Lecture Notes in Mechanical Engineering

Dan Zhang
Bin Wei *Editors*

Mechatronics and Robotics Engineering for Advanced and Intelligent Manufacturing

Springer

Lecture Notes in Mechanical Engineering

About this Series

Lecture Notes in Mechanical Engineering (LNME) publishes the latest developments in Mechanical Engineering—quickly, informally and with high quality. Original research reported in proceedings and post-proceedings represents the core of LNME. Also considered for publication are monographs, contributed volumes and lecture notes of exceptionally high quality and interest. Volumes published in LNME embrace all aspects, subfields and new challenges of mechanical engineering. Topics in the series include:

- Engineering Design
- Machinery and Machine Elements
- Mechanical Structures and Stress Analysis
- Automotive Engineering
- Engine Technology
- Aerospace Technology and Astronautics
- Nanotechnology and Microengineering
- Control, Robotics, Mechatronics
- MEMS
- Theoretical and Applied Mechanics
- Dynamical Systems, Control
- Fluid Mechanics
- Engineering Thermodynamics, Heat and Mass Transfer
- Manufacturing
- Precision Engineering, Instrumentation, Measurement
- Materials Engineering
- Tribology and Surface Technology

More information about this series at http://www.springer.com/series/11236

Dan Zhang · Bin Wei
Editors

Mechatronics and Robotics Engineering for Advanced and Intelligent Manufacturing

Springer

Editors
Dan Zhang
Department of Mechanical Engineering, Lassonde School of Engineering
York University
Toronto, ON
Canada

Bin Wei
Faculty of Engineering and Applied Science
University of Ontario Institute of Technology
Oshawa, ON
Canada

ISSN 2195-4356 ISSN 2195-4364 (electronic)
Lecture Notes in Mechanical Engineering
ISBN 978-3-319-33580-3 ISBN 978-3-319-33581-0 (eBook)
DOI 10.1007/978-3-319-33581-0

Library of Congress Control Number: 2016943792

Printed on acid-free paper

This Springer imprint is published by Springer Nature
The registered company is Springer International Publishing AG Switzerland

Preface

The 2nd International Conference on Mechatronics and Robotics Engineering, ICMRE 2016, was held in Nice, France, during February 18–22, 2016. The aim of ICMRE 2016 is to provide a platform for researchers, engineers, academics as well as industry professionals from all over the world to present their research results and development activities in the area of mechatronics and robotics engineering. This book introduces recent advances and state-of-the-art technologies in the field of robotics engineering and mechatronics for the advanced and intelligent manufacturing. This systematic and carefully detailed collection provides a valuable reference source for mechanical engineering researchers who want to learn about the latest developments in advanced manufacturing and automation, readers from industry seeking potential solutions for their own applications, and those involved in the robotics and mechatronics industry.

This proceedings volume contains 36 papers that have been selected after review for oral presentation. These papers cover several aspects of the wide field of advanced mechatronics and robotics concerning theory and practice for advanced and intelligent manufacturing. The book contains three parts, the first part focuses on the *Design and Manufacturing of the Robot*, the second part deals with the *Mechanical Engineering and Power System*, and the third part investigates the *Automation and Control Engineering*.

We would like to express grateful thanks to our Program Committee members and Organization Committee members of the 2nd International Conference on Mechatronics and Robotics Engineering, special thanks to the keynote speakers: Prof. Alexander Balinsky, Cardiff University, UK, Prof. Farouk Yalaoui, Université de Technologie de Troyes, France, Prof. Dan Zhang, York University, Canada, and Prof. Elmar Bollin, Offenburg University of Applied Sciences, Germany. We would like to express our deep appreciation to all the authors for their significant contributions to the book. Their commitment, enthusiasm, and technical expertise are what made this book possible. We are also grateful to the publisher for supporting this project and would especially like to thank Arumugam Deivasigamani, Anthony Doyle, and Janet Sterritt for their constructive assistance and cooperation,

both with the publishing venture in general and the editorial details. We hope that the readers find this book informative and useful.

Finally, the editors would like to sincerely acknowledge all the friends and colleagues who have contributed to this book.

Toronto, Canada Dan Zhang
Oshawa, Canada Bin Wei
February 2016

Contents

Part I
Design and Manufacturing of the Robot

Critical Review and Progress of Adaptive Controller Design for Robot Arms

Dan Zhang and Bin Wei

Abstract Recent progress of adaptive control, particularly the model reference adaptive control (MRAC) for robotic arm is illustrated. The model reference adaptive controller design issues that researchers face nowadays are discussed, and its recent methodologies are summarized. This paper provides a guideline for future research in the direction of model reference adaptive control for robotic arms.

Keywords Adaptive control · Robot arm · Model reference approach

1 Introduction

In general terms, the robot control problem is formulated as follows, given a desired trajectory, a mathematical model of the manipulator and its interactions with the environment, find the control algorithm which sends torque commands to the actuators so that the robot can achieve expected motion. Control the robot to perform in a certain way is one of the most challenging problems because the robot mechanism is highly nonlinear, i.e. the robot dynamic equation is expressed by nonlinear dynamics that include couplings between the variables, and also the dynamic parameters of the robot vary with position of the joint variables (when the joint moves). Conventional control methods model the manipulator as uncoupled linear subsystems, these methods can produce satisfactory performances at low speeds, but it is not efficient anymore when used for high speed and high accuracy operations. In order to address the above problem, adaptive control can be relied on. Model reference adaptive approach is most popular and established technique.

Adaptive control is the control method used by a controller which must adapt to a controlled system with parameters which vary, or are initially uncertain. For non-adaptive controller, the controller is designed based on the priori information of the system, i.e. one knows the system and designs the controller (e.g. PID controller)

D. Zhang · B. Wei (✉)
University of Ontario Institute of Technology, Oshawa, ON, Canada
e-mail: Bin.Wei@uoit.ca

D. Zhang and B. Wei (eds.), *Mechatronics and Robotics Engineering for Advanced and Intelligent Manufacturing*, Lecture Notes in Mechanical Engineering, DOI 10.1007/978-3-319-33581-0_1

gears to that system and assume there is no change in the system. Whereas for the adaptive controller, the controller does not necessary need to depend on previous information of the system, and if there is sudden change in environment, the controller can cope with it to adapt to the changed conditions. If we consider a system that we know its transfer function, we design a fixed classical controller, that controller will remain fixed parameters as long as it applies to the system, so we say that this controller depends on its structure and designed on a priori information, that is non-adaptive controller. However, if the controller is depending on posteriori information, for example, if one is changing the parameters of the controller, because of the changes of the parameters of the system or because of the disturbances coming from the environment, that controller is called adaptive. If the system is subject to unknown disturbances, or the system is expected to undergo changes in its parameters in a way which is not pre-determined from the beginning, in that case we use adaptive control. However, in some cases we know how the system operating condition will change, for example, for an aircraft, we know that the aircraft controller is determined by its altitude and speed, and we expect that aircraft to fly at specific value for altitude and speed, in that case one can design a controller for each expected operating point and we switch between the different controllers, this is called gain-scheduling. In other cases we know that the parameters of the system change, but we know also a range for the change of every parameter, in that case it is possible to design a fixed controller that can cope with different changes of the parameters, and guarantee the stability and performance, this kind of controller is robust controller.

