

**FOOD SCIENCE AND TECHNOLOGY SERIES**



# **Handbook of Food Science and Technology 2**

*Food Process Engineering  
and Packaging*

**Edited by**

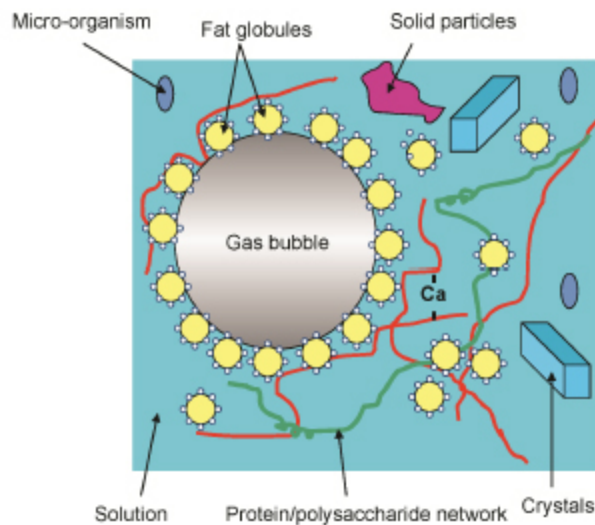
**Romain Jeantet, Thomas Croguennec  
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# Introduction

Food is a complex and heterogeneous system. It often consists of a protein and/or polysaccharide matrix surrounding, sustaining a typically aqueous continuous phase containing soluble hydrophilic compounds (carbohydrates, salts, vitamins, etc.) and some dispersed elements (cells, fat globules, gas bubbles, crystals, etc.) ([Figure I.1](#)). Such a system is thermodynamically, biologically and chemically very unstable.



**Figure I.1.** *Matrix structure of food*

The dispersed elements are subject to forces that cause phase separations either by sedimentation when the density of dispersed elements is greater than that of the dispersing phase, or by creaming when the opposite applies. Chemical potential or pressure gradients exerted on either side of the interfaces can induce the transfer of solutes and structural changes (coalescence, plasmolysis). This physicochemical instability can be increased by mechanical and thermal stresses on products during

storage (refrigeration, freezing) and preparation (defrosting, reheating, etc.).

Protein and polysaccharide polymers that contribute significantly to the structuring of food are likely to reorganise during storage because of the influence of temperature on hydrophobic, ionic and hydrogen interactions. As described in Volume 1 [JEA 16a], recrystallization is sometimes accompanied by the release and migration of water with textural changes (starch retrogradation and bread staling).

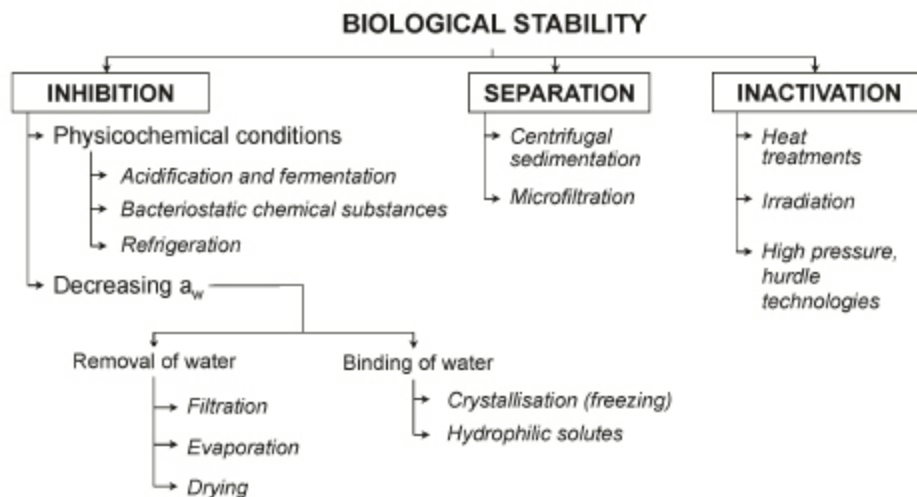
Biological agents, enzymes and microorganisms find suitable conditions for their action and development in most foods both in terms of physicochemical conditions (pH, water activity  $a_w$  and temperature) as well as availability of substrates and growth factors. Lipolysis, proteolysis and oxidation reactions, metabolite production (acid, alcohol, gas), the development of pathogenic flora and the release of toxins are all elements that negatively affect the sensory, safety and nutritional quality of food, as has already been demonstrated in the past. The same applies for chemical reactions (Maillard reaction, lipid autoxidation).

## **I.1. How to ensure biological stability of foods?**

Four strategies can be used to ensure the biological stability of agricultural commodities and food ([Figure I.2](#)):

- inactivation of biological agents either through energy input, which induces a denaturation of enzymes and cell constituents, or through physical, chemical or enzymatic treatments (latter being able to lyse or alter some transport properties of the cell wall);

- separation of biological agents based on density differences or their size;
- inhibition of enzymes, microorganisms and reactions by decreasing water availability (lowering  $a_w$ ), which limits both the transfer of substrates and growth factors as well as the transfer of metabolites or reaction products that accumulate in the reaction environment;
- Inhibition through the creation of limiting physicochemical conditions (pH, oxygen pressure, bacterial inhibitors, etc.).



**Figure I.2.** *Strategies for the biological stability of food*

All these strategies will be detailed in [Part 2](#) of this book.

### **1.1.1. Thermal inactivation of biological agents**

The destruction of microorganisms and the inactivation of enzymes by heat treatments that provide energy for the denaturation of macromolecules (proteins, DNA) were first applied by Appert and Pasteur. Food pasteurization and sterilization have been defined taking into account the thermal sensitivity of pathogenic flora:

- pasteurization destroys a large proportion of undesirable flora (*Listeria*, *Staphylococci*, *Salmonella sp.*, etc.) and extends shelf life in the cold chain;
- sterilization destroys all microorganisms and denatures almost all enzymes (depending on the heat treatment) to allow long-term storage at room temperature.

