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Non-cooperative Stochastic Differential Game Theory of Generalized Markov Jump Linear Systems

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Preface

Differential game refers to a kind of problem related to the modeling and analysis of conflict in the context of a dynamical system. More specifically, a state variable or variables evolved over time according to differential equations. It is a mathematical tool for solving the bilateral or multilateral problems in dynamic continuous conflicts, competition, or cooperation, which has been widely applied in the fields of military, industrial control, aeronautics and astronautics, environmental protection, marine fishing, economic management and the market competition, finance, insurance, etc.

This book is focused on the generalized Markov jump linear systems which is widely used in engineering and social science, using dynamic programming method and the Riccati equation method to study the dynamic non-cooperative differential game problems and its related applications. This book includes the following studies: the stochastic differential game of continuous-time and discrete-time Markov jump linear systems; the stochastic differential game of linear stochastic differential game of generalized Markov jump systems; the stochastic H_2/H_∞ robust control of generalized Markov jump systems; and the risk control of portfolio selection, European option pricing strategy, and the optimal investment problem of insurance companies. In addition, this book created a variety of mathematical game models to derive the explicit expression of equilibrium strategies, to enrich the theory of equilibrium analysis of dynamic non-cooperative differential game of generalized Markov jump systems. It is to analyze and solve the robust control problems of generalized Markov jump systems based on the game theory. The applications of these new theories and methods in finance and insurance fields were presented.

The main content is divided into the following six sections:

1. The introduction and basic knowledge

This section introduces the basic models and the latest research of generalized Markov jump systems, the research content of differential game theory of generalized Markov jump systems, and the related concepts of differential game theory.

2. The stochastic differential game of continuous-time Markov jump linear systems

From the perspective of stochastic LQ problem, this section studied the stochastic optimal control problem of continuous-time Markov jump linear systems, and then to extend study on the two-person Nash stochastic differential game problem, finally to explore the two person Stackelberg stochastic differential game problem, and to achieve the equilibrium solutions of various problems.

3. The stochastic differential game of discrete-time Markov jump linear systems

From the perspective of stochastic LQ problem, this section studied the stochastic optimal control problem of discrete-time Markov jump linear systems, and then to extend study on the two person Nash stochastic differential game problem, finally to explore the two person Stackelberg stochastic differential game problem, and to achieve the equilibrium solutions of various problems.

4. The stochastic differential game of generalized Markov jump linear systems

This part is to establish the following models: two person zero-sum stochastic differential game, two person nonzero-sum game, Nash game, Stackelberg game, to achieve the equilibrium solutions, and to obtain the explicit expressions of the equilibrium strategies.

5. The stochastic H_2/H_∞ control of generalized Markov jump linear systems

Based on Nash game and Stackelberg game, this part is to establish the Markov jump linear systems models, the stochastic H_2/H_∞ control of generalized Markov jump linear systems models, to achieve the mathematical expression of the optimal robust control.

6. The stochastic differential game of generalized Markov jump linear systems in the applications in the fields of finance and insurance

This part is to establish differential game models of the minimal risk control of portfolio selection, option pricing strategy, and the optimal investment of insurance companies. And regarding the probability measurements of the economic environment as a player, regarding the investors as another player, the differential game models are to achieve the optimal control equilibrium strategies by solving two person differential game problems.

The research achievements of this book are sponsored by two foundations: the National Natural Science Foundation of China, which is named “Non-cooperative stochastic differential game theory of generalized Markov jump linear systems and its application in the field of finance and insurance” (71171061); and the Natural Science Foundation of Guangdong Province, which is named “Non-cooperative stochastic differential game theory of generalized Markov jump linear systems and its application in the field of economics” (S2011010000473). All achievements of this research are counting on the assistances and supports of National Nature Science Foundation of China and the Natural Science Foundation of Guangdong Province. Thanks a lot!

A group of members contribute to the accomplishment of this book, which includes the following: Dr. leader Zhang Cheng-ke, who is the professor; the doctoral student supervisor; the dean of School of Economics and Commerce, Guangdong University of Technology; the executive director of Chinese Game Theory and Experimental Economics Association; the executive director of National College Management of Economics Department Cooperative Association; vice chairman of Systems Engineering Society of Guangdong Province; Dr. Zhu Huai-nian, who is the lecturer of School of Economics and Commerce, Guangdong University of Technology; Dr. Bin Ning, who is the lecturer of School of Management, Guangdong University of Technology; and Dr. Zhou Hai-ying, who works in Students' Affairs Division, Guangdong University of Technology. Team members play a team spirit; have close cooperation; work in unity and cooperation; publish a number of papers, which has laid a good foundation for the completion of this book. The achievements of this book presented in front of readers are the collaborative efforts and hard work of all members of the research group!

