Power Systems

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Fundamentals of Electrical Drives

Second Edition

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Foreword

Within one academic lifetime, the electric drive has progressed from the three-machine DC drive called the Ward-Leonard system to today's sophisticated AC drives utilizing PWM inverter power electronics and field orientation or direct torque control. Roughly around the same period, machine theory progressed from the classical "one machine at a time" approach to the generalized or unified approach emphasizing similarities between machine types. This unified theory also utilized much more sophisticated mathematical tools to obtain models applicable to transients as well as steady state. This enabled theoretical modeling a host of important machine problems, but almost always required computer solutions as opposed to more general analytic solutions. This often left one with a feeling of detachment from the physical reality of inrush currents, the whine of spinning rotors, and the smell of over-warm electrical insulation.

Partway through my academic lifetime, I was introduced to the next phase of unified theory; the use of complex notation to model the effective spatial orientation of quantities within a machine. This concept, often called space vector theory, provides a much clearer mathematical picture of what is happening in a machine, but at the expense of another level of abstraction in the model. However, the insights provided to one initiated in the method are so significant that today essentially all work in drive control is presented in this format. And therein lies a problem. To the uninitiated these presentations appear quite unintelligible, and a route to becoming initiated is generally hard to find and often harder to follow once found.

This purpose of this book is to show the theory and notation used, in modern electric drive analysis and design at an introductory level. The authors, bring an exceptional breadth of knowledge to this book making it stand out from other books that only providing mathematical foundation for advanced work. This strong effort is made to present the physical basis for all of the major steps in development, and to give the space vector physical and mathematical meaning. Readers using the book for self-study will find the sets of simulation tutorials at the end of each chapter of special value in mastering the implications and fine points of the material covered in the chapter.

Electric machine theory with its interacting temporal, spatial variations and multi-winding topologies can appear to be a very complicated and difficult subject. The approach followed in this book is, I believe, one that will help eliminate this perception by providing a fundamental, coherent, and user-friendly introduction to electric machines for those beginning a serious study of electric drive systems.

Madison, WI, USA Donald W. Novotny

Preface

Our motivation and purpose for writing this book stems from our belief that there is a practical need for a learning platform which will allow the motivated reader to gain a basic understanding of the modern multidisciplinary principles which govern electrical drives. The book in question should appeal to those readers who have an elementary understanding of electrical circuits and magnetics and who have an interest or need to comprehend advanced textbooks in the field of electrical drives. Consideration has also been given to those interested in using this book as a basis for teaching this subject matter. In this context, a Springer website *Extra Materials* has been set up which contains the simulation examples and tutorials discussed in this book. Furthermore, all the figures in this book are available on the Springer website, in order to assist lecturers with the preparation of electronic "power point" type lectures.

Electrical drives consist of a number of components: the electrical machine, converter, and controller, all of which are discussed at various levels. A brief résumé of magnetic and electrical circuit principles is given in Chap. 1 together with a set of generic building modules which are used throughout this book to represent dynamic models. Chapter 2 is designed to familiarize the reader with the process of building a dynamic model of a coil with the aid of generic modules. This part of the text contains an introduction on phasors as required for steady-state analysis. The approach taken in this and the following chapters is to present a physical model, which is then represented by a symbolic model with the relevant equation set. A generic model is then presented which forms the basis for a set of *build and play* simulations set out in various steps in the tutorial at the end of the chapter.

Chapter 3 introduces a single-phase *ideal transformer* (ITF) which forms the basis of a generic transformer model with leakage and magnetizing inductance. A phasor analysis is given to familiarize the reader with the steady-state model. The *build and play* tutorials at the end of the chapter give the reader the opportunity to build and analyze the transformer model under varying conditions. It is emphasized that the use of these *build and play* sets are essential components of the learning process throughout this book.

Chapter 4 deals with star and delta connected three-phase systems and introduces the generic modules required to model such systems. The space vector-type representation is also introduced in this part of the text. A set of *build and play* tutorials are given which reinforce the concepts introduced in this chapter.

