

Andrew Pollard · Luciano Castillo  
Luminita Danaila · Mark Glauser *Editors*

# Whither Turbulence and Big Data in the 21st Century?

 Springer

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ISBN 978-3-319-41215-3

ISBN 978-3-319-41217-7 (eBook)

DOI 10.1007/978-3-319-41217-7

Library of Congress Control Number: 2016945165

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*Bill George and John Lumley shared a passion for turbulence but had other passionate interests too.*



*John Lumley standing in front of a restored 1951 Mk. VI Bentley, August 4, 2007, which he did in his “spare” time. John took great pride and pleasure as he acted as a chauffeur for weddings of close friends. Photo taken by Shelley Blackler (the groom’s mother and dear friend of the Lumleys). Photo supplied by John’s daughter, Jennifer. John wrote a book *Still Life with Cars: An Automotive Memoir*, McFarland & Co., Jefferson, North Carolina and London, 2005 (ISBN 0-7864-2053-7).*



*Bill George aboard “Wings,” the 42-foot sailboat that he and his wife sailed to Europe from America in 1995 and have lived aboard for extended periods since in Sweden and in various research locations throughout America and Europe. (Photo in Calais, FR, 2009, by Abofazl Shiri, one of many of Bill’s Ph.D. students who wrote all or part of their dissertations aboard)*

# Preface

It is often said that advances in most fields of endeavor result from “standing on the shoulders of giants,” and this meeting is no exception. In 1989, John Lumley, who needs no introduction to readers and researchers interested in turbulence, brought together leading thinkers and doers in turbulence to discuss the then-current controversies in the subject as well as to consider the role of public policy (and therefore funding) decisions that help to steer the field in either a direct way or through decisions which have unintended consequences. The meeting was international in scope and attendance and there resulted from this meeting a volume entitled “Whither Turbulence? Turbulence at the Crossroads.” The present volume summarises the findings and presentations of another meeting that considered the broad question of “Whither Turbulence” in the context of the ubiquitous network of computers and networks. John Lumley was invited to and indeed enthusiastically supported it: “I am honored . . . you have my blessing, for what it is worth.” This simple statement is a testament to his kind demeanor. Unfortunately, John could not attend due to illness and sadly he passed away in late May 2015.

In the intervening years or so between Lumley’s volume and the present one, much has happened and new giants have emerged in this, the oft-said “last unsolved problem in classical physics.” A significant disrupter to and leader in our field is Professor W. K. “Bill” George who was also Lumley’s student. From George, there have emerged many academic children and now grandchildren, each of whom continues to provide leadership and impact on the field. Given his 5-decade long career, the meeting, details of which are provided within these pages, was dedicated to Bill on the occasion of his 70th birthday.

In 25 years, the world of research in turbulence has changed to where computation and simulation has grown to become the third leg of the scientific stool. In fact, with the web/Internet, commodity computing, high-performance computing, and significant advances in experimental tools, especially particle image velocimetry, it could be said that what was a dream in 1989, say active control of turbulence, is



now becoming a reality because the three legs of the stool (theory, simulation, and experiment) have each advanced, and Bill has been leading the charge on at least two of those.

However, it remains to be seen what state we will be in 2040. The meeting in Cargese began 50 years after Gordon Moore predicted the future of the semiconductor: a doubling of computer processing speeds every two years. A look at Bill Reynolds' paper in the Lumley volume (Fig. 1, p. 342) suggests similar growth such that peta-flops are now reachable (as at 2008), with exascale computing on the near horizon (expected by 2020). One can imagine even further ubiquitous computational infrastructure and new and even more exciting methods, algorithms, and most importantly, ideas. But a significant issue now is data and this will continue to grow. In 1989, again with reference to Reynolds' paper, computer memory sizes were of order gigabytes, while in 2015, terabyte drives are ubiquitous and cheap. An example of drivers for increased data storage and bandwidth is the square kilometer array (radio telescope) that will produce about 30 exabytes of data per month, which will require a doubling of the current Internet traffic bandwidth, worldwide! One can imagine that while the turbulence community will continue to push the Reynolds number envelope, it will be in combination with other physicochemical processes (e.g., high-Schmidt-number turbulent mass transfer) over the full spectrum of scales (nano- to full scale, including planetary scale).

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# Acknowledgments

The organizers of the Whither Turbulence meeting would like to extend their thanks to the supporters of the meeting. These are:

The National Science Foundation  
Queen's University at Kingston  
US Office of Naval Research  
Global Office of Naval Research  
CNRS  
Institut P'  
ERCOFTAC  
CORIA  
Association Française de Mécanique

The meeting organization was ably assisted by Gabrielle Whan of Queen's University and Linda Whitehead at Texas Tech University.

