Introduction to

Dynamics and Control in Mechanical Engineering Systems

Cho W. S. To

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INTRODUCTION TO DYNAMICS AND CONTROL IN MECHANICAL ENGINEERING SYSTEMS

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To my uncle
Mei Chang Cai (*a.k.a. Muljanto Tjokro*)

Contents

Series Preface

The Wiley-ASME Press Series in Mechanical Engineering brings together two established leaders in mechanical engineering publishing to deliver high-quality, peer-reviewed books covering topics of current interest to engineers and researchers worldwide.

The series publishes across the breadth of mechanical engineering, comprising research, design and development, and manufacturing. It includes monographs, references and course texts.

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Preface

It is understood that there are many excellent books on system dynamics, control theory, and control engineering. However, the lengths of the majority of these books are of the order of six or seven hundred pages or more. There are, however, very few books that cover sufficient material and are limited to around 300 pages. The present book is aimed at addressing the balance. While it is more concise than those longer books, it does include many detailed steps in the example solutions. The author does believe that the detailed steps in the example solutions are essential in a first course textbook.

This book is based on lecture notes that have been developed and used by the author since 1986. These lecture notes have been employed in courses such as Mechanical Control and Process Control, as well as Dynamics and Control. The first two courses were taught by the author at the University of Western Ontario, London, Ontario, Canada while the third course has been given by the author at the University of Nebraska, Lincoln, Nebraska, USA, since 1996. All three courses have primarily been taken by junior undergraduates with majors in mechanical engineering and chemical engineering. Therefore, the subject matter dealt with in this book covers material for a first course of three credit hours per semester in system dynamics or control engineering. For a course in Mechanical Control or Process Control the material in the entire book, except the second half of Chapter 4, has been used. For a course in Dynamics and Control the material in the entire book except Chapter 11 has been covered. For a four credit hour course, the component of laboratory experiments has been omitted from the present book for two main reasons. First, the inclusion of the laboratory experiments is not feasible in the sense that its inclusion would increase drastically the length of the book. Second, nowadays many laboratory experiments are computer-aided in the sense that major software is required. Exclusion of laboratory experiments in the present book provides freedom for the instructors to select a particular software and allows them to tailor the design of their experiments to the availability of laboratory instrumentation in a particular department or engineering environment.

Under normal conditions, it is expected that the students using the present book have already taken courses in their sophomore year. These courses include linear algebra and matrix theory, a second course in mathematics with Laplace transformation, and engineering dynamics. In addition, students are expected to be able to use MATLAB, which is introduced during their first year or first semester of their sophomore year.

Acknowledgments

Many figures in Chapters 3–10 were drawn by Professor Jing Sun of Dalian University of Technology, Dalian, China. Professor Sun was a senior visiting scholar at the University of Nebraska, Lincoln, during the academic year 2012 to 2013. The author is grateful for Professor Sun's kindness in preparing these figures. Specifically, the latter are: Figures 3.2–3.4; Figures 4.4–4.7, 4.9, 4.11–4.13; Figures 5.1–5.4; Figures 6.1–6.3; Figures 7.2 and 7.3; Figures 8.1–8.9; Figures 9.1, 9.2, 9.4–9.8; and Figure 10.1.

Finally, the author would like to express his sincere thanks to Paul Petralia, Senior Editor, Clive Lawson, Project Editor, Anne Hunt, Associate Commissioning Editor, and their team members for their assistance and effort in the production of this book.

1

Introduction

This book is concerned with the introduction to the *dynamics* and *controls* of *engineering systems* in general. The emphasis, however, is on mechanical engineering system modeling and analysis.

- ***Dynamics*** is a branch of mechanics and is concerned with the studies of particles and bodies in motion.
- The term ***control*** refers to the process of *modifying* the *dynamic behavior* of a system in order to achieve some *desired outputs*.
- A ***system*** is a combination of components or elements so constructed to achieve an objective or multiple objectives.

1.1 Important Difference between Static and Dynamic Responses

The question of why one studies engineering dynamics as well as control, and not statics, is best answered by the fact that in control engineering it is the dynamic behavior of a system that is modified instead of the static one. Furthermore, the most important difference between statics and dynamics from the point of view of a mechanical engineering designer is in the responses of a system to an applied force.

