FOOD SCIENCE AND TECHNOLOGY SERIES



Handbook of Food Science and Technology 2

Food Process Engineering and Packaging

Edited by

Romain Jeantet, Thomas Croguennec Pierre Schuck and Gérard Brulé





Handbook of Food Science and Technology 2

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Introduction

Food is a complex and heterogeneous system. It often consists of a protein and/or polysaccharide matrix surrounding, sustaining a typically aqueous continuous phase containing soluble hydrophilic compounds (carbohydrates, salts, vitamins, etc.) and some dispersed elements (cells, fat globules, gas bubbles, crystals, etc.) (Figure I.1). Such a system is thermodynamically, biologically and chemically very unstable.



Figure I.1. Matrix structure of food

Introduction written by Gérard BRULÉ.

The dispersed elements are subject to forces that cause phase separations either by sedimentation when the density of dispersed elements is greater than that of the dispersing phase, or by creaming when the opposite applies. Chemical potential or pressure gradients exerted on either side of the interfaces can induce the transfer of solutes and structural changes (coalescence, plasmolysis). This physicochemical instability can be increased by mechanical and thermal stresses on products during storage (refrigeration, freezing) and preparation (defrosting, reheating, etc.).

Protein and polysaccharide polymers that contribute significantly to the structuring of food are likely to reorganise during storage because of the influence of temperature on hydrophobic, ionic and hydrogen interactions. As described in Volume 1 [JEA 16a], recrystallization is sometimes accompanied by the release and migration of water with textural changes (starch retrogradation and bread staling).

Biological agents, enzymes and microorganisms find suitable conditions for their action and development in most foods both in terms of physicochemical conditions (pH, water activity a_w and temperature) as well as availability of substrates and growth factors. Lipolysis, proteolysis and oxidation reactions, metabolite production (acid, alcohol, gas), the development of pathogenic flora and the release of toxins are all elements that negatively affect the sensory, safety and nutritional quality of food, as has already been demonstrated in the past. The same applies for chemical reactions (Maillard reaction, lipid autoxidation).

I.1. How to ensure biological stability of foods?

Four strategies can be used to ensure the biological stability of agricultural commodities and food (Figure I.2):

- inactivation of biological agents either through energy input, which induces a denaturation of enzymes and cell constituents, or through physical, chemical or enzymatic treatments (latter being able to lyse or alter some transport properties of the cell wall);

- separation of biological agents based on density differences or their size;

- inhibition of enzymes, microorganisms and reactions by decreasing water availability (lowering a_w), which limits both the transfer of substrates

and growth factors as well as the transfer of metabolites or reaction products that accumulate in the reaction environment;

- Inhibition through the creation of limiting physicochemical conditions (pH, oxygen pressure, bacterial inhibitors, etc.).



Figure I.2. Strategies for the biological stability of food

All these strategies will be detailed in Part 2 of this book.

I.1.1. Thermal inactivation of biological agents

The destruction of microorganisms and the inactivation of enzymes by heat treatments that provide energy for the denaturation of macromolecules (proteins, DNA) were first applied by Appert and Pasteur. Food pasteurization and sterilization have been defined taking into account the thermal sensitivity of pathogenic flora:

- pasteurization destroys a large proportion of undesirable flora (*Listeria*, *Staphylococci*, *Salmonella sp.*, etc.) and extends shelf life in the cold chain;

- sterilization destroys all microorganisms and denatures almost all enzymes (depending on the heat treatment) to allow long-term storage at room temperature.