Thanks to Zhang Chengke's graduate students Cao Ming, Zhu Ying! They have made a lot of work in terms of manuscript input, format correction, and check the formula, etc.

Special thanks for the help and supports of Guo Kaizhong, who is the professor of Guangdong University of Technology; and Cao Bingyuan, who is the professor of Guangzhou University! Owing to their constant encouragements make this book completed and presented to the readers as soon as possible.

Counting on the References to the scholars quoted in the book, which make the fruitful base of our work!

Although we have made a lot of efforts for the completion of this book, due to the limited level, there must be a lot of shortcomings and deficiencies. Please to criticize and correct.

Guangzhou, China

Cheng-ke Zhang
Huai-nian Zhu
Hai-ying Zhou
Ning Bin

Contents

Representation of Symbol

M'	The transpose of any matrix or vector M;
$M > 0$	The symmetric matrix M is positive definite;
$\mathbb{R}^n$	The n-dimensional Euclidean space;
$\mathbb{R}^{n\times m}$	the set of all $n \times m$ matrices;
$\mathcal{S}^n$	the set of all $n \times n$ symmetric matrices;
$\mathcal{S}^n_+$	The subset of all non-negative definite matrices of $\mathcal{S}^n$;
$\mathcal{S}^n_l$	$\underbrace{\mathcal{S}^n \times \cdots \times \mathcal{S}^n}_{l}$;
$\left(\mathcal{S}^n_+\right)^l$	$\underbrace{\mathcal{S}^n_+ \times \cdots \times \mathcal{S}^n_+}_{l}$;
$\mathcal{M}^l_{n,m}$	space of all $A = (A(1), A(2), \cdots, A(l))$ with $A(i)$ being $n \times m$ matrix, $i = 1, 2, \cdots, l$;
$\mathcal{M}^l_n$	$\mathcal{M}^l_{n,n}$;
χ_A	The indicator function of a set A;
$\mathcal{C}(0, T; \mathbb{R}^{n\times m})$	The set of continuous functions $\phi : [0, T] \to \mathbb{R}^{n\times m}$;
$L^p(0, T; \mathbb{R}^{n\times m})$	the set of functions $\phi : [0, T] \to \mathbb{R}^{n\times m}$ such that $\int_0^T \|\phi(t)\|^p dt < \infty (p \in [1, \infty))$;
$L^\infty(0, T; \mathbb{R}^{n\times m})$	the set of essentially bounded measurable functions $\phi : [0, T] \to \mathbb{R}^{n\times m}$;
$\mathcal{C}^1\left(0, T; \mathcal{S}^n_l\right)$	the set of continuously differential functions $\phi : [0, T] \to \mathcal{S}^n_l$.

Content Introduction

This book systematically studied the stochastic non-cooperative differential game theory of generalized linear Markov jump systems and its application in the field of finance and insurance. First, this book was an in-depth research of the continuous-time and discrete-time linear quadratic stochastic differential game, in order to establish a relatively complete framework of dynamic non-cooperative differential game theory. And using the method of dynamic programming principle and Riccati equation, this book derive into all kinds of existence conditions and calculating method of the equilibrium strategies of dynamic non-cooperative differential game. Then, based on the game theory method, this book studied the corresponding robust control problem, especially the existence condition and design method of the optimal robust control strategy. Finally, this book discussed the theoretical results and its applications in the risk control, option pricing, and the optimal investment problem in the field of finance and insurance, enriching the achievements of differential game research.

This book can be used as a reference book for graduate students majored in economic management, science and engineering of universities in learning non-cooperative differential games, and also for engineering technical personnel and economic management cadres.

