Trends in the History of Science

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From Classical to Modern Algebraic Geometry

Corrado Segre's Mastership and Legacy

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Preface

At the origin of the Italian School of Algebraic Geometry, the figure of Corrado Segre (Saluzzo 1863–Torino 1924), celebrated for the excellence of his geometric investigations and his exemplary style of scholar, still offers today an enduring model of scientific education to new generations and an outstanding scientific legacy to contemporary geometry.

Corrado Segre played the role of leader of the above-mentioned School in the decades around the beginning of the twentieth century, for scientific, historical, and biographical reasons. The great British geometer Henry Frederick Baker affirmed that "He could probably be said to be the father of the wonderful, Italian School which has achieved so much in the birational theory of algebraical loci."[1]

The times were favorable for several reasons. Corrado Segre, as well as his students and scientific companions, belonged to the first generations growing up in the new, unified Italy. These scholars and scientists could be described, in some sense, as builders of the nation. They were adding the moral and concrete task of building new scientific institutions for a new, modernized country, to their own scientific interest.

In a more general sense, Segre's School, though rooted within a specific discipline of the domain of science, was nevertheless an open and inclusive community of persons. Through scientific exchange and debate, they were interacting on a larger series of cultural issues: amongst themselves and both with society and the rest of the world. Certainly, in this period the ideas of science, education, and progress had many opportunities to meld; indeed, the model of the Italian School of Algebraic Geometry is a fine example of the consequences of this melding.

Federigo Enriques, a former disciple of Segre, was also certainly aware of this atmosphere and of the significance of Segre's leadership. In 1938, while speaking about mathematical schools and the progress and evolution in mathematics, he

[1]Baker, Henry Frederick, Corrado Segre, *Journal of the London Mathematical Society*, 1 (1926): 263–271, on p. 269.

probably referred back to his personal experiences and memories. His words are the most appropriate to understand the mood and soul of Segre's School:

> Actually the progress of mathematics doesn't depend exclusively on the efforts of individual research, but also on the relationship between researchers and the cultural environment from which they originate. In order to correctly understand what history teaches us, it is necessary to underline the importance of the school in forming the mathematician […]. The experiences and inspirations, together with unsuccessful attempts or glimpsed results and problems, as well as different types of research criteria formulated for practical purposes can only be communicated verbally in the intimacy of conversations between colleagues and friends or even better between master and pupil. The pupil continues the master's thoughts and ideas even after he has more or less knowingly reworked them into a new form. […] Schools have a tendency to grow beyond their original conception and at that point the student will be influenced by the new and different ideas which nurture him. The development of mathematical schools […] gains new life passing from one country to another, almost as if the spirit of the world could participate on a larger scale in this collective work.[2]

The School led by Segre flourished so much as to be directly associated with his name. In the same decades, Corrado Segre was a world-renowned master of geometrical sciences and author of fundamental achievements in the study of algebraic varieties.

On the occasion of the 150th anniversary of his birth, the Academy of Sciences, the University and the Polytechnic of Turin, in collaboration with several other scientific institutions organised the international conference *Homage to Corrado Segre (1863–1924)* and a series of initiatives to commemorate Corrado Segre and to reconstruct in a unified view the different aspects of Segre's scientific legacy.[3]

As a consequence, the conference brought together scholars in different fields, mainly from history of mathematics and algebraic geometry.

This volume recollects the refereed contributions of most of the participants in the conference and a few more invited papers, and naturally relies on two sections, reflecting the historical and the geometrical character of the international meeting.

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[2]Enriques, Federigo, *Le matematiche nella storia e nella cultura*, Bologna: Zanichelli 1938, pp. 180–181: *Invero i progressi delle matematiche non dipendono soltanto dallo sforzo della ricerca individuale, sì anche dai rapporti dei ricercatori fra loro e coll'ambiente di cultura da cui traggono origine. Per bene comprendere questo insegnamento della storia, conviene rilevare l'importanza che ha nella formazione del matematico la scuola […] Le esperienze e le suggestioni che si legano a tentativi non riusciti o a resultati e problemi appena intravisti, tanti criterii di ricerca che non sono formulati in maniera astratta, si comunicano soltanto a voce nell'intimità delle conversazioni fra colleghi ed amici o meglio fra maestro e scolaro. Lo scolaro riprende e continua il pensiero del maestro anche quando più o meno consapevolmente lo ricrea in una nuova forma […] La scuola tende ad allargarsi al di fuori del proprio ambiente di origine, ed allora l'influenza sullo scolaro viene a comporsi con altri motivi diversi che la fecondano […] Lo sviluppo delle scuole matematiche […] si ravviva passando da una nazione ad un'altra, quasi a far partecipare più largamente all'opera comune lo spirito del mondo.*

[3]The Conference was held in Turin from November 28 to 30, 2013. See: http://ricerca.mat.uniroma3.it/GVA/Segre150/segre150.html.

The title of the volume contains the words *from classical to modern algebraic geometry*. They put in evidence the extraordinary influence of Segre and his importance today. This is generally visible in all the contributions, offered by the main specialists on subjects related to the life and work of Corrado Segre.

Historians propose to reconstruct how Segre's leadership became recognised in Italy and abroad taking also in account a great number of unpublished and unknown documents.

The first essay by Alberto Conte and Livia Giacardi offers a picture frame for the following papers. The authors, running through the 36 years (1888–1924) of teaching higher geometry in Turin, show how Segre's courses were a veritable forge for future researchers. The forty handwritten notebooks of his university lectures and other unpublished sources allow them to understand how he stimulated and closely interacted with his Italian and foreign students, and to identify the most salient features of his scientific leadership. Erika Luciano and Silvia Roero in their paper, relying on a very rich documentation, illustrate the complex dynamic of scientific exchanges with the international mathematical community, as well as some aspects of the scientific and personal biography of Segre, related to his institutional, political, and editorial role. The remaining essays are dedicated to a thorough analysis of less studied aspects of Segre's work relating to three different stages of his life. David Rowe shows how line geometry was an excellent starting point for both Segre and Italian algebraic geometry, concentrating his attention on two of Segre's papers dating back to the beginning of the eighties. Paola Gario focuses on the relationship between Segre, Guido Castelnuovo, and Federigo Enriques, referring to the period (1887–1897) of their collaboration on the problem of the resolution of singularities of algebraic surfaces, without overlooking the interpersonal dynamics emerging from their rich correspondence. Finally, Aldo Brigaglia shows the genesis and the historical and scientific relevance of Segre's important works concerning the complex projective geometry and comments on how a genuine recognition of it arrived only later, with Julian Coolidge's work, and above all that by Elie Cartan.

The section is completed by the biographical timeline of Segre livened up by quotations and enriched by the portraits of the mathematician at different ages.

The section dedicated to contributions from the field of algebraic geometry confirms the continuity and the presence of the research themes considered by Corrado Segre. Classical algebraic geometry is sometimes used today as a name for a large active area of research within algebraic geometry. This area appears to be connected in a more direct way to the language, themes, and problems (often concerning concrete examples or special projective varieties), which were familiar to algebraic geometers of Segre's times. The above-mentioned contributions largely fit in this area. The picture emerging from them enlightens a very interesting series of nice geometric problems and new results. Several subjects of classical flavor are touched by this picture, with the use of modern techniques and new methods. All the contributions have a correspondence to Segre's work. We can partially summarize them as follows.