To reduce the risk of pathogen proliferation, treatments are based on the thermal resistance of *Clostridium botulinum*, one of the most heat stable and pathogenic microorganisms. By aiming to destroy microorganisms and inactivate enzymes through thermal denaturation, macromolecules responsible for food texture and biological constituents (vitamins) are also partially or totally denatured. These side effects degrade the sensory and nutritional quality of food. Thus, successfully destroying microorganisms without reducing the food quality too much requires the precise control of heat transfers and momentum during heat treatments. Heat treatment is not always used due to the thermal sensitivity of some products (e.g. egg white) or in order to preserve the raw nature of the product. Other non-thermal treatments (i.e. inducing a temperature increase of less than 25°C) are therefore used, for example ionization, high pressure and pulsed electric fields. These techniques have limited applications mainly due to cost, legislation and consumer acceptability. Hurdle technologies (combination of treatments to minimise energy input in the food product) may be an alternative.

### **1.1.2. Inhibition by decreasing water activity**

Most microbial and biochemical changes that reduce the quality of food occur in the aqueous phase. Water has a dual role:

- as a solvent, it ensures the transfer of substrates, growth factors, biological agents and reaction products thereby creating optimum conditions for reactions;
- as a reaction substrate, it is involved in hydrolysis reactions (proteolysis, lipolysis).

This dual action of water requires its availability, which is characterised by  $a_w$ : any treatment that reduces water availability slows down the reaction rate.  $a_w$  can be lowered by the removal of free water through evaporation or drying, the crystallization of solvent water (freezing) or the addition of highly hydrophilic solutes that bind water molecules through hydrogen or dipolar interactions (salting, sugaring). Inhibition through the decrease of  $a_w$  can be reversed by rehydration, defrosting and dilution. This method of preservation only slightly affects the nutritional and sensory quality if freezing and dehydration are well controlled in terms of heat and mass transfers. The shelf life may however be limited in the case of fat-containing products since reactions involving lipid substrates occur at the interface between the lipid and aqueous phases, and are consequently less dependent on water availability. Conversely, lipid oxidation is increased when  $a_w$  reaches 0, as exposure to oxygen is increased in these conditions.

### **1.1.3. Chemical inhibition**

The kinetics of microbial growth and the rate constants of enzymatic reactions are highly dependent on the pH of the medium. It is possible to slow down biological phenomena by deviating from the optimum reaction pH. As this is around neutral pH for most microorganisms and enzymes, microbial growth can be limited by acidifying the medium either through the addition of acid (acetic acid, lactic acid,

propionic acid, carbonic acid, etc.) or through fermentation. Aerobic microorganism growth can also be controlled by reducing the oxygen pressure through vacuum packaging or modified atmosphere packaging.

Many metabolites and reaction products are themselves inhibitors and, above a certain concentration, can slow down or terminate fermentation (alcohol, lactic acid or propionic acid) or reactions.

The bacteriostatic or bactericidal role of some chemical substances has been shown in ancestral processes: this is the case with nitrate/nitrite in meat curing and phenolic compounds resulting from the thermal degradation of lignin in the smoking process. These technologies generate undesirable substances due to their carcinogenic nature (nitrosamines, benzopyrenes). A good understanding of the biochemical reactions and processing conditions responsible for the formation of these substances can limit their levels in food.

## **1.2. How to ensure physicochemical stability of foods**

The presence of dispersed elements in food (fat globules in the case of emulsions, air bubbles in the case of foams) or the presence of proteins and carbohydrates, which are thermodynamically incompatible, can destabilise the product. The chemical potential gradients at interfaces can induce water and solute transfers between two phases or between a base and a topping or filling (e.g. hydration of a shortbread or puff-pastry base from the topping/filling); all these transfers are able to reduce the sensory quality of food.

The transfer rate  $v$  of dispersed elements is governed by Stokes law:

$$v = \frac{D^2}{18\eta} \Delta\rho g \quad [I.1]$$

with  $D$  the diameter of particles (m),  $\Delta\rho$  the density difference between the dispersed phase and the continuous phase ( $\text{kg m}^{-3}$ ),  $\eta$  the dynamic viscosity of the continuous phase (Pa s) and  $g$  the gravitational acceleration ( $\text{m s}^{-2}$ ).

To stabilise the dispersed elements, the parameters  $D$  and  $\eta$  can be adjusted, or a macromolecular network can be created to limit the movement of particles by entrapping them within the network.

The diameter of dispersed elements like fat globules can be reduced by mechanical treatments such as homogenization: in order to obtain relatively small diameters, it is necessary to lower the interfacial tension that creates resistance to an increase in the interphase surface. This surface tension can be lowered using amphiphilic molecules positioned at the interface, the hydrophobic portion facing the lipid phase and the hydrophilic portion facing the aqueous phase. In addition to creating micro-emulsions, it is also essential that the fat globules do not agglomerate, which could result in coalescence and an increase of the fat globule diameter. To avoid this, ionic and steric constraints must be created on the surface of the globule, which by repulsion limit the coalescence of particles. Reducing  $D$  is not always enough to stabilise the system; it is also possible to increase viscosity by increasing the concentration of solutes (e.g. adding sugars) or by immobilising solvent water by adding carbohydrate or protein macromolecules to induce thickening.

It is more difficult to stabilise a foam than an emulsion since the density difference between the dispersed and the dispersing phases is  $1,000 \text{ kg m}^{-3}$  in a foam whereas it is  $100 \text{ kg m}^{-3}$  in an emulsion. It is therefore necessary to create a macromolecular network that immobilises air



bubbles. Globular proteins, especially from egg whites, have good foaming properties since they position themselves at the [air/water] interface during beating, which creates a dissymmetry in their environment. As a result, denaturation occurs through changes in their tertiary or even secondary structures, which may be accompanied by molecular interactions and help forming a cohesive film ensuring good foam stability.

The dynamics of water and solutes between the various components of food can be controlled by creating hydrophobic interfaces; this is the role of fat in puff or shortcrust pastry. It is also necessary that the physical properties of this fat are not adversely affected by the crystallization of triglycerides during the refrigeration of the products.