To reduce the risk of pathogen proliferation, treatments are based on the thermal resistance of *Clostridium botulinum*, one of the most heat stable and

pathogenic microorganisms. By aiming to destroy microorganisms and enzymes inactivate through thermal denaturation. macromolecules responsible for food texture and biological constituents (vitamins) are also partially or totally denatured. These side effects degrade the sensory and nutritional quality of food. Thus, successfully destroying microorganisms without reducing the food quality too much requires the precise control of heat transfers and momentum during heat treatments. Heat treatment is not always used due to the thermal sensitivity of some products (e.g. egg white) or in order to preserve the raw nature of the product. Other non-thermal treatments (i.e. inducing a temperature increase of less than 25°C) are therefore used, for example ionization, high pressure and pulsed electric fields. These techniques have limited applications mainly due to cost, legislation and consumer acceptability. Hurdle technologies (combination of treatments to minimise energy input in the food product) may be an alternative.

I.1.2. Inhibition by decreasing water activity

Most microbial and biochemical changes that reduce the quality of food occur in the aqueous phase. Water has a dual role:

 as a solvent, it ensures the transfer of substrates, growth factors, biological agents and reaction products thereby creating optimum conditions for reactions;

- as a reaction substrate, it is involved in hydrolysis reactions (proteolysis, lipolysis).

This dual action of water requires its availability, which is characterised by a_w : any treatment that reduces water availability slows down the reaction rate. a_w can be lowered by the removal of free water through evaporation or drying, the crystallization of solvent water (freezing) or the addition of highly hydrophilic solutes that bind water molecules through hydrogen or dipolar interactions (salting, sugaring). Inhibition through the decrease of a_w can be reversed by rehydration, defrosting and dilution. This method of preservation only slightly affects the nutritional and sensory quality if freezing and dehydration are well controlled in terms of heat and mass transfers. The shelf life may however be limited in the case of fat-containing products since reactions involving lipid substrates occur at the interface between the lipid and aqueous phases, and are consequently less dependent on water availability. Conversely, lipid oxidation is increased when a_w reaches 0, as exposure to oxygen is increased in these conditions.

I.1.3. Chemical inhibition

The kinetics of microbial growth and the rate constants of enzymatic reactions are highly dependent on the pH of the medium. It is possible to slow down biological phenomena by deviating from the optimum reaction pH. As this is around neutral pH for most microorganisms and enzymes, microbial growth can be limited by acidifying the medium either through the addition of acid (acetic acid, lactic acid, propionic acid, carbonic acid, etc.) or through fermentation. Aerobic microorganism growth can also be controlled by reducing the oxygen pressure through vacuum packaging or modified atmosphere packaging.

Many metabolites and reaction products are themselves inhibitors and, above a certain concentration, can slow down or terminate fermentation (alcohol, lactic acid or propionic acid) or reactions.

The bacteriostatic or bactericidal role of some chemical substances has been shown in ancestral processes: this is the case with nitrate/nitrite in meat curing and phenolic compounds resulting from the thermal degradation of lignin in the smoking process. These technologies generate undesirable substances due to their carcinogenic nature (nitrosamines, benzopyrenes). A good understanding of the biochemical reactions and processing conditions responsible for the formation of these substances can limit their levels in food.

I.2. How to ensure physicochemical stability of foods

The presence of dispersed elements in food (fat globules in the case of emulsions, air bubbles in the case of foams) or the presence of proteins and carbohydrates, which are thermodynamically incompatible, can destabilise the product. The chemical potential gradients at interfaces can induce water and solute transfers between two phases or between a base and a topping or filling (e.g. hydration of a shortbread or puff-pastry base from the topping/filling); all these transfers are able to reduce the sensory quality of food.

The transfer rate v of dispersed elements is governed by Stockes law:

$$v = \frac{D^2}{18\eta} \Delta \rho g$$
 [I.1]

with *D* the diameter of particles (m), $\Delta\rho$ the density difference between the dispersed phase and the continuous phase (kg m⁻³), η the dynamic viscosity of the continuous phase (Pa s) and g the gravitational acceleration (m s⁻²).

To stabilise the dispersed elements, the parameters D and η can be adjusted, or a macromolecular network can be created to limit the movement of particles by entrapping them within the network.