Hyperquadrics are considered by Laura Costa, Maria Rosa Mirò-Roig, and Joan Pons-Llopis in order to generalize and study, on an odd dimensional hyperquadric, instanton bundles, and their families. A special attention is payed to three-dimensional quadrics and to Hooft bundles on them. The paper by Luca Chiantini and Duccio Sacchi aims to introduce a notion of Hilbert function for subvarieties of Segre products, that is, products of projective spaces. This notion is more sophisticated than the natural one, defined via Segre embedding, and appears to be a starting point for a new theory and advances in the study of the complexity of a general tensor, with applications to several fields. Nodal cubic threefolds with isolated singularities were classified by Segre. In particular, the cubic threefold with maximal number of nodes bears his name as the Segre primal. Igor Dolgachev takes up the study of six nodal cubic threefolds, which is a case of special beauty and interest. The split surface of lines of a six nodal cubic threefold is described in all details. The Segre primal and its ubiquity in geometry are also revisited. The classical, and modern, subject of algebraic surfaces of general type and their moduli is well represented by the paper of Margarida Mendes Lopes and Rita Pardini. In it, some famous surfaces appear, namely Enriques surfaces and Godeaux surfaces. In particular, the authors construct the family of those Enriques surfaces which are quotient of a Godeaux surface by an involution, proving several properties related to this construction. As mentioned, algebraic curves and their linear series are very well present in this volume, due to the lecture notes of Corrado Segre reproduced here. Moreover, a paper by Edoardo Sernesi offers to the reader a self-contained and very interesting reconstruction of the proof of Riemann–Roch theorem for curves, which is essentially due to Castelnuovo and reflects ideas and observations of both Segre and Castelnuovo. Line geometry is a further very important theme where Corrado Segre played a leading role. This theme is taken up by Emilia Mezzetti in the paper "Geometry of lines and degeneracy loci of morphisms of vector bundles." The title reflects the modern point of view and the modern use of vector bundles techniques in view of several applications. Nevertheless, this paper is also an original survey where the deep connections between new methods and Segre's ideas are pointed out. Finally, the Cremona group of birational automorphisms of a projective space is obviously present in this volume. On this subject, new and important progress was made very recently. Moreover, very interesting connections to other fields, for instance, in the different fields of complex dynamics and of algebraic statistics, were deepened. Two papers on the Cremona group complete the series of geometric contributions to this volume. They reflect very well some of the recent changes of the "state of the art" on this subject. One is the paper by Jeremy Blanc on the set of the algebraic elements of the Cremona group of any projective space, which is proved to be a non-closed countable union of closed sets. The other paper, due to Ciro Ciliberto, Maria Angelica Cueto, Massimiliano Mella, Kristian Ranestad, and Piotr Zwiernik, introduces an effective method, with applications, to linearize suitable rational varieties by a sequence of Cremona transformations.

Notably, papers from historical or from geometric sections often converge on the same geometric theme or question, offering different but complementary points of

view. The reader can profit of both. This appears to be an interesting feature of this book and an achievement of the goals of the conference *Homage to Corrado Segre*.

Furthermore, a third part of the volume is enriched by the anastatic print of the unpublished Segre's manuscript *Introduzione alla geometria sugli enti algebrici semplicemente infiniti*. This document, with an introduction by Alberto Conte, is one of the forty manuscripts which recollect the notes of Segre's courses during the academic years. It corresponds to the year 1890–1891. Interestingly, it appears as a preliminary version of the famous memoir on algebraic curves published by Segre in 1894.[4]

This volume is completed by the list of documents of the Segre Archives, due to Livia, Giacardi, Erika Luciano, Chiara Pizzarelli and C. Silvia Roero. The bibliography of the works by Corrado Segre, including all the reports on the papers of his disciples and collaborators, and the list of the handwritten notebooks, closes the volume.

Turin, Italy — Livia Giacardi
Rome, Italy — Alessandro Verra

[4]Segre, Corrado, Introduzione alla geometria sopra un ente algebrico semplicemente infinito, *Annali di Matematica pura ed applicata*, 2, 22 (1894a): 41–142.

Acknowledgements

At the end of this work, which has involved the combined collaboration of mathematicians and historians with the intention of giving us a fuller profile of Segre's work, we wish to thank all those who have provided their collaboration. First of all, heartfelt thanks go to the authors and referees who made it possible to carry out this work, and to Birkhäuser for having included the volume in the series *Trends in the History of Science.* Our most wholehearted thanks also go to the President of the Academy of Science of Turin, Alberto Conte, to the Heads of the Department of Mathematics of the University of Torino, Catterina Dagnino and then Alessandro Andretta for according permission to reproduce the portraits of Segre and the handwritten Segre's notebook, and to all those who in various ways have provided their help, Ciro Ciliberto, Sloan Despeaux, Judith Goodstein, Steven and Beverly Kleiman, Antonio Salmeri, Rosanna Roccia, Norbert Schappacher, and KimWilliams.

Special thanks go to the heirs of Corrado Segre, Daniele, Lorenzo and Silvano Fuà for allowing us to access to the unpublished documents in their possession and for donating them to the University of Turin. Our thanks also go to the directors and personnel of the various archives we explored, Paola Novaria and Giuliana Maria Borghino Sinleber (Archivio Storico, University of Turin), Manuel Onjugaren (Archives, Losanna), Elena Borgi and Lavinia Iazzetti (Accademia delle Scienze, Turin), Laura Bitossi (Biblioteca di Scienze, Firenze), Anna Dagnese, Laura Garbolino, Orietta Piccini, Giulia Scarcia, Giuseppe Semeraro, and Antonella Taragna (Biblioteca Speciale di Matematica "Giuseppe Peano", University of Turin), Angharad Gwilym, Kate Hawke, and Edd Mustill (Special Collections and Archives, University of Liverpool), Barbara Gilbert (The University of Chicago Library) and Tom McCutchon (Columbia University Rare Book and Manuscript Library).

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Contents

Abbreviations

ANL-Castelnuovo	*Archivio Castelnuovo*, Accademia Nazionale dei Lincei, Roma. It can be accessed at the Web site [Gario 2010]: http://operedigitali.lincei.it/Castelnuovo/Lettere_E_Quaderni/menu.htm
ANL-Levi-Civita	*Archivio Levi Civita*, Accademia Nazionale dei Lincei, Roma
ANL-Volterra	*Archivio Volterra*, Accademia Nazionale dei Lincei, Roma
ASUT	Archivio storico dell'Università di Torino
AUL-Young	*Papers of Grace and William Young*, Archives, University of Liverpool
BMFI- Montesano	Fondo Domenico Montesano, Biblioteca del Dipartimento di Matematica, Università di Firenze
BMP	Biblioteca Speciale di Matematica "Giuseppe Peano", Università di Torino
BMP- Fano	*Fondo Fano*, Biblioteca Speciale di Matematica "Giuseppe Peano", Torino
BMP-Segre	*Fondo Segre,* Biblioteca Speciale di Matematica "Giuseppe Peano", Torino It can be accessed at the Web site [Giacardi 2013]: http://www.corradosegre.unito.it/
BMP-Terracini	*Fondo Terracini,* Biblioteca Speciale di Matematica "Giuseppe Peano", Torino
DES papers	David Eugene Smith Professional Correspondence, Columbia University Libraries, New York
EJWP	Ernest Julius Wilczynski Papers, The University of Chicago Library, Chicago
EPFL	École Polytechnique Fédérale de Lausanne
MCT-Mary Cytron Treves	*Fascicolo di Mary Cytron Treves,* H123, *Notizie relative a singoli internati o elenchi di internati*, in Archivio Centrale dello Stato, Ministero dell'Interno, Divisione Generale di Pubblica Sicurezza, Affari Generali e Riservati, A4bis (Stranieri internati), b. 85