The organizers also wish to acknowledge the following individuals for their assistance during the meeting, which ensured that any difficulty was resolved with notable aplomb:

Christophe Latailleur, CORIA  
Amandine Cyr, CORIA  
The Cargese staff: Dominique Donzella, Nathalie Giudicelli, Pierre-Eric Grossi, and Brigitte Cassegrain

Finally, we wish to thank Springer-Verlag for enthusiastically supporting the publication of this volume, especially Michael Luby and Brinda Megasyamalan.



# Overview of Volume

The meeting itself followed a traditional format held over a 5-day period in the delightfully salubrious environment of the Institut d'Etudes Scientifiques de Cargèse, Corsica. The meeting was divided into 15 sessions and two extensive discussion sessions. These were:

Turbulence, Then, Now and Future; Turbulence Control; Turbulent Boundary Layers; Simulations and Fluid Dynamics; Turbulent Structures and Jets; Turbulent Boundary Layers over Rough Surfaces; High Reynolds Numbers; Atmospheric Flows and Theory; Turbulence Theory; Turbulence and Renewable Energy; Wall-Bounded Flows; Large Data; Complex and Industrial Flows; Turbulence; and Simulations and Experiments.

The two discussion sessions were devoted to Challenges in Turbulence in the Twenty-First Century: What Problems We Should Focus On in the Next 20 Years? and Big Data: Opportunities for Collaborations and Dealing with Large Databases, which benefitted from an extensive video conversation between Profs. Hacker and Dyke at Purdue University and the delegates at the meeting. These two discussion sessions were recorded and the transcription of them is included here.

The volume begins with two papers that provide some perspective on the field of turbulence: Christos Vassilicos' delightful article "From Tennekes and Lumley to Townsend and to George: A Slow March to Freedom" and Bill George's "A 50-Year Retrospective and the Future," wherein he once again challenges the community to think differently. Thereafter, we have assembled the remaining contributions according to themes. These are Turbulent Boundary Layers, Jets, Environmental and Wind Energy, Data Manipulation, and General Topics.

The remaining 22 contributions reflect the diversity of the topics considered and extensively discussed at the meeting. These original research contributions are arranged as noted above, and each author has endeavored to place into context the link between their contribution and Big Data.

The Turbulent Boundary Layer section contains four papers. Marusic et al. consider the response of a TBL to different tripping conditions under "fixed and carefully quantified initial conditions" and to determine "under what conditions the

effects of upstream trip and other initial conditions no longer play a role in defining the state of the boundary layer.” They find that after sufficient development length they reach a “converged state.” From their perspective, the data set obtained from such broad spatial and temporal resolution experiments requires a community-type decision on what data should be retained for future use. Soria et al. present DNS of zero pressure gradient and adverse pressure gradient TBLs starting from the same initial condition argue the existence of a self-similar flow regardless of the pressure gradient. Their perspective on Big Data is for our community to recognize that many of the tools required have been and continue to be developed by computer scientists and recommend the need for broader interaction with them. Doosttalab et al. consider transitionally rough TBL using DNS and find that Townsend’s hypothesis, where the characteristics of a turbulent flow are independent of surface roughness beyond about 5 roughness heights, is invalid. They note that the Big Data issue revolves around having access to adequate HPC resources. Shahab et al. perform a quadrant analysis of a TBL that is perturbed by a shock and find that the higher-order correlations inform further those data obtained from lower-order statistical data. They introduce the idea of the 4 Vs of Big Data, volume, velocity, value, and variety, where velocity refers to the timeliness of the data.

The section on Jets begins with deepening fundamental insight on equilibrium similarity and scale-by-scale energy budgets in the near to intermediate field of a round free jet. They comment on data analytics and provide a useful excursion into high-performance computing, networks, and data archiving. Tinney et al. focus on rocket nozzle plumes and noise generation to focus on the “link between the sources of most intense vibro-acoustic loads that form during ignition . . . .” They speculate that in the future there will be a need for “strong synergy between both experimental and numerical disciplines that leverages carefully designed measurements with robust, yet simple, computational models with built-in analytics.” Voivenel et al. consider variable viscosity jets and determine that “the presence of a strong viscosity discontinuity across the jet edge results in an increase in both the scalar spread rate and the turbulent fluctuations.” At this stage of their work, the Big Data issue seems to be manageable with current resources. Hodzic et al. performed stereoscopic PIV in a round jet to test the robustness of Lumley’s projection approach. They noted the sensitivity of the results to spatial resolution. They argue that proper orthogonal decomposition is a useful tool to filter and therefore compress the size of a database to encapsulate the essential information contained in a larger database.