The diameter of dispersed elements like fat globules can be reduced by mechanical treatments such as homogenization: in order to obtain relatively small diameters, it is necessary to lower the interfacial tension that creates resistance to an increase in the interphase surface. This surface tension can be lowered using amphiphilic molecules positioned at the interface, the hydrophobic portion facing the lipid phase and the hydrophilic portion facing the aqueous phase. In addition to creating micro-emulsions, it is also essential that the fat globules do not agglomerate, which could result in coalescence and an increase of the fat globule diameter. To avoid this, ionic and steric constraints must be created on the surface of the globule, which by repulsion limit the coalescence of particles. Reducing D is not always enough to stabilise the system; it is also possible to increase viscosity by increasing the concentration of solutes (e.g. adding sugars) or by immobilising solvent water by adding carbohvdrate or protein macromolecules to induce thickening.

It is more difficult to stabilise a foam than an emulsion since the density difference between the dispersed and the dispersing phases is 1,000 kg m⁻³ in a foam whereas it is 100 kg m⁻³ in an emulsion. It is therefore necessary to create a macromolecular network that immobilises air bubbles. Globular proteins, especially from egg whites, have good foaming properties since they position themselves at the [air/water] interface during beating, which creates a dissymmetry in their environment. As a result, denaturation occurs through changes in their tertiary or even secondary structures, which may be accompanied by molecular interactions and help forming a cohesive film ensuring good foam stability.

The dynamics of water and solutes between the various components of food can be controlled by creating hydrophobic interfaces; this is the role of fat in puff or shortcrust pastry. It is also necessary that the physical properties of this fat are not adversely affected by the crystallization of triglycerides during the refrigeration of the products.

The physicochemical stability of a product must be controlled throughout its shelf life but also during its handling by consumers including defrosting and reheating. Controlling the stability of a complex system that is thermodynamically unstable is one of the major challenges facing food engineers; this issue will be addressed in Part 3 of this book. Nowadays, several functional carbohydrate and protein ingredients are available to limit transfers and destabilization within the food matrix. Part 4 of this book will be devoted to the physicochemical basis of fractionation and related technologies used in the development of functional ingredients.

I.3. Packaging, an essential attribute for food preservation

When responding to market needs and consumer demands with regards to safety, sensory and nutritional quality, offering a service and preserving, where possible, the traditional character of food is a constant challenge for the food industry. Quality control requires a good understanding of the food and its biochemical, chemical and thermodynamic reactivity as well as the processing and storage methods involved. Indeed, food can be transported over long distances, stored, distributed and finally endure a sufficiently long shelf life in the hands of the consumer. Hence, the need for packaging to protect the product is something that satisfies specific functions in terms of food preservation. Packaging could also be described as the last stage of the production chain. These elements will be addressed in Part 5 of this book.

Part 1

Basis of Food Engineering

Transport Phenomena – Basis of Unit Operations

The processing of agricultural commodities into finished food products can be analyzed in different ways. It can be examined by sector or by establishing a detailed description of the different unit operations in each one. Such an approach is not very useful in that the same unit operations exist in different sectors. For example, there is very little difference, with the exception of product characteristics or the type of material used, between pasteurization in a brewery and that in a milk factory, between concentration in a milk factory and that in a sugar refinery or between water extraction of sugar from sugar beet and solvent extraction of fat from oilcake.

Such overlaps can be limited by another approach, which looks at processing at the level of elementary operations (or unit operations) such as filtration, centrifugation, pasteurization or drying.

However, this approach can be further rationalized as such unit operations are fundamentally linked to three types of transport phenomena (or transfers) resulting from a physical difference (temperature, concentration or speed) between two points within a system: heat transfer (transfer of thermal energy from a hot to a cold point), mass transfer (transfer of mass from a concentrated to a diluted point) and momentum

Chapter written by Romain JEANTET.

transfer (transfer of momentum from a moving point at a given speed to a moving point at a lower speed).