Segre *Opere*	Corrado Segre, *Opere*, Roma: Ed. Cremonese, 4 vols., 1957–1963
UMI-Archivio	Archivio Storico della Unione Matematica Italiana, Bologna
UTo-ACS	*Archivi Corrado Segre*, Università di Torino. Most of the documents can be accessed at the Web site: http://users.mat.unimi.it/users/gario/Elenco-Segre.html
//	End of page of the manuscript
[…]	Omitted text
[ ]	Addition of editor
f.	*Folium*
fols.	*Folios*
n. d.	No date
n. e.	No publishing house
n. p.	No place
n. n. p.	Not numbered page
op. cit.	Cited reference
r.	*Recto*
rev.	Reviewer
Tr.	Translation
transl.	Translator
v.	*Verso*

Part I
Corrado Segre' Leadership: From Turin to the International Scene

Segre's University Courses and the Blossoming of the Italian School of Algebraic Geometry

Alberto Conte and Livia Giacardi

"Anyone wishing to evaluate the work of Segre properly cannot disregard the contributions made by his School, whose value is in large part due to him" (*Chi voglia rettamente valutare l'opera del Segre non può far astrazione dai contributi portati dalla sua scuola, ché del merito di questi spetta a lui una parte notevole.*

(Castelnuovo 1924b, 358).

Abstract

One of the greatest architects of the "geometric Risorgimento" (Coolidge 1927, 352) in Italy, Corrado Segre provides a shining example of the role of mentor in the history of mathematics. His university courses were a veritable forge for future researchers. The years between 1891 and the beginning of the 20th century witnessed the launch in Turin, under his guidance, of the Italian School of algebraic geometry, which in a short time would assume an internationally recognised role. Undoubtedly decisive in the formation of the School was the fact that Segre had fostered the onset and consolidation of the following lines of research: hyperspatial projective geometry; research in the foundations of the hyperspatial projective geometry; birational algebraic geometry; enumerative geometry; projective differential geometry; and projective geometry in the complex domain. However, also significant was the role of his university teaching, a valuable record of which is conserved in the forty handwritten notebooks of his university lectures. These not only make it possible to reconstruct the genesis and developments of his scientific research, but also allow us to understand how his own research stimulated and closely interacted

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with that of his students, thus showing the importance of his teaching in the flourishing of the Italian School. Here, after a brief overview of Segre's scientific contributions, we intend to show the close interaction between teaching and research, in order to bring out the influence of his university courses on his students and to pinpoint the moment when his role as leader began to be acknowledged; and make evident the enthusiasm, the nature of the collective work of his group of researchers, at least up to the early years of the twentieth century. We also intend to show how his vision of mathematics education—closely linked to his idea of the objectives of mathematics—was transmitted to the future teachers who attended his courses at the *Scuola di Magistero* (Teachers College) of the University of Turin. Finally we will try to identify the most salient features of his scientific leadership.

1 The Roots of the Italian School of Algebraic Geometry in Segre's Works

Segre's scientific work, as is well known, began when the innovating impulse given to geometric studies by Luigi Cremona (1830–1903) had begun to wane, and is linked to the important research in hyperspatial geometry carried out by Enrico D'Ovidio (1843–1933) and above all Giuseppe Veronese (1854–1917).[1] Research in geometry, as Federigo Enriques wrote, "had appeared to be completely absorbed in the contemplation of objects that it created itself, without any apparent connection with the *great problems*".[2] It was Segre who changed the course of things. A "fertile, ingenious precocity, joined to an amazing maturity of intellect and vastness of culture"[3] were the traits that distinguished the young mathematician. He received his degree before he was 20 years old, in August 1883, with a dissertation assigned to him by Enrico D'Ovidio, entitled "Studio sulle quadriche in uno spazio lineare ad *n* dimensioni ed applicazioni alla geometria della retta e specialmente delle sue serie quadratiche", which was published that same year in two memoirs of the Accademia delle Scienze di Torino.[4] As Castelnuovo wrote:

[1]See for example Scorza (1932), Loria (1924), Terracini (1926), Castelnuovo (1929), Boffi (1986), (Severi 1957), Terracini (1958), B. Segre (1961), Togliatti (1963), Menghini (1986), Brigaglia and Ciliberto (1995), Conte and Ciliberto (2004). More information on Segre's scientific biography can be found in Giacardi (2001a) and Brigaglia (2013).

[2]*Geometría parecía completamente absorta en la contemplación de los objetos formados por ella misma, sin connexión visible con los* grandes problemas (Enriques 1920, 5).

[3][…] *feconda, geniale precocità, accomunata ad una sbalorditiva maturità d'ingegno e vastità di cultura* (B.Segre 1963–64, 8). See also, for example, Pascal (1924, 461), Fano (1930, 43).

[4]In citing Segre's writings (articles and reports), reference will always be made to the *Bibliography of the works by Corrado Segre*, in this present volume. The thesis was published in two notes: (Segre 1883b, 1883c). The manuscript of the thesis is conserved in the BMP-Segre Scritti. 1. On Segre's early works, see the essay by David Rowe in this present volume. See also Ghione and Ottaviani (1992). For details on Segre's academic career see Conte et al. (2013).

> Those who still read today […] the two works, intimately connected, remain surprised by the confidence and vastness of views and means with which that young man, Corrado Segre, treats the broad subject. The dissertation seems due, not to a beginner, but to an experienced mathematician.[5]

In his thesis Segre proposed to study hyperspatial projective geometry, basing it solidly on linear algebra, in contrast to the common approach in Italy, in the conviction that "it opens to mathematicians a limitless field of research full of interest".[6] What was new was the idea of developing it systematically as a geometric science and not as a disguised form of analysis; also new was the application to it of the methods of projection that had shown themselves to be fruitful in ordinary projective geometry.

His geometric approach to the research emerges clearly in these first papers. Segre illustrated it to Leopold Kronecker, who appeared not to have understood it correctly. He confidently explained what he meant by the 'geometric interpretation' of some of Kronecker's and Weierstrass' results:[7]

> I have also seen from your letter that I did not explain myself clearly regarding the geometrical interpretation of your research and that of Mr. Weierstrass on the theory of bilinear and quadratic forms. Perhaps should I not say "geometric interpretation" as these words suggest (and you were thinking, it appears to me) a work that consists only in changes of words. Now I would consider ridiculous a scholar who occupied himself with merely changing analytical terms into geometric terms in analytical results already known. But that's not what I meant to say in my last letter. To explain more clearly, consider the theorems on the conditions under which two quadratic forms can be transformed into two other quadratic forms.
>
> In putting it into geometric terms, one can say that these theorems provide the conditions by which two pairs of second-degree surfaces (in an *n*-dimensional space) are identical from the standpoint of projective geometry. But these conditions remain analytical, because they involve elementary divisors, etc.; what is the geometric meaning of elementary divisors? If to a pair of second-degree surfaces there corresponds a double, triple, etc. root of the determinant of their bundle, these two surfaces touch each other at one or more points, but what difference will there be between these contacts according to different degrees of elementary divisors, that is to say, what *singularities* will the intersection of these two surfaces have for a given system of elementary divisors? That is one of the questions that I set out to resolve and which at first presented me with some difficulties. Thus I was able to establish a geometric *classification* of the intersections of two 2nd-degree surfaces. Similarly, analytical results on bilinear forms gave me, through a geometric study, the classification of *homographies* or *collineations* in any linear space. […] But I tell you again, and

[5] *Chi legge anche oggi* […] *i due lavori, strettamente collegati resta sorpreso della sicurezza e vastità di vedute e di mezzi con cui quel giovane, Corrado Segre, tratta l'ampio soggetto. La dissertazione sembra dovuta non già ad un principiante, ma ad un matematico provetto* (Castelnuovo 1924b, 353).

