

LEARNING MADE EASY



2nd Edition

Basic Math & Pre-Algebra

for
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Discover fractions,
decimals, and percents

Apply math to everyday life
with real world examples

Tackle algebra
word problems

A circular inset image showing a collection of colorful, 3D plastic numbers in various colors (red, yellow, blue, green) scattered on a dark surface.

Mark Zegarelli

Basic Math & Pre-Algebra

for
dummies[®]
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2nd Edition

by Mark Zegarelli

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Basic Math & Pre-Algebra For Dummies®, 2nd Edition

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Introduction

Once upon a time, you loved numbers. This isn't the first line of a fairy tale. Once upon a time, you really did love numbers. Remember?

Maybe you were 3 years old and your grandparents were visiting. You sat next to them on the couch and recited the numbers from 1 to 10. Grandma and Grandpa were proud of you and — be honest — you were proud of yourself, too. Or maybe you were 5 and discovering how to write numbers, trying hard not to print your 6 and 7 backward.

Learning was fun. *Numbers* were fun. So what happened? Maybe the trouble started with long division. Or sorting out how to change fractions to decimals. Could it have been figuring out how to add 8 percent sales tax to the cost of a purchase? Reading a graph? Converting miles to kilometers? Trying to find that most dreaded value of x ? Wherever it started, you began to suspect that math didn't like you — and you didn't like math very much, either.

Why do people often enter preschool excited about learning how to count and leave high school as young adults convinced that they can't do math? The answer to this question would probably take 20 books this size, but solving the problem can begin right here.

I humbly ask you to put aside any doubts. Remember, just for a moment, an innocent time — a time before math-inspired panic attacks or, at best, induced irresistible drowsiness. In this book, I take you from an understanding of the basics to the place where you're ready to enter any algebra class and succeed.

About This Book

Somewhere along the road from counting to algebra, most people experience the Great Math Breakdown. This feels something like when your car begins smoking and sputtering on a 110°F highway somewhere between Noplace and Not Much Else.

Please consider this book your personal roadside helper, and think of me as your friendly math mechanic (only much cheaper!). Stranded on the interstate, you

may feel frustrated by circumstances and betrayed by your vehicle, but for the guy holding the toolbox, it's all in a day's work. The tools for fixing the problem are in this book.

Not only does this book help you with the basics of math, but it also helps you get past any aversion you may feel toward math in general. I've broken down the concepts into easy-to-understand sections. And because *Basic Math & Pre-Algebra For Dummies* is a reference book, you don't have to read the chapters or sections in order — you can look over only what you need. So feel free to jump around. Whenever I cover a topic that requires information from earlier in the book, I refer you to that section or chapter, in case you want to refresh yourself on the basics.

Here are two pieces of advice I give all the time — remember them as you work your way through the concepts in this book:

- » **Take frequent breaks.** Every 20 to 30 minutes, stand up and push in your chair. Then feed the cat, do the dishes, take a walk, juggle tennis balls, try on last year's Halloween costume — do *something* to distract yourself for a few minutes. You'll come back to your books more productive than if you just sat there hour after hour with your eyes glazing over.
- » **After you've read through an example and think you understand it, copy the problem, close the book, and try to work it through.** If you get stuck, steal a quick look — but later, try that same example again to see whether you can get through it without opening the book. (Remember that, on any tests you're preparing for, peeking is probably not allowed!)

Although every author secretly (or not-so-secretly) believes that each word he pens is pure gold, you don't have to read every word in this book unless you really want to. Feel free to skip over sidebars (those shaded gray boxes) where I go off on a tangent — unless you find tangents interesting, of course. Paragraphs labeled with the Technical Stuff icon are also nonessential.

Foolish Assumptions

If you're planning to read this book, you likely fall into one of these categories:

- » A student who wants a solid understanding of the basics of math for a class or test you're taking
- » An adult who wants to improve skills in arithmetic, fractions, decimals, percentages, weights and measures, geometry, algebra, and so on for when you have to use math in the real world

» Someone who wants a refresher so you can help another person understand math

My only assumption about your skill level is that you can add, subtract, multiply, and divide. So to find out whether you're ready for this book, take this simple test:

$$5 + 6 = \underline{\quad}$$

$$10 - 7 = \underline{\quad}$$

$$3 \times 5 = \underline{\quad}$$

$$20 \div 4 = \underline{\quad}$$

If you can answer these four questions, you're ready to begin.