Chapter 5 deals with the concepts of real and reactive power in single-as well as three-phase systems. Additional generic modules are introduced in this part of the text, and tutorial examples are given to familiarize the reader with this material.

Chapter 6 extends the ITF concept introduced earlier to a space vector-type model which is represented in a symbolic and generic form. In addition, a phasor-based model is also given in this part of the text. The *build and play* tutorials are self-contained step-by-step simulation exercises which are designed to show the reader the operating principles of the transformer under steady-state and dynamic conditions. At this stage of the text, the reader should be familiar with building and using simulation tools for space vector-type generic models which form the basis for a transition to rotating electrical machines.

Chapter 7 introduces a unique concept, namely, the *ideal rotating transformer* (IRTF), which is the fundamental building block that forms the basis of the dynamic electrical machine models discussed in this book. A generic space vector-based IRTF model is given in this part of the text which is instrumental in the process of familiarizing the reader with the torque production mechanism in electrical machines. This chapter also explores the conditions under which the IRTF module is able to produce a constant torque output. It is emphasized that the versatility of the IRTF module extends well beyond the electrical machine models discussed in this book. These advanced IRTF-based machine concepts are used in our second book *Advanced Electrical Drives* [2] and also in our third book *Applied Control of Electrical Drives* [10]. The latter-mentioned book has been recently introduced to facilitate the transition to experimental drives by the reader. The *build and play* tutorials at the end of this chapter serve to reinforce the IRTF concept and allow the reader to "play" with the conditions needed to produce a constant torque output from this module.

Chapters 8–9 deal with the implementation of the IRTF module for synchronous and asynchronous machines. In both cases, a simplified IRTF-based symbolic and generic model is given of the machine in question to demonstrate the operating principles. This model is then extended to a "full" dynamic model as required for modeling standard electrical machines. A steady-state analysis of the machines is also given in each chapter. In the sequel of each chapter, a series of *build and play* tutorials are introduced which take the reader through a set of simulation examples which steps up from a very basic model designed to show the operating principles, to a full dynamic model which can be used to represent the majority of modern AC electrical machines in use today.

Chapter 10 dealt with the DC machine, for which a dynamic model is introduced. In addition, the steady-state torque/speed characteristics of this machine with either PM or field excitation are discussed.

Chapter 11 deals with the converter, modulation, and control aspects of the electrical drive at a basic level. Both half- and full-bridge converter concepts are discussed together with the pulse width modulation (PWM) strategies that are in use in modern drives. A model-based current control algorithm is presented in combination with a DC machine. The *build and play* tutorials in the sequel of this chapter clearly show the operating principles of PWM-based current-controlled electrical drives.

The purpose, content, and approach of our book have been presented above. On the basis of this material, the following set of unique points are presented below in response to the question as to why prospective readers should purchase this book:

- The introduction of an *ideal rotating transformer* (IRTF) module concept is a basic didactic tool for introducing the elementary principles of torque production in electrical machines to the uninitiated reader. The apparent simplicity of this module provides the reader with a powerful tool which can be used for the understanding and modeling of a very wide range of electrical machines well beyond those considered in this book.
- The application of the IRTF module to AC machines provides a unique insight into their operation principles. The book shows the transitional steps needed to move from a very basic IRTF model to a full IRTF-based dynamic model usable for representing the dynamic and steady-state behavior of most machines in use today. In addition the IRTF based module can be readily extended to include more specific machine effects such as “skin effect” in asynchronous machines. Furthermore, the IRTF module can be extended to machine models outside the scope of this book. Examples which appear in the book *Advanced Electrical Drives* by the authors of this text are the salient pole PM machine and the single-phase IRTF-based induction machine.
- This text is designed to bridge the gap between advanced textbooks covering electrical drives and textbooks at either a fundamental electrical circuit level or more generalized mechatronic books. Our text is accompanied by a set of tutorials which are located in the *Extra Materials section* at the Springer website. This book should fit well into the undergraduate curriculum for students who have completed first or second year and who have an interest in seeking a career in the area of electrical drives. The book should also appeal to engineers with a non-drive background who have a need to acquire a better understanding of modern electrical drive principles.
- The use of *build and play*-type tutorials is of fundamental importance to understanding the theory presented in the text. The didactic role of modern simulation tools in engineering cannot be overestimated, and it is for this reason that extensive use is made of generic modules which are in turn used to build complete models of the drive. Such an approach allows the reader to visualize the complex equation set which is at the basis of these models. The simulation tool used in these tutorials is “PLECS®” which can be used with MATLAB/SIMULINK or (as is the case in this book) as “stand-alone” software. The said tutorials are linked directly to the generic modules discussed in the corresponding chapter and are included in the *Extra Material, Springer website:extras.springer.com* linked to this book.
- A series of “demonstration” laboratories are introduced which are used to experimentally verify key theoretical concepts/models introduced in this book.