[6] [...] *essa apre ai cultori della matematica un campo sconfinato di ricerche piene di interesse* (Segre 1883b, *Opere* 3, 27).

[7] Segre's aim was to give a geometric interpretation of the elementary divisors and it was not new. This was the subject of Klein's thesis in 1868, but Segre introduced the new notion of "characteristic", which would be used hereinafter (until now); see Brechenmacher (2006, . 475–477). See also Segre (1883–84c).

> I hope to convince you fully when my work is printed, that it is not just a change of terms that produces these geometric results but rather they are the result of more difficult reasonings. (C. Segre to L. Kronecker, Turin 25 December 1883)[8]

Another aspect that deserves to be underlined is the abstract formulation that Segre gives to his treatment, which emerges from the very beginning, when he emphasises how the geometry of a space of arbitrary dimension had by that time assumed an important place in mathematics, even when a point of such a space is considered as an entity whose nature remains undetermined. For the pure mathematician, he noted, the lack of a meaningful representation of the objects under study is irrelevant (Segre 1883b, *Opere* 3, 26). After Segre, the concept of abstract geometry became 'an ordinary tool for working' for the Italian School of algebraic geometry. As Federigo Enriques would write many years later:

> In fact nothing is more fruitful than the multiplication of our intuitive powers deriving from this principle: it is almost as though beside the mortal eyes, which we are given to examine a figure in a certain respect, we had got a thousand spiritual eyes to contemplate many different transformations [of this figure]; while the unity of the object shines in our mind so enriched, that it allows us to pass easily from one form to another.[9]

[8] *J'ai vu aussi par votre lettre que je ne m'étais pas expliqué clairement à propos de l'interprétation géométrique de vos recherches et de celles de M. Weierstrass sur la théorie des formes bilinéaires et quadratiques. Peut-être ne devrais-je pas dire « interprétation géométrique » car ces mots font penser (et vous ont fait penser, à ce que je vois) à un travail qui consiste seulement dans de changements de mots. Or je considérerais comme ridicule un savant qui ne s'occupât que de changer les mots analytiques en mots géométriques dans de résultats analytiques déjà connus. Mais ce n'est pas là ce que j'entendais dire dans ma dernière lettre. Pour m'expliquer avec plus de clarté, prenons les théorèmes sur les conditions afin que deux formes quadratiques puissent se transformer dans deux autres formes quadratiques. En mettant des mots géométriques, on peut dire que ces théorèmes donnent les conditions pour que deux couples de surfaces du* 2^e *degré (dans un espace à n dimensions) soient identiques du point de vue de la géométrie projective. Mais ces conditions restent analytiques, car il y entre des diviseurs élémentaires, etc.; quelle est donc la signification géométrique des diviseurs élémentaires? Si à un couple de surfaces du* 2^e degré correspond une racine double, triple, etc. *du déterminant de leur faisceau, ces deux surfaces se toucheront mutuellement en un ou plusieurs points, mais quelle différence y aura-t-il entre ces contacts suivant les divers degrés des diviseurs élémentaires, c'est-à-dire quelles* singularités *aura l'intersection de ces deux surfaces pour un système donné de diviseurs élémentaires? Voilà l'une des questions que j'ai taché de résoudre et qui n'a pas laissé de me présenter au premier abord des difficultés. C'est ainsi que j'ai pu établir une* classification *géométrique des intersections de deux surfaces du* 2^e *degré. De même les résultats analytiques sur les formes bilinéaires m'ont donné par un étude géométrique la classification des* homographies *ou* collinéations *dans un espace linéaire quelconque.* […] *Mais je vous le répète et j'espère vous en convaincre à peine mes travaux seront imprimés, ce n'est pas un simple changement de mots qui donne ces résultats géométriques mais bien une suite de raisonnements plus difficiles.* The letter can be accessed at http://users.mat.unimi.it/users/gario/Segre-Ancona/lettereAscienziati.pdf, and it is also transcribed in the essay by Luciano and Roero in this present volume.

[9] *Infatti nulla è più fecondo che la moltiplicazione dei nostri poteri intuitivi recata da codesto principio: pare quasi che agli occhi mortali, con cui ci è dato esaminare una figura sotto un certo rapporto, si aggiungano mille occhi spirituali per contemplarne tante diverse trasfigurazioni; mentre l'unità dell'oggetto splende alla ragione così arricchita, che ci fa passare con semplicità dall'una all'altra forma* (Enriques 1922, 140).

This was an 'abstraction' supported by geometrical intuition, which was at that time a necessary tool to overcome technical difficulties and to obtain new results.

Segre's correspondence of the years 1883–1884 with internationally acclaimed mathematicians such as Klein,[10] Arthur Cayley, Thomas A. Hirst, Theodor Reye, Leopold Kronecker, Karl Weierstrass, and Oscar Schlömilch,[11] show us a young man who enthusiastically read the most recent publications, asked for explanations, made comments, but also illustrated his own research, explaining his approach and seeking to make his results known internationally. For example, he wrote to Kronecker: "I am young and I have great need and desire to learn: I am thus quite grateful to the scholars who are willing to be my teachers for a time" (*Je suis jeune et j'ai beaucoup de besoin et d'envie d'apprendre: je suis donc bien reconnaissant aux savants qui veulent pour quelques moments être mes maîtres.* C. Segre to L. Kronecker, Turin 25 December 1883).

In these two years he wrote no fewer than fifteen articles and notes, of which four were accepted by the *Mathematische Annalen* and two by the *Journal für die reine und angewandte Mathematik*, very high-level journals. His 'precocious maturity' (Fano 1930, 43) and the innovative character of these early research works earned him the award in 1884 of the Mathematics Prize of the Società Italiana delle Scienze. The jury praised Segre "for the generality of the research contained […], for the refinement of various theories treated previously by diverse eminent geometers, for the many and important new results obtained, and finally for the very broad field of research to which he opened the way".[12]

Winner of a competitive examination (*concorso*), in 1888 Segre was called to the chair of higher geometry at the University of Turin, a chair that he would hold until his death. It was his *maestro* D'Ovidio who gave up the professorship of higher geometry and moved to that of higher analysis leaving the field open for his student (Conte et al. 2013, 29–30, 101). His first works regarded above all hyperspatial geometry. Skilfully making use of the recent algebraic results of Weierstrass and Ferdinand Georg Frobenius (Hawkins 2013), Segre was able to provide a geometric and analytical formulation for hyperspatial projective geometry, developing it to such a level as to make it a tool for further research for the Italian School of algebraic geometry.

As his student Gino Fano affirms in the handwritten notes, probably for a lecture given in 1923 in Aberystwyth:

> He [Segre] became so, just in the moment in which Cremona's scientific activity had completely ceased, the new leader of Ital. geometry the founder of a new school. He was also able to learn, to

[10] See Luciano and Roero (2012).