In the Environmental and Wind Energy section, we begin with Armenio’s consideration of spatial scales: from the laboratory to the real world. He focuses on physics of turbulence at laboratory scale which then informs the real-world environmental fluid mechanics issues. Hangan et al. introduce a novel wind engineering facility called the WINDEEE dome, which is “a hexagonal chamber of 25 m in diameter surrounded by a ‘return circuit’ of the same hexagonal shape of 40 m in diameter with the aim to create a wide variety of wind systems (e.g., tornadoes, downburst, all kind of gusts and currents, shear winds and boundary layers, etc.) at large scales and Reynolds numbers.” Clearly, the data deluge from real-world simulations and experiments as considered by Armenio and Hangan et al.

is challenging! Martin et al. consider supervisory control and data acquisition (SCADA) data gathered over an 18-month period from a 67-wind-turbine farm. From these data, they identified four wake effects and provided insight into wind farm performance that is richer than simple power curve analysis. It is clear that the time and spatial scales involved produce huge “Big Data” issues that the SCADA approach may alleviate. Ali et al. delve more deeply into wind turbine wakes through probing with hot wires of the wake of a wind turbine array in a wind tunnel.

Big Data suggests volumes of data rather than necessarily the other 3 Vs introduced by Shahab et al. In this section which is devoted to data manipulation, various compression and filtering approaches are introduced so that inherently each author recognizes the Big Data issue and have decided to address it from a different perspective. Magstadt et al. consider various jet flows that produce pressure, velocity, acoustic, and other data and “different levels of granularity, density (or sparseness)/distribution (uniform, checkered, lattice, random, etc.) and span in space and time to develop a holistic systems-level understanding”. They apply Big Data analyses/modeling tools (the right filters) to identify patterns and predictive models rather than just a posteriori trends, statistics and distributions. Bai et al. consider the use of machine learning where the “overarching goal is to reduce the burden of data acquisition and processing.” Buchave and Velte recognize the limits of data obtained in the temporal domain, and they propose a method to eliminate the need to impose Taylor’s frozen eddy hypothesis and find a way to obtain spatial gradients from both hot wire and laser Doppler signals.

The papers to be found in the General Aspects of Turbulence are listed in alphabetical order. Barros et al. consider blowing as a control strategy and deduce a 30% recovery in base pressure for a simplified blunt vehicle. Fureby explores the state-of-the-art large eddy simulation in the context of large-scale engineering flows with reliance on DNS and a variety of other thermophysical models. He provides an extensive bibliography culminating in petascale DNS of combustion. Grinstein et al. consider under-resolved velocity fields (and initial conditions) as a prelude to accurate predictions and quantifiable uncertainty in turbulent material mixing and relate these ideas to Bill George’s ideas on initial conditions. They conclude that “Ensemble averaging over a suitably complete set of realizations covering the relevant IC (initial condition, ed.) variability is a data reduction strategy of choice”. Meldi and Sagaut consider homogeneous, isotropic turbulence using an eddy-damped quasi-normal Markovian (EDQNM) model and provide insight into the time evolution of  $C_\epsilon$ . Orlandi et al. return to a DNS of the minimal flow unit (channel flow) to explore why an inertial range forms and to further inform the separate effects of large- and small-scale structures. Rahbari and Scalo consider compressible channel flow (DNS) with a general impedance boundary condition to effect flow control. They introduce the idea of “Small Data” and “Big Data” and emphasize the need for both. Tardu rounds out the individual papers by addressing the important problem of dissipation and particularly how the “palm” statistics (“palm statistics are the statistics of a given quantity under the condition that another stochastic process crosses a fixed level”) that help inform mean dissipation conditioned by level crossings of the spanwise velocity seems independent of y

(essentially the buffer layer region, ed.) using a DNS of channel flow at  $Re_\tau = 1100$ . He argues that for computational turbulence scientists should agree on common data formats given the multi-terabyte databases being generated.

The transcription of the 90-minute Discussion Session 1 provides a broad exploration of Whither Turbulence, and the reader will note many different threads. Of course, given the 60+ attendees, you can imagine there are at least 61+ arguments to consider! It became clear that cross-laboratory collaboration is an important theme that will enable large-scale experiments and computations to take place in a collaborative manner. There are many internationally competitive and unique experimental facilities across the world that are willing to share access; of course funding issues remain, including transportation and subsistence costs for enabling this mobility.

