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2nd Edition

Springer

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To Federica Paola

To Renan-Abhinav

Preface to the Second Edition

The first edition of this book appeared at the end of the last century and, since then, it has been used as a textbook in several graduate courses, summer schools, and a preconference tutorial workshop on Nonlinear Systems. Thanks to the experience gained in these activities, several modifications appeared to be appropriate. The chapter on modeling was enlarged and it now includes results on standard realization of nonlinear systems. This is motivated by the importance of state-space representations in the analysis and synthesis of nonlinear control systems in the current literature. The focus of the book remained about structural properties that do not involve stability issues and, for this reason, those issues are only marginally considered. The chapter on systems structure has been enlarged by adding more material on system inversion and by adding motivational examples. A new chapter on output feedback has been included at the end of the book, in view of the major importance it has in practical applications. Various supporting practical examples borrowed from robotics, mechanics, and other application areas have been added throughout the book. A few exercises complete the chapters.

The introduction to the differential algebraic approach has been deleted because new monographs on this topic are available.

Finally, solutions to the problems can be found on the following website: http://www.springer.com/1-84628-594-1

Ancona, May 2006, *Giuseppe Conte and Annamaria Perdon*
Nantes, May 2006, *Claude H. Moog*

Preface to the First Edition

The theory of nonlinear control systems owes a large part of its modern development and success to the systematic use of differential geometric methods and tools. One of the first problems to be considered from the point of view of differential geometry was, at the beginning of the 1970s, that of analyzing the controllability of a nonlinear system. Early works on that topic ([111, 154, 155, 9]) highlighted the power and the potentiality of the differential geometric approach and motivated the interest of many researchers.

During the 1980s, the possibilities offered by the use of differential geometric techniques in the study of nonlinear control systems were largely exploited. One of the underlying leading ideas (see [75, 87]) was that of generalizing, to the greatest possible extent, the so-called geometric approach which had been first developed in the linear case (see [6, 160]). The research effort produced, in that period, many important results and it provided effective solutions to several control problems, such as disturbance decoupling problems, noninteracting control problems, and model matching problems. Excellent and comprehensive descriptions of the methodology and of the results achieved, together with meaningful examples of applications, can be found in [86] and in [126].

In the second half of the 1980s the limits of the differential geometric approach started to be explored and to become known. In particular, it became clear that problems such as system inversion or the synthesis of dynamic feedbacks could hardly be tackled with the already well-established differential geometric methods.

In the same period, the introduction of differential algebraic methods in the study of nonlinear control systems ([49]) offered a way to circumvent a number of difficulties encountered up to that time. The use of differential algebraic concepts characterized, through the work of several authors, a novel approach, which has essentially an algebraic nature, and, at the same time, it provided additional tools for investigating old and new problems. Further results on problems pertaining to inversion, noninteracting control, realization and reduction to canonical forms were obtained in the following years, and others were made achievable.

Today, the use of an algebraic point of view in nonlinear control problems has gained popularity and diffusion. This motivates the present book, whose aim is to give an account of the algebraic approach to nonlinear system theory and of its development in recent years. Together with a number of results which are scattered in the literature, the reader will find in it a self contained, comprehensive description of techniques and tools that can enrich his equipment as a control theorist and can provide a solution to otherwise not easily tractable control problems.

One of the distinctive characteristics that makes the algebraic approach interesting and useful is its inherent simplicity. In comparison with the mathematical background needed for profitably employing differential geometric methods, the knowledge required for using the tools described in this book is very limited. A significant example of this is offered by the way in which the notion of accessibility and the problem of linearization are dealt with. In both cases, a single tool, based on elementary differentiation of a function, namely, the notion of relative degree, gives the key for carrying on a deep analysis and for characterizing relevant dynamic properties. From a didactic point of view, simplicity renders the algebraic approach a practicable and valid choice in teaching engineering courses on nonlinear control. The book emphasizes this aspect and is usable as a teaching aid. In addition, simplicity facilitates the development of efficient algorithmic procedures that are relevant in solving concrete analysis and synthesis problems.