Chapter 1
Introduction

1.1 Research and Development Status of Generalized Markov Jump Linear System Theory

1.1.1 Basic Model of Generalized Markov Jump Linear Systems

The research of switched systems is mainly carried out with the research of hybrid systems [1–5]. A hybrid system is a dynamic system that exhibits both continuous and discrete dynamic behavior–a system, such as manufacturing systems, weather forecast systems, power systems, biological systems, as well as option pricing models in financial engineering, insurance surplus distribution models, multi-sector fixed asset dynamic input-output models, etc., that can both flow (described by a differential equation) and jump (described by a state machine or automaton). In the process of its operation, a hybrid system often suffers from a sudden change in the environment, internal connection changes between each subsystem in a large system, changes of nonlinear objects, damages of the system components and random mutations, such as human intervention. These phenomena can be seen as a response of the system driven by a class of random events. In general, the state of such a system is defined by the values of the continuous variables and a discrete mode. The state changes either continuously, according to a flow condition, or discretely according to a control graph. Continuous flow is permitted as long as so-called invariants hold, while discrete transitions can occur as soon as given jump conditions are satisfied. Discrete transitions may be associated with events. Such systems are often called hybrid systems in control theory.

When the discrete event of hybrid systems is characterized by discrete switching signals, such important systems are called jump systems. This kind of systems can be described by finite subsystems or dynamic models, and at the same time there is a switch law, which makes the switching between various subsystems.

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A stochastic jump system can usually be described by the following state equations:

$$\begin{cases} \dot{x}(t) = f(t, x(t), u(t), r(t)), \\ r(t) = \varphi(t, x(t), r(t^-), u(t)). \end{cases} \tag{1.1.1}$$

where $x(t) \in \mathbb{R}^n$ is a continuous variable, $u(t) \in \mathbb{R}^m$ is an external signal of continuous control input or continuous dynamic systems, $r(t)$ is a piece-wise constant function valued in a finite set $\Xi = \{1, \cdots, l\}$, usually referred as "switch signals", or "switching strategy" of the system. $r(t^-)$ indicates that $r(t)$ is a piece-wise constant right hand continuous function. When $r(t)$ takes different values, the system (1.1.1) corresponds to different subsystems. $f(\cdot,\cdot,\cdot,\cdot)$ reflects continuous state variables changes of the system, $\varphi(\cdot,\cdot,\cdot,\cdot)$ is the transition function of discrete states, which reflects dynamic changes of logic strategies or discrete events of systems. Obviously, when the switching strategy $r(t) \in \Xi = \{1\}$, the random jump system is degraded as a simple stochastic system. So, a simple random system is a special case of the stochastic jump systems (1.1.1).

A generalized stochastic jump system is usually described by the following state equations:

$$\begin{cases} E\dot{x}(t) = f(t, x(t), r(t), u(t)), \\ r(t) = \varphi(t, x(t), r(t^-), u(t)), \end{cases} \tag{1.1.2}$$

where $E \in \mathbb{R}^{n \times n}$ is a known singular matrix with $0 < \text{rank}(E) = k \le n$, $x(t) \in \mathbb{R}^n$, $u(t) \in \mathbb{R}^m$, $r(t)$, $r(t^-)$, $f(\cdot,\cdot,\cdot,\cdot)$, $\varphi(\cdot,\cdot,\cdot,\cdot)$ are the same as system (1.1.1).

This book is focused on a kind of special jump systems with Markov switching parameters, which is known as Markov jump systems. In such systems, the switching rules determine which corresponding subsystem the system would be switched to at each moment, and the state of the system would be switched to the corresponding state at the corresponding moment. But during the process of the system switching from one mode to another mode, there is no switching rule to obeying, and the switching process between different modes is random. This kind of random switching accords with some certain statistical properties—the transformation among various regime of the discrete event finite set of the system is a Markov jump process, therefore, it can be also regarded as a special case of stochastic systems, called stochastic Markov switching systems (also known as stochastic Markov jump systems, or stochastic Markov modulation systems).

A Markov jump system is constructed by two parts. One part of the system is the state of the system, and the other part is the system mode, which depends on the Markov process, deciding the execution of the subsystem at a certain moment, in order to control and coordinate the normal operation of the whole system.