Consider a lightly damped, simple, single degree-of-freedom (dof) system that is subjected to a unit step load. The dynamic response is shown in Figure 1.1. Note that the largest peak or overshoot is about 1.75 units, while the magnitude of the input is 1.0 unit. Owing to the positive damping in the system, the dynamic response approaches asymptotically to its steady-state (s.s.) value of unity. If one looks at the largest *mean square value* for the dynamic response, it is about 3.06 units squared. On the other hand, the mean square value for the s.s. or static

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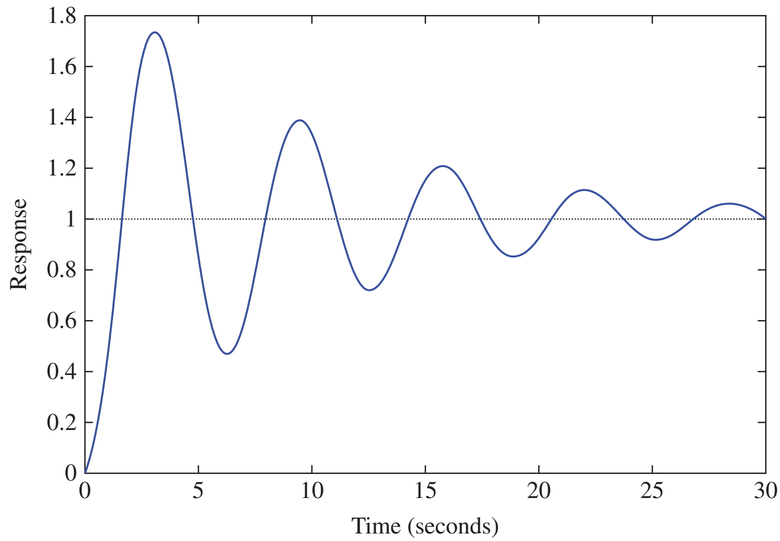

Figure 1.1 Dynamic response of a single dof system under unity input

response is 1.0 unit squared. Thus, the largest mean square value, which is the main design parameter, for the dynamic case is about 306% that of the static case, indicating the importance of dynamic response compared with that of the static case.

1.2 Classification of Dynamic Systems

This book deals with the study of dynamic and control systems in the engineering or physical world. In the latter many phenomena are *nonlinear* and *random* in nature, and therefore to describe, study, and understand such phenomena one has to formulate these phenomena in the conceptual or mathematical world as nonlinear differential equations. The latter, apart from some special cases, are generally very difficult to solve mathematically, and therefore in many situations these nonlinear differential equations are simplified to *linear* differential equations such that they may be solved analytically or numerically.

The meaning of a *linear* phenomenon may better be understood by considering a simple uniform cantilever beam of length L under a dynamic point load $f(t)$ applied transversely at the tip as shown in Figure 1.2. If the tip deflection $y(L,t)$, or simply written as y, satisfies the condition that

$$y \leq \pm \frac{5}{100} L$$

then y is said to be linear, and therefore a linear differential equation can be used to describe the deflection y. If the deflection y is larger than 5% of the length L of the beam, a nonlinear differential equation has to be employed instead. The word *random* mentioned in the foregoing means that *statistical analysis* is required to study such phenomena, instead of the usual *deterministic* approaches that are employed throughout in this book.

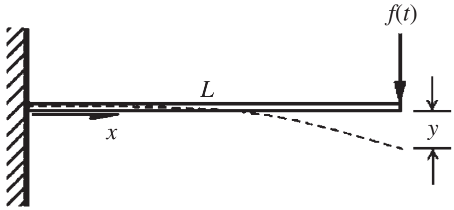

Figure 1.2 Cantilever beam with a point load

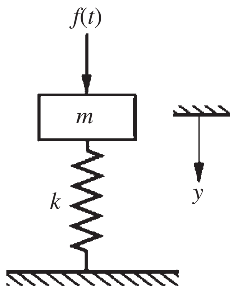

Figure 1.3 A lumped-parameter model of a massless cantilever beam

For the cantilever beam shown in Figure 1.2, the transverse deflection y at any point x along the length of the beam is a function of space x and time t, and therefore the differential equation required to describe the deflection is a *partial differential equation* (p.d.e.). Such a system is referred to as *continuous*. Continuous systems are also known as *distributed parameter* models and they possess an infinite number of dof.

