Analysing
and
Interpreting
the
Yield Curve

Second Edition

MOORAD CHOUDHRY

Foreword by

CHRISTOPHER WESTCOTT
Former Divisional Treasurer
Royal Bank of Scotland

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Analysing and Interpreting the Yield Curve

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Analysing and Interpreting the Yield Curve

Second Edition

MOORAD CHOUDHRY

*With contributions from Polina Bardaeva, Ken Kortanek,
Kevin Liddy, Wolfgang Marty and Vladimir Medvedev*

WILEY

For Lindsay
Ultimate Yummy Mummy

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Foreword

It is an honour to be asked to contribute a few words at the beginning of this publication.

Moorad and I first met back in 2010. We were both attending the Group Balance Sheet Management Committee at The Royal Bank of Scotland, our employer at the time. Although I forget the specific theme of the discussion, I remember our desire, incidentally which was not necessarily shared by the other committee members, to investigate and better understand a trend in the balance sheet of our organisation and in the wider economy.

Respecting each other's viewpoint, we quickly became good friends. However, we are like "chalk and cheese". In Belbin's terminology, Moorad possesses many of the qualities of the "Plant" and "Resource Investigator", being creative, imaginative and free-thinking as well as outgoing and enthusiastic, whereas I am most comfortable in the role of the "Completer Finisher".

Although our shared passion for the financial markets became evident at our first meeting, it was only later I realised Moorad also possesses a burning desire to pass on his knowledge, understanding and perceptive insights to others, by either delivering lectures and presentations or through the written word.

I first became aware of Moorad's teaching skills in 2014 when he invited me to speak to students studying for the Certificate of Bank Treasury Risk Management. This is a practitioner-oriented professional qualification in bank asset-liability management, which is delivered to a global audience and was developed by Moorad and the team from WBS Training Ltd. There are also his numerous short course and conference performances I could highlight, all with financial risk management as the enduring theme.

When it comes to writing, Moorad is both versatile and prolific. By my counting, this updated edition of *Analysing and Interpreting the Yield Curve* is his third book of the year so far. However, there's still a way to go to catch Corin Tellado, a renowned Spanish author who published over 4,000 novels in her lifetime. The comparison is probably unfair as it is doubtless a more difficult challenge to combine risk management theory, practice and current market developments in a captivating read, whilst also pursuing a full-time career in banking!

Moorad is a past master at simplifying complex ideas and communicating them in an easy going and engaging manner. In these respects, this book is no different to all his other publications. Compared to the first edition, sections have been added for the latest developments in the financial markets, including: the multi-currency yield curve, the SONIA curve, the interpretation of negative yield curves, a post-crash discounting technique for the swap curve and how to use the theoretical and observed US Treasury curve as a means of identifying relative value in bond spread trades. These concepts will be of interest to anyone working in the bond markets, be they a trader, sales-person, fund manager, research analyst, investor or issuer, or, like me, simply a student of the financial markets.

I commend this book to you, hope you enjoy the read and leave you with the words of Benjamin Franklin:

An investment in knowledge pays the best interest.

Chris Westcott
Former Treasurer, Retail and Wealth Division
The Royal Bank of Scotland
1 May 2018

Preface

The yield curve, and everything about it, was my first and most intense love in finance. It probably still is. I could talk about it for hours, at any time, day or night. I think it is the most significant topic in banking, the very foundation of finance. But it's interesting to me to observe how the perception, and indeed the requirement, for technical excellence in banking changes subtly the more senior the level of practitioner. The higher one rises in the profession, the less it would appear that one needs to know about subjects such as the curve, how to interpret it, how best to interpolate it, and how to understand and make sense of what it's trying to tell us. What I thought was the most important and vital issue in finance, something that absolutely everyone had to know about, turns out to be just one more arcane specialism that is not discussed that often at the bank's asset-liability committee (ALCO), hardly at all at the executive management committee (EXCO), and fewer times still at the Board.