For example:

- heat transfers play a major role in the elementary operations of pasteurization, sterilization and concentration;

- mass transfers mainly govern extraction operations by diffusion.

However, such transfer processes are often connected and interdependent, for example in a drying operation, heat transfer causes the mass transfer (of moisture) from a product to the environment.

It is therefore essential to first examine these before studying unit operations, which combined together form the basis for food production overall.

There are three different mechanisms of heat transfer:

- conduction (or diffusion): in this case, the matrix in which heat transfer takes place, can be considered motionless with regard to the direction of transfer. For example, heat transfer in a solid does not involve motion of the material itself. In a liquid, heat transfer may be conductive if the energy is transferred by molecular agitation, apart from the random motion of molecules in the direction of the heat transfer;

– convection: in this case, transfer processes involve movement of part of a matrix that carries a quantity of heat, mass or momentum from one point to another. For example, central heating is convective on two levels: heat produced by the boiler is transported to radiators by the forced circulation of water in the pipes, and from the radiators to the room by the convective movement of air;

– radiation: this type of transfer, which is specific to heat transfers, occurs by electromagnetic wave propagation and does not require material support. For example, browning of foods can be carried out by infrared radiation.

These three transport mechanisms can coexist. Radiation may be considered dominant in the case of very high temperatures. However, in most cases in the food industry, the temperatures used (between -20° C and

 $+150^{\circ}$ C) are relatively low and radiation can be disregarded. As such, conduction and convection processes will be the focus of attention in this chapter.

1.1. Transfer processes in conduction

In conduction, transfer rates are linked to the characteristics of the particular system and are directly proportional to the exchange surface area and the potential gradient.

As a result, there is a close similarity between the equations describing heat, mass and momentum transfers.

1.1.1. Heat transfers: Fourier's law

Fourier's law is expressed by the vector relationship:

$$\overline{\phi_{q}} = -\lambda \operatorname{grad}(\theta)$$
[1.1]

 ϕ_q is the heat flux density vector (W m⁻²), i.e. the scalar value $\frac{1}{dA}\frac{dQ}{dt}$ (dA: elementary surface exchange; m²) multiplied by the normal unit vector at a constant-temperature surface:

$$\phi_{q} = \frac{1}{dA} \frac{dQ}{dt} = -\lambda \frac{d\theta}{dx}$$
[1.2]

The proportionality factor λ between the heat flux density and the temperature gradient is the thermal conductivity (W m⁻¹ K⁻¹), characteristic of a given product.

1.1.1.1. Establishing steady state conditions at an infinite plate

Generally, in a system, physical quantities vary with time from one point to another. This system is in a non-steady state (or transient state). However, if the environment external to the system is invariant (constant), the system, after a certain time, tends towards a state of equilibrium, where at any given point the physical quantities (in this case temperature) become invariant (constant) over time. Such a system has reached a steady state (or a stationary state). The geometric model most often used to describe such a system is the semi-infinite plate (small thickness relative to length and width) which is assumed to be a homogeneous and isotropic solid, bounded by two parallel planes. The model includes the wall through which thermal energy is exchanged via a plate heat exchanger, which can be considered infinite as the edges have a negligible effect on heat transfer.

In this model, constant-temperature surfaces are parallel to the sides of the plate, and equation [1.2] is integrated for the entire heat exchange surface area A in:

$$\phi_{q} = \frac{1}{A} \frac{dQ}{dt} = -\lambda \frac{d\theta}{dx}$$
[1.3]

 $\frac{dQ}{dt}$, heat flux or power, can both be noted $\overset{\bullet}{Q}$.