Icons Used in This Book

Throughout the book, I use four icons to highlight what's hot and what's not:



REMEMBER

This icon points out key ideas that you need to know. Make sure you understand before reading on! Remember this info even after you close the book.



TIP

Tips are helpful hints that show you the quick and easy way to get things done. Try them out, especially if you're taking a math course.



WARNING

Warnings flag common errors that you want to avoid. Get clear about where these little traps are hiding so you don't fall in.



TECHNICAL
STUFF

This icon points out interesting trivia that you can read or skip over as you like.

Beyond the Book

In addition to the material in the print or e-book you're reading right now, remember that (as they say on those late-night infomercials) "There's much, much more!" To view this book's Cheat Sheet, simply go to www.dummies.com and search for "Basic Math & Pre-Algebra For Dummies Cheat Sheet" in the Search box for a set of quick reference notes on converting between English and metric measurement units; using the order of operations (also called order of

precedence); working with the commutative, associative, and distributive properties; converting among fractions, decimals, and percents; and lots, lots more.

In addition, www.Dummies.com contains a set of related material on topics like how to use factor trees to find the greatest common factor (GCF) of two or more numbers; how to use the percent circle, a helpful tool for solving percent problems; how to calculate the probability of getting certain rolls in the casino game of craps, and more.

And remember that in math, practice makes perfect. The *Basic Math & Pre-Algebra Workbook For Dummies* includes hundreds of practice problems, each group with a brief explanation to help you get started. And if that's not enough practice, *1,001 Practice Problems in Basic Math & Pre-Algebra For Dummies* provides lots more. Check them out!

Where to Go from Here

You can use this book in a few ways. If you're reading this book without immediate time pressure from a test or homework assignment, you can certainly start at the beginning and keep going to the end. The advantage to this method is that you realize how much math you *do* know — the first few chapters go very quickly. You gain a lot of confidence, as well as some practical knowledge that can help you later, because the early chapters also set you up to understand what follows.

If your time is limited — especially if you're taking a math course and you're looking for help with your homework or an upcoming test — skip directly to the topic you're studying. Wherever you open the book, you can find a clear explanation of the topic at hand, as well as a variety of hints and tricks. Read through the examples and try to do them yourself, or use them as templates to help you with assigned problems. Here's a short list of topics that tend to back students up:

- » Negative numbers (Chapter 4)
- » Order of operations (Chapter 5)
- » Word problems (Chapters 6, 13, 18, and 23)
- » Factoring of numbers (Chapter 8)
- » Fractions (Chapters 9 and 10)

Generally, any time you spend building these five skills is like money in the bank as you proceed in math, so you may want to visit these sections several times.

1

Getting Started with Basic Math and Pre-Algebra

IN THIS PART . . .

See how the number system was invented and how it works.

Identify four important sets of numbers: counting numbers, integers, rational numbers, and real numbers.

Use place value to write numbers of any size.

Round numbers to make calculating quicker.

Work with the Big Four operations: adding, subtracting, multiplying, and dividing.

IN THIS CHAPTER

Finding out how numbers were invented

Looking at a few familiar number sequences

Examining the number line

Understanding four important sets of numbers

Chapter 1

Playing the Numbers Game

One useful characteristic about numbers is that they're *conceptual*, which means that, in an important sense, they're all in your head. (This fact probably won't get you out of having to know about them, though — nice try!)

For example, you can picture three of anything: three cats, three baseballs, three cannibals, three planets. But just try to picture the concept of three all by itself, and you find it's impossible. Oh, sure, you can picture the numeral 3, but the *threeness* itself — much like love or beauty or honor — is beyond direct understanding. But when you understand the *concept* of three (or four, or a million), you have access to an incredibly powerful system for understanding the world: mathematics.

In this chapter, I give you a brief history of how numbers came into being. I discuss a few common *number sequences* and show you how these connect with simple math *operations* like addition, subtraction, multiplication, and division.