No matter, I still think that it's a very important topic and it's a pity that it isn't viewed in this way by everyone in banking. But that's their look out. The fact that you are reading this book shows that you agree that it's a worthwhile topic to get to grips with!

So, fully 15 years after the first edition, is there anything new to write about on the curve? As it happens, yes, a fair bit. The global financial markets are a very different beast today compared to what they were in 2003. Of course, the fundamentals of yield curve analysis, interpolation, and interpretation remain unchanged. But the behaviours of curves are different in various nuanced ways (and some not so nuanced, and in fact very much "in your face" – for example, the negative interest rate curves that are a commonplace in some countries in the eurozone). That's why I think this book was worth updating, so that we might cover issues such as multi-currency curves, the overnight index swap (OIS) curve, and key factors in post-crash discounting of the swap curve. And speaking of interest-rate swaps, it is routine to see (for example, in US dollar markets) the swap curve trading through the sovereign bond curve. That would have been inexplicable when I was working as a government bond primary dealer in the 1990s . . . but then again, negative interest rates as routine would also have been inexplicable. Plenty for us to be getting on with then. And in an era of ever more intensive regulator and

compliance burden, having to deal with a purely technical subject may even come across as a breath of fresh air to some practitioners!

I always try to emphasise the practical and the user-friendly in all my writing. There is such a disconnect between academia and practice in finance that there would be little value in me expounding purely on the theoretical. Unfortunately (or fortunately, depending on your point of view), the yield curve is one of those topics that it is difficult to leave the technical out of. That said, I hope the contents of the book are of relevance and practical value to the practitioner in banking and finance. This is not intended to be a textbook describing nothing that actually takes place in a bank, unlike some finance textbooks I have encountered over the years. Rather, it is meant for those who need to update the curve for use in internal funds transfer pricing, or to estimate the value to be derived from purchasing one bond in preference to another bond, or to price a new issue private placement structured product, or to have an idea of what the market thinks the state of the economy is. In other words, this book is for anyone that is using the yield curve for one or more of its myriad different practical applications in the financial markets.

Hence this second edition, which I hope you find of interest and of some use. As Ian MacDonald said in the preface to his final update of the majestic *Revolution In The Head*, no further editions will be forthcoming. Or as the last *Oi!* album proclaimed, "That's yer lot!".

Comments on the text are welcome and should be sent to me via John Wiley & Sons Limited.

All the best.

Moorad Choudhry
Surrey, England
19 December 2018
A Solid Bond In Your Heart

Preface to the First Edition

As Sir Arthur Conan Doyle would have put it, so elementary a form of literature as the textbook on financial economics hardly deserves the dignity of a preface. It is possible though, to bring some instant clarity to the purpose of such a book if we open with a few words here.

In my book *The Bond and Money Markets*, I try to explain, from first principles, just how important the global debt market is, and describe the various participants that interact with each other in this market. Given the importance of the global bond market, one can never learn too much about it. But this is not a book about the bond market; rather it is about a very specific, and important part of the bond markets. In developed markets, as well as a fair number of developing ones, there is usually a large number of bonds trading at one time, at different yields and with varying terms to maturity. Investors and traders incessantly examine the relationship between the yields on bonds that are in the same class; plotting yields of bonds that differ only in their term to maturity produces what is known as the *yield curve*. The yield curve represents the bond market. It is sometimes referred to as the *term structure of interest rates*, but as we shall see later in this book, this expression refers to only one specific type of yield curve. There are lots of different yield curves. We shall examine them all in detail later.

Much of the analysis and pricing activity that takes place in the bond markets revolves around the yield curve. The primary yield curve in any domestic capital market is the government bond yield curve, so for example, in the US market it is the US Treasury yield curve. So in this book we will talk mainly, but not exclusively, about the government yield curve. And because the author spent over five years as a United Kingdom government bond trader (or gilt-edged market maker), most of the examples will be from the gilt market. But the principles remain the same. It is the importance of the yield curve to just about every aspect of finance that has been the motivation behind writing this book. Our objective is to:

- describe what the yield curve is;
- explain what it tells us;
- try to explain why it assumes certain shapes;
- show how we can use it;
- introduce how it is modelled;
- show how it is fitted from market rates.