On the other hand, for simplicity, if one approximates the uniform cantilever beam as massless such that the elasticity of the beam may be considered as a spring of constant coefficient $k = 3EI/L^3$, where E is the Young's modulus of elasticity of the material and I the second moment of cross-sectional area of the beam, and the mass of the beam m is considered concentrated at the tip of the beam, then the dynamic deflection of this *discrete* or *lumped-parameter* model, shown in Figure 1.3, can be described by an *ordinary differential equation* (o.d.e.).

1.3 Applications of Control Theory

It is believed that the first use of automatic control in Western civilization dated back to the period of 300 BC [1]. In the Far East the best-known automatic control in ancient China is the south-pointing chariot [1].

Fast forward to 1922, when Minorsky [2] introduced his three-term controller for the steering of ships, thereby becoming the first to use the proportional, integral, and derivative (PID) controller. In this publication [2] he also considered nonlinear effects in the closed-loop system (to be defined in Chapter 8). In modern times the theory of control has been applied in many fields. The following representative applications are important examples.

- The theory of control has been employed by economists, medical personnel, financial experts, political scientists, biologists, chemists, and engineers, to name but a few.
- In automobile engineering, many components of a car, such as the steering system, and the driverless car that has already appeared in the testing and refined design phase, employ many feedback control devices.
- Within the field of mechanical engineering, the speed control and maintenance of a turbine, and the heating system and water heater in a house, or the heating, ventilation and air conditioning (HVAC) system in a modern building, employ automatic control systems.
- In aerospace, the control of aircraft, helicopters, satellites, and missiles requires very sophisticated advanced control systems [3].
- In shipbuilding industries, control systems are often employed for steering and navigation [4].

1.4 Organization of Presentation

This book consists of 12 chapters. After this introduction, Chapter 2 is concerned with a brief review of Laplace transforms. The emphasis is on their applications in the analysis and design of dynamic and control systems. Use of the software MATLAB [5] provides several examples.

Chapter 3 presents the formulations and dynamic behaviors of hydraulic and pneumatic systems. A simple nonlinear system together with the linearization technique is included.

Chapter 4 deals with the formulations and dynamic behaviors of mechanical oscillatory systems. The focus in this chapter is on the formulation and analysis of linear single dof and many degree-of-freedom (mdof) vibration systems. Modal analysis of mdof systems is introduced in this chapter. Simple distributed-parameter models or continuous systems are included. Many solved problems are presented in this chapter.

The formulations and dynamic behaviors of thermal systems are introduced in Chapter 5. Dynamic equations of simple systems as well as the three-capacitance oven model are derived and investigated.

For completeness, the most basic electrical elements, laws, and networks, their corresponding dynamic equations, and derivations of transfer functions for various representative electromechanical systems are presented in Chapter 6.

The basic dynamic characteristics, theories, and operating principles of sensors or transducers are included in Chapter 7. The emphasis in this chapter is, however, on applications and derivations of dynamic equations of motion and their interpretations. Examples included in this chapter are accelerometers, microphones, and a piezoelectric hydrophone.

Chapter 8 is concerned with the fundamentals of engineering control systems. Transfer functions for open-loop and closed-loop feedback control systems are considered. System transfer functions of dynamic systems by block diagram reduction are illustrated with examples.

Modeling and analysis of engineering control systems are presented in Chapter 9. The time domain response of a unity feedback control system is developed and explained. Control types, such as the PID controls, s.s. error analysis, performance indices, and sensitivity functions are considered in this chapter.

The stability analysis of feedback control systems is introduced in Chapter 10. The focus in this chapter is the application of the Routh-Hurwitz stability criterion. For illustration, various examples are worked out in detail.

Chapter 11 is concerned with graphical methods in control systems. The methods introduced include the root locus method and root locus plots, polar and Bode plots, the Nyquist stability criterion and Nyquist diagrams, gain, phase margins in relative stability analysis, contours of magnitude, phase of system frequency response, the so-called *M* and *N* circles, and the Nichols chart. Various questions are solved by employing MATLAB at the end of this chapter. These questions are selected to show the powerful capability of MATLAB in the context of response computation.