Figure 1.1. Temperature profile in an infinite plate in a transient state. Temperature gradient is indicated by arrows at the x and x+dx sides

If, starting from a uniform temperature θ_0 (Figure 1.1), one side of this plate is rapidly brought to the temperature θ_1 , heat transfer takes place under the influence of the temperature gradient in a direction normal to the plate. After a certain time, the temperature profile is that shown as in Figure 1.1. From equation [1.3], the quantity of heat dQ that enters a section of

thickness dx in a time interval dt through the constant-temperature surface at the x side is evidently greater than that leaving through the constant-temperature surface at the x+dx side. Heat therefore accumulates in this section, which results in a temperature increase in the plate element. This continues until that which enters x is equal to that which leaves x+dx, that is until $\frac{d\theta}{dx}$ is the same for all constant-temperature surfaces or until the temperature profile in the plate is linear (Figure 1.2).



Figure 1.2. Temperature profile in an infinite plate in a steady state

1.1.1.2. Heat transfer in an infinite plate in a steady state

As we have just seen, heat transfer in a steady state infinite plate is characterized by a linear temperature gradient (Figure 1.2). Equation [1.3] is integrated on the plate thickness l in the following way:

$$\dot{\mathbf{Q}} = \mathbf{A} \ h \,\Delta\theta$$
 [1.4]

with $\Delta \theta = \theta_1 - \theta_0 > 0$ and:

$$h = \frac{\lambda}{l}$$
[1.5]

h, the heat transfer coefficient (W m⁻² K⁻¹), characterises the ability of the plate to allow heat to pass through since it incorporates both thermal conductivity λ and thickness *l*.

1.1.1.3. Heat transfer in a steady state composite plate

A wall is often composed of a several barriers to the heat transfer: the heat exchange plate itself, fouling deposits, boundary layers of fluid in contact with the surfaces, etc. The composite plate model is useful therefore, given its multiple applications.

In a steady state, the heat flux density reaches a constant value overall and in each of the sub-plates comprising the composite plate (Figure 1.3):

$$\phi_{q} = \frac{1}{A} \frac{dQ}{dt} = \begin{cases} \lambda_{1} \frac{\theta_{1} - \theta_{2}}{l_{1}} = h_{1}(\theta_{1} - \theta_{2}) \\ \lambda_{2} \frac{\theta_{2} - \theta_{3}}{l_{2}} = h_{2}(\theta_{2} - \theta_{3}) \\ \lambda_{3} \frac{\theta_{3} - \theta_{4}}{l_{3}} = h_{3}(\theta_{3} - \theta_{4}) \end{cases}$$
[1.6]

This is equivalent to:

$$\Leftrightarrow \phi_{q} = \frac{1}{\frac{1}{h_{1}} + \frac{1}{h_{2}} + \frac{1}{h_{3}}} \Delta \theta \qquad [1.7]$$

The heat flux density is therefore proportional to the temperature difference and the transfer coefficient relating to all sub-plates, denoted by h_g (W m⁻² K⁻¹), also known as the overall heat transfer coefficient, which is as follows:

$$\begin{cases} \frac{1}{h_g} = \frac{1}{h_1} + \frac{1}{h_2} + \frac{1}{h_3} \\ \phi_q = h_g \Delta \theta \end{cases}$$
[1.8]

More generally, for a stack of successive *i* layers (thickness l_i , thermal conductivity λ_i):

$$\frac{1}{h_g} = \sum_i \frac{1}{h_i} \text{ with } \frac{1}{h_i} = \frac{l_i}{\lambda_i}$$
[1.9]

The terms $\frac{1}{h} = \frac{l}{\lambda}$ are referred to as thermal resistance. In fact, by analogy with electrical resistance, the overall thermal resistance of a system is the sum of the individual thermal resistances that constitute the system.



Figure 1.3. Temperature profile in a succession of infinite plates joined together in a steady state

1.1.1.4. Heat transfer in a non-steady state

In the study of heat transfer, the transient phase is often neglected, focusing mainly on unit processing equipment (pasteurizers, sterilizers, vacuum evaporators etc.) under steady state conditions. However, it is sometimes necessary to study transfer processes under transient state (dynamic) conditions. An illustrative example is the calculation of sterilization rates, which are based on temperature change at the core of a product over time (see Chapter 4).

To simplify our approach, we will keep the geometric model of the semiinfinite plate (Figure 1.1). The initial conditions are:

- the entire plate is at a uniform temperature θ_0 ;

- at 0 time, one side is rapidly brought to temperature θ_1 , which is then assumed to be constant over time, and the other side is maintained at θ_0 .

To evaluate heat transfer, we simply apply the qualitative description in section 1.1.1.1. The amount of heat that enters a section of thickness dx in a time interval dt through the constant-temperature surface at the x side is:

$$-\lambda A \frac{\mathrm{d}\theta}{\mathrm{d}x} \mathrm{d}t$$

The amount of heat that leaves in the same time interval through the constant-temperature surface at the x + dx side is:

$$-\lambda A\left(\frac{\mathrm{d}\theta}{\mathrm{d}x} + \frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{\mathrm{d}\theta}{\mathrm{d}x}\right)\mathrm{d}x\right)\mathrm{d}t$$

The amount of heat that accumulates in this section is therefore:

$$\lambda A \frac{d^2 \theta}{dx^2} dx dt$$
 [1.10]

In addition, this accumulation of heat in the section of thickness dx, volume $dV = A \cdot dx$, and mass $dm = A \cdot dx \cdot \rho$, results in a temperature variation $d\theta$, which satisfies the relationship:

$$\operatorname{dm} C_{p} \operatorname{d} \theta = \operatorname{A} \operatorname{dx} \rho C_{p} \operatorname{d} \theta \qquad [1.11]$$

where C_p (J kg⁻¹ K⁻¹) is the specific heat capacity of the product. Hence, by equating expressions [1.10] and [1.11]:

$$\frac{\mathrm{d}\theta}{\mathrm{d}t} = \frac{\lambda}{\rho C_p} \frac{\mathrm{d}^2\theta}{\mathrm{d}x^2} = \mathrm{D}_{\mathrm{q}} \frac{\mathrm{d}^2\theta}{\mathrm{d}x^2}$$
[1.12]

with $D_q = \frac{\lambda}{\rho C_p}$ [1.13]

 D_q is the thermal diffusivity (m² s⁻¹).

This equation provides analytical solutions for simple geometric shapes such as infinite plates, infinite cylinders or spheres. Resulting charts allow the calculation of heat transfers and temperature changes, and show dimensionless temperature θ^* (so-called reduced temperature, representing the transient temperature distribution) as a function of the Fourier number Fo at a given position of the shape in question (Figures 1.4, 1.5 and 1.6). θ^* and Fo are functions of temperature and time, expressed non-dimensionally:

$$\theta^* = \frac{\theta_1 - \theta}{\theta_1 - \theta_0} \tag{1.14}$$

$$Fo = \frac{\lambda}{\rho C_p} \frac{t}{l^2}$$
[1.15]

where *l* is a characteristic length of a given geometric shape. It corresponds to the half-thickness of an infinite plate or to the radius of an infinite cylinder or sphere, commonly denoted as Δx and R_{max} , respectively. Note that θ^* ranges between 1 (t₀) and 0 (t ∞), regardless of the values of θ_0 and θ_1 .

Using these graphs, it is possible to determine the change in temperature in finite solids formed by invoking these infinite shapes (Newman expression). For example, a point in a food may be visualized as the intersection of an infinite plate and an infinite cylinder. In this case, the dimensionless temperature at a point in the finite solid is obtained by multiplying the dimensionless temperatures at the same point in the infinite solids that comprise it.



Figure 1.4. Non-stationary heat or mass transfers in an infinite plate



Figure 1.5. Non-stationary heat or mass transfers in an infinite cylinder



Figure 1.6. Non-stationary heat or mass transfers in a sphere

Time and space discretization methods offer more general solutions to solving this equation. Bimbenet and Loncin [BIM 95] provide a very good overview of these methods.