Another positive quality of the algebraic approach is its wide applicability in the field of dynamic systems and control. Although only continuous-time systems are considered in the book, the tools and methods described apply successfully to a number of control problems concerning discrete-time nonlinear systems, as shown in [4, 68]. Applications to time-varying systems are also possible, and recent results have been obtained in dealing with time-delay systems (see [13, 120]). With respect to other general methodologies, then, the algebraic approach appears to be more versatile and capable of going to the heart of the problem.

Only a basic knowledge of systems and control theory is required for reading the book, whose material is arranged in a self-explicatory way. The general setting and the fundamental notions are described and illustrated in the first part, entitled Methodology. Mathematical preliminaries are presented including notations from exterior differentiation. The system analysis completes this part and deals with fundamental properties as accessibility and observability. The structure algorithm and a canonical decomposition of the system are given as well.

In the second part, entitled Applications to Control Problems, the tools and techniques of the algebraic approach are employed for solving a number of basic control problems that are of practical interest in fields such as robotics and control of general mechanical systems, as well as in process control. The solution of the feedback linearization problem is given in terms of the accessibility filtration $\{\mathcal{H}_k\}$ introduced in Chapter 3. The disturbance decoupling problem is solved using the subspace $\mathcal{X} \cap \mathcal{Y}$, namely, the subspace that is observable independently from the input (Chapter 4). In the noninteracting control problem and in the model matching problem, we use the output filtration $\{\mathcal{E}_k\}$ and the structure algorithm (Chapter 5).

Finally, in the third part, entitled Differential Algebra, differentially algebraic tools and concepts are introduced in an elementary, but comprehensive, way, and the results of the first parts are revisited and analyzed from a differentially algebraic point of view. The notions described in the third part may contribute to expand not only the technical knowledge of the reader, but also his comprehension of the key ideas of the algebraic approach. A conceptually powerful way of approaching the theory of nonlinear systems has been proposed at the end of the 1980s in [49, 52, 54], introducing the use of differential algebra and differential-algebraic methods. In comparison with other approaches which employ differential geometric methods (see [86, 126] for a comprehensive description) or Volterra or generating series (see [48]), this one appears in particular capable of removing some drawbacks present in the notions of rank and it allows us to characterize general invertible compensators. In addition, it provides useful insight and results in connection with various analysis and synthesis problems (see [44, 52, 54, 64, 136]). Although differential algebraic methods are better suited for dealing with systems described by polynomials or rational functions, extensions to more general cases (as suggested, for instance, in [50, 141]) are possible. Here, we give a brief introduction to the principal notions of differential algebra and we introduce a general notion of dynamic systems in differential algebraic terms. Related system theoretical properties are described and the principal results obtained by the differential algebraic approach are mentioned without entering into the detailsto help the reader in establishing a connection with the notions studied in the previous parts of this book.

The authors acknowledge the NATO for its financial support of a joint research project under grant CRG 890101. Several results of this project are reported in this book.

Finally, the authors would like to thank J.W. Grizzle, and M.D. di Benedetto for some joint research work which inspired this book. Valuable discussions with M. Fliess, A. Isidori, A. Glumineau, E. Aranda, J.B. Pomet, R. Andiarti, Ü. Kotta, and Y.F. Zheng are acknowledged as well as the careful reading by L.A. Márquez Martínez and R. Pothin, and the help of E. Le Carpentier.

Ancona, December 1998, *Giuseppe Conte and Annamaria Perdon*
Nantes, December 1998, *Claude H. Moog*

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Part I Methodology

Part I

Methodology

1

Preliminaries

In very general terms, a dynamic system is a mathematical object that models, in some way, the evolution over time of a given physical phenomenon. Many authors, in the recent history of system and control theory, have provided concrete refinements and specializations of the above informal definition, from the axiomatic characterization given in [99], to the development of the so-called behavioral point of view [133, 159], or to the module theoretical characterization proposed in [54].

Here, we do not enter into this debate, but, assuming the classical control theoretical point of view, we view dynamic systems as objects described by a system of first-order differential equations of the form

$$\Sigma = \begin{cases} \dot{x} = f(x) + g(x)u \\ y = h(x) \end{cases} \tag{1.1}$$

where the independent variable $t \in I\!R$ denotes time; the state $x(\cdot)$ belongs to $I\!R^n$; the input $u(\cdot)$ belongs to $I\!R^m$; the input $y(\cdot)$ belongs to $I\!R^p$; and the entries of f, g, h are functions in a sufficiently general class.