From Fig. 1, one can see that for non-adaptive control, firstly when one needs to improve the performance error, the modelling accuracy will also be increased, secondly it cannot improve itself, and thirdly it is assumed that future will be much like present, ignoring environment changes and change in dynamics. So adaptive controller is needed to address the above problem. Now for the adaptive control, it improves itself under unforeseen and adverse conditions, and it achieves a given system performance asymptotically, it does not trade performance for modelling accuracy, as shown in Fig. 1.

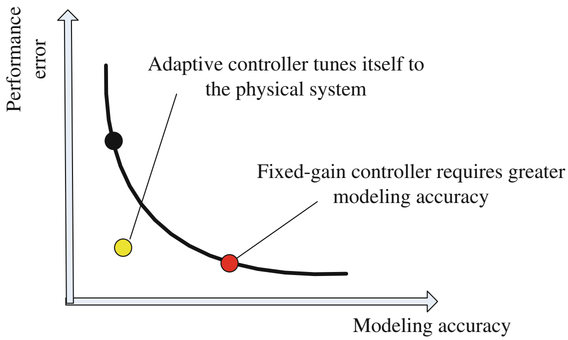


Fig. 1 Adaptive control

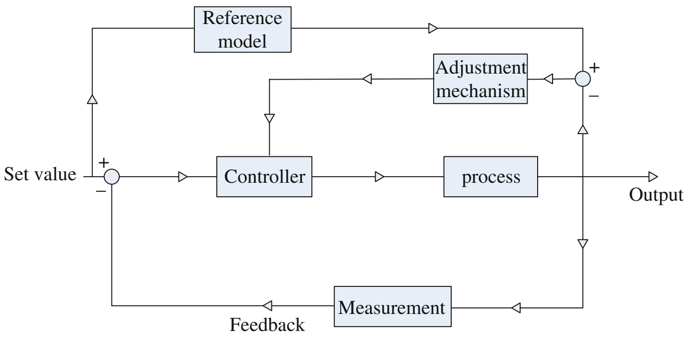


Fig. 2 Diagram of MRAC system

The adaptive control can be categorized into the following, model reference adaptive control, self-tuning adaptive control and gain-scheduled control. With the model-reference adaptive control, an accurate model of the system is developed. The set value is used as an input to both the actual and the model systems, and difference between the actual output and the output from the model is compared. The difference in these signals is then used to adjust the parameters of the controller to minimize the difference, as shown in Fig. 2.

Compared to other control methods, adaptive control is possible to achieve good performance over a wide range of motions and payloads. The advantage of the model reference adaptive control is that the plant parameters need not be fully known, instead, estimates of the plant parameters are used and the adaptive controller utilizes past input/output information to improve these estimates. However there are two drawbacks to MRAC. Stability analysis of the system is critical as it is not easy to design a stable adaptive law. The other problem is that MRAC relies on cancellation of the non-linear terms by the reference model (Sutherland 1987). In reality, exact cancellation cannot be expected, but the non-linear terms may be made so small so as to be negligible. Model reference adaptive control method was initially introduced in Whitaker et al. (1958), when they considered adaptive aircraft flight control systems, using a reference model to obtain error signals between the actual and desired behavior. These error signals were used to modify the controller parameters to attain ideal behavior in spite of uncertainties and varying system dynamics. The goal of an adaptive control system is to achieve and maintain an acceptable level in the performance of the control system in the presence of plant parameter variations. Whereas a conventional feedback control system is mainly dedicated to the elimination of the effect of disturbances upon the controlled variables. An adaptive control system is mainly dedicated to the elimination of the effect of parameter disturbances/variations upon the performance of the control system.

2 General Adaptive Control

In traditional control system, feedback is used to reject the disturbance effect that are acting on the controlled variables in order to bring the controlled variables back to their desired value. To do that, the variables are measured and compared to the desired values and the difference is fed into the controller. In these feedback systems, the designer adjusts the parameters of the controller so that a desired control performance is achieved. This is done by having a priori knowledge of the plant dynamics. When the parameters of the plant dynamic models change with time due to disturbances, the conventional control cannot deal with it anymore as the control performance will be degraded. At this time, one needs to resort to the adaptive control. A structured approach for the design of distributed and reconfigurable control system is presented in Valente and Carpanzano (2011). Distributed architectures are conceived as interconnected independent modules with standard interfaces which can be modified and reused without affecting the overall control structure. Whereas for the centralized control architectures, any change of the machine structure requires an extensive replacement of the control system. In RMS, modular and distributed architecture is essential to guarantee the capability of each single module or portions of the control to be adapted when a hardware reconfiguration occurs. But the paper did not explain in details on how the distributed and adaptive controller have been designed.

In Valentea and Mazzolinib (2015), a control approach is developed which consists of control conceptual design, application development and evaluation of solution robustness. In order to enable the control system reconfiguration, an essential feature of the control architecture is the modularity and distribution of the control decisions across various entities. The control system should be conceived as a set of independent and distributed control modules, capable of nesting one to each other.

The basic concept of adaptive control and several kinds of categories are introduced in Landau (2011), i.e. open-loop adaptive control, direct adaptive control, indirect adaptive control, robust control, and conventional control, etc. The design of a conventional feedback control is oriented to the elimination of the effect of disturbances on the controlled variables, controlled variables are, for examples, temperature if one controls the temperature, position if one controls the position of the end-effector, etc.; whereas the design of adaptive control is oriented to the elimination of effect of parameter disturbances on the performance of the control system. Simply put, the adaptive control can be seen as a conventional feedback control system but where the controlled variable is the performance index. So there are two loops for the adaptive control, one is the conventional feedback loop and the other is the adaptation loop.

The neural networks is used in Wilson and Rock (1995) for the control reconfiguration design for a space robot. The traditional controller was presented, and by using the neural networks, the traditional controller is updated to a reconfigurable controller. Two neural-network-control were developed to achieve quick adaptation controller. Firstly, a fully-connected architecture was used that has the ability to incorporate an a priori approximate linear solution instantly, this permits quick stabilization by an approximate linear controller. Secondly, a back-propagation learning method was used that allows back-propagation with discrete-valued functions. This paper presents a new reconfigurable neural-network-based adaptive control system for the space robot, but it did not explain in details.