> Hence, it is hoped that the critical reader will be convinced that the material presented in this book is applicable to actual electrical drives.

The second edition of this book has been tailored to the text *Advanced Electrical Drives* by the same authors. Notably some changes have been made to ease the readers' transition to our textbooks *Advanced Electrical Drives* as well as *Applied Control of Electrical Drives*. Notably the new edition makes use of so-called "amplitude invariant" space vectors, which is in line with the approach used in *Applied Control of Electrical Drives*. Specifically, Chap. 3 has been extensively revised to introduce the so-called "universal-oriented model approach" at an early stage. Furthermore, Chap. 10 on DC machines has been simplified. Finally, in Chap. 11 the term "incremental flux" has been omitted and replaced by the variable *average voltage per sample* given its use in our other books. The said chapter has also been extended to cover "H-bridge" operation.

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Acknowledgments

The process of writing this book has not been without its difficulties. That this work has come to fruition stems from a deep belief that the material presented in this book will be of profound value to the engineering community as a whole and the educational institutions in particular.

The content of this second edition reflects upon the collective academic and industrial experience of the authors concerned. In this context, the input of students in general and other colleagues cannot be overestimated. In particular, the authors wish to thank the staff and students of the Institute for Power Electronics and Electrical Drives (ISEA), RWTH Aachen University, Germany.

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Contents

Chapter 1
Introduction

1.1 Why Use Electro-Mechanical Energy Conversion?

Electric motors are around us everywhere. Generators in power plants are connected to a three-phase power grid of alternating current (AC), pumps in your heating system, refrigerator, and vacuum cleaner are connected to a single phase AC grid and switched on or off by means of a simple contactor. In cars a direct current (DC) battery is used to provide power to the starter motor, windshield wiper motors, and other utilities. These motors run on direct current and in most cases they are activated by a relay switch without any control.

Many applications driven by electric motors require more or less advanced control. Lowering the speed of a fan or pump can be considered relatively simple. Perhaps one of the most difficult ones is the dynamic positioning of a tug in a wafer-stepper with nanometer accuracy while accelerating at several g's. Another challenging controlled drive is an electric crane in a harbor that needs to be able to move an empty hook at high speed, navigate heavy loads up and down at moderate velocities, and make a soft touchdown as close as possible to its intended final position. Other applications such as assembly robots, electric elevators, electric motor control in hybrid vehicles, trains, streetcars, or CD-players can, with regard to complexity, be situated somewhere in between.

Design and analysis of all electric drive systems requires not only knowledge of dynamic properties of different motor types, but also a good understanding of the way these motors interact with power electronic converters and their loads. These power converters are used to control motor currents or voltages in various manners.