[11] The letters of these mathematicians constitute part of the archives of the Fuà Family (Gario 1989a) and can be accessed online at http://users.mat.unimi.it/users/gario/Elenco.html. They are kept now in the *Archivi Corrado Segre*, University of Turin. See also the essay by Luciano and Roero.

[12] […] *per la generalità delle ricerche contenute* […], *pel perfezionamento* […] *a varie teorie trattate precedentemente da diversi valenti Geometri, per i molti ed importanti risultati nuovi ottenuti, ed infine per il campo vastissimo di ricerche cui esso apre la via*. (RAPPORTO relativo al conferimento del premio nelle Matematiche, dalla Società Italiana delle Scienze, per l'anno 1884, *Memorie di Matematica e Fisica della Società Italiana delle Scienze* (3), VII, 1890, XXXIV–XXXVI, at p. XXXVI. The members of the jury were three great mathematicians of the Italian Risorgimento: Enrico Betti, Eugenio Beltrami and Giuseppe Battaglini.

> make his own, and to let estimate by his pupils all that, for the development of his programme, was to be got from the most important foreign mathematicians (Klein, Noether, Lie, Cayley, Zeuthen, Darboux,…); and by means of his 35 years of teaching, about all most various branches of geometry, diff. and enumerative geom. (abzähl. Geom.) included, he had a very great influence on the development of all geometry in Italy.[13]

Even many years later, in a lecture given in Lausanne in 1944, Fano would say:

> Fortunately the Italian School reacted well against this possible trend, which Ovidio called 'tick-tock geometry', while *C. Segre* (1863–1924), one of the great Italian masters, always warned his students against these kind of works, which lead to the degeneration of scientific development.[14]

Beginning in 1886 Segre's works show a broadening of his horizons under the influence, on the one hand, of the new approach of the German School of Alexander Brill and Max Noether, and on the other hand, of the ideas expounded by Klein in his celebrated *Erlangen Program*. With regard to Brill and Noether, in 1904 Segre wrote, "An entire school of Italian geometers recognises in the Memoir by Brill and Noether [Über die algebraischen Funktionen und ihre Anwendung in der Geometrie, 1873] its point of departure".[15] As for Klein, Segre encouraged the translation into Italian of Klein's *Erlangen Program*, entrusting it to his student Fano, "for the benefit of the Italian geometers who know very little of it".[16] We will say more about this below. Presenting the translation, he underlined the importance of transformation groups for the development of geometrical research, which he himself had realised as early as 1885.[17] Therefore in Segre's research we can see his progressive detachment from a restricted projective view in order to arrive at the study of the properties invariant under birational transformations. The first signals of this shift of interest can be traced back to a note of 1886 on "Remarques sur les transformations uniformes des courbes elliptiques en elles-mêmes" (Segre 1886a), but it is above all in the memoir on algebraic ruled surfaces, published in two parts in the *Mathematische Annalen*,[18] that the line of research becomes clearer. In the second of these memoirs, "Recherches générales sur les courbes et les surfaces réglées algébriques" Segre takes time to explain to the international scientific community the advantages of the Italian approach:

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[13]Cf. Fano n. d., [*Appunti vari*], f. 63r. This is Fano's original English text. See also Fano (1923) and the typewritten notes kept in the Archives of the University of Liverpool, that the authors intend to publish.

[14]*Heureusement l'Ecole Italienne a bien réagi contre cette possible tendance, que d'Ovidio a appelée 'tic tac géométrie' tandis que* C. Segre (1863–1924) *un des grands maîtres Italiens, a toujours mis-en garde ses élèves contre ces productions, conduisant à une dégénération du développement scientifique* (Fano n. d., [*Appunti vari*], f. 58r).

[15]*Tutta una scuola di geometri italiani riconosce nella Memoria di BRILL e NOETHER [Über die algebraischen Funktionen und ihre Anwendung in der Geometrie, 1873] il suo punto di partenza.* (Segre 1905a, *Opere* 4, 462). See also the presentation by Segre of the important memoir of Brill and Noether (Segre 1894–95).

[16][...] *pour l'avantage des géomètres italiens qui ne le connaissent presque pas*. See Segre's letter to Klein dated Turin 19 November 1889 in: Luciano and Roero (2012, 151). All the letters from Segre to Klein cited in what follows are found in Luciano and Roero (2012).

[17]See Hawkins (1994, 187), Hawkins (2000, 251–260).

[18]See Segre (1887b, 1889a).

> It is not a question (I add this for those who may not be aware of the progress that this branch of mathematics is making, especially in Italy) of easy extensions to higher spaces of results for ordinary space that are already known. […] in introducing the spaces of all dimensions one has not only the advantage of greater generality, but can avail himself of all the power of a tool that one who limits himself to ordinary space does not have: that is to say, the consideration of entities of a space as a projection of those of higher spaces.[19]

In a very brief note of 1887, "Sui sistemi lineari di curve piane algebriche di genere p" (Segre 1887d) emerges one of the fundamental concepts of classic algebraic geometry, that of *characteristic series*[20] of a linear system of plane curves. With regard to this, Fano wrote:

> I may say that, in studying geometry on an algebraic manifold, *it is the fundam. concept of what we may call the Italian or geometrical method,* to study a (simple) lin. syst. of M_{k-1} on M_k by reducing it to the hyperplanar sections of a new (?) M_k. It involves necessarily more-dims. methods. Particularly: for lin. series of groups of points on a curve, a new curve, on which the groups are determined by hyperplanes, etc. It was stated firstly by *Segre* on the case of lin. syst. of planes curves & rational surfaces; and shortly afterwards applied by himself & *Castelnuovo* to groups of points on alg. curves – later still, in other cases.[21]

The culminating and synthesising work of this period is the important memoir entitled "Introduzione alla geometria sopra un ente algebrico semplicemente infinito" (Segre 1894a), which also includes the research carried out in Turin by Castelnuovo, and which, according to Severi, contains the 'roots' of Italian algebraic geometry. Here:

> […] the geometry of linear series on a curve is expounded according the hyperspatial method, underlining the fact that it needs neither considerations of functions nor algebraic developments, and that the algebraic nature of the entities comes into play only through Chasles's principle of correspondence! The synthesis in this area has achieved its utmost effectiveness. For example the proofs of the Riemann-Roch theorem and of the Cayley-Brill principle of correspondence are admirable.[22]

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[19] *Il ne s'agit pas (j'ajoute cela pour celui qui ne serait pas au courant des progrès que cette branche des mathématiques est en train de faire, surtout en Italie) de faciles extensions aux espaces supérieurs de résultats qui pour l'espace ordinaire soient déjà connus.* […] *en introduisant les espaces de toutes les dimensions on n'a pas seulement l'avantage de la plus grand généralité, mais encore celui de pouvoir se servir dans toute sa force d'un instrument que ne possède pas celui qui veut se borner à l'espace ordinaire: c'est-à-dire la considération des êtres d'un espace comme projection de ceux des espaces supérieurs.* (Segre 1889a, *Opere* 1, 125–126).

[20] It would be Castelnuovo who gave it this name in (Castelnuovo 1892).

[21] Cf. Fano n. d., [*Appunti vari*], f. 80r. This is Fano's original English text. See also the paper by Edoardo Sernesi in this present volume.