3 Adaptive Control for Robotic Manipulators

Non-adaptive controller designs often ignores the nonlinearities and dynamic couplings between joint motions, when robot motions require high speed and accelerations, it greatly deteriorate its control performance. Furthermore, non-adaptive controller designs requires the exact knowledge and explicit use of the complex system dynamics and system parameters. Uncertainties will cause dynamic performance degradation and system instability. There are many uncertainties in all robot dynamic models, model parameters such as link length, mass and inertia, variable payloads, elasticities and backlashes of gear trains are either impossible to know precisely or varying unpredictably. That is why adaptive control is needed to address the above problem.

Model reference adaptive control and its usage to robotic arms were introduced in Neuman and Stone (1983) and Amerongen (1981). Some design problems in adaptive robot control are briefly stated. Dubowsky and Desforges (1979) is the first one that applies the model reference adaptive control in the robotic manipulator. The approach follows the method in Donalson and Leondes (1963). A linear, second-order, time-invariant differential equation was used as the reference model for each degree of freedom of the manipulator arm. The manipulator was controlled by adjusting the position and velocity feedback gains to follow the model. A steepest-descent method was used for updating the feedback gains. Firstly the reference model dynamics was written, but the paper did not explain how the author had the reference model dynamic equation, subsequently the nonlinear manipulator (plant) dynamic equation was written, but how this equation is related to the Lagrange equation is not clear, thirdly an error function was written and the paper follows the method of steepest descent and derived the a set of equations for the parameter adjustment mechanism, which will minimize the difference between the actual closed-loop system response and the reference model response.

An adaptive algorithm was developed in Horowitz and Tomizuka (1986) for serial robotic arm for the purpose of compensating nonlinear term in dynamic equations and decoupling the dynamic interaction among the joints. The adaptive method proposed in this paper is different from Dubowsky's approach (Dubowsky and Desforges 1979). Three main differences are concluded as follows: firstly, in Horowitz's paper, the overall control system has an inner loop model reference adaptive system controller and an outer loop PID controller, whereas the control system in Dubowsky's method is entirely based on the model reference adaptive controller; secondly, in Dubowsky's paper, the coupling among joints and non-linear terms in the manipulator equations are ignored whereas this is considered in Horowitz's method; thirdly, in Horowitz's paper, the design method is based on the hyper-stability method whereas the adaptive algorithm design in Dubowsky and Desforges (1979) is based on the steepest descent method.

Model reference adaptive control, self-tuning adaptive control and linear perturbation adaptive control are briefly reviewed in Hsia (1986). For the model reference adaptive control, the main idea is to synthesize/design a control signal u to the robot dynamic equation, which will force the robot to behave in a certain manner specified by the reference model, and the adaptive algorithm is designed based on the Lyapunove stability criterion.

The MRAC methods presented in Srinivasan (1987) is based on the theory of partitioning control, which makes them capable of compensating for non-linear terms in the dynamic equations and also to decouple the dynamic interactions between the links. It followed and used Horowitz's method (Horowitz 1983) and Sutherland's method (Sutherland 1987). Future research would focus on further simplification of MRAC schemes since the implementation of MRAC methods for the real time control of manipulators has proven to be a challenging task. There is no contribution in this thesis as it just followed and summarized the Horowitz's method and Asare and Wilson's method (Asare and Wilson 1987), and it did not propose its own method or theory.

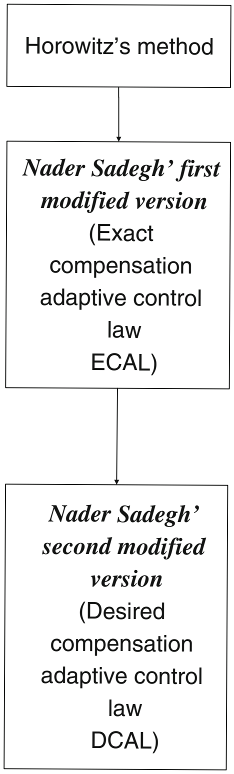
A MRAC system of 3-DOF serial robotic manipulator was presented in Horowitz (1983), but derivation for the adaptive algorithm is not explained. It was concerned with the application of MRAC to mechanical manipulators. Due to the dynamic equations of mechanical manipulators are highly nonlinear and complex, and also the payload sometimes varies or unknown, the author applied the MRAC to the mechanical manipulators. An adaptive algorithm was developed for compensating nonlinear terms in the dynamic equations and for decoupling the dynamic interactions. Finally a 3-DOF serial manipulator was used as computer simulation and the results show that the adaptive control scheme is effective in reducing the sensitivity of the manipulator performance to configuration and payload variations. The core content of Horowitz's method can be concluded as four steps: first step, deterministic nonlinearity compensation and decoupling control. Because one needs to calculate the inertia matrix Mp and nonlinear term V, the second step is proposed, i.e. adaptive nonlinearity compensation and decoupling control, which is to adaptively adjust the inertia matrix Mp and nonlinear term V instead of calculating them, and the adaptive algorithm was developed; final step, complete the overall control system by adding the feedback gain Kp, Kv and KI. In Horowitz (1983), it did not entirely use the Landau's hyperstability design (Landau 1979), he used some part of it, and he himself developed the adaptive algorithm. Because according to Hsia (1986), Horowitz's method was separated from the Landau's hyperstability design. And also from Sutherland (1987), it is stated that "While Landau's method replied on a pre-specified parameter matrix for a model and continuous adaptation of the plant parameters, it will be seen later that it is possible to estimate the model parameters and adapt them continuously", from this statement, it is obvious that Horowitz has his own theory to derive the adaptive algorithm, he did not use Landau's method to derive the adaptive algorithm, but how the adaptive algorithm was derived was not explicitly addressed. In Sutherland (1987), it used the same approach with Horowitz's to a 2-DOF serial robotic manipulator and a flexible manipulator.

In Tomizuka et al. (1986) and Tomizuka and Horowitz (1988), the experiment on the continuous time and discrete time adaptive control on 1-DOF test stand robot arm and Toshiba TSR-500 V robot were briefly conducted. Horowitz et al. (1987) is the continuation of Tomizuka et al. (1986) on a single axis direct drive robotic arm. It applies to a two axis direct drive robotic arm.

In Tomizuka et al. (1985), it presented the experiment evaluation of model reference adaptive controller and robust controller for positioning of a robotic arm under variation of payload. The results show that both method can be insensitive of the payload variation. Four adaptive control methods for the robotic arm were summarized in Jarnali (1989), i.e. computed torque technique, variable structure systems, adaptive linear model following control, and adaptive perturbation control, and the adaptive nonlinear model following control was proposed subsequently, which combines the self-tuning regulator and the model reference adaptive control.