(1) Mathematical Model of Continuous Generalized Markov Jump Systems

The continuous generalized stochastic Markov jump linear system is described as:

$$E\dot{x}(t) = A(r(t))x(t) + B(r(t))u(t), \tag{1.1.3}$$

where $E \in \mathbb{R}^{n\times n}$, $x \in \mathbb{R}^n$, $u(t) \in \mathbb{R}^m$ are the same as system (1.1.1), and the "switch signals" or "switching strategy" of the system $r(t) \in \Xi = \{1, \cdots, l\}$ is a Markov chain with finite state. Ξ is the state space. Define $\Pi = [\pi_{ij}]_{l\times l}$ as the transition matrix of Markov process $r(t)$, and the transition probability could be written as:

$$\Pr\{r(t+\Delta) = j | r(t) = i\} = \begin{cases} \pi_{ij}\Delta + o(\Delta), & \text{if } i \neq j, \\ 1 + \pi_{ii}\Delta + o(\Delta), & \text{else}, \end{cases} \tag{1.1.4}$$

where π_{ij} represents the transition probability from mode i to mode j, with $\pi_{ij} \geq 0$, $\sum_{j=1}^{l} \pi_{ij} = 1$, and $o(\Delta)$ is the higher order infinitesimal. The matrice $A(r(t))$ and $B(r(t))$ are the functions of the stochastic process $r(t)$, and for each $r(t) = i \in \Xi$, $A(r(t))$ and $B(r(t))$ are real matrices with appropriate dimension.

(2) Mathematical Model of Discrete Generalized Markov Jump Systems

The discrete generalized stochastic Markov jump linear system is described as:

$$Ex(k+1) = A(r(k))x(k) + B(r(k))u(k), \tag{1.1.5}$$

where $E \in \mathbb{R}^{n\times n}$, $x(t) \in \mathbb{R}^n$, $u(t) \in \mathbb{R}^m$ are the same as system (1.1.3), and the elements of the transition probability matrix $\Lambda = [\lambda_{ij}]_{l\times l}$ of the system switching track $r(k) \in \Xi = \{1, \cdots, l\}$ are given by:

$$\lambda_{ij} = \Pr\{r(k+1) = j | r(k) = i\}, \tag{1.1.6}$$

where λ_{ij} represents the transition probability from mode i to mode j, which satisfies that $\lambda_{ij} \geq 0$, $\sum\limits_{j=1}^{l} \lambda_{ij} = 1$.

(3) Applications of Generalized Stochastic Markov Jump Linear Systems

As a special kind of stochastic jump systems, the Markov jump system has practical applications with engineering background. Such as the influence of sudden changes of environment on the behavior of the system, changes of interconnected subsystems, changes of nonlinear system operations, etc., can all be considered as random switching between multimodal systems. Economic system, aircraft control system, robot manipulator system, large space flexible structure system and stochastic decision-making and continuous control systems all have such kinds of system models. Especially in the field of finance and insurance, for example, in 1973, Black and Scholes used geometric Brownian motion to simulate the price of risk assets of options at time t, that is

$$dX(t) = \mu X(t)dt + \sigma X(t)dw(t), \tag{1.1.7}$$

where μ is the rate of return, σ is the disturbance rate, $w(t)$ is the Brownian motion, reflecting the changes of financial market. Although Black used (1.1.7) to give an almost perfect formula of option pricing, the model still had many defects, such as: (a) it failed to depict the discontinuous change of stock price; (b) the empirical analysis showed that the stock volatility was not constant. So many scholars tried to improve the model. On one hand, Merton (1976) put forward a jump diffusion model, which adding a jump process on the model (1.1.7) to characterize the discontinuous changes in stock price [6]. On the other hand, some researchers proposed to let the coefficient of the geometric Brownian motion depends on some hidden Markov chain, that is to say, assuming risk assets are satisfied that:

$$dX(t) = \mu(r(t))X(t)dt + \sigma(r(t))X(t)dw(t), \tag{1.1.8}$$

which $r(t)$ is a Markov chain with finite state, and assuming its state space is $\Xi = \{1, \cdots, l\}$, the infinitesimal operators is $\Pi = [\pi_{ij}]_{l\times l}$. In economics, the state of $r(t)$ is usually called regime-switching or Markov regime-switching, and $X(t)$ is called a process of geometric Brownian motion with Markov regime-switching. The state of $r(t)$ can be interpreted as economic condition structure changes, the regime's replacement, alternating macro news, and economic cycles, etc. There are many literature discussing model (1.1.8), for instance, when $l = 2$, Guo (2000) [7] studied Russia's options pricing problems with Markov modulated geometric Brownian motion model. Guo (2001) [8] further studied an explicit solution to an optimal stopping problem with regime switching, and Jobert (2006) [9] extended the result of Guo into option pricing with finite state Markov-modulated dynamics. Recently, Elliott (2007) [10] studied a class of pricing options under a generalized Markov-modulated jump-diffusion model, assuming that asset price followed:

$$dX(t) = \mu(r(t))X(t)dt + \sigma(r(t))X(t)dw(t) + X(t^-)\int_{\mathbb{R}^+} zN(dt, dz), \tag{1.1.9}$$

where $N(dt, dz)$ is the possion measure.