Compared to other drive systems such as steam engines (still used for aircraft launch assist), hydraulic engines (famous for their extreme power per volume),

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pneumatic drives (famous for their simplicity, softness, and hissing sound), combustion engines in vehicles, or turbo-jet drives in helicopters or aircrafts, electric drive systems have a very wide field of applications thanks to some strong points:

- Large power range available: actuators and drives are used in a very wide range of applications from wrist watch micro-watt level to machines at the multi-megawatt level, e.g., as used in coal mines, steel industry, and ship propulsion systems.
- Electrical drives are capable of full torque at standstill, hence no clutches are required.
- Electrical drives can provide a very large speed range, usually gearboxes can be omitted.
- Clean operation, no oil-spills to be expected.
- Safe operation is possible in environments with explosive fumes (pumps in oil-refineries).
- Immediate use: electric drives can be switched on immediately.
- Low service requirement: electrical drives do not require regular service as there are very few components subject to wear, except the bearings. This means that electrical drives can have a long life expectancy, typically in excess of 20 years.
- Low no-load losses: when a drive is running idle, little power is dissipated since no oil needs to be pumped around to keep it lubricated. Typical efficiency levels for a drive are in the order of 85 %. In some cases this may be as high as 98 %. The higher the efficiency the more costly the drive technology, in terms of initial costs.
- Electric drives produce very little acoustic noise compared to combustion engines.
- Excellent control ability: electrical drives can be made to conform to precise user requirements. This may, for example, be in relation to realizing a certain shaft speed or torque level.
- "Four-quadrant operation": Motor and braking mode are both possible in forward or reverse direction, yielding four different quadrants: forward motoring, forward braking, reverse motoring, and reverse braking. Positive speed is called forward, reverse indicates negative speed. A machine is in motor mode when energy is transferred from the power source to the shaft, i.e., when both torque and speed have the same sign.

1.1.1 Modes of Operation

When a machine is in motoring mode, most of the energy is transferred from the electrical power source to the mechanical load. Motoring mode takes place in quadrants 1 and 3 (see Fig. 1.1b). If the shaft torque and shaft speed are in opposition, then the flow of energy is reversed, in which case the drive is in the so-called braking mode.

Braking comes in three "flavors." The first is referred to as "regenerative" braking operation, where most of the mechanical energy from the load is returned to

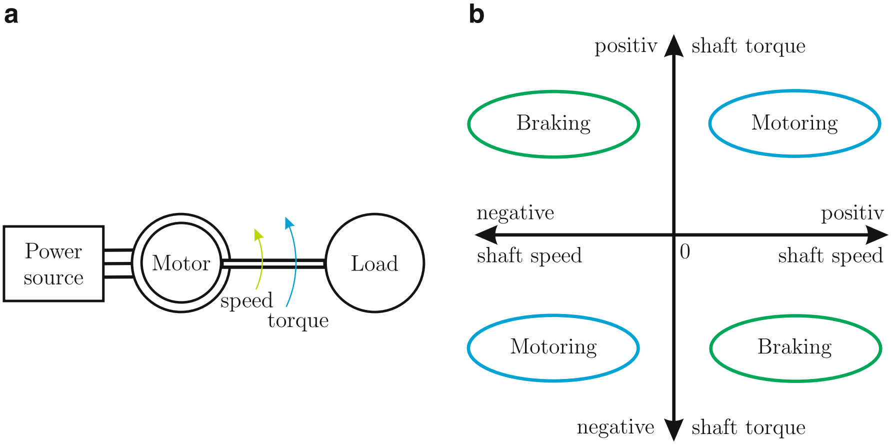


Fig. 1.1 Motoring and braking operation. (**a**) Motor with power supply. (**b**) Operating modes

the power source. Most drives which contain a converter (see Sect. 1.2) between motor and supply use a diode rectifier as a front end, hence power can only flow from the AC power grid to the DC-link in the drive and not the other way around. In such converters regenerative operation is only possible when the internal DC-link of the drive is shared with other drives that are able to use the regenerated power immediately. Sharing a common rectifier with many drives is economic and becoming standard practice. Furthermore, attention is drawn to the fact that some power sources are not able to accept any (or only a limited amount) of regenerated energy.

The second option is referred to as “dissipative” braking operation. Typically, this method is used to dissipate irreversibly the kinetic energy of the mechanical load system in an external brake-chopper-resistor. A brake-chopper can burn away a substantial part of the rated power for several seconds, designed to be sufficient to stop the mechanical system in a fast and safe fashion. One can regard such a brake-chopper as a big Zener-diode that prevents the DC-link voltage in the converter from rising too high. Brake-choppers come in all sizes, in off-shore cranes and locomotives power levels of several megawatts are common practice.

The third braking mode is the one where mechanical power is completely returned to the motor, while at the same time some electrical power may still be delivered, i.e., both mechanical and electrical input power are dissipated in the motor. Think of a permanent magnet motor being shorted, or an induction motor that carries a DC current in its stator, acting as an eddy-current-brake.

Of course there are also disadvantages when using electrical drive technology, a few of these are briefly outlined below.

- Low torque/force density compared to combustion engines or hydraulic systems. This is why aircraft control systems are still mostly hydraulic. However, there is an emerging trend in this industry to use electrical drives instead of hydraulic systems.

- High complexity: A modern electrical drive encompasses a range of technologies as will become apparent in this book. This means that it requires highly skilled personnel to repair or modify such systems.

1.2 Key Components of an Electrical Drive System

The "drive" shown in Fig. 1.1a is in fact only an electrical machine connected directly to a power supply. This configuration is widely in use but one cannot exert very much control in terms of controlling torque and/or speed. Such drives are either on or off with rather wild starting dynamics. The drive concept of primary interest in this book is capable of what is referred to as "adjustable speed" operation [8] which means that the machine can be made to operate over a wide speed range. A simplified structure of an adjustable speed drive is shown in Fig. 1.2. A brief description of the components is given below:

- Load: This component is central to the drive in that the purpose of the drive is to meet specific mechanical load requirements. It is emphasized that it is important to fully understand the nature of the load and the user requirements which must be satisfied by the drive. The load component may or may not have sensors to measure either speed, torque, or shaft angle. The sensors which can be used are largely determined by the application. The nature of the load may be translational or rotational and the drive designer must make a prudent choice whether to use a direct-drive with a large motor or geared drive with a smaller but faster one. Furthermore, the nature of the load in terms of the need for continuous or intermittent operation must be determined.
- Motor: A limited range of motor types is presently in use. Among these are the so-called classical machines, which have their origins at the turn of the nineteenth century. This classical machine set has displaced a large assortment

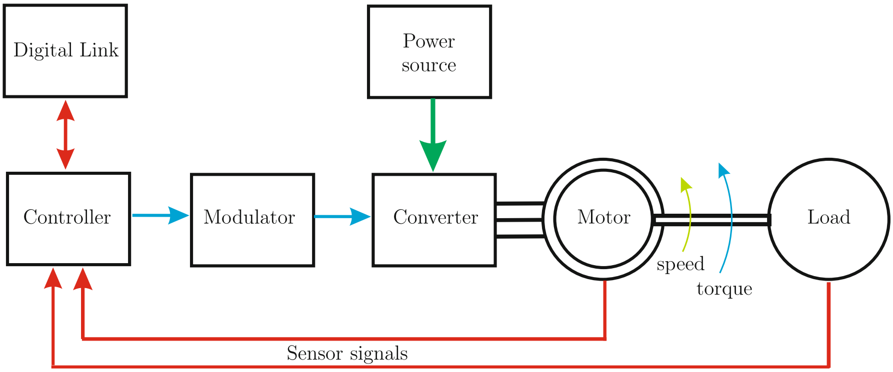


Fig. 1.2 Typical drive setup

of "specialized" machines used prior to the introduction of power electronic converters for speed control. This classical machine set contains the DC (Direct Current) machine, asynchronous (induction) machine, synchronous machine, and "variable reluctance" machine. Of these the "variable reluctance" machine will not be discussed in this book. A detailed discussion of this machine appears in the textbook *Advanced electrical drives* written by the authors of this book.

The term "motor" refers to a machine which operates as a motor, i.e., energy flows from the motor to the load. When the energy flows in the opposite direction a machine is said to operate as a generator.