Paper (Sadegh and Horowitz 1987) proposed a modified version of Horowitz's method and the assumption that matrix M and N is constant during adaptation can be removed by modifying the control law and parameter adaptation law. It is demonstrated that by modifying the control law (i.e. making the Coriolis and centripetal acceleration compensation controller a bilinear function of the joint and model reference velocities instead of a quadratic function of the joint velocities) and by modifying the parameter adaptation law (i.e. decomposing the nonlinear parameters in the manipulator dynamic equations into the product of two quantities: one constant unknown quantity, which includes the numerical values of the masses and moments of inertia of the links and the payload and the link dimensions, and the other a known nonlinear function of the manipulator structural dynamics. The nonlinear functions are then assumed to be known and calculable. The parameter adaptation law is only used to estimate the unknown constant quantities), the assumption that matrix M and N is constant during adaptation can be removed. Finally the stability of the above adaptive control law is proved. The above called "exact compensation adaptive control law (ECAL)". In the conclusion, the author found that in order to implement the adaptive controller, one needs to calculate the elements of W(xp, xv, xv) (Sadegh and Horowitz 1987), this procedure is excessively time consuming since it involves computations of highly nonlinear functions of joint position and velocities, to overcome this difficulty, later in Sadegh and Horowitz (1990) and Sadegh (1987), he proposed further modified version. The modification consists in utilizing the desired joint positions and velocities in the computation of the nonlinearity compensation controller and the parameter adaptation law instead of the actual quantities, this is known as "desired compensation adaptive control law (DCAL)" The above whole modification process is shown in Fig. 3.

Fig. 3 Modification process



Nader Sadegh applied Craig's method (Craig et al. 1986) to the Horowitz's method, so the condition M and N assumed constant during adaptation can be removed.

Craig's method is re-parametrization, i.e. decompose the manipulator dynamic equation's nonlinear parameters into the product of two quantities: one constant unknown quantity, which includes the numerical values of the masses and moments of inertia of the links and the payload and link dimensions, and a known nonlinear function of the manipulator structural dynamics. The nonlinear functions are assumed to be known and calculable. The parameter adaptation law is only used to estimate the unknown constant quantities.

One method of reparametrizing the manipulator's dynamic equations consists in decomposing each element of the matrices M(x), N(x)'s and the vector g(x) into products of unknown constant terms and known functions of the joint displacement vector. Or a second method consists in the re-parametrization of dynamic equation into the product of unknown constant vector, and a matrix formed by known functions of joint position.

4 Conclusion

Recent progress of model reference adaptive control for robotic arm is presented. The model reference adaptive controller design issues are discussed, and its recent methodologies are summarized. This paper provides a guideline for future research in the direction of model reference adaptive control for robotic arms.

Acknowledgments The authors would like to thank the financial support from the Natural Sciences and Engineering Research Council of Canada (NSERC) and Canada Research Chairs program.

References

Amerongen, J. (1981). MRAS: Model reference adaptive systems. *Journal A, 22*(4), 192–198.

Asare, H., & Wilson, D. (1987). Evaluation of Three Model reference adaptive control algorithms for robotic manipulators. Proceedings of IEEE International Conference on Robotics and Automation. pp. 1531–1542.

Craig. J. J., Hsu. P., & Sastry. S. S. (1986). Adaptive control of mechanical manipulators. In *Proceedings of the 1986 IEEE International Conference on Robotics and Automation*, San Francisco, April.

Donalson, D., & Leondes, T. (1963). A model referenced parameter tracking technique for adaptive control systems. *IEEE Transactions on Applications and Industry, 82*(68), 241–252.

Dubowsky, S., & Desforges, D. (1979). The application of model-referenced adaptive control to robotic manipulators. *Journal of Dynamic Systems, Measurement, and Control, 101*, 193–200.

Horowitz, R. (1983). Model reference adaptive control of mechanical manipulators. PhD thesis, University of California.

Horowitz, R., & Tomizuka, M. (1986). An adaptive control scheme for mechanical manipulators—Compensation of nonlinearity and decoupling control. *Journal of Dynamic Systems, Measurement, and Control, 108*(2), 1–9.

Horowitz, R., Tsai, M. C., Anwar, G., & Tomizuka, M. (1987). Model reference adaptive control of a two axis direct drive manipulator arm. In *Proceedings of 1987 IEEE International Conference on Robotics and Automation*, pp. 1216–1222.

Hsia, T. (1986). Adaptive control of robot manipulators—A review. In *Proceedings of 1986 IEEE International Conference on Robotics and Automation*, pp. 183–189.

Jarnali, H. (1989). Adaptive control methods for mechanical manipulators: A comparative study. Master thesis, Naval Postgraduate School.

Landau, I. D., et al. (2011). "Introduction to Adaptive Control" in adaptive control, communications and control engineering. Springer-Verlag London Limited.

Landau, Y. (1979). *Adaptive control—The model reference approach*. CRC Press.

Neuman, C. P., & Stone, H. W. (1983). MRAC control of robotic manipulators. In K. S. Narendra (Ed.), *3rd Yale Workshop on Applications of Adaptive Systems Theory*. Yale University, New Haven, CT, pp. 203–210.

Sadegh, N. (1987). Adaptive control of mechanical manipulators: Stability and robustness analysis. PhD thesis, University of California, 1987.

Sadegh, N., & Horowitz, R. (1987). Stability analysis of an adaptive controller for robotic manipulators. In *Proceedings of 1987 IEEE International Conference on Robotics and Automation*, pp. 1223–1229.

Sadegh, N., & Horowitz, R. (1990). Stability and robustness analysis of a class of adaptive controllers for robotic manipulators. *International Journal of Robotics Research, 9*(3), 74–92.

Srinivasan, R. (1987). Adaptive control for robotic manipulators. Master thesis, Carleton University.

Sutherland, J. (1987). Model reference adaptive control of a two link manipulator. Master thesis, Carleton University.

Tomizuka, M., & Horowitz, R. (1988). Implementation of adaptive techniques for motion control of robotic manipulators. *Journal of Dynamic Systems, Measurement, and Control, 110*(1), 62–69.

Tomizuka, M., Horowitz, R., & Anwar, G. (1986). Adaptive techniques for motion controls of robotic manipulators, in Japan—USA Symposium on Flexible Automation, Osaka, Japan, pp. 217–224.

Tomizuka, M., Horowitz, R., & Teo, C. L. (1985). Model reference adaptive controller and robust controller for positioning of varying inertia. In *Proceedings of Conference on Applied Motion Control*, University of Minnesota, pp. 191–196.

Valente, A., Carpanzano, E., & Brusaferri, M. (2011). Design and implementation of distributed and adaptive control solutions for reconfigurable manufacturing systems. In *CIRP Sponsored ICMS*. International Conference on Manufacturing Systems.

Valentea, A., Mazzolinib, M., & Carpanzanoa, E. (2015). An approach to design and develop reconfigurable control software for highly automated production systems. *International Journal of Computer Integrated Manufacturing*, *28*(3), 321–336.

Whitaker, H. P., Yamron, J., & Kezer, A. (1958). *Design of model reference adaptive control systems for aircraft*. Report R-164, Instrumentation Laboratory, M. I. T. Press, Cambridge, Massachusetts.