[22] [...] *la geometria delle serie lineari sopra una curva viene appunto esposta secondo il metodo iperspaziale, sottolineando che non occorrono in essa né considerazioni funzionali né sviluppi algebrici e che l'algebricità degli enti interviene soltanto attraverso il principio di corrispondenza di Chasles! La sintesi in questo terreno ha raggiunto la sua efficienza massima. Mirabili ad esempio le dimostrazioni del teorema di Riemann-Roch e del principio di corrispondenza di Cayley-Brill* (Severi 1957, X). See also in this present volume the presentation by Alberto Conte of the handwritten notebook that forms the basis of the memoir itself.

This memoir, in fact, served as a spur for important research by his students Castelnuovo, Enriques and Severi, research in which Segre participated rather laterally, as will be shown below, leaving the field open for his students.[23]

However, of particular importance is the introduction in 1896 (Segre 1895–96) of one of the most important invariants of an algebraic surface, today known as the 'Zeuthen-Segre invariant', which Segre had already presented to his students in the course of 1893–94. No less significant is the research carried out on questions related to the singularity of algebraic surfaces (Segre 1897b).

Projective geometry, which had aroused Segre's youthful enthusiasm and which constitutes a sort of *leitmotiv* that runs through his entire scientific research, again attracted his attention between 1889 and 1891; in particular, it was K.G.C. von Staudt's theory of imaginary elements that reawakened his interest. In 1887 Segre had invited Mario Pieri[24] to translate von Staudt's *Geometrie der Lage*; the translation came out in 1889, prefaced by a valuable bio-bibliographical essay written by Segre himself.[25] Extending the German mathematician's field of research, he enlarged the group of projective transformations by adding those that he called 'anti-projectivities', that is, correspondences in which the cross-ratios of four elements are transformed in their conjugate. Segre developed a complete theory of such correspondences and opened the way to a new field of geometric research, that of hyperalgebraic entities. His results, presented in four articles in the *Atti della R. Accademia delle Scienze di Torino*—and later, from a different point of view, in a memoir published in the *Mathematische Annalen*—[26]would be reprised and used many years later, above all by Elie Cartan[27]even if Baker considered them only "an interesting exercise in algebra" (Baker 1926, 270). In the memoir appeared in the *Mathematische Annalen* (Segre 1891d) Segre constructed various real 'models' of a projective space defined in the complex domain, the most simple of these is Segre's variety.[28] This research was completed in 1898 by Gerrit Mannoury and became the starting point of significant works by Wilhelm Wirtinger and William Hodge.[29] In the same paper Segre presented a careful study on anti-projectivity and hyperalgebraic entities and introduced the bicomplex numbers. On this research, in 1906 Eduard Study, who based himself on Segre's results, commented:

[23]See for example Castelnuovo (1924b, 355), Terracini (1961, 12–13), B. Segre (1963-64, 15), Brigaglia and Ciliberto (1995, Sect. 1.3).

[24]See the letter of C. Segre to M. Pieri, Turin 11 October 1887, in Arrighi (1997, 113), and the letter of C. Segre to F. Klein, Turin 14 October 1887.

[25]See the correspondence between C. Segre and A. Papellier and between C. Segre and K. Rudel about the life of von Staudt , in UTo-ACS, VII and http://users.mat.unimi.it/users/gario/Segre-Ancona/lettereRicevute.pdf.

[26]See Segre (1889–90, 1890–91, 1891d).

[27]Hawkins (1994, 200–204). See also the chapter by Aldo Brigaglia in this present volume.

[28]In a brief note of 1891 (Segre 1891c) Segre defined for the first time the product of two projective spaces, now referred to as 'Segre variety', a concept "that had important repercussions for the geometry of the twentieth century" (Severi 1957, XI).

[29]See B. Segre (1963–64, 16).

> SEGRE, however, restricted himself to the consideration of algebraic entities, and made large use of geometric reasoning. (It appears that for this reason the memoir by Segre is not as well-known as it should be).[30]

In letter addressed to Adolf Hurwitz in 1894 Segre himself complained that the importance of this research had not yet been grasped.[31]

Aware of the importance of consolidating his relationships with the European scientific milieu, in the summer of 1891 Segre undertook a journey to Germany with the aim of visiting the principal institutes and libraries in a country that was, at the time, on the cutting edge of mathematical research, and meeting in person those who had influenced his research. He visited Frankfurt a. M., Göttingen, Berlin, Leipzig, Dresden, Nuremberg, and Munich, and was in contact, among others, with Kronecker, Weierstrass, Nöther, Reye, Karl Rohn, Rudolf Sturm, Moritz Cantor, and Klein, some of whom he had been in correspondence with since 1883. His enthusiasm emerges from what he wrote to Castelnuovo:

> No one who hasn't been here can imagine what breed of man Klein is, and what kind of organisation he was able, with a skill that no one else possesses, to impose on mathematical studies in this university: it is something that has made an extraordinary impression on me. And I have already had many extremely vivid impressions of scientists during this journey![32]

In the same year Segre published in the *Rivista di matematica* the article "Su alcuni indirizzi nelle investigazioni geometriche. Osservazioni dirette ai miei studenti" (Segre 1891a), which, as it is well known, became the starting point of the dispute[33] with the director of the journal, Giuseppe Peano, regarding the way scientific research should be conceived. In this paper Segre offers a vivid picture of the recent achievements of algebraic geometry and of the open questions, underlining the importance of using both synthetic and analytic approaches. The English translation (Segre 1904) contributed to make Italian geometry widely known. With regard to this article Lucien Godeaux wrote that "Studying it was a revelation for me, and that's how I came to know Italian geometry". (*Son étude fut pour moi une révélation et c'est ainsi que je connus la Géométrie italienne,* Godeaux 1964, 24).

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[30] *Il* SEGRE *però si è ristretto alla considerazione degli enti algebrici, ed ha fatto largo uso di ragionamenti geometrici. (Pare che per queste ragioni la Memoria del Segre non sia conosciuta quanto meriterebbe).* (Study 1906, 345). See also Castelnuovo (1924b, 356).

[31] C. Segre to A. Hurwitz, Turin 29 June 1894, in Luciano and Roero (2012, 166).

[32] *Chi non è stato qui non può immaginare che razza d'uomo è Klein e che specie d'organizzazione egli ha saputo, con abilità che nessun altro può avere, imporre agli studi matematici in questa Università: è una cosa che m'ha fatto un'impressione straordinaria. E sì che d'impressioni vivissime da parte degli scienziati ne ho già avute parecchie in questo viaggio!* (C. Segre to G. Castelnuovo, Göttingen 30 June 1891, in ANL-Castelnuovo, in Gario (2010). All the letters from Segre to Castelnuovo cited in what follows, are conserved in ANL-Castelnuovo and can all be accessed on the website (Gario 2010); if they have been published, the specific reference will be indicated.

[33] This debate has been discussed several times. See, for example, Manara and Spoglianti (1977), Giacardi (2001a, Sect. 3), Avellone et al. (2002, Sect. 3), Roero (2004, 138–144), Luciano (2006, 65–71).