We begin with some basic description of bonds and bond mathematics, just to set the scene. We assume a basic knowledge and familiarity with bonds and market institutions, and concentrate on the yield curve. It is an arcane, specialist topic but well worth getting familiar with. We explain term structure theory, describe the most popular mathematical approaches used to model the yield curve, and show how to fit the yield curve using econometric techniques. This knowledge is of great use to just about anyone involved in the bond markets: traders, bond salespersons, fund managers, research analysts, issuers of bonds . . . in fact issuers, investors, and all the middlemen in between. Investors in the equity markets can also benefit from an understanding of the yield curve, as it enables one to gain a better insight into market sentiment.

We must necessarily be quite focused and specialist in our discussion of the yield curve. Hopefully the more technical material is presented in good order so that it remains accessible. There are any number of textbooks available for the complete beginner, which are recommended in end-chapter reading lists, along with further reading.

Moorad Choudhry
Surrey, England
30 June 2003

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Thanks to everyone at Wiley, including Stephen Mullaly, Syd Ganaden, Jean-Karl Martin, Debbie Scott, Sandra Glue, Banurekha Venkatesan, Elisha Benjamin, Katy Smith and Aida Ferguson.

Big thanks to my co-authors, Polina Bardaeva, Ken Kortanek, Kevin Liddy, Wolfgang Marty, and Vladimir Medvedev. It's a privilege to work with you.

Thanks to Marc Dodd and everyone at King & Shaxson Ltd, and Martin Ward at One Savings Bank for helping to make my latest return to the markets so enjoyable.

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Thanks to everyone at The BTRM. What a genuinely great bunch of people I am privileged to work with.

Thanks to Philip Curtis-Evans at Bloomberg for his assistance and instant responses whenever I requested screen print permissions.

For very kind and very much appreciated comments on Linked In, all the more touching as I have never actually met them, a very special thanks to Brian Twomey, Donald Van Deventer and David Harper. It meant a lot to me, thank you gentlemen.

And a very big, big thanks to Mike Kirsopp and everyone at Cambridge & Counties Bank. And I mean everyone! The bank has at least one thing in common with the New Zealand All Blacks. . .

About the Author

Moorad Choudhry is Head of ALM at Cambridge & Counties Bank in Leicester.

He was previously a gilt-edged market-maker and money markets trader at Hoare Govett Securities Limited (later ABN Amro Hoare Govett Limited) and a sterling bond proprietary trader at Hambros Bank Limited. He subsequently traded money markets, asset-backed commercial paper, and structured finance repo at KBC Financial Products (a subsidiary of KBC Bank N.V.), and was latterly Treasurer, Corporate Banking Division at The Royal Bank of Scotland.

Analysing and Interpreting the Yield Curve

But don't forget the songs
That made you cry,
And the songs that saved your life,
Yes, you're older now
And you're a clever swine,
But they were the only ones who ever stood by you.

——The Smiths, *Rubber Ring* (Rough Trade Records, 1985)

PART I

Introduction to the Yield Curve
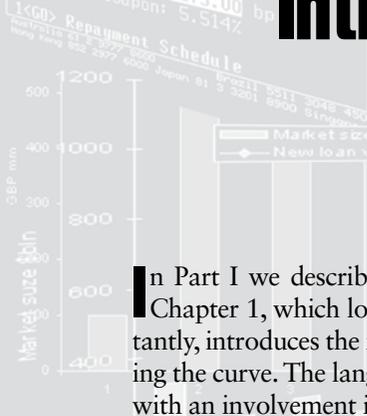
In Part I we describe the yield curve itself. The bulk of the discussion is in Chapter 1, which looks at the different types of yield curve and, more importantly, introduces the main theories of the yield curve. We also look at interpreting the curve. The language is non-specialist and should be accessible to anyone with an involvement in the financial markets. This is followed by a look at spot and forward rates, and the derivation of such rates from market yields.