Wilson, E., & Rock, S. (1995). Reconfigurable control of a free-flying space robot using neural networks. In *Proceedings of the 1995 American Control Conference*, *2*, 1355–1359.

Stiffness Analysis of a Planar 3-RPS Parallel Manipulator

Bo Hu, Chunxiao Song and Bo Li

Abstract This paper studied the stiffness model and characteristics of a planar 3-RPS PM with 3-DOF. The 6 × 6 form stiffness matrix of the planar 3-RPS PM is derived with both active and constrained wrenches considered. To characteristic the stiffness of the planer 3-RPS PM, two decomposition methods including the eigen-screw decomposition and the principle axes decomposition are applied to the stiffness matrix. The stiffness matrix decomposition provides a physical interpretation and allows the identification of the compliant axes of the planar 3-RPS PM.

Keywords Planar parallel manipulator · Stiffness · Eigenscrew decomposition · Principle axes decomposition · Compliant aixs

1 Introduction

In recent years, the planar 3 degree of freedom (DOF) parallel manipulators (PMs) have attracted much attention (Angeles 2014). Merlet et al. (1998) presented some definitions such as constant orientation workspace, reachable workspace and dexterous workspace for the planar PMs. Binaud et al. (2010) compared the sensibility of five 3-DOF planar PMs including the 3-RPR, 3-RPR, 3-RRR, 3-RRR and 3-PRR PMs. Mejia et al. (2015) derived a mathematical closed-form solution to obtain the maximum force with a prescribed moment in 3-DOF planar mechanisms. Kucuk (2009) performed dexterity comparison for seven 3-DOF planar PMs with two kinematic chains using genetic algorithms and indicated that the PPR planar

B. Hu (✉) · C. Song · B. Li
Parallel Robot and Mechatronic System Laboratory of Hebei Province,
Yanshan University, Qinhuangdao 066004, Hebei, China
e-mail: hubo@ysu.edu.cn

B. Hu · C. Song · B. Li
Key Laboratory of Advanced Forging and Stamping Technology and Science
of Ministry of National Education, Yanshan University,
Qinhuangdao 066004, Hebei, China

D. Zhang and B. Wei (eds.), *Mechatronics and Robotics Engineering for Advanced and Intelligent Manufacturing*, Lecture Notes in Mechanical Engineering, DOI 10.1007/978-3-319-33581-0_2

robot manipulator is the best configuration with the best dexterous maneuverability among the others. Dong et al. (2016) proposed a piezoelectric actuated 3-RPR planar micro-manipulator with orthogonal structure and developed its prototype.

Stiffness analysis plays an important role in design of planar 3-DOF PMs. In this aspect, Gosselin (1990) derived general $n \times n$ stiffness matrix for n-DOF PMs by only considering the elastic deformation of actuator factor. Wu et al. (2010) compared the stiffness performance of 4-RRR, 3-RRR and 2-RRR PMs. Zhao et al. (2007) investigated the stiffness performance of planar parallel 3-RRR mechanism with flexible joints.

Most of the stiffness model of planar PMs only considered the actuator factor while the constraint factors were not considered. Recently, the stiffness model considered both active and constrained wrenches has been established for various spatial lower mobility PMs (Li and Xu 2008; Hu and Lu 2011; Hu et al. 2014). Due to the consideration of constraints, this stiffness model is more suitable for the lower mobility PMs. However, up to now, the stiffness models of planar PMs with both active and constrained wrenches considered have not been studied.

Stiffness characteristic analysis is also an important research content for the planar PMs. To investigated the stiffness characteristics of PMs, some researchers proposed effective approaches for the stiffness matrix decomposition (Loncaric 1987; Huang and Schimmels 2000; Chen et al. 2015). Huang and Schimmels (2000) proposed an alternative synthesis algorithm for realization of an arbitrary spatial stiffness matrices, which has been widely used in stiffness characteristic analysis. Chen et al. (2015) presented an alternative decomposition of stiffness matrices, which can be used in both Plucker's ray and axis coordinates. And the compliant axis proposed by Patterson and Lipkin (1993a) is also a better way to explain the characteristic of stiffness.

For the above reasons, the stiffness model and characteristic of a novel planar 3-RPS PM which have constrained forces is studied in this paper.

2 Stiffness Model of the Planar 3-RPS PM

2.1 Kinematics Description

The planar 3-RPS PM includes a base B, a moving platform m, three identical RPS (revolute joint-active prismatic joint-spherical joint)-type leg. Here, B is a regular triangle with O as its center and A_i (i = 1, 2, 3) as its three vertices. m is a regular triangle with o as its center and a_i (i = 1, 2, 3) as its three vertices. For the planar 3-RPS PM, the three R joints are perpendicular with B (see Fig. 1).

Let $\perp$ be a perpendicular constraint and $\|$ be a parallel constraint. Let $\{B\}$ be a frame O-XYZ attached on B at O, $\{m\}$ be a frame o-xyz attached on m at o. Some geometrical conditions ($X \parallel A_1A_3$, $Y \perp A_1A_3$, $Z \perp B$, $x \parallel a_1a_3$, $y \perp a_1a_3$, $z \perp m$) for O-XYZ and o-xyz are satisfied.

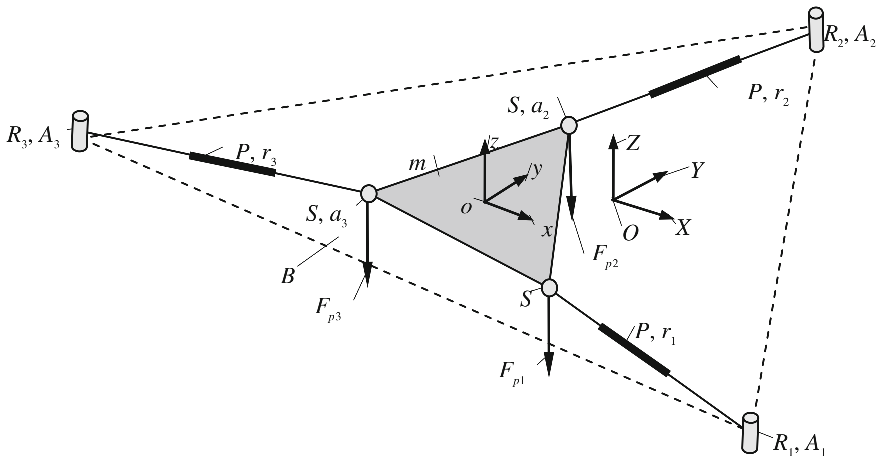


Fig. 1 Sketch of the planar 3-RPS PM

For the planar 3-RPS PM, the unit vectors $\boldsymbol{R}_{\mathrm{i}}$ of R_{i} (i = 1, 2, 3) in $\{B\}$ can be expressed as following:

$$\boldsymbol{R}_1 = \boldsymbol{R}_2 = \boldsymbol{R}_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \tag{1}$$

The position vectors $\boldsymbol{A}_i$ (i = 1, 2, 3) of three vertices A_i in $\{B\}$ can be expressed as follows:

$$\boldsymbol{A}_1 = \frac{1}{2}\begin{bmatrix} qL \\ -L \\ 0 \end{bmatrix}, \quad \boldsymbol{A}_2 = \begin{bmatrix} 0 \\ L \\ 0 \end{bmatrix}, \quad \boldsymbol{A}_3 = -\frac{1}{2}\begin{bmatrix} qL \\ L \\ 0 \end{bmatrix}, \quad q = \sqrt{3}, \tag{2a}$$

where, L denotes the distance from the center of O to A_i.