The final chapter, Chapter 12, deals with modern control system analysis. The state space or vector space method is presented. The relationship between the Laplace transformed state equation and transfer function of a feedback control system is derived. The concepts of *controllability, observability, stabilizability, and detectability* are introduced, so as to provide a foundation for studies of multiple input and multiple outputs (MIMOs) feedback control systems. Various approximated system responses are obtained by employing MATLAB.

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2

Review of Laplace Transforms

Laplace transformation [1–3] is one of several powerful transformations that can be applied to the analysis of signals and dynamic engineering problems. In the context of dynamic and control system analysis, Laplace transforms are applied to obtain the transfer functions and, in turn, the block diagram representation, and the solutions of linear differential equations. Of course, they can be applied to obtain the solutions of partial differential equations (p.d.e.).

While the method of Laplace transformation can be applied to the solution of ordinary differential equations (o.d.e.) and p.d.e., in this book it is applied to obtain the solutions of o.d.e. since application of Laplace transformation to the solutions of p.d.e. is beyond the scope of this book. The process of solution by application of Laplace transforms has the following three stages:

- The given equation of motion is transformed into a *subsidiary* equation.
- The subsidiary equation is solved by *purely algebraic* manipulations.
- The solution of the subsidiary equation is transformed back (that is, taking the inverse Laplace transform) to provide the solution of the given problem.

The above solution of the o.d.e. by *algebraic operations* instead of *calculus operations* is referred to as *operational mathematics* [2].

This chapter begins with the definition of Laplace transforms and reviews of various important concepts and theorems. These topics are dealt with in Sections 2.1–2.6. The Laplace transforms of periodic functions and partial fraction method are considered in Sections 2.7 and 2.8, respectively. Section 2.9 is concerned with representative solved questions. These illustrative examples are included to demonstrate the solution of o.d.e. by the use of Laplace transforms. Applications of the software package MATLAB [4] for various problems are presented in Section 2.10.

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2.1 Definition

The Laplace transform $F(s)$ of $f(t)$ is defined as [1,2]:

$$F(s) = \int_0^\infty f(t)e^{-st}\,dt \tag{2.1}$$

in which the function $f(t)$ is defined for all $t \geq 0$.

The inverse of $F(s)$ or inverse Laplace transform of $F(s)$ is represented as:

$$f(t) = \mathcal{L}^{-1}[F(s)],$$

where the symbol $\mathcal{L}^{-1}[.]$ is applied to denote the inverse Laplace transform of the enclosing quantity. In this book the uppercase represents the Laplace transform of the lower case function, unless stated otherwise.

- The function $f(t)$ in Equation (2.1) is linear. Thus, for example, one cannot operate:

$$\int_0^\infty [f(t)]^3 e^{-st}\,dt.$$

- If, however, $[f(t)]^3$ is convergent and has no *multiple values* then the Laplace transform of $[f(t)]^3$ can be evaluated by the so-called *multi-fold* or *multi-dimensional Laplace transform*. For the present course, this is not considered.

Some commonly applied Laplace transforms and properties of Laplace transforms are included, respectively in Tables 2.1 and 2.2. More Laplace transforms and properties may easily be found in a mathematical handbook [5,6].

Table 2.1 Functions and their Laplace transforms

$f(t)$, $t \geq 0$	$F(s)$
1. $\delta(t)$, unit impulse at $t = 0$	1
2. $u(t)$, unit step	$1/s$
3. t^n	$n!/s^{n+1}$
4. e^{-at}	$1/(s + a)$
5. $t^{n-1} e^{-at}/(n - 1)!$	$1/(s + a)^n$
6. $1 - e^{-at}$	$a/[s(s + a)]$
7. $(e^{-at} - e^{-bt})/(b - a)$	$1/[(s + a)(s + b)]$
8. $[(c - a)e^{-at} - (c - b)e^{-bt}]/(b - a)$	$(s + c)/[(s + a)(s + b)]$
9. $\sin at$	$a/(s^2 + a^2)$
10. $\cos at$	$s/(s^2 + a^2)$
11. $e^{-at} \sin bt$	$b/[(s + a)^2 + b^2]$
12. $e^{-at} \cos bt$	$(s + a)/[(s + a)^2 + b^2]$