The physicochemical stability of a product must be controlled throughout its shelf life but also during its handling by consumers including defrosting and reheating. Controlling the stability of a complex system that is thermodynamically unstable is one of the major challenges facing food engineers; this issue will be addressed in [Part 3](#) of this book. Nowadays, several functional carbohydrate and protein ingredients are available to limit transfers and destabilization within the food matrix. [Part 4](#) of this book will be devoted to the physicochemical basis of fractionation and related technologies used in the development of functional ingredients.

### **I.3. Packaging, an essential attribute for food preservation**

When responding to market needs and consumer demands with regards to safety, sensory and nutritional quality, offering a service and preserving, where possible, the

traditional character of food is a constant challenge for the food industry. Quality control requires a good understanding of the food and its biochemical, chemical and thermodynamic reactivity as well as the processing and storage methods involved. Indeed, food can be transported over long distances, stored, distributed and finally endure a sufficiently long shelf life in the hands of the consumer. Hence, the need for packaging to protect the product is something that satisfies specific functions in terms of food preservation. Packaging could also be described as the last stage of the production chain. These elements will be addressed in [Part 5](#) of this book.

Introduction written by Gérard BRULÉ.

**PART 1**  
**Basis of Food Engineering**

# 1

## **Transport Phenomena - Basis of Unit Operations**

The processing of agricultural commodities into finished food products can be analyzed in different ways. It can be examined by sector or by establishing a detailed description of the different unit operations in each one. Such an approach is not very useful in that the same unit operations exist in different sectors. For example, there is very little difference, with the exception of product characteristics or the type of material used, between pasteurization in a brewery and that in a milk factory, between concentration in a milk factory and that in a sugar refinery or between water extraction of sugar from sugar beet and solvent extraction of fat from oilcake.

Such overlaps can be limited by another approach, which looks at processing at the level of elementary operations (or unit operations) such as filtration, centrifugation, pasteurization or drying.

However, this approach can be further rationalized as such unit operations are fundamentally linked to three types of transport phenomena (or transfers) resulting from a physical difference (temperature, concentration or speed) between two points within a system: heat transfer (transfer of thermal energy from a hot to a cold point), mass transfer (transfer of mass from a concentrated to a diluted point) and momentum transfer (transfer of momentum from a moving point at a given speed to a moving point at a lower speed).

For example:

- heat transfers play a major role in the elementary operations of pasteurization, sterilization and concentration;
- mass transfers mainly govern extraction operations by diffusion.

However, such transfer processes are often connected and interdependent, for example in a drying operation, heat transfer causes the mass transfer (of moisture) from a product to the environment.

It is therefore essential to first examine these before studying unit operations, which combined together form the basis for food production overall.

There are three different mechanisms of heat transfer:

- *conduction (or diffusion)*: in this case, the matrix in which heat transfer takes place, can be considered motionless with regard to the direction of transfer. For example, heat transfer in a solid does not involve motion of the material itself. In a liquid, heat transfer may be conductive if the energy is transferred by molecular agitation, apart from the random motion of molecules in the direction of the heat transfer;
- *convection*: in this case, transfer processes involve movement of part of a matrix that carries a quantity of heat, mass or momentum from one point to another. For example, central heating is convective on two levels: heat produced by the boiler is transported to radiators by the forced circulation of water in the pipes, and from the radiators to the room by the convective movement of air;
- *radiation*: this type of transfer, which is specific to heat transfers, occurs by electromagnetic wave propagation and does not require material support. For example,

browning of foods can be carried out by infrared radiation.

These three transport mechanisms can coexist. Radiation may be considered dominant in the case of very high temperatures. However, in most cases in the food industry, the temperatures used (between  $-20^{\circ}\text{C}$  and  $+150^{\circ}\text{C}$ ) are relatively low and radiation can be disregarded. As such, conduction and convection processes will be the focus of attention in this chapter.

## 1.1. Transfer processes in conduction

In conduction, transfer rates are linked to the characteristics of the particular system and are directly proportional to the exchange surface area and the potential gradient.

As a result, there is a close similarity between the equations describing heat, mass and momentum transfers.

### 1.1.1. Heat transfers: Fourier's law

Fourier's law is expressed by the vector relationship:

$$\vec{\phi}_q = -\lambda \vec{\text{grad}}(\theta) \quad [1.1]$$

$\vec{\phi}_q$  is the heat flux density vector ( $\text{W m}^{-2}$ ), i.e. the scalar value  $\frac{1}{dA} \frac{dQ}{dt}$  ( $dA$ : elementary surface exchange;  $\text{m}^2$ ) multiplied by the normal unit vector at a constant-temperature surface:

$$\phi_q = \frac{1}{dA} \frac{dQ}{dt} = -\lambda \frac{d\theta}{dx} \quad [1.2]$$

The proportionality factor  $\lambda$  between the heat flux density and the temperature gradient is the thermal conductivity

(W m<sup>-1</sup> K<sup>-1</sup>), characteristic of a given product.

### **1.1.1.1. Establishing steady state conditions at an infinite plate**

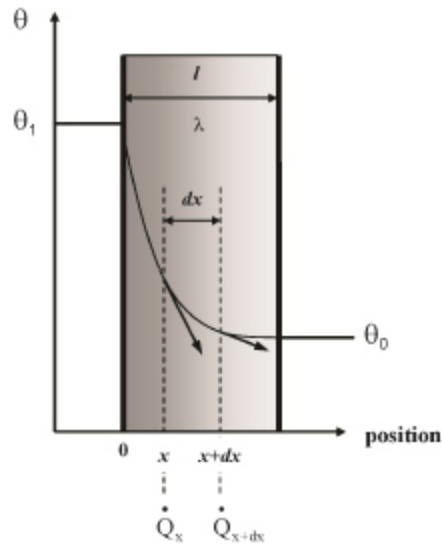
Generally, in a system, physical quantities vary with time from one point to another. This system is in a non-steady state (or transient state). However, if the environment external to the system is invariant (constant), the system, after a certain time, tends towards a state of equilibrium, where at any given point the physical quantities (in this case temperature) become invariant (constant) over time. Such a system has reached a steady state (or a stationary state).