The coordinate a_i (i = 1, 2, 3) in $\{m\}$ can be expressed as following:

$${}^{m}\boldsymbol{a}_1 = \frac{1}{2}\begin{bmatrix} ql \\ -l \\ 0 \end{bmatrix}, \quad {}^{m}\boldsymbol{a}_2 = \begin{bmatrix} 0 \\ l \\ 0 \end{bmatrix}, \quad {}^{m}\boldsymbol{a}_3 = -\frac{1}{2}\begin{bmatrix} ql \\ l \\ 0 \end{bmatrix} \tag{2b}$$

where, l denotes the distance from the center of o to a_i.

The coordinate a_i in $\{B\}$ can be expressed as following:

$${}^{B}\boldsymbol{a}_i = \begin{bmatrix} X_{ai} \\ Y_{ai} \\ Z_{ai} \end{bmatrix} = {}_{m}^{B}\mathbf{R}\,{}^{m}\boldsymbol{a}_i + \boldsymbol{o}, \quad {}_{m}^{B}\mathbf{R} = \begin{bmatrix} c_\alpha & -s_\alpha & 0 \\ s_\alpha & c_\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad {}^{B}\boldsymbol{o} = \begin{bmatrix} X_o \\ Y_o \\ Z_o \end{bmatrix} \tag{2c}$$

Here, α denotes the angle between B and m.

From Eqs. (2a), (2b) and (2c), the inverse solution can be formulated as following:

$$\begin{aligned}
r_1^2 &= (qlc_\alpha/2 + ls_\alpha/2 + X_o - qL/2)^2 + (qls_\alpha/2 - lc_\alpha/2 + Y_o + L/2)^2 \\
r_2^2 &= (-ls_\alpha + X_o)^2 + (lc_\alpha + Y_o - L)^2 \\
r_3^2 &= (-qlc_\alpha/2 + ls_\alpha/2 + X_o + qL/2)^2 + (-qls_\alpha/2 - lc_\alpha/2 + Y_o - L)^2
\end{aligned} \tag{3}$$

Here r_i (i = 1, 2, 3) is the length of ith leg.

Based on the geometrical approach for determining the constrained forces/torques (Hu et al. 2014), one constrained force F_{pi} (i = 1, 2, 3) which is parallel with R_i and passes through the center of S joint in each RPS type leg can be determined.

As the constrained forces/torques do not work to m, it leads to

$$\begin{bmatrix} \boldsymbol{Z}^T & (\boldsymbol{d}_i \times \boldsymbol{Z})^T \end{bmatrix} \begin{bmatrix} \boldsymbol{v} \\ \boldsymbol{\omega} \end{bmatrix} = 0, \quad \boldsymbol{Z} = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}^T, \quad \boldsymbol{d}_i = \boldsymbol{a}_i - \boldsymbol{o} \tag{4a}$$

where, $\boldsymbol{f}_i$ denotes the unit vector of F_{pi}, $\boldsymbol{a}_i$ (i = 1, 2, 3) and $\boldsymbol{o}$ denote the coordinates of a_i and o respect to O, respectively.

From Eq. (4a) and Hu et al. (2014), it leads to

$$\mathbf{V}_r = \mathbf{J}_{6\times6} \begin{bmatrix} \boldsymbol{v} \\ \boldsymbol{\omega} \end{bmatrix}, \quad \mathbf{J}_{6\times6} = \begin{bmatrix} \boldsymbol{\delta}_1^T & (\boldsymbol{d}_1 \times \boldsymbol{\delta}_1)^T \\ \boldsymbol{\delta}_2^T & (\boldsymbol{d}_2 \times \boldsymbol{\delta}_2)^T \\ \boldsymbol{\delta}_3^T & (\boldsymbol{d}_3 \times \boldsymbol{\delta}_3)^T \\ \boldsymbol{Z}^T & (\boldsymbol{d}_1 \times \boldsymbol{Z})^T \\ \boldsymbol{Z}^T & (\boldsymbol{d}_2 \times \boldsymbol{Z})^T \\ \boldsymbol{Z}^T & (\boldsymbol{d}_3 \times \boldsymbol{Z})^T \end{bmatrix}, \quad \boldsymbol{V}_r = \begin{bmatrix} v_{r1} \\ v_{r2} \\ v_{r3} \\ 0 \\ 0 \\ 0 \end{bmatrix}, \quad \boldsymbol{\delta}_i = \frac{\boldsymbol{a}_i - \boldsymbol{A}_i}{|\boldsymbol{a}_i - \boldsymbol{A}_i|} \tag{4b}$$

Here, $\boldsymbol{v}$ and $\boldsymbol{\omega}$ denote the linear and angular velocities of m, respectively, and $\mathbf{J}_{6\times6}$ is the Jacobian matrix of the planar 3-RPS PM.

2.2 Stiffness Matrix Establishment

Let $\boldsymbol{F}_o = [F_x\ F_y\ F_z]^{\mathrm{T}}$ and $\boldsymbol{T}_o = [T_x\ T_y\ T_z]^{\mathrm{T}}$ be the forces and torques applied on m at o, respectively. Let F_{ri} and F_{pi} (i = 1, 2, 3) be the active force and constrained force of r_i, respectively. Using the principle of virtual work, we obtain

$$\boldsymbol{F}_r^T \boldsymbol{V}_r + \begin{bmatrix} \boldsymbol{F}_o^T & \boldsymbol{T}_o^T \end{bmatrix} \begin{bmatrix} \boldsymbol{v} \\ \boldsymbol{\omega} \end{bmatrix} = 0 \tag{5a}$$

Here, $\boldsymbol{F}_r = [F_{r1}\ F_{r2}\ F_{r3}\ F_{p1}\ F_{p2}\ F_{p3}]^{\mathrm{T}}$.

From Eqs. (4b) and (5a), it leads to

$$\boldsymbol{F}_r = -(\mathbf{J}_{6\times6}^{-1})^T \begin{bmatrix} \boldsymbol{F}_o \\ \boldsymbol{T}_o \end{bmatrix}, \begin{bmatrix} \boldsymbol{F}_o \\ \boldsymbol{T}_o \end{bmatrix} = \mathbf{J}_{6\times6}^T \boldsymbol{F}_r \tag{5b}$$

In the RPS type leg, the active force F_{ri} (i = 1, 2, 3) produces a flexibility deformations along r_i and the constrained force F_{pi} (i = 1, 2, 3) produces a bending deformation which is perpendicular with r_i.