After that, I describe how some of these ideas come together with a simple yet powerful tool: the *number line*. I discuss how numbers are arranged on the number line, and I also show you how to use the number line as a calculator for simple arithmetic. Finally, I describe how the *counting numbers* (1, 2, 3, . . .) sparked the

invention of more unusual types of numbers, such as *negative numbers*, *fractions*, and *irrational numbers*. I also show you how these *sets of numbers* are *nested* — that is, how one set of numbers fits inside another, which fits inside another.

Inventing Numbers

Historians believe that the first number systems came into being at the same time as agriculture and commerce. Before that, people in prehistoric, hunter-gatherer societies were pretty much content to identify bunches of things as “a lot” or “a little.”

But as farming developed and trade between communities began, more precision was needed. So people began using stones, clay tokens, and similar objects to keep track of their goats, sheep, oil, grain, or whatever commodity they had. They exchanged these tokens for the objects they represented in a one-to-one exchange.

Eventually, traders realized that they could draw pictures instead of using tokens. Those pictures evolved into tally marks and, in time, into more complex systems. Whether they realized it or not, their attempts to keep track of commodities led these early humans to invent something entirely new: *numbers*.

Throughout the ages, the Babylonians, Egyptians, Greeks, Romans, Mayans, Arabs, and Chinese (to name just a few) all developed their own systems of writing numbers.

Although Roman numerals gained wide currency as the Roman Empire expanded throughout Europe and parts of Asia and Africa, the more advanced system that the Arabs invented turned out to be more useful. Our own number system, the Hindu–Arabic numbers (also called decimal numbers), is closely derived from these early Arabic numbers.

Understanding Number Sequences

Although humans invented numbers for counting commodities, as I explain in the preceding section, they soon put them to use in a wide range of applications. Numbers were useful for measuring distances, counting money, amassing an army, levying taxes, building pyramids, and lots more.

But beyond their many uses for understanding the external world, numbers have an internal order all their own. So numbers are not only an *invention*, but equally a *discovery*: a landscape that seems to exist independently, with its own structure, mysteries, and even perils.

One path into this new and often strange world is the *number sequence*: an arrangement of numbers according to a rule. In the following sections, I introduce you to a variety of number sequences that are useful for making sense of numbers.

Evening the odds

One of the first facts you probably heard about numbers is that all of them are either even or odd. For example, you can split an even number of marbles *evenly* into two equal piles. But when you try to divide an odd number of marbles the same way, you always have one *odd*, leftover marble. Here are the first few even numbers:

2 4 6 8 10 12 14 16 . . .

You can easily keep the sequence of even numbers going as long as you like. Starting with the number 2, keep adding 2 to get the next number.

Similarly, here are the first few odd numbers:

1 3 5 7 9 11 13 15 . . .

The sequence of odd numbers is just as simple to generate. Starting with the number 1, keep adding 2 to get the next number.

Patterns of even or odd numbers are the simplest number patterns around, which is why kids often figure out the difference between even and odd numbers soon after learning to count.

Counting by threes, fours, fives, and so on

When you get used to the concept of counting by numbers greater than 1, you can run with it. For example, here's what counting by threes, fours, and fives looks like:

Threes:	3	6	9	12	15	18	21	24 . . .
Fours:	4	8	12	16	20	24	28	32 . . .
Fives:	5	10	15	20	25	30	35	40 . . .



TIP

Counting by a given number is a good way to begin learning the multiplication table for that number, especially for the numbers you're kind of sketchy on. (In general, people seem to have the most trouble multiplying by 7, but 8 and 9 are also unpopular.) In Chapter 3, I show you a few tricks for memorizing the multiplication table once and for all.

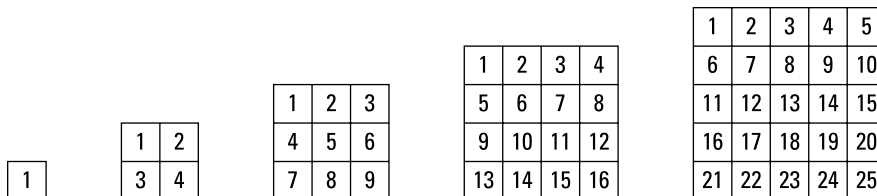
These types of sequences are also useful for understanding factors and multiples, which you get a look at in Chapter 8.

Getting square with square numbers

When you study math, sooner or later you probably want to use visual aids to help you see what numbers are telling you. (Later in this book, I show you how one picture can be worth a thousand numbers when I discuss geometry in Chapter 16 and graphing in Chapter 17.)