By the beginning of the 1890s Segre had already acquired a notable reputation even outside Italy. In 1893 he received a letter from Chicago asking him to promote participation at the International Congress of Mathematicians, which was to take place at the end of August in conjunction with the World's Columbian Exposition in that city; on that occasion he invited Castelnuovo to present a brief report on hyperspatial geometry in Italy, something which, however, he did not do.[34] In 1897 Segre was invited to be vice-president of the geometry section of the International Congress of Mathematicians in Zurich. On that occasion he wrote to his wife Olga: "The nomination pleased me, because […] there were many other geometers older than me who could be nominated".[35] His student Fano gave one of the six talks during the section, while Enriques gave one in the section of algebra. Just before leaving for Zurich Segre wrote to Volterra: "I believe that if I were unable to go, later I would feel regretful, as having missed the occasion to see men of value, and singular encounters".[36]

The following year, 1898, the jury commission for the Royal Prize for mathematics awarded by the Accademia dei Lincei, composed of Eugenio Beltrami, Luigi Bianchi, Valentino Cerruti, Luigi Cremona and Enrico D'Ovidio, assigned Segre half of the prize, shared equally with Vito Volterra, with a very flattering statement in which they cited, along with the novelty and importance of the results, the elegance of the method that associates "with rare ability geometric procedures with analytic procedures, grasping their intimate relationships", and explicitly acknowledging his role as leader of a School.[37]

At the basis of a group of Segre's works regarding problems of differential projective geometry dating to the years 1907–1913, there is the first volume of Gaston Darboux's *Leçons sur la théorie générale des surfaces* (1887), which Segre used in his course of 1903–04, as well as his contacts and the encounter with Ernest Wilczynski.[38] The first paper devoted expressly to differential projective geometry of hyperspaces dates back to 1907 (Segre 1906–07b); it is, however, in a later work of 1910 (Segre 1910a) that Segre lays the foundations for a systematic construction of such a geometry, which would receive a great impetus first from his student Alessandro Terracini and later from Enrico Bompiani. The brief note of 1908, "Complementi alla teoria delle tangenti coniugate di una superficie" (Segre 1908), which refers instead to ordinary space, marks a noteworthy step forward in the general theory of surfaces. Here Segre, generalising the concept of conjugate

[34]C. Segre to G. Castelnuovo,Turin 28 June 1893.

[35]C. Segre to Olga Michelli Segre, [Zurich] 10 August 1897, UTo-ACS, II: *La nomina mi ha fatto piacere perché* […] *vi sarebbero stati tanti altri geometri più anziani di me da nominare.*

[36]*Io credo che se non potessi andarci, dopo ne proverei rammarico, come d'un'occasione perduta di vedere uomini di valore, e riunioni singolari* (C. Segre to V. Volterra, Ancona 31 July 1897, ANL-Volterra).

[37][...] *con rara abilità i procedimenti geometrici agli analitici, cogliendone le intime relazioni.* (Relazione sul concorso al Premio Reale per la Matematica, pel 1895, 1901, 367).

[38]See Coolidge (1927, 355). Cf also Sect. 4 in Ciliberto and Sallent (2012), and the essay by Luciano and Roero in this present volume.

tangents, is led to introduce, among other things, the particular triad of tangent lines coming out of a point on a surface, today known as 'Segre tangents', the differential equations of which would be set forth by Guido Fubini. Also worthy of note, in the context of differential geometry, is the invariant, known as Wölffing–Mehmke-Segre invariant, relative to a pair of mutually tangent curves.

While it is true that these scientific contributions of Segre's provided the stimulus for the future research of the Italian School of algebraic geometry, it is also true that, in parallel with these, a significant role was played by his university teaching. We have valuable testimony of this in the forty handwritten notebooks[39] in which each summer he carefully developed the topics of the course that he would teach the following autumn. A shining example of the profound interaction between research and teaching, the notebooks not only allow us to reconstruct the genesis and developments of Segre's scientific research, of which they sometimes constitute 'a preliminary stage', sometimes a 'reflection' (Terracini 1953a, 261), but also allow us to understand the importance of his teaching in the birth of the Italian School of algebraic geometry. In his courses Segre presented his students with the most recent research, suggested topics to be studied, addressed problems that were still unsolved, all in the principal aim of directing the most promising young people towards scientific research; sometime, preparing classes, he himself arrived at posing new problems.[40]

2 The 'Geometric Orgies' of Turin

In the 1880s and 1890s Segre was able to create around him a climate of wildly enthusiastic work, of friendly collaboration, of scientific dialogue, lively and fertile. The fruit of this atmosphere would be felt throughout Italy. Castelnuovo recalls the period that he spent in Turin by speaking of 'geometric orgies' of Turin,[41] while Fano, speaking of the research in hyperspatial projective geometry, which represented the core of the 1880–1890 research, wrote: "it was *indispensable* that everything be treated and digested, that it became the blood of our blood, that we

[39]The handwritten notebooks of Segre's university lectures begin with 1888–89, the year in which Segre held the chair in higher geometry, and conclude in 1924, the year of his death. Of these, thirty-four develop topics of higher geometry, three are of mathematical physics and correspond to the years 1895–1897 in which Segre was charged with teaching that subject, and the two remaining lesson notebooks contain respectively brief mentions of various questions of analysis and of geometry, and the lectures given at the Scuola di Magistero. The last one includes, among other things, the lists of the students who attended Segre's courses from 1883 to 1892, with indications of the scores given to them.

[40]See, for example, C. Segre to G. Castelnuovo, Turin 1 June 1899.

[41]G. Castelnuovo to F. Amodeo, 6 February 1893, in Palladino F. & N. (2006, 304).

had it at our fingertips in order to be able to use it in the most advanced research … Fecundity!"[42]

Segre's university courses became a veritable breeding ground for future researchers. Many students wrote their dissertations on the most advanced topics of research under his supervision; among the most brilliant were Gino Fano (1892), Beppo Levi (1896), Alberto Tanturri (1899), Severi (1900), Giovanni Zeno Giambelli (1901), Alessandro Terracini (1911) and Eugenio Togliatti (1912). There were also many newly-graduated mathematicians, Italian and foreign, who, attracted by his fame, came to Turin to attend his lectures and carry out their post-graduate studies. These included Castelnuovo (1887–1891), Federico Amodeo (1890–1891), Federigo Enriques (November 1892, November 1893–January 1894), Gaetano Scorza (1899–1900), as well as the English husband and wife William H. Young and Grace Chisholm Young (1898–99), Julian Coolidge (1903–04), and some years later, Charles Hershel Sisam (1908–09) from the United States, and others. There was also a group of young mathematicians who, after having earned their degrees under Segre, or having collaborated with him as assistants, or having simply attended his lectures, published their first articles under his influence, such as Francesco Palatini, Umberto Perazzo, and Pilo Predella.[43] Some of the young people in Segre's entourage later turned their interest to topics that were characteristic of the School of Peano, or collaborated with him. The best known among these are Mario Pieri and Beppo Levi, but there were others as well, such as Luigia Viriglio and Matteo Bottasso, who exerted a certain influence in mathematics education.[44]

The relationship that was closest and most fertile was undoubtedly that with Guido Castelnuovo (Venice 1865–Rome 1952). The two first came into contact in July 1885 when Castelnuovo, who had not yet received his degree, sent Segre an article to read. In the letters that followed, Segre gave advice, proposed topics for research, and suggested articles to read. He came to appreciate his correspondent to such an extent that in October 1887 he offered him the position of assistant in the course taught by D'Ovidio, a position of 'an honorific nature' because it was assigned each year to the most outstanding graduate.[45] Segre himself held it in 1883–84. In that same letter, Segre suggested that Castelnuovo study linear systems of curves, and indicated a line of research.[46] As Segre put it, they were "two young men who placed their ideals of goodness, honesty and love of science before any

[42][…] *era* indispensabile *che fossero trattate e digerite, che diventassero sangue del ns sangue, averle sulla p. delle dita, p. valersene in ricerche + elevate … Fecondità!* (Fano n. d., [*Appunti vari*], fls 69r and 69v).