Discussion Session 2 considers Big Data. This session, which was two hours long, included a video link with Purdue University and Professor Tom Hacker of the Department of Computer and Information Technology and Professor Shirley Dyke of Mechanical and Civil Engineering. They are connected to the National Earthquake Engineering Simulation (<http://nees.org>) and provided a basis for the extensive discussions on Big Data, cyberinfrastructure, and collaborative environments. The perspective they brought to the general discussion triggered many parallel ideas on data archiving, transmission, and sharing. As a result of these discussions, Prof. Hacker and Dyke were invited to provide a written contribution to this volume. Furthermore, the editors invited Profs. Meneveau and Marusic (of Johns Hopkins and Melbourne, respectively), the former being unable to join the meeting directly; Dr. Sillero and Prof. Jimenez of Universidad Politecnica de Madrid, who could not participate directly; and Prof. Menon and Ranjan of Georgia Tech and Dr. Oefelein of Sandia National Labs, who were approached after the meeting to contribute their ideas and perspectives on Big Data. As the reader will appreciate, there are manifold interpretations of what Big Data means. The emergence of big databases for turbulence research will continue to evolve, and the community will continue to find efficacious ways to both create and interface with them. The invited contributors address the Big Data issue from very different perspectives, and with the other perspectives mentioned, we hope that they will stimulate the turbulence and fluid dynamics community to work with the broader Big Data communities and agencies before they force us into a position that is not in our best interest.

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**Part I**  
**Historical Perspectives**

# Chapter 1

## From Tennekes and Lumley to Townsend and to George: A Slow March to Freedom

J.C. Vassilicos

### 1.1 Introduction

The spring of 2015 has been a landmark for the turbulence research community. In April, Bill George turned 70 and a conference was held in Cargese, Corsica, in his honour. A little more than a month later, on May 30th, John Lumley passed away. John was a central reference figure in turbulence research in the second half of the twentieth century and also the PhD supervisor of Bill George. The spring of 2015 has therefore brought some pause for thought and reflection on the ups and downs, attempts, failures and achievements, vigorous debates, agreements and disagreements on various aspects of the turbulence problem(s) over the past 50 years.

I know Bill George far better than I have known John Lumley. The last time I talked to John Lumley was in the summer of 2000 at the European Turbulence Conference in Barcelona. He came up to me when we were heading towards a social function organised by the conference and said: ‘do you mind if I stick with you, I do not know anyone here any longer’! I knew instantly that I would never forget this comment. Perhaps it meant that the European turbulence community did not interact much with the American one at the time? But it surely also signalled the advent of a new generation and with it, perhaps, a shift of emphasis on what is worth researching. The other two landmarks that come to my mind whenever I think of this episode are June 2nd 1986 when Stan Corrsin passed away, and March 1989 when the meeting ‘Whither Turbulence? Turbulence at the Crossroads’ was held in Cornell, having been organised by John Lumley. I have often had the sense that a way was lost in turbulence research after these two dates’. Turbulence at

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the Crossroads' was perhaps a prescient warning. A decade later one of the central figures in turbulence did not recognise his own research community.

Bill George often recounts how, at a turbulence workshop in Monte Verita, Switzerland, in 1998, John Lumley approached him, and referring to Chap. 4 'Boundary-free shear flows' of the classic 1972 textbook 'A first course on Turbulence' (that John had co-authored with H. Tennekes), told him: 'I now teach it your way'. This is a good example of the vigorous debates, agreements and disagreements that gripped the turbulence community through the time which spanned the careers of these two men and I have chosen to concentrate my contribution to this book on boundary-free shear flows. This book is a memento of the Cargese April 2015 meeting which celebrated the achievements of Bill George but which also intentionally echoed John Lumley's 1989 meeting through its title 'Whither turbulence and Big Data in the 21st Century'; I guess, perhaps trying to find our way again.

## 1.2 Self-preserving Turbulent Wake: Townsend [13] and Tennekes and Lumley [12]

The particular boundary-free turbulent shear flow which I discuss here for the purpose of concrete illustration is the self-preserving turbulent wake. Usually one assumes that a planar or axisymmetric turbulent wake becomes self-preserving/self-similar at some distance downstream where the details of the wake-producing obstacle have lost their influence on the flow. The problem is to know how the turbulence wake's mean velocity deficit and mean width vary with streamwise distance.

Following Townsend [13], Tennekes and Lumley [12] present a solution to this problem which is based on the Reynolds-averaged streamwise momentum equation. Concentrating attention on the axisymmetric case to more concretely ground thoughts, the Reynolds-averaged streamwise momentum equation is

$$U_\infty \frac{\partial}{\partial x}(U_\infty - U) = -\frac{1}{r} \frac{\partial}{\partial r} r \langle u_x u_r \rangle \quad (1.1)$$

where  $U_\infty$  is the uniform free stream velocity and  $U$  is the streamwise mean flow velocity at streamwise distance  $x$  from the obstacle and radial distance  $r$  from the streamwise centreline. The Reynolds shear stress is  $\langle u_x u_r \rangle$  in usual understandable notation.