Dynamic systems of the above kind arise naturally in modeling, at least locally, many physical phenomena by first principles. Moreover, techniques based on these models have proved quite effective in analyzing and controlling objects such as machines, robots, vehicles as well as industrial, economic, and biological processes [101, 147]. These considerations respond to two primary concerns about the choice of our models, namely, generality and usefulness. However, to make the picture more precise, it remains to specify in which class of functions f, g, and h are taken. The following discussion will help us to make a motivated choice.

Modeling involves approximation and some degree of uncertainty in dealing with dynamic systems, we are naturally interested in properties whose validity in nominal situations may imply validity in almost all situations, that is, in almost all situations except, so to say, in pathological ones. A way to capture this feature in mathematical terms is that of considering *generic* properties, that is, properties that hold on open and dense subsets of suitable domains of

definition, provided they hold at some point of such domains. This, however, imposes some restriction on the class of mathematical objects we want to deal with because the concept of generic properties must make sense for them.
To understand this fact and its consequences better, let us consider, for instance, the role of the vector field $g(x)$ in (1.1). Using a model of the form (1.1), we may clearly ask that independent inputs have, in general, independent effects on the system. Otherwise, simpler models, in which the dimension of the input vector u is reduced, could be used, at least locally. Since this fact depends on the rank of the vector field $g(x)$ at different points x of the space of states, it is useful to require that the property of having maximal rank is generic for $g(x)$. In particular, this implies that vector fields $g(x)$ whose components are C^∞ functions are not admissible in (1.1), since there are C^∞ functions, e.g., $f(x) = \begin{cases} e^{-1/x^2}, & \text{if} x < 0 \\ 0, & \text{if} x \geq 0 \end{cases}$, that, being neither generically zero nor generically different from zero, could give rise to vector fields whose rank is neither generically maximal nor generically lower than the maximum. In other terms, the notion of generic property does not make sense, in general, for systems defined by C^∞ functions. The situation is different if we restrict our attention to systems defined by analytic functions, and also meromorphic ones, and this, as stated in Section 1.2, motivates our choice throughout the book.

1.1 Analytic and Meromorphic Functions

To specify the class of functions we will deal with in our models, let us introduce the following definition.

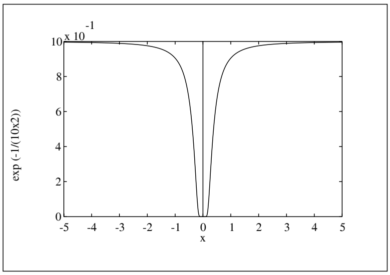

Fig. 1.1. Graph of $\exp(-1/10x^2)$

Definition 1.1. *Let $I \subseteq I\!R$ be an open interval. A function $f : I \to I\!R$ is analytic at a point $x_0 \in I$ if it admits a Taylor series expansion in a neighborhood of x_0. If f is analytic at every point of $I \subseteq I\!R$, we say that f is analytic in I.*

Examples of analytic functions are given by polynomial functions, as well as by the trigonometric functions $\sin x$ and $\cos x$. Rational functions are analytic at any point in their domain of definition.
The function $f(x) = \begin{cases} e^{-1/x^2}, & \text{if} x \neq 0 \\ 0, & \text{if} x = 0 \end{cases}$ is C^∞, but it is not analytic at $x = 0$ (see Figure 1.1).

The basic property of analytic functions we are interested in is stated in the following proposition.

Proposition 1.2. *Let $I \subseteq I\!R$ be an open interval, and let $f : I \to I\!R$ be an analytic function on I; then either*

(i) $f \equiv 0$ in I, or
(ii) the zeros of f in I are isolated.