The geometric model most often used to describe such a system is the semi-infinite plate (small thickness relative to length and width) which is assumed to be a homogeneous and isotropic solid, bounded by two parallel planes. The model includes the wall through which thermal energy is exchanged via a plate heat exchanger, which can be considered infinite as the edges have a negligible effect on heat transfer.

In this model, constant-temperature surfaces are parallel to the sides of the plate, and [equation \[1.2\]](#) is integrated for the entire heat exchange surface area A in:

$$\phi_q = \frac{1}{A} \frac{dQ}{dt} = -\lambda \frac{d\theta}{dx} \quad [1.3]$$

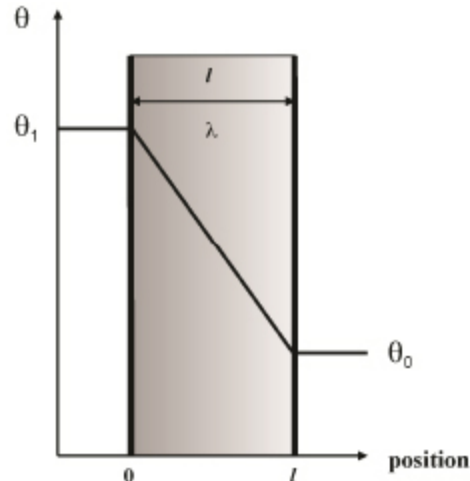
$\frac{dQ}{dt}$ , heat flux or power, can both be noted  $\dot{Q}$ .



**Figure 1.1.** *Temperature profile in an infinite plate in a transient state. Temperature gradient is indicated by arrows at the  $x$  and  $x+dx$  sides*

If, starting from a uniform temperature  $\theta_0$  ([Figure 1.1](#)), one side of this plate is rapidly brought to the temperature  $\theta_1$ , heat transfer takes place under the influence of the temperature gradient in a direction normal to the plate. After a certain time, the temperature profile is that shown as in [Figure 1.1](#). From [equation \[1.3\]](#), the quantity of heat  $dQ$  that enters a section of thickness  $dx$  in a time interval  $dt$  through the constant-temperature surface at the  $x$  side is evidently greater than that leaving through the constant-temperature surface at the  $x+dx$  side. Heat therefore accumulates in this section, which results in a temperature increase in the plate element. This continues until that which enters  $x$  is equal to that which leaves  $x+dx$ , that is until  $\frac{d\theta}{dx}$  is the same for all constant-temperature surfaces or until the temperature profile in the plate is linear ([Figure 1.2](#)).





**Figure 1.2.** *Temperature profile in an infinite plate in a steady state*

### **1.1.1.2. Heat transfer in an infinite plate in a steady state**

As we have just seen, heat transfer in a steady state infinite plate is characterized by a linear temperature gradient ([Figure 1.2](#)). [equation \[1.3\]](#) is integrated on the plate thickness  $l$  in the following way:

$$\dot{Q} = A h \Delta\theta \quad [1.4]$$

with  $\Delta\theta = \theta_1 - \theta_0 > 0$  and:

$$h = \frac{\lambda}{l} \quad [1.5]$$

$h$ , the heat transfer coefficient ( $\text{W m}^{-2} \text{K}^{-1}$ ), characterises the ability of the plate to allow heat to pass through since it incorporates both thermal conductivity  $\lambda$  and thickness  $l$ .

### **1.1.1.3. Heat transfer in a steady state composite plate**

A wall is often composed of a several barriers to the heat transfer: the heat exchange plate itself, fouling deposits,

boundary layers of fluid in contact with the surfaces, etc. The composite plate model is useful therefore, given its multiple applications.

In a steady state, the heat flux density reaches a constant value overall and in each of the sub-plates comprising the composite plate ([Figure 1.3](#)):

$$\phi_q = \frac{1}{A} \frac{dQ}{dt} = \begin{cases} \lambda_1 \frac{\theta_1 - \theta_2}{l_1} = h_1(\theta_1 - \theta_2) \\ \lambda_2 \frac{\theta_2 - \theta_3}{l_2} = h_2(\theta_2 - \theta_3) \\ \lambda_3 \frac{\theta_3 - \theta_4}{l_3} = h_3(\theta_3 - \theta_4) \end{cases} \quad [1.6]$$

This is equivalent to:

$$\Leftrightarrow \phi_q = \frac{1}{\frac{1}{h_1} + \frac{1}{h_2} + \frac{1}{h_3}} \Delta\theta \quad [1.7]$$

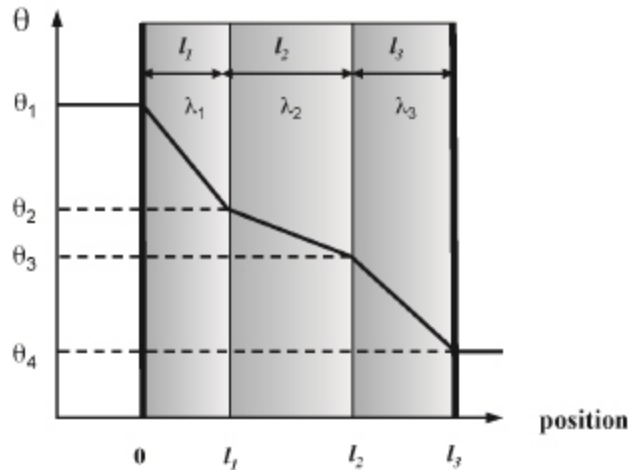
The heat flux density is therefore proportional to the temperature difference and the transfer coefficient relating to all sub-plates, denoted by  $h_g$  ( $\text{W m}^{-2} \text{K}^{-1}$ ), also known as the overall heat transfer coefficient, which is as follows:

$$\begin{cases} \frac{1}{h_g} = \frac{1}{h_1} + \frac{1}{h_2} + \frac{1}{h_3} \\ \phi_q = h_g \Delta\theta \end{cases} \quad [1.8]$$

More generally, for a stack of successive  $i$  layers (thickness  $l_i$ , thermal conductivity  $\lambda_i$ ):

$$\frac{1}{h_g} = \sum_i \frac{1}{h_i} \quad \text{with} \quad \frac{1}{h_i} = \frac{l_i}{\lambda_i} \quad [1.9]$$

The terms  $\frac{l}{\lambda}$  are referred to as thermal resistance. In fact, by analogy with electrical resistance, the overall thermal resistance of a system is the sum of the individual thermal resistances that constitute the system.