The self-preserving solutions considered by Townsend [13] and Tennekes and Lumley [12] are of the form

$$U_\infty - U = u_0(x) f[r/\delta(x)] \quad (1.2)$$

and

$$\langle u_x u_r \rangle = u_0^2 g[r/\delta(x)]. \quad (1.3)$$

Note the strong assumption, later relaxed by Townsend [14] and George [2], that the Reynolds shear stress scales with the square of the centreline mean velocity deficit  $u_0$ . With this assumption, the problem to find the  $x$ -dependencies of  $u_0(x)$  and the wake width  $\delta(x)$  is simply solved by noting that (1.1) admits solutions of the form (1.2) and (1.3) under the conditions

$$\frac{d}{dx}\delta(x) \sim \frac{u_0}{U_\infty} \text{ and } u_0\delta^2 \sim U_\infty\theta^2 = \text{Const}$$

where  $\theta$  is the conserved momentum thickness. There are therefore two conditions for two unknowns,  $u_0$  and  $\delta$ , leading to the solution (where  $x_0$  is a virtual origin)

$$u_0/U_\infty \sim \left(\frac{x-x_0}{\theta}\right)^{-2/3} \quad (1.4)$$

and

$$\delta/\theta \sim \left(\frac{x-x_0}{\theta}\right)^{1/3} \quad (1.5)$$

which can be found in many turbulence textbooks, in particular Townsend [13, 14], Tennekes and Lumley [12], Mathieu and Scott [6] and Pope [9].

### 1.3 Townsend [14]

With Townsend [14] starts the ‘slow march to freedom’ mentioned in the title of this contribution. In the 1976 revision of his 1956 book, Townsend relaxed the self-preservation ansatz of the Reynolds shear stress and replaced (1.3) with

$$\langle u_x u_r \rangle = R_0 g[r/\delta(x)]. \quad (1.6)$$

where  $R_0$  is an extra free parameter and not necessarily proportional to  $u_0^2$ . This extra freedom leads to the two conditions

$$\frac{d}{dx}\delta(x) \sim \frac{R_0}{U_\infty u_0} \text{ and } u_0\delta^2 \sim U_\infty\theta^2 = \text{Const}$$

which are necessary for (1.2) and (1.6) to be consistent with (1.1). These two conditions now involve three, rather than two, unknowns, namely  $u_0$ ,  $\delta$  and  $R_0$  and are therefore not conclusive by themselves. This is effectively a closure problem,

as is so typically the case in turbulence, and Townsend [14] introduced the idea of using the turbulent kinetic energy equation and the Taylor–Kolmogorov turbulence dissipation scaling to close this problem. However he still over-restricted the problem by assuming that the turbulent kinetic energy’s self-preservation form is

$$K = R_0 k[r/\delta(x)]. \quad (1.7)$$

The result of Townsend’s [14] approach was identical to Townsend’s [13] and Tennekes and Lumley’s [12] equations (1.4) and (1.5). Allowing more freedom to the problem led to the same  $x$ -dependencies of the wake’s mean flow velocity deficit and width. If this was not enough, Townsend’s [14] approach returned  $R_0 \sim u_0^2$  by itself. The extra freedom Townsend [14] gave was not appreciated by the problem which naturally returned to the same conclusions and even assumptions of Townsend [13] and Tennekes and Lumley [12]. The one extra gain of Townsend’s [14] approach was the scaling  $K = R_0 k[r/\delta(x)] \sim u_0^2 k[r/\delta(x)]$ .

## 1.4 George [2]

George [2] took an extra, crucial, step in this slow march to freedom. Like Townsend [14] he left  $R_0$  unconstrained but unlike Townsend [14] he assumed

$$K = K_0 k[r/\delta(x)]. \quad (1.8)$$

and did not constrain  $K_0$  to be proportional to  $R_0$ . He left  $K_0$  as a free parameter.

George’s [2] approach is based on the momentum equation (1.1) and associated self-preservation forms (1.2) and (1.6) as well as the turbulence kinetic equation

$$U_\infty \frac{\partial}{\partial x} K = -\langle u_x u_r \rangle \frac{\partial}{\partial r} U + T - \epsilon \quad (1.9)$$

and associated self-preservation forms (1.8),

$$T = T_0 t[r/\delta(x)]. \quad (1.10)$$

for the transport and pressure terms and

$$\epsilon = D_0 e[r/\delta(x)]. \quad (1.11)$$

for the turbulence dissipation rate. This is similar to Townsend [14] except that Townsend used a slightly different form of the turbulent kinetic energy equation and took  $K_0$  to be proportional to  $R_0$ .

George’s [2] theory leads to five conditions for six unknowns, namely  $u_0$ ,  $\delta$ ,  $R_0$ ,  $K_0$ ,  $T_0$  and  $D_0$  and, again, requires closure to provide answers. This closure is achieved as in Townsend [14] by using the Taylor–Kolmogorov scaling

$$D_0 \sim K_0^{3/2} / \delta. \quad (1.12)$$



Readers who are novices in turbulence research might be surprised to read at this stage that all this extra freedom given by George [2] is in some sense inconsequential because the results of George's [2] theory are (1.4) and (1.5) again and therefore identical to those of Townsend [13], Tennekes and Lumley [12] and Townsend [14]. Furthermore, Townsend [14] and George [2] both predict  $K_0 \sim u_0^2$ . It would seem that however much freedom you allow into the problem, the predictions are always the same. Even so, John Lumley felt it necessary to tell Bill George in 1998 that 'I now teach it your way', meaning in the way of this section rather than Sect. 1.2.