So, in the field of engineering systems as well as the social and economic systems, such as the option pricing problem in financial engineering, investment insurance dividend distribution problems, multi-sectoral dynamic input-output model of fixed assets, and actual economic system models. All these systems can all be described by the mathematical model of generalized stochastic Markov jump linear systems.

1.1.2 *Research Status of Generalized Markov Jump Systems*

(1) **Research on the theory of generalized Markov jump systems**

Concrete model of stochastic Markov jump linear systems was first put forward by Krasovskii and Lidskii [11] and Florentin [12] as a numerical example of mathematical analysis. Many researches mainly focused on the stability and stabilization controller design of stochastic jump systems in recent 10 years [13–20]. Professor Mao, one of the famous international scholars in the field of stochastic analysis, issued the asymptotic stability results and numerical methods of stochastic jump systems in his monograph published in 2006 [21]. Professor Mao and his coauthor Dr. Huang studied the stability of singular stochastic Markov jump systems. There are too many researches about the application in engineering and social economy of Markov jump systems, and we can't list them all in a limited space. Our analysis focused on optimal control problem of stochastic Markov jump system (i.e., problem of single stochastic Nash differential game) and the robust control problem which are closely related to this book.

Sworder (1969) first discussed optimal control problem of hybrid linear systems with Markov jump parameters from the perspective of stochastic maximum principle and applied it to the actual control problems [22]. Then, Wonham (1971) proposed the dynamic programming problem of stochastic control system, and successfully applied it to the optimal control of linear jump systems [23]. Fragoso and Costa (2010) gave the separation principle for LQ problems of stochastic Markov jump system in continuous time setting [4]. Görges et al. (2011) proposed the optimal control problems and solution methods of generalized jump systems [5].

Boukas et al. (2001) studied LQR problem of controlled jump rate [24].One of the Chinese scholars named Sun (2006) conducted the control and optimization problem of jump systems, systematically [25]. Mahmoud et al. (2007) gave the analysis results and synthesis of uncertain switched systems in discrete-time setting [26]. Zhang (2009) studied the stability and stabilization of Markov jump linear systems with partly transition probabilities [27–30]. Guo and Gao studied the jump structure control of singular Markov jump systems with time delay [31]. Dong and Gao gave the analysis and control of generalized bilinear Markov jump systems [32]. Zhang and Zhang studied the control theory and application about nonlinear differential algebraic system (including generalized bilinear systems), systematically [33]. Obviously, the research on the singular (or non-singular) stochastic Markov jump linear quadratic optimal control problem (i.e. LQ problem) has relatively obtained a number of achievements, which lay a solid foundation for studying the non-cooperative game theory of generalized stochastic Markov jump systems. But at present, the research results on the LQ non cooperative differential game theory of the generalized stochastic Markov jump system are less, so we put forward the research of LQ non-cooperative differential game theory of generalized stochastic Markov jump systems.

(2) Research on Non-cooperative differential game theory driven by ordinary differential equations and stochastic differential equations

The study of game theory has also made abundant achievements, among which, there are many researches on dynamic non-cooperative differential game theory, where the system dynamics are described by differential equations, which includes saddle point equilibrium theory of zero-sum game, Nash equilibrium nonzero-sum game, Stackelberg leader-follower game theory and incentive theory.