[43]See Appendix 5 at the end of this article, also in Giacardi (2013).

[44]The connections between the two Schools of Peano and Segre deserves a thorough treatment.

[45]C. Segre to G. Castelnuovo, Turin 6 October 1887.

[46]*The important problem to solve is to find all the systems of a given genus p of minimum order to which all the others can be reduced with Cremona transformations; up to now only the cases of p = 0, 1 have been discussed completely* (*La gran questione da risolvere è lo stabilire tutti i sistemi di dato genere p d'ordine minimo ai quali tutti gli altri posson ridursi con trasformazioni cremoniane; solo i casi di p = 0, 1 sono stati finora discussi completamente.* C. Segre to Castelnuovo, Turin 6 October 1887 and 28 July, 13 August 1890).

Philistine egotism";[47] at the time, Segre was 24 years old, Castelnuovo 22. Castelnuovo accepted the position of assistant[48] and remained in Turin until 1891, when he obtained the professorship in Rome. It was Segre who, in August 1890, had convinced him to take part in the competition for the chair, telling him also of the favourable opinions of Eugenio Bertini and D'Ovidio, and at the same time encouraging him to pursue the research on linear systems of curves of a given genus, suggesting points to develop and letting him know that the journal *Mathematische Annalen* was amenable to accepting work in Italian.[49] In 1892 Castelnuovo published his important paper entitled "Ricerche generali sopra i sistemi lineari di curve piane" (Castelnuovo 1892), the germ of which, as Castelnuovo himself affirms, was drawn from a paper in which Segre mentioned the advantages that the theory of linear systems could derive from the geometry of the curve (Segre 1887d). In his long and detailed report Segre underlined the importance of Castelnuovo's paper for the results, the new lines of research that it opened, the new concepts, and the questions that it highlighted.[50] The two entities used by Castelnuovo to translate the results from one theory to the other were 'characteristic series' and 'adjoint system'.

During the period he spent in Turin, Castelnuovo published no fewer than sixteen articles, and the long and intense dialogues with Segre led to the creation of the Italian line of research on the theory of curves, and laid the foundations for all of Italian algebraic geometry. After Segre's death, Castelnuovo wrote:

> To him I owe a good part of what I know; in those long conversations, which we had two or three times a day during my stay of 4 years in Torino, I learned more than in my university courses. To him I owe the incentive for my first works, and the advice and aid arising from his experience and his knowledge.[51]

After leaving Turin Castelnuovo maintained a close, uninterrupted correspondence with Segre,[52] who never ceased to be generous with advice on research and on teaching; for example, he encouraged him to write a work on plane involutions

[47][…] *due ragazzi che al di sopra dell'egoismo dei filistei ripongono i loro ideali di bontà, di onestà e di culto della scienza* (C. Segre to G. Castelnuovo, Turin 12 November 1891).

[48]In Turin Castelnuovo also taught, in an extracurricular setting, a course in projective geometry from 1889–90 to 1893–94, and in the academic year 1889–90 he taught at the military academy as well.

[49]C. Segre to G. Castelnuovo, Turin 13 August 1890.

[50]See Segre's report in the *Atti dell'Accademia delle Scienze di Torino* (1890–91, 602), available in the section *Relazioni* of the website (Giacardi 2013).

[51]*A Lui devo buona parte di quel che so; in quelle lunghe conversazioni, che avevamo due o tre volte al giorno durante la mia permanenza di quattro anni a Torino, ho imparato più che nei miei corsi universitari. A Lui devo l'incitamento ai miei primi lavori, e i consigli e gli aiuti della Sua esperienza e della Sua sapienza* (G. Castelnuovo to Olga Michelli Segre, Rome 25 May 1924, UTo-ACS, II).

[52]There are 255 existing letters from Segre to Castelnuovo from 1885 al 1905, in Gario (2010). Part of these have been published in Bottazzini et al. (1996, 669–678). The words that Segre addressed to his friend immediately after Castelnuovo's transfer to Rome show the profound friendship and scientific solidarity that had arisen between them (C. Segre to G. Castelnuovo,

(28 August 1892) and reread the lecture notes for the course that he was giving in Rome (21 February 1892 and 22 June 1892). Thanks to the letters that he sent to his friend (an average of thirty a year at first) it is possible to follow not only the thread of the scientific work of the two mathematicians, but also Segre's relationships with other collaborators or students, and the academic world in general, as well as the most important events of his personal life.[53] There emerges the figure of a teacher concerned about the training and the future of the young researchers under his guidance, and of the prestige of his faculty: a teacher who devoted time and energy to the preparation of his courses, to the revision of the works of his students, and to promoting Italian research on the international scene,[54] but also a teacher who was lavish with advice on teaching methods.[55] When the need arose, he could be severe, and selective.

This would soon be well understood by the Neapolitan Federico Amodeo (Avellino 1859–Naples 1946), who, as the winner of several competitive examinations for a teaching position in secondary schools, chose the Istituto Tecnico Sommeiller in Turin because he was attracted by Segre's growing fame; he intended to attend Segre's lectures. With a letter of introduction from his teacher Achille Sannia, who was already in correspondence with Segre,[56] Amodeo arrived in Turin in December 1890. He became part of the group of young mathematicians who orbited around Segre and Peano, and which had given rise to a kind of scientific community that was baptised *Pitareide*.[57] Their gathering point was first "Bergia's little rooms",[58] at the corner of Via Lagrange and Corso Vittorio Emanuele II and then the American Bar in the Galleria Nazionale. Amodeo had been in correspondence with Segre since May 1888, and their epistolary relationship would continue[59] even after, in the summer of 1891, Amodeo returned to

(Footnote 52 continued)
Turin, 12 November 1891). On the relationship between Segre and Castelnuovo, see the chapter by Paola Gario in this present volume.

[53]Particular attention is given to his marriage to Olga Michelli (25 and 26 March 1893), and to the birth of his daughters Elena (14 March 1894) and Adriana (28 October 1897). See *Corrado Segre. Biographical Timeline* in this present volume.

[54]See, for example, C. Segre to G. Castelnuovo, 15 July, 29 July, 8 August, 7 September, 23 November 1891; 8 June, 4 September, 28 January, 16 November 1892; 4 February, 27 May, 28 June, 26 September, 7 December, 11 December 1893; 29 January 1894; 5 December 1896; 26 February, 6 June 1899; 12 April 1901, etc.

[55]See, for example, C. Segre to G. Castelnuovo, 23 November 1891, 10 February, 21 February, 24 February, 22 June 1892.

[56]The correspondence between Sannia and Amodeo shows how Sannia continually asked Segre's opinions for the second edition of his *Lezioni di geometria proiettiva*; see Palladino F. and N. (2006); see also the chapter by Aldo Brigaglia in this present volume.

[57]See, for example, the following letters: C. Segre to F. Amodeo, Turin 24 November 1891, G. Castelnuovo to F. Amodeo, Rome 30 November 1891, in Palladino F. and N. (2006, 185 and 283), and C. Segre to G. Castelnuovo, Turin 28 November 1891.

[58]See C. Segre to G. Castelnuovo, Turin, 28 November 1891.

[59]The correspondence between Segre and Amodeo consists in thirty-four letters from 1888 to 1893, published in Palladino F. and N. (2006).