There is in fact one important difference between George [2] and the other theories. Whereas for Townsend [13], Tennekes and Lumley [12] and Townsend [14]  $R_0 \sim u_0^2$ , George [2] predicts  $R_0 \sim U_\infty u_0 \frac{d}{dx} \delta$ . The scalings (1.4) and (1.5) imply that  $R_0 \sim u_0^2$  and  $R_0 \sim U_\infty u_0 \frac{d}{dx} \delta$  return the same  $x$ -dependence for  $R_0$ . However, if the  $x$ -scalings of  $u_0$  and  $\delta$  were different, as can happen if the dissipation's scalings differ from (1.12), then it would be possible to distinguish between  $R_0 \sim u_0^2$  and  $R_0 \sim U_\infty u_0 \frac{d}{dx} \delta$ .

Dissipation scalings different from (1.12) have indeed been discovered recently in various turbulent flows, and in the axisymmetric turbulent wake in particular (see [1, 3, 8, 15]). This has created an opportunity for the distinction between George [2] and the other theories of self-preserving axisymmetric turbulent wakes [12–14] to become manifest and meaningful.

## 1.5 Turbulence Dissipation Scalings

The Taylor–Kolmogorov dissipation law first proposed by Taylor [11] and then given a theoretical underpinning by Kolmogorov [5] is

$$D_0 \sim K_0^{3/2}/L \quad (1.13)$$

where  $L$  is an integral length-scale. Kolmogorov's [5] theoretical justification of (1.13) is given in the framework of his equilibrium cascade theory where the interscale energy flux at length-scales comparable to  $L$  scales as  $K_0^{3/2}/L$  and balances the turbulence dissipation at the smallest, viscosity-dominated, length-scales (see [15]). Townsend [13, 14] presents arguments in support of  $L \sim \delta$  which allows (1.13) to be written as (1.12), the form which provides the closure leading to the wake laws (1.4) and (1.5).

It is intriguing that there is another way to obtain (1.12), and this is via the strong self-preservation theorem proved by Johansson et al. [4] in the Appendix of their paper. These authors showed that if one uses all individual component Reynolds stress equations (neglecting viscous terms for high enough Reynolds number) instead of the single turbulent kinetic energy equation used by Townsend [14] and George [2] and if one assumes that every single term in each one of these equations is self-preserving (strong self-preservation), then (1.12) follows directly. There is no need to assume the validity of the dissipation scaling (1.12) in this strong self-preservation scenario, it just follows.

Conversely, the strong self-preservation theorem implies that if (1.12) is violated then there have to be some terms in some of the individual component Reynolds stress equations which are not self-preserving. Recent wind tunnel experiments of high Reynolds number axisymmetric turbulent wakes [1, 8] have shown that in a region extending in the streamwise direction between about ten and at least fifty times the size  $L_B$  of the wake-generating obstacle, the dissipation scalings are given by

$$D_0 \sim U_\infty L_B K_0 / \delta^2 \quad (1.14)$$

which is very different from (1.12). This new dissipation law characterises non-equilibrium interscale energy exchanges because one can still expect the interscale energy flux at length-scales comparable to  $L$  to scale as  $K_0^{3/2}/L$  (see arguments based on the Karman–Howarth equation in Vassilicos [15]), in which case the turbulence dissipation at small scales clearly does not balance this large-scale flux. It is remarkable that forms equivalent to (1.14) seem to hold in a variety of turbulent flows (see [3, 15]), suggesting some universality to non-equilibrium turbulence.

Replacing the dissipation relation (1.12) with the non-equilibrium relation (1.14) in the theory of George [2] leads to

$$u_0/U_\infty = C_1 \left( \frac{x-x_0}{\theta} \right)^{-1} \theta/L_B \quad (1.15)$$

and

$$\delta(x)/L_B = C_2 \left( \frac{x-x_0}{\theta} \right)^{1/2} (L_B/\theta)^{1/2}. \quad (1.16)$$

Extensive hot wire anemometry measurements have confirmed these two scalings in a streamwise region between  $x \approx 5L_B$  and at least  $x = 50L_B$  [1, 7, 8]. The use of systematically different plates with same surface area  $A$  has made it clear that the dimensionless constants  $C_1$  and  $C_2$  depend on the geometrical details of the wake generator [1, 7, 8].