- Converter: This unit contains a set of power electronic (semiconductor) switches which are used to manipulate the energy transfer between power supply and motor. The use of switches is important given that no power is dissipated (in the ideal case) when the switches are either open or closed. Hence, theoretically, the efficiency of such a converter is 100 %, which is important particularly for large converters given the fact that semiconductor devices cannot operate at high temperatures. Hence, it is not possible to absorb high losses which inevitably appear in the form of heat. A large range of power electronic switches is available to the designer to meet a wide range of applications.
- Modulator: The switches within the converter are controlled by the modulator which determines which switches should be on, and for what time interval, normally on a micro-second timescale. An example is the pulse width modulator that realizes a required pulse width at a given carrier-frequency of a few kHz.
- Controller: The controller, typically a digital signal processor (DSP), or micro-controller (MCU) in combination with programmable logic devices, contains a number of software based control loops which control and protect, for example, the currents in the converter and machine. In addition, torque, speed, and shaft angle control loops may be present within this module. Shown in the diagram are the various sensor signals which form the key inputs to the controller together with a number of user set-points (not shown in the diagram). The output of the controller is a set of control parameters which are used by the modulator.
- Digital link: This unit serves as the interface between the controller and an external computer. With the aid of this link drive set-points and diagnostic information can be exchanged with a remote user.
- Power supply: In most cases the converter requires a DC voltage source (DC voltage link). The power can be obtained directly from a DC power source, in case one is available, for example, batteries in electric vehicles. However, in most cases the DC power requirements are met via a rectification process, which makes use of the single or three-phase AC (referred to as the "grid") power supply as provided by the utility grid.

1.3 What Characterizes High Performance Drives?

Prior to moving to a detailed discussion of the various drive components it is important to understand the reasons behind the ongoing development of drives. Firstly, an observation of the drive structure (see Fig. 1.2) shows that the drive has components which cover a very wide field of knowledge. For example, moving from load to controller one needs to appreciate the nature of the load, have a thorough understanding of the motor, and comprehend the functioning of the converter and modulator. Finally, one needs to understand the control principles involved and how to implement (in software) the control algorithms into a micro-processor or DSP. Hence, there is a need to have a detailed understanding of a very wide range of topics which is perhaps one of the most challenging aspects of working in this field. The development of electrical machines occurred, as was mentioned earlier, more than a century ago. However, the step to a high performance adjustable speed drive took considerably longer and is in fact still ongoing. The main reasons as to why drive technology has improved over the last decades are briefly outlined below:

- Availability of fast and reliable power semiconductor switches for the converter: A range of switches is available to the user today to design and build a wide range of converter topologies. The most commonly used switching devices for motor drives are MOSFETs for low-voltage applications and IGBTs for medium (kW) and higher (MW) powers. In addition GCTs are available for medium-voltage and high-voltage applications.
- Availability of fast computers for (real time) embedded control: the controller needs to provide the control input to the modulator at a sampling rate which is typically in the order of 100 μs. Within that time frame the computer needs to acquire the input data from sensors and user set-points and apply the control algorithm in order to calculate the control outputs for the next cycle. The presence of low cost fast micro-processors or DSPs since the mid-eighties has been of key importance for drive development.
- Better sensors: A range of reliable and low cost sensors is available to the user which provides accurate inputs for the controller such as LEMs, incremental encoders, and Hall-effect sensors.
- Better simulation packages: The availability of sophisticated so-called finite-element computer aided design (CAD) packages for motor design has been instrumental in gaining a better understanding of machines. Furthermore, they have been and continue to be used for designing machines and for optimization purposes. In terms of simulating the entire drive structure there are simulators with graphical user interfaces, such as, among others, MATLAB/Simulink® and PLECS® which allow the user to analyze a detailed dynamic model of the entire system. This means that one can analyze the behavior of such a system under a range of conditions and explore new control techniques without the need of

actually building the entire system. This does not mean that implementing real life systems is no longer required. The proof of the pudding is in the eating, and only experimental validation can prove that the supposedly exact models are indeed valid for a real drive system, which was our motivation for writing our latest book *Applied Control of Electrical Drives* [10].

Simulation and experiment are never exactly the same. When the models are not able to describe the drive system under certain conditions, it might be useful to enhance the simulation model to incorporate some of the found differences. As engineers, we should be aware of the fact that drive systems are often closed-loop systems that are able to tolerate (to some extent) deviations in parameters and unknown load torques without any problem. To paraphrase Einstein, “A simulation model should be as simple as possible, but no simpler” is the key to a successful simulation. This means that essential dynamics or non-linearities, found in the real world system, need to be implemented in the (physics based) simulation model in order to study extreme situations with acceptable accuracy.

The simulation model used depends on what needs to be studied. Simulating pulse width modulated outputs requires a very short simulation time-step, in the order of sub-μs or so, while the overall mechanical system and the motor’s response can be calculated at a hundred times larger time-step with negligible loss of accuracy, as long as the power converter is regarded as a non-switching controlled voltage source. Another extreme example is the study of thermal effects on the motor. In that case only the average power dissipation in terms of seconds or even minutes is of interest.

- Better materials: The availability of improved magnetic, electrical, and insulation materials has provided the basis for efficient machines capable of withstanding higher temperatures, thereby offering long application life and low life-cycle costs.

1.4 Notational Conventions

1.4.1 Voltage and Current Conventions

The conventions used in this book for the voltage and current variables are shown with the aid of Fig. 1.3. The diagram shows the variables: voltage u and current i, which are specifically given in “lower case” notation, because they represent instantaneous values, i.e., a function of time. The voltage and current “arrows” shown in Fig. 1.3 point to the negative terminal of the respective circuit, i.e., motoring arrow system, in which positive power $p = ui$ means power absorbed by the electrical circuit (load).

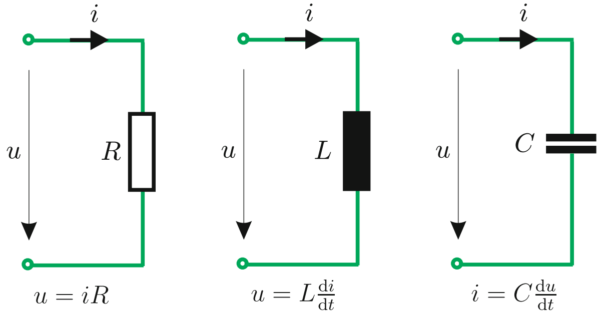
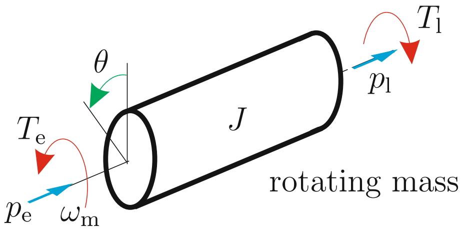


Fig. 1.3 Notation conventions used for electrical quantities

Fig. 1.4 Notation conventions used for mechanical quantities



1.4.2 Mechanical Conventions

The mechanical conventions used in this book are shown with the aid of Fig. 1.4. The electro-magnetic torque T_e produced by the machine corresponds with a power output $p_e = T_e\omega_m$, where ω_m represents the rotational speed, otherwise known as the angular frequency. The load torque T_l is linked to the power delivered to the load $p_l = T_l\,\omega_m$. Ignoring bearing and windage losses in the machine, the torque difference $T_e - T_l$ results in an acceleration $J{}^{d\omega_m}/_{dt}$ of the total rotating mass, which is characterized by its inertia J. This rotating structure is represented as a lumped mass formed by the rotor of the motor, motor shaft, and load. The corresponding mechanical equation which governs this system is of the form

$$J\frac{d\omega_m}{dt} = T_e - T_l \tag{1.1}$$

The angular frequency may also be written as $\omega_m = {}^{d\theta}/_{dt}$ where θ represents the rotor angle.

Figure 1.4 shows the machine operating as a motor, i.e., $T_e > 0$ and $\omega_m > 0$. These motor conventions are used throughout this book.