Table 2.2 Properties of Laplace transforms

$f(t)$	$F(s)=\int_0^{\infty} e^{-st}f(t)\ dt$
1. $af(t) + bg(t)$	$aF(s) + bG(s)$
2. $\frac{df}{dt}$	$sF(s) - f(0)$
3. $\frac{d^2f}{dt^2}$	$s^2F(s)-sf(0)-\dot{f}(0)$, $\dot{f}(0)=\left.\frac{df}{dt}\right\vert_{t=0}$
4. $\frac{d^nf}{dt^n}$	$s^nF(s)-\sum_k^n s^{n-k}f_{k-1}^{(k-1)}(0)$, $f_{k-1}^{(k-1)}(0)=\left.\frac{d^{k-1}f(t)}{dt^{k-1}}\right\vert_{t=0}$
5. $\int_0^t f(t)dt$	$\frac{1}{s}F(s)$
6. $g(t)=\begin{cases}0 & t<0\\ f(t-a) & t\geq a\end{cases}$	$G(s)=e^{-as}F(s)$
7. $e^{-at}f(t)$	$F(s+a)$
8. $f\left(\frac{t}{a}\right)$	$aF(as)$
9. $h(t)=\int_0^t f(\tau)g(t-\tau)d\tau=\int_0^t f(t-\tau)g(\tau)d\tau$	$H(s)=F(s)\ G(s)$

Before leaving this section it may be appropriate to mention that, strictly speaking, for a divergent function the integral defined by Equation (2.1) may not exist, so its Laplace transform cannot be established. For example, if the function $f(t)$ is the exponential e^{at} where a is a positive constant parameter such that Equation (2.1) becomes:

$$F(s)=\int_0^{\infty} e^{at}e^{-st}dt=\int_0^{\infty} e^{(a-s)t}dt$$

then $F(s)$ does not exist when $a > s$ since the integrand grows with time, and therefore the integral is not defined. Of course, one can use the shifting theorem to obtain the same result.

On the other hand, if one starts with the exponentially decaying function e^{-at} then the Laplace transform does exist. Having found the Laplace transform of the exponentially decaying function, one can then replace a in the resulting expression with $-a$. The result is then the Laplace transform of the exponentially rising function e^{at}. In other words, the Laplace transform of e^{at} is $\frac{1}{s-a}$.

Before leaving this section, it should be mentioned that there are many functions which do not have their Laplace transforms. An example of such a function frequently encountered in statistical analysis is $f(t) = e^{t^2}$.

2.2 First and Second Shifting Theorems

If f(t) has the transform F(s), where s > k, then $e^{at}f(t)$ *has the transform F(s − a), where s − a > k with k being a constant.* This is known as the *first shifting theorem*.

Symbolically,

$$\mathcal{L}[e^{at}f(t)] = F(s-a). \tag{2.2}$$

If one takes the inverse Laplace transform on both sides, one obtains

$$e^{at}f(t) = \mathcal{L}^{-1}[F(s-a)]. \tag{2.3}$$

The second shifting theorem can be stated as follow. *If f(t) has the transform F(s), then the "shifting function"*

$$\tilde{f}(t) = f(t-a)u(t-a) = \begin{cases} 0 & \text{if } t<0 \\ f(t-a) & \text{if } t>a \end{cases} \tag{2.4}$$

has the transform $e^{-as}F(s)$. That is,

$$\mathcal{L}[f(t-a)u(t-a)] = e^{-as}F(s). \tag{2.5}$$

If one takes the inverse Laplace transform on both sides of Equation (2.5), one has:

$$f(t-a)u(t-a) = \mathcal{L}^{-1}[e^{-as}F(s)]. \tag{2.6}$$

In the foregoing $u(t)$ is the *unit step function* which is also known as the *Heaviside function* such that $u(t-a)$ is defined as:

$$u(t-a) = \begin{cases} 0 & \text{if } t<a \\ 1 & \text{if } t>a \end{cases} \tag{2.7}$$

in which $a \geq 0$.

2.3 Dirac Delta Function (Unit Impulse Function)

The Laplace transform of the so-called generalized function,

$$\mathcal{L}[\delta(t-a)] = e^{-as}, \tag{2.8}$$