Let δr_i (i = 1, 2, 3) denotes the flexibility deformations along r_i produced by the active force F_{ri}, it leads to

$$F_{r_i} = k_{r_i}\delta r_i, \quad k_{r_i} = \frac{ES_i}{r_i} \tag{6a}$$

Here, E is the modular of elasticity and S_i denotes the ith leg's cross section of RPS type leg.

Let δd_i (i = 1, 2, 3) denotes the bending deformation of r_i produced by the constrained forces F_{pi}. It leads to,

$$F_{p_i} = k_{p_i}\delta d_i, \quad k_{p_i} = \frac{3EI}{r_i^3} \tag{6b}$$

where, I is the moment of inertia.

From Eqs. (6a) and (6b), it leads to

$$\boldsymbol{F}_r = \mathbf{K}_p \begin{bmatrix} \delta \boldsymbol{r} \\ \delta \boldsymbol{d} \end{bmatrix}, \quad \delta \boldsymbol{r} = \begin{bmatrix} \delta r_1 \\ \delta r_2 \\ \delta r_3 \end{bmatrix}, \quad \delta \boldsymbol{d} = \begin{bmatrix} \delta d_1 \\ \delta d_2 \\ \delta d_3 \end{bmatrix},$$
$$\mathbf{K}_p = \begin{bmatrix} k_{r1} & 0 & 0 & 0 & 0 & 0 \\ 0 & k_{r2} & 0 & 0 & 0 & 0 \\ 0 & 0 & k_{r3} & 0 & 0 & 0 \\ 0 & 0 & 0 & k_{p1} & 0 & 0 \\ 0 & 0 & 0 & 0 & k_{p2} & 0 \\ 0 & 0 & 0 & 0 & 0 & k_{p3} \end{bmatrix} \tag{7}$$

Let $\delta \boldsymbol{p}$ and $\delta \boldsymbol{\Phi}$ be the position and orientation deformation of m, respectively. By using the principle of virtual work, the following equation can be derived:

$$\boldsymbol{F}_r^T \begin{bmatrix} \delta \boldsymbol{r} \\ \delta \boldsymbol{d} \end{bmatrix} = -\begin{bmatrix} \boldsymbol{F}_o^T & \boldsymbol{T}_o^T \end{bmatrix} \begin{bmatrix} \delta \boldsymbol{p} \\ \delta \boldsymbol{\Phi} \end{bmatrix} \tag{8}$$

From Eqs. (5b), (7) and (8), it leads to

$$\begin{bmatrix} \boldsymbol{F}_o \\ \boldsymbol{T}_o \end{bmatrix} = \mathbf{K} \begin{bmatrix} \delta \boldsymbol{p} \\ \delta \boldsymbol{\Phi} \end{bmatrix}, \quad \mathbf{K} = \mathbf{J}_{6\times 6}^{\mathrm{T}} \mathbf{K}_p \mathbf{J}_{6\times 6} \tag{9}$$

Here, $\mathbf{K}$ is the stiffness matrix of the planar 3-RPS PM.

3 Stiffness Characteristics Analysis

To characteristic the stiffness of the planer 3-RPS PM, the eigenscrew decomposition and the principle axes decomposition approaches are applied to the stiffness matrix. Loncaric (1987) proposed that by using the decomposition, the stiffness matrix can be realized by several parallel simple or screw springs, which is a direct correspondence between the mechanism realization and physical appreciation of a spatial stiffness matrix. In addition, the compliant axis of the planer 3-RPS PM are also studied in this section to reversal the characteristic of this PM.

3.1 The Eigenscrew Decomposition of Stiffness Matrix

The eigenscrew problem mentioned by Patterson and Lipkin (1993a) of the spatial stiffness matrix can be expressed as following:

$$\mathbf{K}\Delta \boldsymbol{e} = \lambda \boldsymbol{e} \tag{10}$$

where λ and the corresponding $\boldsymbol{e}$ are the eigenvalue and eigenvector of $\mathbf{K}\Delta$, respectively. The transformation matrix Δ interchanges the first and last three components of a screw, which can be expressed as following:

$$\boldsymbol{\Delta} = \begin{bmatrix} \mathbf{0}_{3\times 3} & \mathbf{I}_{3\times 3} \\ \mathbf{I}_{3\times 3} & \mathbf{0}_{3\times 3} \end{bmatrix} \tag{11}$$

The eigenscrew decomposition proposed by Huang and Schimmels (2000) of spatial stiffness matrix can be expressed as:

$$\mathbf{K} = \sum_{i=1}^{6} k_i \boldsymbol{w}_i \boldsymbol{w}_i^T, \quad k_i = \frac{\lambda_i}{2h_i}, \quad h_i = \frac{1}{2} \boldsymbol{w}_i^T \Delta \boldsymbol{w}_i \tag{12}$$

where, spring wrench $\boldsymbol{w}_i$ is the unitization of $\boldsymbol{e}_i$ (i = 1, …, 6), h_i is the pitch of $\boldsymbol{w}_i$ and $\boldsymbol{w}_i$ can be defined as:

$$\boldsymbol{w}_i = \begin{bmatrix} \boldsymbol{n}_i \\ \boldsymbol{\rho}_i \times \boldsymbol{n}_i + h_i \boldsymbol{n}_i \end{bmatrix} \tag{13}$$

Here, $\boldsymbol{n}_i$ and $\boldsymbol{\rho}_i$ (i = 1, …, 6) are the direction and position vectors of the *i*th spring, respectively.

3.2 *The Principle Axes Decomposition of Spatial Stiffness Matrix*

In the principle axes decomposition (Chen et al. 2015), the wrench $\underline{\boldsymbol{F}}$ and $\delta\underline{\boldsymbol{P}}$ are expressed in axis coordinate. The relation between ray and axis coordinate can be expressed as following:

$$\underline{\boldsymbol{F}} = \boldsymbol{\Delta F}, \quad \delta\underline{\boldsymbol{P}} = \boldsymbol{\Delta}\delta\boldsymbol{P} \tag{14}$$

From Eq. (11), it leads to

$$\Delta\Delta = \mathbf{E} \tag{15}$$

here $\mathbf{E}$ is an identity matrix.

The relation of stiffness matrices between these two systems can be derived from Eqs. (14) and (15) as following,

$$\underline{\mathbf{K}} = \Delta\mathbf{K}\Delta = \begin{bmatrix} \underline{\mathbf{A}} & \underline{\mathbf{B}} \\ \underline{\mathbf{B}}^T & \underline{\mathbf{C}} \end{bmatrix} \tag{16}$$

where the symmetric 3 × 3 block matrices $\underline{\mathbf{A}}$ and $\underline{\mathbf{C}}$ denote the rotational and translational parts, and $\underline{\mathbf{B}}$ denote the coupling part.