Given that the wake scalings in this region are different from (1.4) and (1.5), this is the ideal ground where to test the George [2] approach against those of Townsend [13, 14] and Tennekes and Lumley [12]. The direct numerical simulations (DNS) of Dairay et al. [1] gives clear support for George's

$$R_0 \sim U_\infty u_0 \frac{d}{dx} \delta(x) \quad (1.17)$$

and clearly invalidates  $R_0 \sim u_0^2$  which is an assumption for Townsend [13] and Tennekes and Lumley [12] and a prediction for Townsend [14]. George's [2] footsteps in this slow march to freedom have therefore been indispensable, at least for the region where the non-equilibrium dissipation law rules and (1.15) and (1.16) hold too. The DNS of Redford et al. [10] suggest that (1.4) and (1.5)

actually hold very much further downstream where the local Reynolds number has dropped to much lower values and, presumably, the dissipation scalings have changed from (1.14) to (1.12).

## 1.6 $K_0 \sim u_0^2$ ? The March to Freedom is Not Over

As mentioned in Sect. 1.2, the justification often given for self-preservation of flow profiles is that the details of the wake-producing obstacle lose their influence on the flow at some distance far enough downstream. It is clear from the self-preservation theorem of Johansson et al. [4] that there cannot be strong self-preservation in the region where (1.14) holds instead of (1.12). Recent wind tunnel experiments and DNS by Nedic et al. [7] and Dairay et al. [1] have shown that profiles such as those of the mean flow velocity, the Reynolds shear stress, the turbulent kinetic energy and the turbulence dissipation are self-preserving in this region whereas other flow profiles are not. Their data also invalidate  $K_0 \sim u_0^2$  in this region and strongly suggest

$$K_0 \sim R_0 \tag{1.18}$$

instead. This is important because both theories which can make a prediction on  $K_0$ , those of Townsend [14] and George [2], predict  $K_0 \sim u_0^2$ .

The failure of  $K_0 \sim u_0^2$  points to a failure of (1.9) in the context of George's [2] approach because approximating the turbulence production by  $-\langle u_x u_r \rangle \frac{\partial}{\partial r} U$  is essential for obtaining  $K_0 \sim u_0^2$ . The DNS of Dairay et al. [1] shows that the turbulence production is dominated by normal stress terms on and around the centreline and that these normal stress terms are not negligible off centreline either. It also shows that the profiles of these normal stresses are not self-preserving at least till  $x = 100L_B$  which is consistent with the strong self-preservation theorem and the failure of the Taylor–Kolmogorov dissipation law in this region.

There is therefore a need to grant new freedoms by not constraining all terms in the energy equation to be self-preserving. Dairay et al. [1] have therefore been led to propose a new approach which is based on the momentum equation (1.1) and associated self-preservation forms (1.2) and (1.6) as well as the turbulence kinetic equation

$$U_\infty \frac{\partial}{\partial x} K = P + T - \epsilon \tag{1.19}$$

where only  $K$  and  $\epsilon$  are self-preserving but not  $P$  and  $T$  (though it of course follows from (1.19) that  $P + T$  is self-preserving). The self-preservation forms of  $K$  and  $\epsilon$  are again given by (1.8) and (1.11), respectively, and the problem to determine the a priori independent quantities  $u_0, R_0, \delta, K_0$  and  $D_0$  requires more information to be closed. The information is enough as it is to give (1.17), very much like in George [2], but not more. If one assumes that the turbulence is out of two-point

equilibrium and obeys (1.14), then this extra piece of information allows (1.15) and (1.16) to follow, in agreement with experimental and numerical data. Hence we have a consistent theoretical framework in agreement with observations, at least for the region where the turbulent dissipation scales as (1.14). More details and discussion can be found in Dairay et al. [1].

## 1.7 Conclusion

This slow march to freedom has taken about 60 years and does not seem to have fully ended yet. The important progress brought about by the seminal contributions of Townsend [14] and George [2] has led us to new questions. For example, given that the dimensionless constants  $C_1$  and  $C_2$  in (1.15) and (1.16) depend on the details of the wake-generating body, thereby indicating a clear dependence of the flow on initial conditions, why is it that some flow profiles are self-preserving in the region where (1.15) and (1.16) hold? What is the root cause of self-preservation?

Secondly, what happens at the far downstream point where the wake laws (1.15) and (1.16) cease to hold and the traditional wake laws (1.4) and (1.5) take over as the DNS of Redford et al. [10] would suggest? Are there similar transitions from one scaling to another in other boundary-free turbulent shear flows? And what does such a transition imply for self-preservation?

And finally, what is the reason for (1.18) and what are the cascade physics behind the non-equilibrium dissipation law (1.14)? How do these physics change to give rise to the expected scaling (1.12) at some point downstream?

Big data sets generated by experimental measurements and DNS have helped make progress beyond Townsend [14] and George [2] and will definitely play an important role in answering these new questions. But most important of all, one conclusion of the 2015 meeting ‘Whither turbulence and Big Data in the 21st Century’ must surely be that one must ask the right questions in the first place.

**Acknowledgements** This may be the right place to record my deepest gratitude to Myriam Scheel Larsen for her very careful, caring and conscientious teaching. She has been a link between Bill George and myself decades before we knew it. There must have been days 35 or so years ago in Denmark, when she would come to school to teach introductory physics and chemistry to my class in the morning and then have dinner in the evening with Bill George who was already Professor and who was visiting her husband, Professor Poul Scheel Larsen.

## References

1. T. Dairay, M. Obligado, J.C. Vassilicos, Non-equilibrium scaling laws in axisymmetric turbulent wakes. *J. Fluid Mech.* **781**, 166–195 (2015)
2. W.K. George, The self-preservation of turbulent flows and its relation to initial conditions and coherent structures, in *Advances in Turbulence*, ed. by W.K. George, R. Arndt (Hemisphere Publishing Corp., New York, 1989), pp. 39–73

3. S. Goto, J.C. Vassilicos, Energy dissipation and flux laws for unsteady turbulence. *Phys. Lett. A* **379**(16–17), 1144–1148 (2015)
4. P.B.V. Johansson, W.K. George, M. Gourlay, Equilibrium similarity, effects of initial conditions and local Reynolds number on the axisymmetric wake. *Phys. Fluids* **15**(3), 603–617 (2003)
5. A.N. Kolmogorov, Dissipation of energy in locally isotropic turbulence. *Dokl. Akad. Nauk. SSSR* **32**, 16–18 (1941)
6. J. Mathieu, J. Scott, *An Introduction to Turbulent Flow* (Cambridge University Press, Cambridge, 2000)
7. J. Nedic, J.C. Vassilicos, B. Ganapathisubramani, Axisymmetric turbulent wakes with new non-equilibrium similarity scalings. *Phys. Rev. Lett.* **111**(14), 144503 (2013)
8. M. Obligado, T. Dairay, J.C. Vassilicos, Non-equilibrium scalings of turbulent wakes. *Phys. Rev. Fluids* (2016, to appear)
9. S.B. Pope, *Turbulent Flows* (Cambridge University Press, Cambridge, 2000)
10. J.A. Redford, I.P. Castro, G.N. Coleman, On the universality of turbulent axisymmetric wakes. *J. Fluid Mech.* **710**, 419–452 (2012)
11. G.I. Taylor, Statistical theory of turbulence. *Proc. R. Soc. Lond. A* **151**, 421–444 (1935)
12. H. Tennekes, J.L. Lumley, *A First Course in Turbulence* (MIT Press, Cambridge, 1972)
13. A.A. Townsend, *The Structure of Turbulent Shear Flow* (Cambridge University Press, Cambridge, 1956)
14. A.A. Townsend, *The Structure of Turbulent Shear Flow* (Cambridge University Press, Cambridge, 1976)
15. J.C. Vassilicos, Dissipation in turbulent flows. *Ann. Rev. Fluid Mech.* **47**, 95–114 (2015)

# Chapter 2

## A 50-Year Retrospective and the Future

William K. George

### 2.1 Big Data

Experimental turbulence research has always been about “Big Data”—and usually never enough of it. Part of the reason has been because of the need to use statistical measures. Data records measured in thousands of time integral scales are necessary to make even the simplest estimators converge, sometimes even tens and hundreds of thousands of integral scales in length for probability density functions and correlations at large lags. As a general rule the time (or length of record required) for a given statistical error is proportional to the rms fluctuations of the statistical quantity being estimated divided by the square root of the number of effectively independent realizations of it. Note that the variance of *the quantity being measured* is not the same as the variance of the underlying process. For example, if a second moment is to be measured its variance is  $\langle [u^2 - \langle u^2 \rangle]^2 \rangle$ , which for a Gaussian process is  $3 \langle u^2 \rangle^2$ , and can be much larger for non-Gaussian processes which are common in turbulence. The pre-multiplying factor for simple powers of the variance increases rapidly with the order of the moment, so demands on data length can increase very rapidly (v. [15, 29, 30], or appendices of my turbulence notes available at [www.turbulence-online.com](http://www.turbulence-online.com)). The same is true for attempts to measure events of decreasing probability (like the tails of a pdf), since the lower the probability of it being observed, the more “statistically independent” data that must be acquired to measure it. Fractional statistical error, or variability, is the rms fluctuation of the quantity being measured divided by its average or expected value, or the variability of the quantity desired itself. So the higher the variability of the process, the more independent samples are required to estimate it. Quantities with zero mean will always have infinite variabilities, but finite errors. Many a student has thrown away

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