve practical purposes, they were much more inclined to justify their work on the grounds that it contributed to wisdom, or improved one's character, or led to a greater appreciation of the beauty of the universe and understanding of its creator. It seemed not to occur to these men and women that their efforts might hold the key to material progress. The work was its own reward, an end in itself, not a means to an end. The life of a scholar, the life of 'contemplation', was considered to be an exquisitely happy one. Doctors, whose efforts were intended to have more everyday practical value, were apt to differentiate themselves entirely from the 'philosophers'.

Related to this mindset that sees intellectual exercise as an end in itself is a perspective in which *how* to solve a problem is equally as interesting as actually solving it, often more so. This attitude arose partly out of necessity,

for Greek and Hellenistic scholars were avidly interested in questions that they lacked the technology to answer definitively. Perhaps we can best acclimatize ourselves to the ancient way of thinking by recalling doing mathematics in school. Presented with the problem 'If you ride your bicycle at an average of 30 miles per hour, and it takes you 10 minutes to get to school, how far is school?' you do not immediately start quibbling that 30 miles per hour is not an accurate measurement of the speed you normally ride, that it actually takes you 12 minutes to get to school, and that this exercise isn't going to end with anyone knowing how far your school really is. No. What everyone is interested in is your showing that you understand how to solve the problem. Move back a step and imagine that it was also up to you to invent the method for solving it – that no one, in fact, had ever even thought it possible to calculate the distance to your school, and that you couldn't ride there to measure it directly – and you have put yourself a little in the shoes of Eratosthenes and other scholars of the Hellenic and Hellenistic world. It is an attitude which allows, indeed encourages, the formation of hypotheses, sometimes out of thin air, statements such as 'We don't know that this is true, but let's assume for a moment that it is, and see where that gets us.' Or even such a statement as 'We know that this is *not* true, but let's pretend for the moment that it is and ask "what then?" ' To criticize the results of an exercise like that by saying the results are 'wrong' (i.e., do not accord with twentieth-century findings) is to miss the point.

Does this mindset in which the pursuit of knowledge was valued quite apart from any practical spin-off – where the method was often thought more important than the result, where hypotheses, even those based on false assumptions, were encouraged – provide a clue to Eratosthenes's success? Perhaps. After all, the Hellenistic world had no practical need to know the circumference of the Earth.

However, that attitude predated Eratosthenes by several centuries. What's more, Eratosthenes's results were remarkably *accurate* by twentieth-century standards. Did his success have something to do with the widening of mental horizons and the mixture of knowledge from many cultures that followed the campaigns of Alexander? It did, but, again, Eratosthenes was not the only inventive man to enjoy that legacy. And the notion that the Earth isn't flat was nothing new either, nor was the idea of calculating its circumference. There had been estimates in Aristotle's time. But it was Eratosthenes who did the measurement and got it right – or so close to right that his calculation impresses us to this day – using a line of thought that we can easily agree was correct.

Eratosthenes, 'son of Aglaos', was born not in Egypt or Greece but in the ancient city of Cyrene, west of Egypt on the northern coast of Africa. Citizens of Crete and Santorini had founded Cyrene some 350 years earlier and it had become one of the most cultured cities of the Hellenistic world, though still subordinate to the Egypt of the Ptolemys. Cyrene counted some distinguished figures among its citizenry. Besides Eratosthenes there was Aristippos, who founded the Cyrenaic school there. He was a pupil of Socrates. Aristippos's daughter Arete followed him as head of the school. Her son Aristippos II succeeded her. He was nicknamed Metrodidactos, which translates as 'mother-taught'.

The date given for Eratosthenes's birth is the '126th Olympiad', referring to the Olympic games that took place every four years. In modern dating, that puts it between 276 and 273 BC. He received most of his education in Athens at the feet of eminent scholars of the New Academy and the Lyceum. Plato and Aristotle had originally founded these schools (in Plato's day the New Academy had been simply the Academy) and, though much had changed about

them by the time Eratosthenes arrived, one still couldn't do better by way of an education.

By the middle of the century Eratosthenes had written a few philosophical and literary works and some of these had come to the attention of Ptolemy III Euergetes. The 'brain-drain' from Athens being in the general direction of Alexandria, Eratosthenes in about 244 BC agreed to move there and become a fellow of the museum and tutor to the prince, Philopator. (It isn't to Eratosthenes's credit that his pupil, though a patron of arts and learning, gained a reputation for dissipation and crimes equal to Nero's and Caligula's later in Rome.)

In the course of time Eratosthenes became a senior (alpha) fellow of the museum and upon the death of the chief librarian took over that post - an absolutely unparalleled vantage point from which to keep up with everything that was going on in the intellectual world.