$\underline{\mathbf{K}}$ can be represented in a reduced form $\underline{\mathbf{K}}_O$ by applying a pure rotation $\mathbf{R} = \mathbf{Q}^{\mathrm{T}}$ to the current frame in order to translate $\underline{\mathbf{C}}$ to a diagonal form $\underline{\mathbf{C}}_O$, where $\mathbf{Q}$ represents a 3 × 3 orthogonal matrix whose columns are just the eigenvectors of $\underline{\mathbf{C}}$. Then the stiffness matrix can be decomposed into two sets of rank-1 symmetric stiffness matrices as following (Chen et al. 2015):

$$\begin{aligned} &\underline{\mathbf{K}}_O = \begin{bmatrix} \underline{\mathbf{A}}_O & \underline{\mathbf{B}}_O \\ \underline{\mathbf{B}}_O^T & \underline{\mathbf{C}}_O \end{bmatrix} = \underline{\mathbf{K}}_{OS} + \underline{\mathbf{K}}_{OT} = \sum_{i=1}^{3} k_i \boldsymbol{w}_i \boldsymbol{w}_i^T + \sum_{j=4}^{6} k_j \boldsymbol{w}_j \boldsymbol{w}_j^T, \\ &\underline{\mathbf{A}}_O = \mathbf{Q}^T \underline{\mathbf{A}} \mathbf{Q}, \quad \underline{\mathbf{B}}_O = \mathbf{Q}^T \underline{\mathbf{B}} \mathbf{Q}, \quad \underline{\mathbf{A}}_{OT} = \underline{\mathbf{A}}_O - \underline{\mathbf{B}}_O \underline{\mathbf{C}}_O^{-1} \underline{\mathbf{B}}_O^T, \\ &\boldsymbol{w}_i = \begin{bmatrix} \frac{1}{k_i} \boldsymbol{b}_i^T & \boldsymbol{e}_i^T \end{bmatrix}^T, \quad \boldsymbol{w}_j = \begin{bmatrix} \boldsymbol{\alpha}_i^T & \boldsymbol{0}_{3\times 1}^T \end{bmatrix}^T, \end{aligned} \tag{17}$$

where, $\underline{\mathbf{K}}_{OS}$ and $\underline{\mathbf{K}}_{OT}$ are the principal components corresponding to the screw and torsional springs, respectively. k_i (i = 1, 2, 3) and k_j (j = 1, 2, 3) are the ith eigenvalue of $\underline{\mathbf{C}}$ and $\underline{\mathbf{A}}_{OT}$, respectively. $\boldsymbol{e}_i$ (i = 1, 2, 3) denotes the unit vector associated with the coordinate axis of $\{O\}$, namely $\boldsymbol{e}_1 = [1, 0, 0]^T$, $\boldsymbol{e}_2 = [0, 1, 0]^T$, $\boldsymbol{e}_3 = [0, 0, 1]^T$, $\boldsymbol{b}_i$ represents the ith column of $\underline{\mathbf{B}}_O$, $\boldsymbol{a}_i$ represents the ith eigenvector of $\underline{\mathbf{A}}_{OT}$, and $\boldsymbol{w}_i$ (i = 1, 2, 3) is the ith wrench-compliant axis of this elastic system.

From Eq. (17), any spatial stiffness matrix can be uniquely realized by three screw and three torsional springs connected in parallel, and the screw springs and torsional springs are orthogonal to each other, respectively.

Let $\{C\}$ be a frame C-$X_QY_QZ_Q$ with the direction of X_Q, Y_Q and Z_Q-axis are along each row of $\boldsymbol{Q}$, respectively. Then $\underline{\mathbf{K}}$ can be expressed in $\{C\}$ as following:

$$\begin{gathered}\underline{\mathbf{K}}_C = \begin{bmatrix} \underline{\mathbf{A}}_* & 0 \\ 0 & 0 \end{bmatrix} + \begin{bmatrix} \underline{\mathbf{B}}_*\underline{\mathbf{C}}\underline{\mathbf{B}}_* & \underline{\mathbf{B}}_*\underline{\mathbf{C}} \\ \underline{\mathbf{C}}\underline{\mathbf{B}}_* & \underline{\mathbf{C}} \end{bmatrix}, \\ \underline{\mathbf{A}}_* = \underline{\mathbf{A}} - \underline{\mathbf{B}}\underline{\mathbf{C}}^{-1}\underline{\mathbf{B}}^T, \quad \underline{\mathbf{B}}_* = \frac{1}{2}(\underline{\mathbf{B}}\underline{\mathbf{C}}^{-1} + \underline{\mathbf{C}}^{-1}\underline{\mathbf{B}}^T)\end{gathered} \tag{18}$$

Equation (18) is referred to as the central principle frame, and C is also called the center of stiffness. $\underline{\mathbf{K}}_C$ is the simplest form of the spatial stiffness matrices, which decouples rotational and translational aspects of stiffness to a certain extent. In (18), there only exists three 3 × 3 symmetric blocks $\underline{\mathbf{A}}_*$, $\underline{\mathbf{B}}_*$, $\underline{\mathbf{C}}$, which correspond to the rotational, coupling and translational parts, respectively.

The homogeneous transformation matrix is given by,

$$g_K = \begin{bmatrix} \mathbf{Q} & \boldsymbol{p} \\ \mathbf{0}_{3\times 1}^T & 1 \end{bmatrix}, \quad \hat{\boldsymbol{p}} = \frac{1}{2}(\underline{\mathbf{B}}_O\underline{\mathbf{C}}^{-1} - \underline{\mathbf{C}}^{-1}\underline{\mathbf{B}}_O) \tag{19}$$

where $\boldsymbol{p}$ is the coordinate of C respected to the original reference frame $\{B\}$.

Based on the above analysis, the stiffness matrix of planar 3-RPS PM can be decomposed into two sets of three rank-1 symmetric matrices, which can also identify the elastic system's force-deflection behavior of planar 3-RPS PM.

3.3 Compliant Axis and Center of Compliance

For a compliant axis (Patterson and Lipkin 1993b), a force produces a parallel liner deformation and a rotational deformation produces a parallel couple. The compliant axis exists if and only if there are two collinear eigenscrews with eigenvalues of equal magnitude and opposite sign. Thus, not all the elastic system exhibits compliant axes. Wrench-compliant and twist-compliant axes are the basic of a compliant axis hierarchy, and most elastic systems exhibit the wrench-compliant/twist-compliant axes. Wrench-compliant axis exists when a wrench produces a parallel linear deformation, and a twist-compliant axis exits when a twist produces a parallel couple. Such kinds of the force-deflection behavior can be interpreted as following: