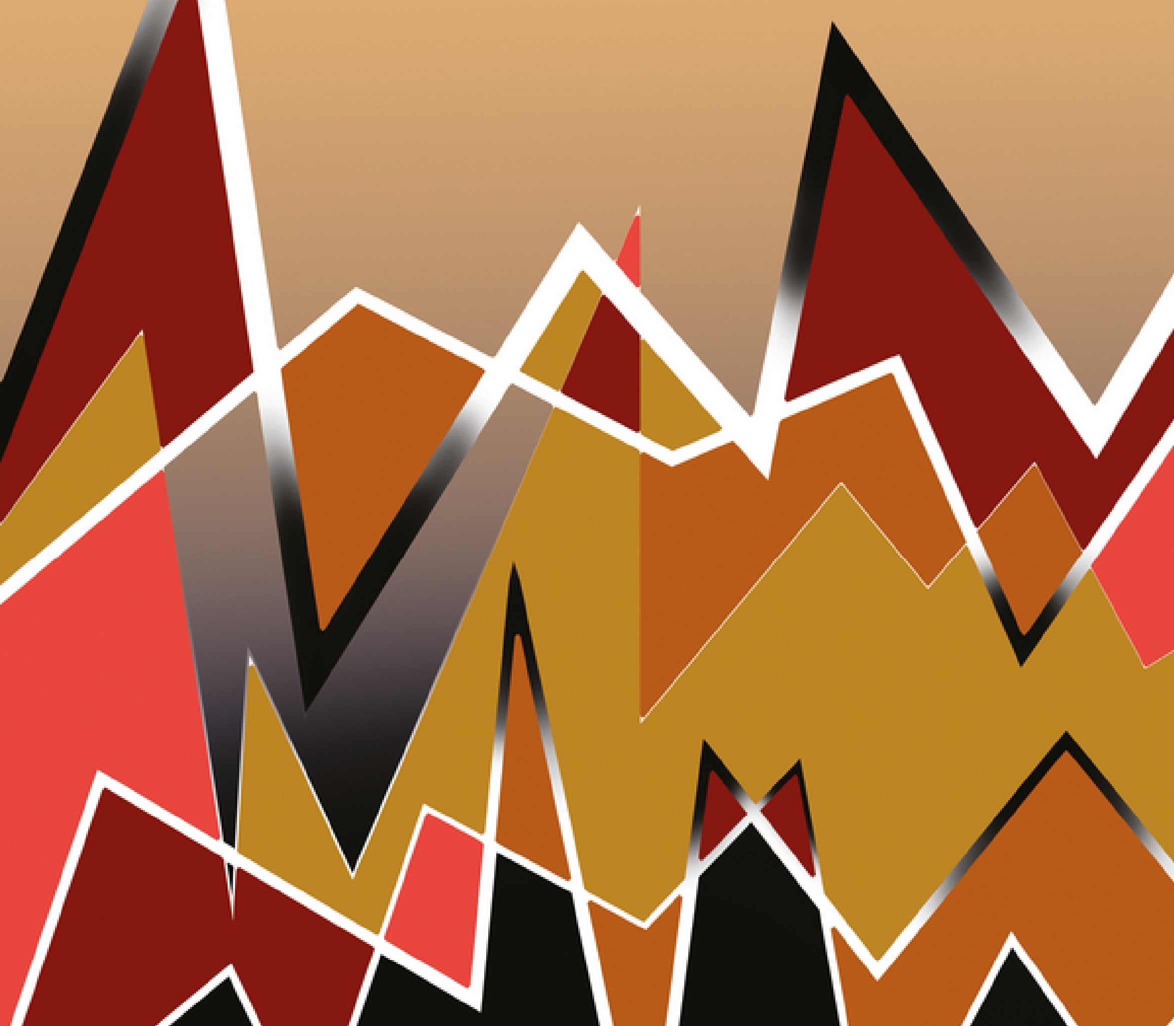
INSIGHT INTO FUZZY MODELING

Vilém Novák, Irina Perfilieva and Antonín Dvořák



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To our children David, Vitalik, Martin, Anna, and Jaroslav

CONTENTS IN BRIEF

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PREFACE

Fuzzy modeling is a special branch of mathematical modeling that has two goals: (i) to construct models based on information that can be given not only in numbers but also, imprecisely, usually in a form of expressions of natural language; (ii) to construct models with less computational demands, which are more robust, that is, little sensitive to changes in the input data.

In comparison with classical models, the fuzzy ones are closer to human way of thinking. For example, when processing images, classical methods work with single pixels. People, however, do not see pixels but larger and usually imprecisely delineated parts of the image. This is the main reason why it is so difficult to develop methods that are as powerful as the human eye. It happens quite often that some objective measure says that a given image is good but the human eye sees it differently and says “no”.

The fuzzy modeling methods are developed with the idea to capture the way how people grasp and manipulate available information. Therefore, fuzzy models make it possible to solve highly nontrivial problems. On one side, these problems come from areas where mathematics has not yet (or very little) contributed, for example, psychology, geography, or geology. The fuzzy modeling methods, however, are successful also in solving classical problems, such as control, time series forecasting, image processing, or classical mathematical problems such as approximation of functions, solution of differential equations, or signal processing. In all cases, these methods manifest the above-mentioned properties — less computational demands and robustness.

Our book is specific from several points of view. First of all, the reader will find in it a consistent and well-established notation and terminology. Furthermore, we made maximum efforts to explain the basic ideas of the presented methods and to avoid misleading terminology occurring in some older books (e.g., we avoid erroneous terms such as “Mamdani implication”).

One of the main contributions of our book is that it contains original results obtained both in the theory as well as in the applications. These results cover especially newly developed theories and also original and, in our opinion, more precise explanation of older known results (this concerns especially relational interpretation of the widely used fuzzy IF-THEN rules).

A special focus is placed on two original theories: the fuzzy natural logic, which, besides others, includes mathematical model of special parts of linguistic semantics, and the theory of fuzzy transform. None of these theories has been yet published in the book form. Our aim is to demonstrate the power of both theories and their increasing potential in applications. We therefore split the book into two parts.

The first part contains extensive presentation of the theory of fuzzy modeling. Let us remark that the development of this theory is in large extent motivated by necessity to solve concrete problems.

The second part presents selected applications in three important areas: control and decision-making, image processing, and time series analysis and forecasting. It is interesting that the latter application combines both above-mentioned theories.

Recall that the book is focused on fuzzy modeling — the essential part of soft computing. Therefore, methods based on fuzzy set theory are preferred. We deliberately omitted other well-known and important theories that are usually mentioned in connection with soft computing, namely, the theory of artificial neural nets is missing because it is deeply elaborated in numerous specialized books and papers that we suggest to read. We think that peripheral presentation of this theory in one chapter of the book focused on fuzzy modeling would be more confusing than beneficial. For the same reasons, we omitted optimization techniques such as evolutionary algorithms or particle swarm optimization that can also be found in numerous specialized books and papers.

We want to thank our colleagues for their help when preparing this book: Martin Dyba, Michal Holčapek, Petr Hurtík, Viktor Pavliska, Marek Vajgl, Radek Valášek and Pavel Vlašánek. Last but not least, we also thank the workers of John Wiley & Sons publishing house.

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ABOUT THE COMPANION WEBSITE

This book is accompanied by a companion website:

www.wiley.com/go/novak/fuzzy/modeling

The website includes:

- executable files with programs realizing methods described in the book
- manuals to these programs
- selected demonstration problems
- updates

PART I

FUNDAMENTALS OF FUZZY MODELING

This part consists of six chapters in which we first explain the role of indeterminacy in human life, which led to the development of special mathematical theories such as probability theory, fuzzy set theory, and fuzzy logic. In Chapter 2, we give an overview of the basic notions of the latter two theories. The main contribution of this part is contained in the three subsequent chapters in which we explain the theory of fuzzy IF-THEN rules, show that they have two possible interpretations enabling different kinds of applications, and explain the theory of fuzzy transform. A strong accent is put to the description of the model of semantics of a special class of linguistic expressions that are used on many places in this book. The theoretical part is finished by brief description of the main principles of fuzzy cluster analysis.

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WHAT IS FUZZY MODELING

1.1 INDETERMINACY IN HUMAN LIFE

Fuzzy modeling is a group of special mathematical methods that make it possible to include in the model imprecise or vaguely formulated expert information that is often characterized using natural language. The developed models (we call them fuzzy models) are very successful because they provide solution in situations when traditional mathematical models fail—either due to their non-adequacy, or due to their inability to utilize the full available information.

Note that the idea to include imprecise information in our models contradicts to what has always been required: as high precision as possible. There is, however, a good reason for doing it, namely, we face a discrepancy between relevance and precision. The so-called *principle of incompatibility* formulated by L. A. Zadeh in [149] says the following:

> As a complexity of system increases, our ability to make absolute, precise, and significant statements about the system's behavior diminishes. At some moment, there will be trade-off between precision and relevance. Increase in precision can be gained only through decrease in relevance; increase in relevance can be gained only through the decrease in precision.

For example, from the description of an enterprise in several sentences, we may learn about its main activity, size, total number of its employees, its business successes, and problems. But we will know nothing about individual people, specific machines, and their parts. To describe everything in detail, we would need much

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more sentences, numbers, tables, and so on. But then the amount of information exponentially increases. We would thus learn more, but any detail would concern only a small part of the enterprise. The requirement to describe the whole enterprise in full detail would lead to a big pile of thick books that, however, nobody would be able to read. And if yes, to understand the content, he/she would need natural language, which means that he/she would have to return to imprecise characterization. Otherwise, he/she would be lost in the abundance of irrelevant details.

We can see that to express relevant information, we need natural language. This is the only and very accomplished tool that enable us to work effectively with vague concepts.

Is full precision achievable? We argue that full precision is only our illusion and is not achievable, even in principle. Otherwise, we could obtain the same result independently on the chosen precision. But this is, in general, impossible. For example, let us compare two containers according to their volume. If their volume is absolutely the same, then we obtain the same number independently if we measure in m^3, mm^3, or in arbitrary fractions such as billionths, quadrillionths, 10^{-120} of m^3, and so on. But this is impossible because at the level of atoms or even elementary particles, we would not be able to distinguish which of the latter belongs to the body of the container and which does not. We conclude that the struggle for limit precision brings us to contradiction.

Let us emphasize that vagueness is inseparable feature of the semantics of natural language. We argue that it is not its weakness but its strength. Natural language is used in almost any human activity. For example, if we want to learn driving a car, we need a teacher who explains us—in natural language—what should we do, for example, "slow down a little", "now accelerate but not too much", and so on. Though such commands are vague, they are sufficient for us to be able to learn driving.

The main theories applied in fuzzy modeling are (mathematical) fuzzy logic and the fuzzy set theory. When facing vagueness, we may ask why we speak about fuzzy sets and fuzzy logic and do not consider techniques of probability and statistics?

The probability theory provides a mathematical model of uncertainty that is met when considering an event that has not yet occurred and we do not know whether it will indeed occur or not. Such an event can be, for example, a result of an experiment we are going to realize. Uncertainty is thus a lack of information about occurrence of some event.[1]

The basic concept in probability theory is a *probability distribution*. This gives us information about occurrence of events from more to less likely ones. Further important concept is *independence of events*. If they are independent, then the probability of their simultaneous occurrence is equal to *product* of their respective probabilities.

On the other hand, let us consider, for example, a cupboard full of red dresses. Then to answer whether the given dress is "red" requires to characterize *truth* of the statement "the color of the given dress is red". This cannot be probability because to be red color is a property, not an occurring event. Moreover, the class of all wave lengths representing red color cannot be a set because we are not able to specify precisely the borderline between "redness" and "non-redness".

[1]This holds also in the case when we know that some event has occurred but we do not know which one.

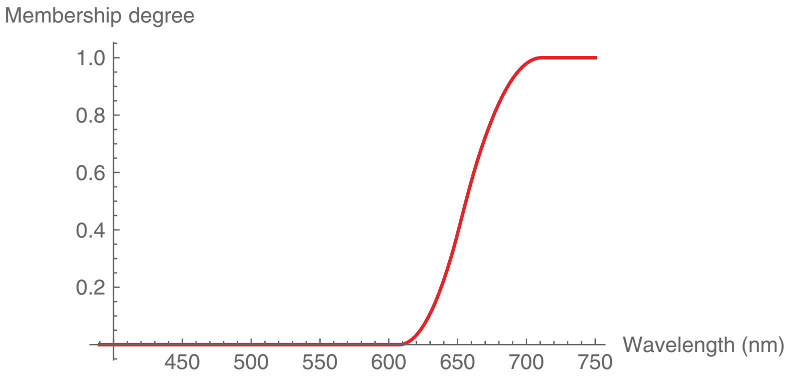

Figure 1.1 Fuzzy set modeling the meaning of "red color".

We can model the meaning of "red" using the concept of a fuzzy set. A fuzzy set A is a function

$$A : U \longrightarrow [0, 1],$$

where U is a set called *universe*. Each element $x \in U$ is assigned a membership degree $A(x) \in [0, 1]$ which is a *truth value*[2] of the proposition saying that $x \in A$. The value $A(x) = 1$ means that x *belongs* to A ($x \in A$). The value $A(x) = 0$ means that x *does not belong* to A ($x \notin A$). All other values mean only partial belonging to the fuzzy set A. To stress that A is a fuzzy set on U, we often write $A \underset{\sim}{\subset} U$.

If now we want to model what does it mean "red", we first define the universe of wave lengths that cover visible spectrum of light. People are able to see wave lengths from the interval $[390, 750]$ nm. Then "red" can be modeled by a fuzzy set $R \underset{\sim}{\subset} [390, 750]$ depicted in Figure 1.1. This means that light of wave length shorter than 600 nm is not red at all. Then the degree of "redness" increases with the increase in wavelengths up to full redness.

Of course, one may ask what is the probability of taking a red dress out of the cupboard. In this case, we face a combination of uncertainty and vagueness because the considered event is vaguely specified. We can thus summarize that there is a general concept called *indeterminacy*.[3] It has at least two distinguished facets: *vagueness* and *uncertainty*. Vagueness can be mathematically modeled using the fuzzy set theory, while uncertainty is mathematically modeled using the probability theory.[4] Of course, in reality, we often face both these facets together. For example, we can ask: "What is the probability that a tall man will come to our party?"[5]

Let us emphasize that indeterminacy cannot be removed. On one hand, it turns out that laws of nature inherently include uncertainty and it is not possible even in

[2]The interval $[0, 1]$ of truth values can be replaced by any other ordered set having the greatest and the smallest elements.

[3]One can also find this concept under the name "uncertainty in the wider sense".

[4]Note that there are also other theories of uncertainty, for example, possibility or belief theory.

[5]We can paraphrase this discussion as follows: the probability theory gives us an answer to the question "what event will occur?" and the fuzzy set theory gives us an answer to the question "what is the occurring event?"

principle to know all aspects causing occurrence of some event. On the other hand, vagueness is related to our way of regarding the world around us and its properties.

We argue that the presence of vagueness is the only way to familiarize with a new situation, or to communicate. Imagine, for example, that when parking a car, we would have at disposal instructions such as "turn the steering wheel by $19°25'32''$ to the left and move by 368.1256 mm back". Following such instructions would require great effort to make sufficiently precise measurements and to move accordingly. However, this would, in fact, be wasting of time because in practice, we do not need so precise parking position. It is sufficient to follow only vague instructions such as "turn the steering wheel a little to the left and move slightly back". Finally, note that we always face imprecision even when high precision is required, for example, when programming precise manipulating robots; the difference is only in the considered scale, that is, "small" could mean, for example, values around 1.3 mm or less.

1.2 FUZZY MODELING: WITH AND WITHOUT WORDS

The attempt to utilize the imprecise information in mathematical models led to the development of fuzzy modeling techniques. Recall that mathematical models manipulate with variables. In traditional models, values of the considered variable are taken from some set of numbers called a universe. Traditional mathematical models manipulate directly with its elements. In a fuzzy model, however, variables may represent fuzzy subsets of the universe. Hence, fuzzy models require partitioning of the universe into parts, for which it is specific that they need not be precisely formed and can overlap.

One of the very important modeling methods is *cluster analysis*. Its idea is the following: for a given set V of some elements, find its partition into c sets of subsets $R_j \subset V$, $j = 1, \ldots, c$, called *clusters*, in such a way that if two objects x, y belong to the same cluster R_j, then they are similar, while if they belong to different clusters, then they are not similar. For example, sizes of shoes represent subsets of lengths of human feet; the length of feet of people having, for example, size 6 is between 241 and 250 mm, for size 7 it is between 251 and 259 mm, and so on.

The classical cluster analysis provides partitioning into disjoint clusters, that is, we require that

$$U = R_1 \cup \cdots \cup R_c \quad \text{and} \quad R_i \cap R_j = \emptyset \quad \text{if } i \neq j.$$

This is often not realistic, because, as everybody knows, people often fit to more than one size of shoes. To cope with problems like this, we need generalization of the classical cluster analysis to the fuzzy one where the crisp clusters are replaced by fuzzy (possibly overlapping) ones. The fuzzy cluster analysis is described in Chapter 6.

The most important tool in fuzzy modeling are *fuzzy IF-THEN rules*. These are special expressions, which characterize relations among parts of two or more universes. For example, let us consider an electric boiler and two universes: values of electric current (A) and temperature (°C). Then the following is a typical fuzzy IF-THEN rule:

$$\mathscr{R} : \ \textsf{IF} \text{ electric current is very strong } \textsf{THEN} \text{ temperature is high} \qquad (1.1)$$