**Figure 1.3.** *Temperature profile in a succession of infinite plates joined together in a steady state*

#### **1.1.1.4. Heat transfer in a non-steady state**

In the study of heat transfer, the transient phase is often neglected, focusing mainly on unit processing equipment (pasteurizers, sterilizers, vacuum evaporators etc.) under steady state conditions. However, it is sometimes necessary to study transfer processes under transient state (dynamic) conditions. An illustrative example is the calculation of sterilization rates, which are based on temperature change at the core of a product over time (see [Chapter 4](#)).

To simplify our approach, we will keep the geometric model of the semi-infinite plate ([Figure 1.1](#)). The initial conditions are:

- the entire plate is at a uniform temperature  $\theta_0$ ;
- at 0 time, one side is rapidly brought to temperature  $\theta_1$ , which is then assumed to be constant over time, and

the other side is maintained at  $\theta_0$ .

To evaluate heat transfer, we simply apply the qualitative description in [section 1.1.1.1](#). The amount of heat that enters a section of thickness  $dx$  in a time interval  $dt$  through the constant-temperature surface at the  $x$  side is:

$$-\lambda A \frac{d\theta}{dx} dt$$

The amount of heat that leaves in the same time interval through the constant-temperature surface at the  $x + dx$  side is:

$$-\lambda A \left( \frac{d\theta}{dx} + \frac{d}{dx} \left( \frac{d\theta}{dx} \right) dx \right) dt$$

The amount of heat that accumulates in this section is therefore:

$$\lambda A \frac{d^2\theta}{dx^2} dx dt \quad [1.10]$$

In addition, this accumulation of heat in the section of thickness  $dx$ , volume  $dV = A \cdot dx$ , and mass  $dm = A \cdot dx \cdot \rho$ , results in a temperature variation  $d\theta$ , which satisfies the relationship:

$$dm C_p d\theta = A dx \rho C_p d\theta \quad [1.11]$$

where  $C_p$  ( $J \text{ kg}^{-1} \text{ K}^{-1}$ ) is the specific heat capacity of the product. Hence, by equating expressions [\[1.10\]](#) and [\[1.11\]](#):

$$\frac{d\theta}{dt} = \frac{\lambda}{\rho C_p} \frac{d^2\theta}{dx^2} = D_q \frac{d^2\theta}{dx^2} \quad [1.12]$$

with

$$D_q = \frac{\lambda}{\rho C_p} \quad [1.13]$$

$D_q$  is the thermal diffusivity ( $\text{m}^2 \text{s}^{-1}$ ).

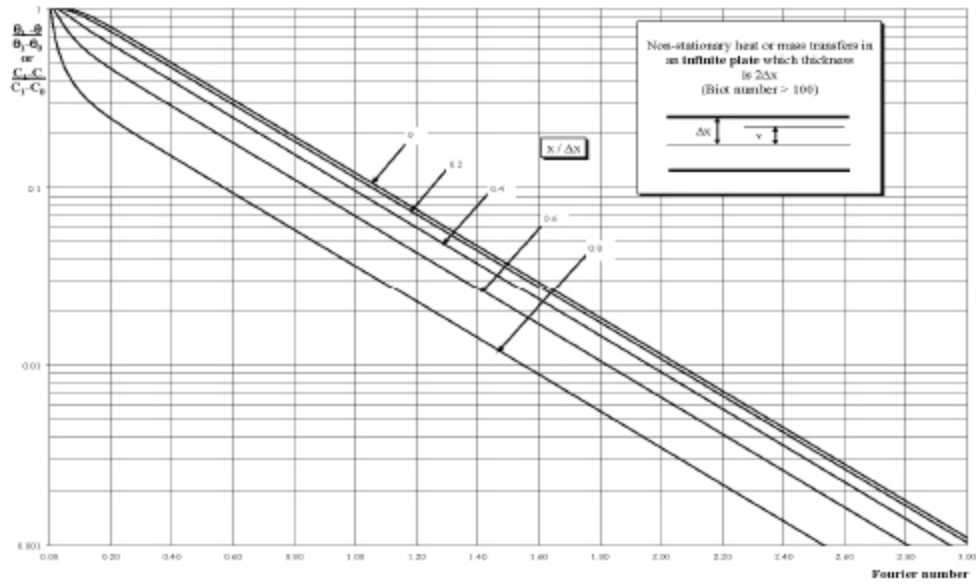
This equation provides analytical solutions for simple geometric shapes such as infinite plates, infinite cylinders or spheres. Resulting charts allow the calculation of heat transfers and temperature changes, and show dimensionless temperature  $\theta^*$  (so-called reduced temperature, representing the transient temperature distribution) as a function of the Fourier number  $Fo$  at a given position of the shape in question ([Figures 1.4](#), [1.5](#) and [1.6](#)).  $\theta^*$  and  $Fo$  are functions of temperature and time, expressed non-dimensionally:

$$\theta^* = \frac{\theta_1 - \theta}{\theta_1 - \theta_0} \quad [1.14]$$

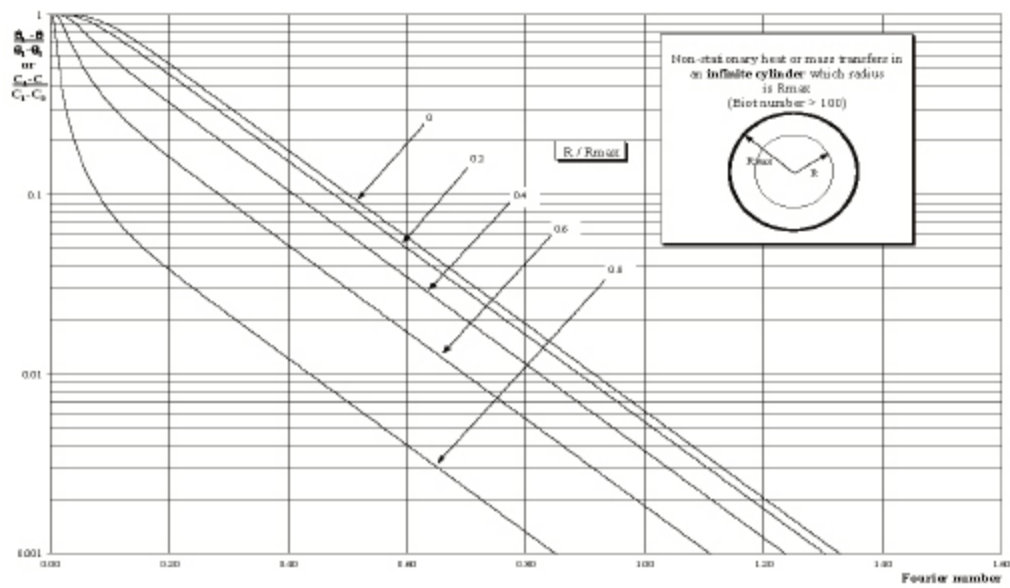
$$Fo = \frac{\lambda}{\rho C_p} \frac{t}{l^2} \quad [1.15]$$

where  $l$  is a characteristic length of a given geometric shape. It corresponds to the half-thickness of an infinite plate or to the radius of an infinite cylinder or sphere, commonly denoted as  $\Delta x$  and  $R_{\text{max}}$ , respectively. Note that  $\theta^*$  ranges between 1 ( $t_0$ ) and 0 ( $t_\infty$ ), regardless of the values of  $\theta_0$  and  $\theta_1$ .

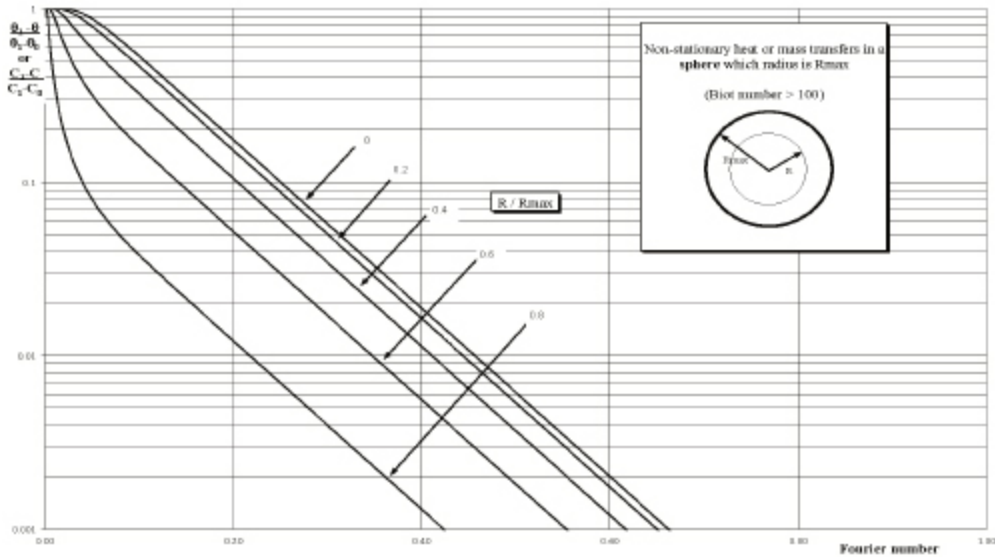
Using these graphs, it is possible to determine the change in temperature in finite solids formed by invoking these infinite shapes (Newman expression). For example, a point in a food may be visualized as the intersection of an infinite plate and an infinite cylinder. In this case, the dimensionless temperature at a point in the finite solid is obtained by multiplying the dimensionless temperatures at the same point in the infinite solids that comprise it.



**Figure 1.4.** Non-stationary heat or mass transfers in an infinite plate



**Figure 1.5.** Non-stationary heat or mass transfers in an infinite cylinder



**Figure 1.6.** *Non-stationary heat or mass transfers in a sphere*

Time and space discretization methods offer more general solutions to solving this equation. Bimbenet and Loncin [BIM 95] provide a very good overview of these methods.

### 1.1.2. Mass transfer: Fick's law

Fick's law is expressed by the vector relationship:

$$\vec{\phi}_m = -D_m \vec{\text{grad}}(C) \quad [1.16]$$

where  $\vec{\phi}_m$  is the mass flux density vector ( $\text{kg s}^{-1} \text{m}^{-2}$ ), i.e. the scalar value  $\frac{1}{dA} \frac{dm}{dt}$  ( $dA$ : elementary surface exchange;  $\text{m}^2$ ) multiplied by the normal unit vector at a constant-temperature surface:

$$\phi_m = \frac{1}{dA} \frac{dm}{dt} = -D_m \frac{dC}{dx} \quad [1.17]$$

The proportionality factor  $D_m$  between the mass flux density and the concentration gradient ( $\text{kg m}^{-3}$ ) is the

diffusion coefficient ( $\text{m}^2 \text{s}^{-1}$ ), characteristic of a given product.

There is a similarity between [equations \[1.16, 1.17\]](#) and [equations \[1.1, 1.2\]](#): Fourier's law describes heat exchange due to a temperature gradient, and Fick's law the change in mass due to a concentration gradient. In other words,  $Q$  can be replaced by  $m$ ,  $\lambda$  by  $D_m$  and  $\theta$  by  $C$  to switch from Fourier's law to Fick's law.