For normal systems (such as deterministic and stochastic systems), Basar (1995) [34] summarized the dynamic non-cooperative differential game theory and its application results described by ordinary differential equations and stochastic differential equations in his monograph, systematically (see [34] and cited literatures). Xu and Mizukami have studied the saddle point equilibrium, Nash equilibrium, Stackelberg game theory and incentive theory of linear singular systems, systematically, (see [35–40] and cited literatures). Dockner et al. (2000) described the non-cooperative differential games with its applications, including the capital accumulation, public goods investment, marketing, global pollution control, financial and monetary policy, international trade and other issues of differential games, and this monograph is known as Bible study of differential games [41]. Erickson (2003) introduced the differentia game model of advertising competition, systematically [42]. Zhukovskiy (2003) introduced Lyapunov method in the field of stochastic differential games, in this book, his mainly use the technique of dynamic programming and optimization vector [43]. Jorgensen and Zaccour (2004) mainly studied the differential game in marketing, and introduced the application of differential games in the pricing-making, advertising, marketing channels and other fields [44]. And they had published many research papers with high citations of differential game theory and applications in recent years. Engwerda [45] (2005) introduced the LQ differential game problems and its application examples in economics and management science, and studied the mathematical skills of how to solving the Riccati equations associated with differential games, systematically (Engwerda 2000, 2003; Engwerda and Salmah 2009) [46–48]. Hamadene (1999) studied the nonzero-sum LQ stochastic differential game of BSDEs [49]. The main analytical tools used in these studies are still variation principle, the maximum principle and dynamic planning. In domestic, Academician Zhang Siying's book (1987) [50] "Differential Game" and Professor Li Dengfeng's book(2000) [51] "Differential Game and Its Applications" are the early related literature, but these two books mainly focus on differential games' applications in military and control problems, and pay little attention on applications in economics and management. Because of published influential papers about two zero-sum differential game with impulse control, professor Yong has been highly praised by Berkovitz who is the editor of American Mathematical [52]. Professor Liu also gave the application of leader-follower game to linear multi-sector dynamic input-output of generalized linear system [53]. Wang et al. (2007) studied on the linear quadratic nonzero-sum stochastic differential game under partially observable information [54]; Wu and Yu (2005, 2008) studied the linear quadratic nonzero-sum stochastic differential game problem with stochastic jump, also studied BSDEs differential game with jump and its application in financial engineering [55, 56]; Luo studied the indefinite linear quadratic differential games and indefinite stochastic linear quadratic optimal control problem with Markov jump parameters [57]. In the application of differential games, there is also a growing number of scholars who applying differential game to option pricing [58] (Zheng 2000), the optimal investment in consumption [59, 60] (Liu et al. 1999; Wu and Wu 2001), fisheries resource allocation [61, 62] (Zhang et al. 2000; Zhao et al. 2004), advertising competition and supply chain [63–67] (Zhang and Zhang 2005, 2006; Fu and Zeng 2007, 2008; Xiong etc., 2009), dynamic pricing with network externalities [68] (Liu et al. 2007) and other areas.

(3) **Research on robust control of generalized Markov jump systems**

The results of jump robust control systems are relatively poor. Hespanha (1998) [69] studied the H_∞ control of jump systems. After that, much attention have been paid on H_∞ control. Xu and Chen proposed the H_∞ control of uncertain stochastic bilinear systems with Markov jumps in discrete-time setting [70]. Ting et al. (2010) [71] studied the mixed H_2/H_∞ Robust control problems of stochastic systems with Markov jumps and multiplicative noise in discrete- time setting. All the scholars above used the Lyapunov method (including linear matrix inequality (LMI) method), in this book, we are going to study robust control of stochastic Markov jump linear systems based on game theory. Pioneering work using game theory to study in robust performance controller was first given in the 1960s by Doroto et al. [72], but it did not arouse enough attention due to the need of solving the differential mini-max problem. Since 1990, this design was thought to be used as a powerful weapon to robust design, and the basic idea was transforming the corresponding robust control problem into a two person differential game of saddle point equilibrium or Nash equilibrium. Basar and Limebeer et al. [73, 74] contributed the representative work. And Limebeer et al. converted the mixed H_2/H_∞ control of linear systems into a Nash equilibrium game, and obtained the optimal control strategies. But for the stochastic Markov jump linear systems, there are few results of robust control with various performance based on game theory.

(4) **Research on applications of generalized Markov jump systems**

There are many applications of Markov jump systems in the field of engineering, such as the automatic control of driving shifting systems, traffic management systems and electrical systems, and so on [75]. While application in the field of social science and economic science (in social science and economic science, Markov jump systems are usually referred to Markov switching systems or Markov regime-switching systems) including, ①: risk asset pricing model and the surplus model of an insurer in finance and insurance (detailed description were covered in ref. [76] and reference therein). In terms of VaR measure of risk management in financial market, there exists a fact that the state of financial time series or