For this second edition we have relegated the introductory chapter on bond yield measurement to the main Appendix.

After a couple of months, his patriotic zeal got on my nerves so much I began to question whether I agreed with him about communism being evil. I agreed it was a bad idea but no longer felt so sure it would ruin the planet. I began to consider the danger of blind faith in, or blind hatred of, a single idea, any idea.

——Robert Wideman, *Unexpected Prisoner: Memoir of a Vietnam POW,* 2016

The Yield Curve

The main measure of return associated with holding bonds is the *yield to maturity* (YTM) or *gross redemption yield* (GRY). In developed markets there is usually a large number of bonds trading at one time, at different yields and with varying terms to maturity. Investors and traders frequently examine the relationship between the yields on bonds that are in the same class. Plotting yields of bonds that differ only in their term to maturity produces the *yield curve*. The yield curve is an important indicator and knowledge source of the state of a debt capital market.[1] It is sometimes referred to as the *term structure of interest rates*, but strictly speaking this is not correct, as this expression should be reserved for the zero-coupon yield curve only. We shall examine this in detail later.

Much of the analysis and pricing activity that takes place in the bond markets revolves around the yield curve. The yield curve describes the relationship between a particular redemption yield and a bond's maturity. We should be aware that the GRY of a bond is only ever the actual yield one receives during the period one holds the bond if certain specific, and generally unrealistic, conditions are met. However, we will leave the discussion of this for later.

Plotting the yields of bonds along the maturity term structure will give us our yield curve. It is very important that only bonds from the same class of issuer or with the same degree of liquidity are used when plotting the yield curve. For example, a curve may be constructed for UK gilts or for AA-rated sterling Eurobonds, but not a mixture of both, because gilts and Eurobonds are bonds from different class issuers. The primary yield curve

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[1] The author was tickled to read this description from someone obviously as excited about the yield curve as he is; Ryan (1997) writes:

"The future . . . of the global economy . . . may well rest on the success of how we finance the [yield] curve. God bless the Treasury Yield Curve!"

in any domestic capital market is the government bond yield curve, so for example, in the US market it is the US Treasury yield curve. With the advent of the euro currency in 11 (ultimately 19) countries of the European Union, in theory any euro-currency government bond can be used to plot a default-free euro yield curve. In practice, only bonds from the same government are used, as for various reasons different bonds within euro-currency countries trade at different yields. Outside the government, bond markets yield curves are plotted for Eurobonds, money market instruments, off-balance sheet instruments, in fact virtually all debt market instruments. Therefore it is always important to remember to compare like-for-like when analysing yield curves across markets.

In this chapter, we look at the yield to maturity yield curve as well as other types of yield curves that may be constructed. We also consider how to derive spot and forward yields from a current redemption yield curve. The main emphasis though, is on interpreting the shape of the yield curve, and explaining why it assumes the shapes it does. Later in the book we examine more advanced techniques for fitting, analysing, and interpreting the yield curve.

First though, we introduce the yield curve for beginners, of course experienced practitioners and graduate students may skip this part.

THE YIELD CURVE FOR BEGINNERS

This section is a summary of the importance and application of the yield curve. It was originally written with private investors in mind, so market practitioners may wish to skip this part.

What is the Yield Curve?

The yield curve is a graph that plots the yield of various bonds against their term to maturity. In other words, it is a snapshot of the current level of yields in the market. It is not an historical graph, that is, it does not show the level of yields over time. That would be an historical price (or yield) chart.

Yield curves are like football . . . it is very easy to grasp the basics, but difficult to become expert at (to continue the football analogy, akin to being good enough to play on a Sunday-morning park football team and being good enough to play on a team that included David Beckham, Steven Gerrard, and Michael Owen). Let us imagine that we looked up gilt yields in the *Financial Times* on a day in August 2002 and saw the following:

TABLE 1.1 Gilt yields

Gilt	Red Yield[2]
Tr 8% 03	3.79
Tr 5% 04	4.00
Tr 7.25% 07	4.62
Tr 5% 12	4.70
Tr 8.75% 17	4.74
Tr 8% 21	468
Tr 4.25% 32	4.52

Table 1.1 shows the yields for gilts of 1-, 2-, 5-, 10-, 15-, 20-, and 30-year maturity. (The 8% 2021 gilt is slightly under 20 years maturity but it will do for our purposes – there is no gilt that matures in 2022 at the time we are looking at this.)

We open up Microsoft Excel[3] and write down two columns, one for "maturity" and one for "yield". The years to maturity column forms the x-axis of the graph, while the yield forms the y-axis. Then we use the Excel "chart wizard" and it plots our graph for us! The result is as shown in Figure 1.1.

This curve looks about right. Intuitively, we expect that yields increase the greater the maturity. If we lend one person an amount of money for one year and another person the same amount of money for 10 years, we would not charge them both the same rate of interest (assuming both had the same credit risk) – we would most likely charge a higher rate to the 10-year borrower, for two reasons:

i. inflation will erode the value of the loan over the longer term, and;
ii. while the longer-dated borrower may be the same credit quality as the short-dated borrower, there are other risks. For example, the long-dated borrower may not be around in 10 years' time. Therefore, as a lender, we need a higher return to compensate for the greater risk the further out into the future we lend money.

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[2] "Red" means redemption yield.
[3] The author has no hesitation in endorsing Microsoft Excel as a superior product. He remembers having to use Lotus 1-2-3 in DOS when he first started work in the City . . .

FIGURE 1.1   Creating a yield curve in Excel.

So this gives us the *positively sloping* yield curve we see in Figure 1.1. However, if that is the case, why does the curve not continue to slope upwards, all the way to the 30-year mark? The rate of upward movement declines after the five-year mark, and then actually decreases to the 30-year point. This is a peculiarity of many markets – the 30-year bond, commonly called the *long bond*, is usually in such great demand among institutional investors, such as pension funds, that this demand outstrips supply. As a result, the price of this bond is forced upwards, and this moves the yield down to below what it should be.

Constructing a yield curve in the wholesale markets is a little more involved than what we have described above, and a ever-so-slightly complicated branch of mathematics is employed to derive the models used to fit yield curves. We consider this in later chapters.

Using the Yield Curve

The yield curve tells us where the bond market is trading now. It also implies the level of trading for the future, or at least what the market thinks will be happening in the future. In other words, it is a good indicator of the future level of the market. It is also a much more reliable indicator than any other used by private investors, and we can prove this empirically.

Let us consider the main uses of the yield curve. All participants in the debt capital markets have an interest in the current shape and level of the yield curve, as well as what this information implies for the future. The main uses are introduced below:

▪ **Setting the yield for all debt market instruments.** The yield curve essentially fixes the cost of money over the maturity term structure.

The yields of government bonds from the shortest-maturity instrument to the longest set the benchmark for yields for all other debt instruments in the market, around which all debt instruments are analysed. Issuers of debt (and their underwriting banks) therefore use the yield curve to price bonds and all other debt instruments. Generally the zero-coupon yield curve is used to price new issue securities, rather than the redemption yield curve.

■ **Acting as an indicator of future yield levels.** As we discuss later in this chapter, the yield curve assumes certain shapes in response to market expectations of the future interest rates. Bond market participants analyse the present shape of the yield curve in an effort to determine the implications regarding the future direction of market interest rates. This is perhaps one of the most important functions of the yield curve, and it is as much an art as a science. The yield curve is scrutinised for its information content, not just by bond traders and fund managers, but also by corporate financiers as part of project appraisal. Central banks and government treasury departments also analyse the yield curve for its information content, not just regarding forward interest rates, but also with regard to expected inflation levels.

■ **Measuring and comparing returns across the maturity spectrum.** Portfolio managers use the yield curve to assess the relative value of investments across the maturity spectrum. The yield curve indicates the returns that are available at different maturity points and is therefore very important to fixed income fund managers, who can use it to assess which point of the curve offers the best return relative to other points.

■ **Indicating relative value between different bonds of similar maturity.** The yield curve can be analysed to indicate which bonds are cheap or dear to the curve. Placing bonds relative to the zero-coupon yield curve helps to highlight which bonds should be bought or sold either outright or as part of a bond spread trade.

■ **Pricing interest rate derivative securities.** The price of derivative securities revolves around the yield curve. At the short end, products such as Forward Rate Agreements are priced off the futures curve, but futures rates reflect the market's view on forward three-month cash deposit rates. At the longer end, interest rate swaps are priced off the yield curve, while hybrid instruments that incorporate an option feature such as convertibles and callable bonds also reflect current yield curve levels. The "risk-free" interest rate, which is one of the parameters used in option pricing, is the T-bill rate or short-term government repo rate, both constituents of the money market yield curve.

YIELD TO MATURITY YIELD CURVE

The most commonly occurring yield curve is the yield to maturity yield curve. The equation used to calculate the yield to maturity is given in the Appendix. The curve itself is constructed by plotting the yield to maturity against the term to maturity for a group of bonds of the same class. Three different examples are shown at Figure 1.2. Bonds used in constructing the curve will only rarely have an exact number of whole years to redemption, however, it is often common to see yields plotted against whole years on the x-axis. This is because once a bond is designated the *benchmark* for that term, its yield is taken to be the representative yield. For example, the then 10-year benchmark bond in the UK gilt market, the 5.75% Treasury 2009, maintained its benchmark status throughout 1999 and into 2000, even as its term to maturity fell below 10 years. The yield to maturity yield curve is the most commonly observed curve simply because yield to maturity is the most frequent measure of return used. The business sections of daily newspapers, where they quote bond yields at all, usually quote bond yields to maturity.

As we might expect, given the source data from which it is constructed, the yield to maturity yield curve contains some inaccuracies. We have already come across the main weakness of the yield to maturity measure, which is the assumption of a constant rate for coupon reinvestment during the bond's life at the redemption yield level. Since market rates fluctuate over time, it is not possible to achieve this (a feature known as *reinvestment risk*). Only zero-coupon bondholders avoid reinvestment risk as no coupon is paid during the life of a zero-coupon bond.
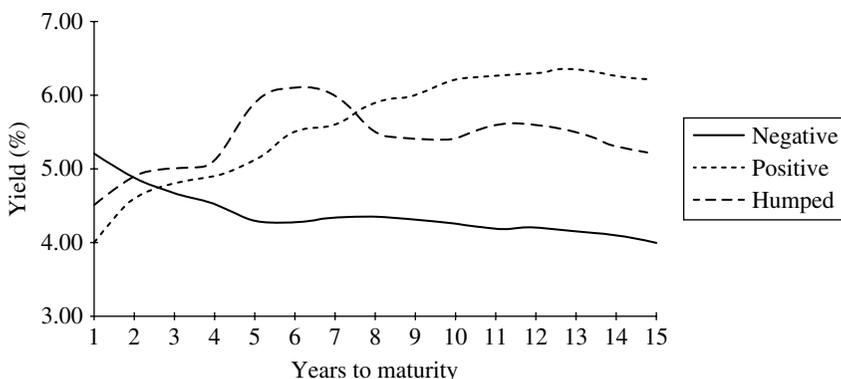
FIGURE 1.2   Yield to maturity yield curves.