As a result, Fourier's law and Fick's law are interchangeable for transient and steady states for a given geometry. In the non-stationary state, the change in concentration, at a given point, as a function of time is:

$$\frac{dC}{dt} = D_m \frac{d^2C}{dx^2} \quad [1.18]$$

An identical approach to that proposed for heat transfers can provide solutions to this equation (see section 1.1.4): [Figures 1.4, 1.5](#) and [1.6](#) give reduced concentration ( $C^*$ ) at a given time ( $Fo$  value) and position for each of the geometric shapes.

In the case of the semi-infinite plate (thickness  $l$ ) and under steady state conditions, integration of [\[1.16\]](#) results in:

$$\dot{m} = A k \Delta C \quad [1.19]$$

with  $\Delta C > 0$  and:

$$k = \frac{D_m}{l} \quad [1.20]$$

$k$  being the mass transfer coefficient ( $\text{m s}^{-1}$ ).

### **1.1.3. Momentum transfer**

In this case, the concept of conduction is comparable to the transport of momentum by friction between parallel layers:




this momentum transfer occurs perpendicularly to the flow direction, excluding the random transfer of liquid elements in this direction. This type of flow, known as laminar flow, occurs for example in a cylindrical tube: each ring-shaped liquid layer rubs against the two adjacent layers, and the momentum transfer takes place from the centre of the pipe to the walls.

Laminar flow is therefore fundamentally different to turbulent flow, which is characterized by a chaotic motion of fluid elements.

### 1.1.3.1. *Newton's law*

The model that underlies this law consists of two parallel plates, one fixed and the other separated by a distance of  $dx$ , with an area  $A$  and moving at a constant velocity under the influence of a tangential force  $F$  ([Figure 1.7](#)). It induces the laminar flow of a fluid present between these plates.

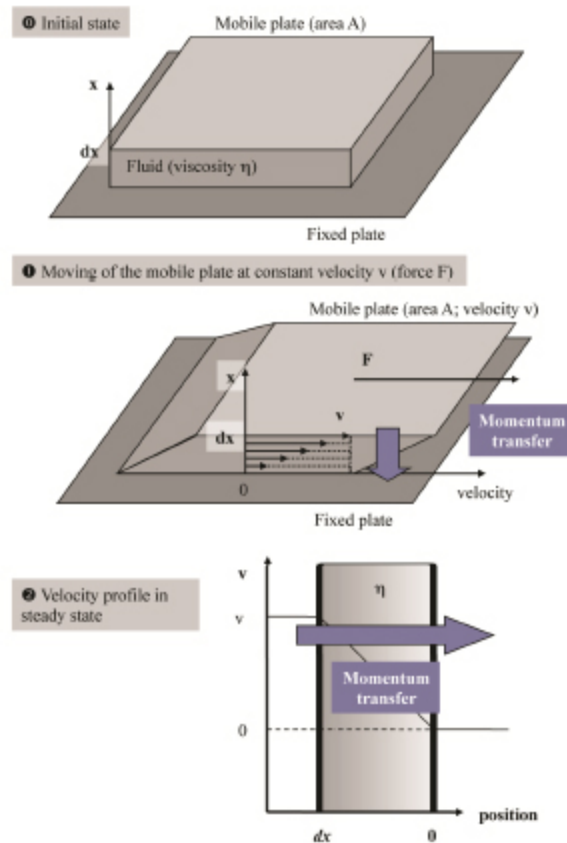
By analogy with previous discussion on heat and mass transfer, this model can also be represented in the steady state by a constant shear rate between the two plates ([Figure 1.7](#); )

Newton's law is expressed by the vector relationship:

$$\vec{\phi}_{mv} = -\eta \vec{\text{grad}}(v) \quad [1.21]$$

$\vec{\phi}_{mv}$  is the momentum flux density vector ( $\text{kg m}^{-1} \text{s}^{-2}$ ), i.e. the scalar value  $\frac{1}{dA} \frac{d(mv)}{dt}$  ( $dA$ : elementary exchange surface) multiplied by the normal unit vector at the isovelocity surface:

$$\phi_{mv} = \frac{1}{dA} \frac{d(mv)}{dt} = -\eta \frac{dv}{dx} \quad [1.22]$$



**Figure 1.7.** *Newton's model and velocity profile in an infinite plate in a steady state*

The proportionality factor  $\eta$  (Pa s) is defined as the dynamic viscosity.

Given that:

$$\frac{d(m v)}{dt} = m \frac{dv}{dt} = m a = F$$

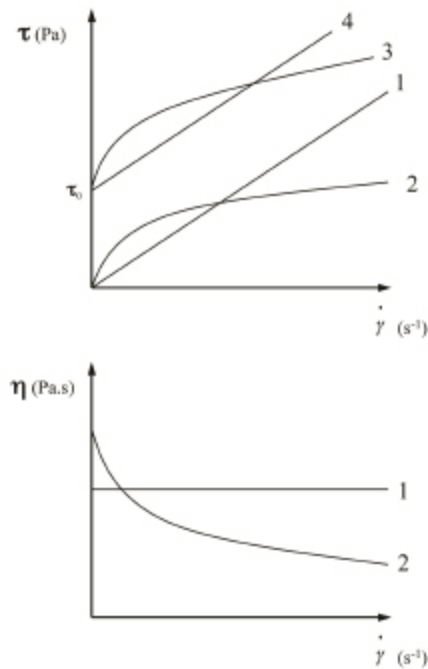
Using [equation \[1.22\]](#), it is therefore possible to explain the force involved in the flow of a liquid in laminar state. This force is proportional to the surface A and to the shear rate (or velocity gradient)  $\frac{dv}{dx}$  ( $s^{-1}$ ):

$$F = -A \eta \frac{dv}{dx} \quad [1.23]$$

The force per unit area  $\frac{F}{A}$  is the shear stress  $\tau$  (Pa) and the shear rate is typically denoted as  $\dot{\gamma}$  ( $s^{-1}$ ):

$$\tau = -\eta \dot{\gamma} \quad [1.24]$$

Newton's model assumes that  $\eta$  is constant regardless of shear rate, which characterises a Newtonian liquid ([Figure 1.8](#) - curve 1): in this case, the shear stress is proportional to the shear rate. However, in the food sector, many liquids display non-Newtonian behaviour. The study of such behaviour comes under rheology, a science that studies the deformation of a body based on the stress applied to it. Some typical fluid models are described in Figure 10, and explained in more detail in Volume 1 [JEA 16a].