Eratosthenes's colleagues gave him two nicknames: *pentathlos* and *beta*. The word *pentathlos* came from athletics. It was a name for those who entered the 'pentathlon', which required five skills: jumping, discus throwing, running, wrestling and either boxing or javelin throwing. Eratosthenes was no athlete. The nickname for him implied a jack-of-all-trades. *Beta* means 'B' or number two, or second. Put those together and you get 'jack-of-all-trades and master of none'. Whether these names were fondly or snidely given isn't clear. Probably snidely. It seems a bit odd that in an era following so closely on the heels of Plato and Aristotle, who had their fingers in just about every intellectual pie around, a man should be mocked for being a jack-of-all-trades. Perhaps in the three-quarters of a century between Aristotle's death and Eratosthenes's arrival at Alexandria, scholarship had become more specialized and specialists had begun to sneer at those who were not specialists. Eratosthenes was evidently old-fashioned to be such a polymath, but he had

been educated that way . . . and how could a man focus very narrowly when he was head of the library, *the* repository of knowledge and ideas on every subject, holding a job which made him responsible for helping the Ptolemys add to that collection. Eratosthenes was bombarded daily by new thoughts and discoveries. Modern scholars compare the breadth of his knowledge to that of Aristotle and Leonardo da Vinci. Whatever criticism Eratosthenes endured, his eclecticism served him well. 'Beta' is remembered while some who dubbed him that have long since been forgotten.

Unfortunately, none of Eratosthenes's many works have survived except in fragments. It's not even certain that all the fragments attributed to him are genuine. Most information about him comes through reports and references of others. However, there is enough to tell that Eratosthenes's measurement of the Earth, and also his motive for attempting it, were rooted in his eclectic and far-ranging knowledge and interests. Eratosthenes was a man of the world, in the real sense of those words. He refused to categorize people as Greeks opposed to Barbarians, adopting a more cosmopolitan attitude which differed from the mindset of Greece in earlier centuries but was not uncommon in Hellenistic times. Perhaps it isn't inaccurate to use a modern term and call this a global point of view. Eratosthenes not only thought that way, but he followed through by collecting information about the people, products and geography of far-flung areas. He wrote about the history of geographical measurement, recalling old ideas going back to Homer about the size, shape and geographical lay-out of the Earth. In fact, he did nothing less than pull together virtually all the geographical knowledge that had been accumulating up until his own time.

Over the centuries, this material had taken a variety of forms. It came from traders, explorers, travellers – as well

as mathematicians and philosophers – and it ranged from fantastic tales to more straightforward reporting, from speculation to measurements and estimates resting on what were probably recognized as shaky assumptions. Among the more reliable sources were eye-witness accounts of Alexander the Great's expeditions and the measurements and records of distances covered on those marches. There were itineraries of coastal voyages and maps and charts connected with them. There was a treatise on harbours by Timosthenes, the admiral of the Ptolemaic fleet, who also studied the winds. There was a book entitled *On the Ocean* by the merchant sea-captain Pytheas, who in about 320 BC sailed north along the coast of Spain and France and reached Cornwall, then continued all the way up to the Orkneys and the Shetlands to latitudes near those of the midnight sun. Pytheas took bearings throughout his voyage and recorded them in his book, which also has descriptive passages:

The barbarians showed us where the Sun keeps watch at night, for around these parts the night is exceedingly short, sometimes two and sometimes three hours, so that only a short interval passes after the Sun sets before it rises once more.

Eratosthenes respected Pytheas's information, though it must have seemed almost as fantastic as Homer's, while many other scholars were contemptuous and disbelieving. Living as Eratosthenes did in Hellenistic Egypt, he may also have known of centuries-old and astoundingly accurate Egyptian geographical calculations.

Eratosthenes's expertise on longitude and latitude surpassed any other of his day or earlier. His predecessors had divided the map into zones. He took that work several steps further by improving on a map devised about 25 years before his birth by a man named Dicaearchus of

Messene. Dicaearchus had divided the known world by using two lines or bands that intersected one another – one running east–west, the other north–south. On Eratosthenes’s revised map the two lines crossed at Rhodes, a little to the east of where Dicaearchus’s lines had met. The horizontal line passed near Gibraltar (then known as the Pillars of Hercules), ran the length of the Mediterranean and then followed the Taurus chain of mountains in southern Turkey (Toros Daglari on modern maps). That line is remarkably near to following what we call the 37th parallel – an impressive achievement for men without the benefit of the mathematical and astronomical knowledge that would go into later mapmaking. It was not yet possible to figure out latitudes with very great precision, and it was virtually impossible to determine longitude (which would prove to be a problem in Eratosthenes’s measurement of the Earth). On Eratosthenes’s map the vertical line followed the Nile, which doesn’t line up so perfectly with Rhodes on modern maps. He added six further lines drawn vertically at intervals between the western and eastern boundaries of the inhabited world, and six more horizontal lines drawn at intervals between its northern and southern boundaries, and he established and measured geographic zones, dividing the world horizontally between the tropical region, the temperate region and the polar circles.

Eratosthenes was also well-acquainted with state-of-the-art geometry, both from Euclid’s profound summing-up about 25 years before Eratosthenes’s birth and from his association with Archimedes, one of the towering creative geniuses that Greek and Hellenistic civilization produced and also one of history’s greatest eccentrics. Most schoolchildren have heard the tale of Archimedes solving a mathematical problem in his bath, leaping from the water, and running naked through the streets shouting ‘Eureka!’ This avid mathematician eventually lost his life when

Roman troops sacked Syracuse. Archimedes, so the story goes, was drawing a mathematical figure in the sand when a Roman soldier (who had missed hearing an order from his superiors to respect the person of this famous old man) asked him to pack up and move along. Archimedes unwisely told the soldier not to interrupt his thought process.

The Hellenistic world revered Archimedes as an inventor (though he himself dismissed such practical achievements as unworthy of notice) and a useful man to have around in a war. Legend has it that he destroyed an entire Roman fleet by using burning mirrors. The Middle Ages thought of him as an engineer and a wizard and credited him with the invention of the Staff of Archimedes. This device was a stick with a small flat disc that could be run up and down it. An observer held the stick up to the Sun and moved the disc along it until it appeared to cover the Sun, then noted on a scale the distance from disc to eye, thus deriving the Sun's apparent diameter.

Modern history and mathematics books recall Archimedes as a brilliant mathematician and geometer who contributed significantly to the understanding of circles and spheres. Archimedes was in the habit of sharing his discoveries and his methods with Eratosthenes and even dedicated his greatest work, *Method*, to him. Eratosthenes must have welcomed, another scholar who was almost as eclectic as he was himself.

Eratosthenes's thoughts stretched to the horizon in all directions. Perhaps it follows that he would have longed to know not only what was beyond those horizons but how far 'beyond' was? Mapping and systematizing things geographically was his bent. Would he not have been unusually curious about how large the total map was? How remarkable if it really should turn out to be, as Aristotle speculated, 'a sphere of no great size'! Eratosthenes's thoughts often took a historical turn, and he was aware of

previous attempts to measure the Earth or estimate that measurement. Would he not have wanted to try his own hand at it, using Euclid's and Archimedes's newer understanding of geometry?

There is still one circumstance to be mentioned – a simple, trivial matter, yet Eratosthenes's successful measurement of the circumference of the Earth would not have taken place without it. A happenstance, perhaps, that such a small gem of information reached the ears of this man who realized what it meant and what could be done with it. It is true that the fact that this snippet of news reached him *did* have something to do with the broadened mental horizons of the world, with improved communications from remote areas, with Eratosthenes's own world centring on northern Africa, and with his habit of keeping his ears and eyes open and wanting to know everything and anything. He was indeed the right man in the right time and place. Perhaps there was no other so likely to run across this back-page news and recognize its worth:

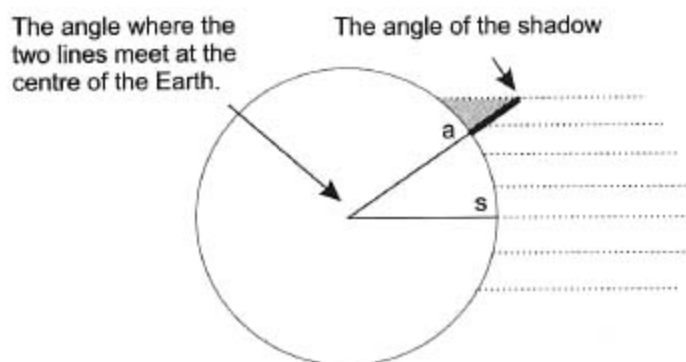
In a well located at Syene (near modern Aswan), on the day of the summer solstice, a shaft of sunlight penetrated all the way to the bottom of the well.

To Eratosthenes there was nothing trivial about this information. It meant that the Sun was shining directly down at Syene, not at an angle, and he knew this showed that Syene was on the tropic. A stick set up at noon at Syene on the day of the summer solstice would not cast a shadow. A stick set up at Alexandria (which he thought was the same longitude as Syene) *would*. Accordingly, Eratosthenes set up a stick at Alexandria on the day of the summer solstice and measured the angle of its shadow when that shadow was at its shortest.

[Figure 1.1](#) below shows the stick at Alexandria and its shadow, and what is meant by 'the angle of the shadow'. The figure illustrates how that must also be the angle

‘subtended’ by the arc Syene–Alexandria at the centre of the Earth. To put that a bit more simply: if we draw a straight line from the point marked Alexandria (where the stick is casting a shadow) to the centre of the Earth, and a second straight line from the point marked Syene (where the stick casts no shadow) to the centre of the Earth, those lines will of course meet at the centre of the Earth. We want to know the angle between those two lines where they meet. Geometry tells us, as it told Eratosthenes, that the angle at the centre of the Earth and the angle of the shadow at Alexandria will be the same angle.

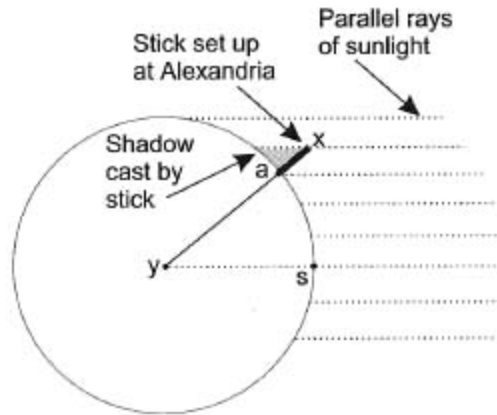
Figure 1.1



Because the Sun's rays are running parallel as they strike the Earth, if a line is drawn from Alexandria (a) where the stick casts a shadow, to the centre of the Earth, and a second line from Syene (s) where there is no shadow, to the centre of the Earth, the angle where those two lines meet will be the same as the angle of the shadow at Alexandria.

[Figure 1.2](#) illustrates Eratosthenes's measurement.

Figure 1.2



Because the sunlight shone all the way to the bottom of the well at Syene (s), Eratosthenes knew that the Sun was shining straight down on the Earth there.

He set up a stick at Alexandria (a), where the Sun wasn't shining straight down, and he measured the angle (x) of the shadow cast by the stick. He knew that because the Sun's rays all run parallel as they strike the Earth, the angle (y) where a line drawn straight down from Alexandria and a line drawn straight down from Syene would meet at the centre of the Earth would be the same angle as the angle of the shadow cast by the stick (x). If Syene is due south of Alexandria, then the distance between Syene and Alexandria must be the same fraction of the Earth's total circumference as the angle at x or y is of 360°

Eratosthenes found that the shadow angle at Alexandria was $7\frac{1}{5}^\circ$, and so he knew that the angle between the 'Syene-Alexandria lines' (meeting at the centre of the Earth) was also $7\frac{1}{5}^\circ$. A circle has 360° , and it is a simple process to find out how many of the Syene-Alexandria angles ($7\frac{1}{5}^\circ$) it will take to make 360° . Think of the cross-section of the Earth as a pie and the two lines coming from Syene and Alexandria as cutting out a wedge of pie. How many wedges of that size can you cut from the whole pie? Divide 360 by $7\frac{1}{5}$, and it comes out to 50 wedges. If we say (as Eratosthenes did) that the distance between Syene and Alexandria at the surface of the Earth (at the pie-crust edge of the pie) is '5,000 stades', then we can multiply 5,000 by 50 and conclude that the distance all the way around the Earth – the circumference of the Earth – is 250,000 stades. Eratosthenes later fine-tuned this to 252,000 stades.

What is this odd unit of measurement, the stade? That question brings up a problem in evaluating Eratosthenes's

result. Whether or not that result matches modern measurements for the circumference of the Earth depends on the length of 'stade' he was using, and it isn't known exactly what that length was. If there are 157.5 metres in a stade, Eratosthenes's result comes to 39,690 kilometres or 24,608 miles for the circumference of the Earth. That is very near the modern calculation - 24,857 miles (40,009 kilometres) around the poles and 24,900 miles (40,079 kilometres) around the equator. After he had found the circumference, Eratosthenes calculated the diameter of the Earth as 7,850 miles (12,631 kilometres), close to today's mean value of 7,918 miles (12,740 kilometres).

Another way of figuring a stade was as $\frac{1}{8}$ or $\frac{1}{10}$ of a Roman mile, and that would make Eratosthenes's result too large by modern standards. There was one additional small difficulty. Eratosthenes assumed that Syene lay on the same line of longitude as Alexandria. Actually, it does not.

But this is nit-picking! No apology need be made for Eratosthenes. First of all, he arguably came astonishingly near to matching the modern measurement. Second, he was probably, for all his curiosity about the world, enough a man of his time to find the puzzle of how to solve this problem by the imaginative use of geometry at least as interesting as the actual measurement. The *method* is ingenious and it is correct. If the numerical result is a little fuzzy because of a lack of agreement about the length of a stade and the impossibility of determining longitude precisely, that does not prevent our recognizing what a brilliant achievement this was or appreciating the intellectual leap involved in recognizing that it *could* be done and *how* it could be done.

Eratosthenes didn't focus his thoughts only on the Earth. He also raised them above the horizon to consider astronomical questions of his day. When it came to measuring the distances to the Sun and the Moon, he must have realized that he had no tool at his fingertips to equal