The tastiest visual aids you'll ever find are those little square cheese-flavored crackers. (You probably have a box sitting somewhere in the pantry. If not, saltine crackers or any other square food works just as well.) Shake a bunch out of a box and place the little squares together to make bigger squares. Figure 1-1 shows the first few.

FIGURE 1-1:
Square numbers.



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Voilà! The square numbers:

1 4 9 16 25 36 49 64...



TIP

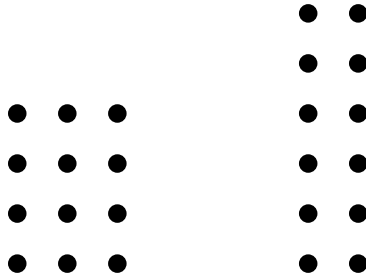
You get a *square number* by multiplying a number by itself, so knowing the square numbers is another handy way to remember part of the multiplication table. Although you probably remember without help that $2 \times 2 = 4$ you may be sketchy on some of the higher numbers, such as $7 \times 7 = 49$. Knowing the square numbers gives you another way to etch that multiplication table forever into your brain, as I show you in Chapter 3.

Square numbers are also a great first step on the way to understanding exponents, which I introduce later in this chapter and explain in more detail in Chapter 4.

Composing yourself with composite numbers

Some numbers can be placed in rectangular patterns. Mathematicians probably should call numbers like these “rectangular numbers,” but instead they chose the term *composite numbers*. For example, 12 is a composite number because you can place 12 objects in rectangles of two different shapes, as in Figure 1-2.

FIGURE 1-2:
The number 12
laid out in two
rectangular
patterns.



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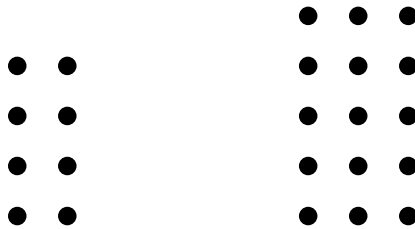
As with square numbers, arranging numbers in visual patterns like this tells you something about how multiplication works. In this case, by counting the sides of both rectangles, you find out the following:

$$3 \times 4 = 12$$

$$2 \times 6 = 12$$

Similarly, other numbers such as 8 and 15 can also be arranged in rectangles, as in Figure 1-3.

FIGURE 1-3:
Composite
numbers, such as
8 and 15, can
form rectangles.



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As you can see, both these numbers are quite happy being placed in boxes with at least two rows and two columns. And these visual patterns show this:

$$2 \times 4 = 8$$

$$3 \times 5 = 15$$

The word *composite* means that these numbers are *composed of* smaller numbers. For example, the number 15 is composed of 3 and 5 — that is, when you multiply these two smaller numbers, you get 15. Here are all the composite numbers from 1 to 16:

4 6 8 9 10 12 14 15 16

Notice that all the square numbers (see “Getting square with square numbers”) also count as composite numbers because you can arrange them in boxes with at least two rows and two columns. Additionally, a lot of other nonsquare numbers are also composite numbers.

Stepping out of the box with prime numbers

Some numbers are stubborn. Like certain people you may know, these numbers — called *prime numbers* — resist being placed in any sort of a box. Look at how Figure 1-4 depicts the number 13, for example.

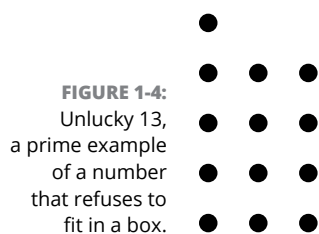


FIGURE 1-4:

Unlucky 13, a prime example of a number that refuses to fit in a box.

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Try as you may, you just can't make a rectangle out of 13 objects. (That fact may be one reason the number 13 got a bad reputation as unlucky.) Here are all the prime numbers less than 20:

2 3 5 7 11 13 17 19

As you can see, the list of prime numbers fills the gaps left by the composite numbers (see the preceding section). Therefore, every counting number is either prime or composite. The only exception is the number 1, which is neither prime nor composite. In Chapter 8, I give you a lot more information about prime numbers and show you how to *decompose* a number — that is, break down a composite number into its prime factors.