**Figure 1.8.** *Rheological models*

If viscosity decreases with shear rate, the behaviour is known as shear thinning ([Figure 1.8](#) - curve 2). Materials in which deformation occurs above the yield stress  $\tau_0$  ([Figure 1.8](#) - curves 3 and 4) demonstrate viscoplastic

behaviour. The model described by curve 4, which displays Newtonian behaviour beyond  $\tau_0$  applies to a “Bingham plastic” material.

$$\tau - \tau_0 = -\eta \frac{dv}{dx} \quad [1.25]$$

### 1.1.3.2. Calculation of laminar flow: Poiseuille’s law

Knowing the relationships that describe the flow of liquids in different types of ducts is necessary to measure fluid transfer through pipe sections, pumps, etc. In addition, these relationships apply to control systems fundamental to the optimization of process flow. For example, consider the case of laminar flow in a cylindrical pipe. If we isolate a coaxial cylindrical liquid element moving through this pipe, the sum of forces on this element yields the following results:

$$\begin{cases} \tau_x = \frac{\Delta P}{L} \frac{x}{2} \\ \tau_0 = 0 \\ \tau_w = \frac{\Delta P}{L} \frac{R}{2} \end{cases} \quad [1.26]$$

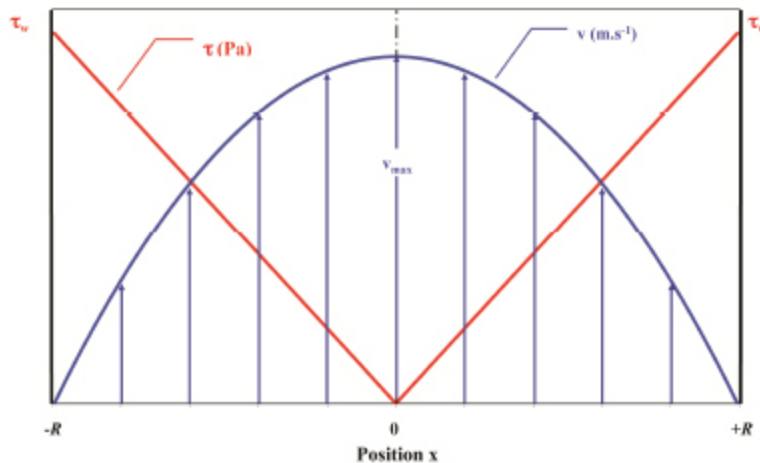
These relationships apply as long as flow is laminar in the gap between the pipe wall and the cylindrical liquid element, even if the flow inside it is turbulent. As before, by combining [equations \[1.24\]](#) and [\[1.26\]](#), we obtain:

$$dv = -\frac{\Delta P}{2 \eta L} x dx$$

This expression integrates from  $x$  to  $R$ , with maximum velocity at  $x = 0$  ( $V_{\max}$ ), equal to  $v_x$  at  $x$  and zero at the wall ( $V_R = 0$ ):

$$\begin{cases} v_x = \frac{\Delta P}{4 \eta L} (R^2 - x^2) \\ v_{\max} = \frac{\Delta P}{4 \eta L} R^2 \end{cases} \quad [1.27]$$

[Figure 1.9](#) gives  $v$  and  $\tau$  as a function of position  $x$ . According to [\[1.26\]](#),  $\tau$  is maximum at the wall ( $x = \pm R$ ) and zero at  $x = 0$ .



**Figure 1.9.** Velocity  $v$  and shear stress  $\tau$  profiles for laminar flow in a cylindrical pipe

Using the expression of local velocity, it is possible to determine the volumetric flow rate  $\dot{V}$  and the average velocity  $\bar{v}$  of the liquid flowing in the pipe:

$$\dot{V} = \frac{\Delta P}{8 \eta L} \pi R^4 \quad [1.28]$$

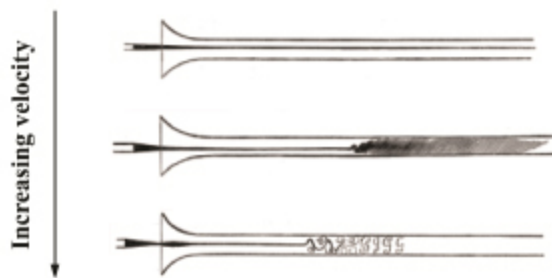
$$\bar{v} = \frac{\Delta P}{8 \eta L} R^2 = \frac{v_{\max}}{2} \quad [1.29]$$

## 1.2. Convective transfer processes

### 1.2.1. Introduction to geometric and physical similarity: Reynolds' experience

The work of Osborne Reynolds, the English physicist (1842–1912), focused mainly on the study and characterization of liquid flow in cylindrical pipes. In 1883, he visualized the flow behaviour of water in cylindrical glass tubes using a dye ([Figure 1.10](#)). He noted that the transition between laminar and turbulent flow under various experimental conditions (fluid velocity  $v$ , pipe diameter  $D$ , fluid viscosity and density respectively  $\eta$  and  $\rho$ )

systematically occurs at the same value for the ratio  $\frac{D \rho v}{\eta}$ , that is approximately equal to 2,000, now referred to as the Reynolds number (Re). It is the ratio of inertial forces to viscous forces (thus dimensionless), and helps characterise flow behaviour. This number forms the basis of studies on geometric and physical similarities between systems: the tubes studied by Reynolds were all cylindrical and assumed to be infinite (small diameter compared to the length), and flow behaviour was the same for identical ratios of inertial to viscous forces. This kind of ratio makes it possible to easily scale up the systems.



**Figure 1.10.** *Reynolds' experiment*

### **1.2.2. Importance of similarity**

It is easy to obtain a description of physical phenomena during heat transfer by conduction, since the basic laws are known. However, in the case of heat transfer by convection, these laws no longer apply, and with the current state of