Proof. Let $Z(f)$ denote the set of zeros of f in I. Since f is analytic in I, for every point $\bar{x} \in Z(f)$, there exists a neighborhood $D(\bar{x}, r) = \{x \in I\!R$ such that $\mid x - \bar{x} \mid < r\} \subseteq I$ such that

$$f(x) = \sum_{n=0}^{\infty} c_n (x - \bar{x})^n$$

for $x \in D(\bar{x}, r)$. Now, two cases are possible: either $c_n = 0$ for $n = 0, 1, 2, \ldots$, or there exists a minimal positive integer m such that $c_m \neq 0$ and $c_n = 0$ for $n < m$. In the latter case, we can write

$$f(x) = (x - \bar{x})^m f_1(x) \tag{1.2}$$

where

$$f_1(x) = \sum_{n=0}^{\infty} c_{n+m} (x - \bar{x})^n \text{ and} f_1(\bar{x}) \neq 0 \tag{1.3}$$

By continuity, $f_1(x) \neq 0$ in a neighborhood $D(\bar{x}, r_1)$ and also $f(x) \neq 0$ for $x \neq \bar{x}$ in $D(\bar{x}, r_1)$, then $\bar{x}$ is an isolated zero. Alternatively, in the first case, $f \equiv 0$ in $D(\bar{x}, r)$ and, hence, $\bar{x}$ is an interior point of $Z(f)$. Then, $Z(f)$ consists of points that are either isolated or interior. In particular, assume that there exists at least one point $\bar{x}$ in $Z(f)$ which is interior and let A be the connected component of $Z(f)$ that contains $\bar{x}$. If the point $sup\{A\}$ belongs to I, by continuity of f, it belongs to $Z(f)$ and hence, since it is not isolated but, necessarily, interior, it must coincide with $sup\{I\}$. Coincidence obviously holds also if $sup\{A\}$ does not belong to I. Then, we get $sup\{A\} = sup\{I\}$ and, in the same way, $inf\{A\} = inf\{I\}$. Therefore, $Z(f) = I$ and $f \equiv 0$ in I. ∎

A polynomial function has a finite number of zeros which are isolated. The function $f(x) = \sin x$ has an infinite number of isolated zeros located at $x = k\pi$ for any positive or negative integer k. A typical example of a nonanalytic continuous function whose zeros are not isolated is the following.

Example 1.3. The function $f(x)$, defined by

$$f(x) = \begin{cases} \sin(1/x), & \text{if} x \neq 0 \\ 0, & \text{if} x = 0 \end{cases}$$

is not analytic since $x = 0$ is a point of accumulation for the zeros of f (see Figure 1.3).

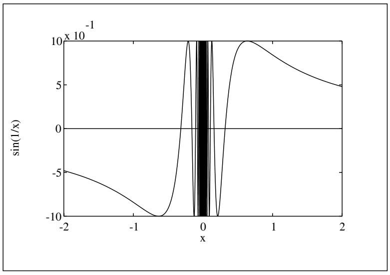

Fig. 1.2. Graph of $\sin(1/x)$

Example 1.4. The function $f(x)$, defined by

$$f(x) = \begin{cases} e^{-1/x^2} sin(1/x), & \text{if} x \neq 0 \\ 0, & \text{if} x = 0 \end{cases}$$

is not analytic since $x = 0$ is a point of accumulation for the zeros of f. However, it is a C^∞ function.

In the multivariable case, one can analogously consider an open domain $D \subset \mathbb{R}^n,\ n \in \mathbb{N}$ and the following Definition.

Definition 1.5. *A function $f : D \to \mathbb{R}$ is said analytic in D if it coincides with its Taylor expansion in the neighborhood of every point $x_0 \in D$.*

The generalization of Proposition 1.2 becomes

Proposition 1.6. *Let $D \subseteq \mathbb{R}$ be a convex, open domain and let $f : D \to \mathbb{R}$ be an analytic function on D, then either*

(i) $f \equiv 0$ on D, or
(ii) the set of zeros of f in D has an empty interior.

Proof. Given any point $x_1 \in D$, take a point x_0, if any exists, in the interior of the set of zeros of f in D. The straight line L that passes through x_1 and x_0 contains an interval whose points are zeros of f and, henceforth, of the restriction of f to L. Since the restriction of f to L is analytic, it is zero by Proposition 1.2, and hence $f(x_1) = 0$. ∎

Example 1.7. Let $f : \mathbb{R}^2 \to \mathbb{R}$, be defined by $f(x_1, x_2) = x_1 - 1$. It is easily seen that f is analytic. For any point $P \in \mathbb{R}^2$ which belongs to the set $\mathcal{Z}$ of zeros of f, there does not exist any $\mathbb{R}^2$-neighborhood included in $\mathcal{Z}$ (see Figure 1.7).

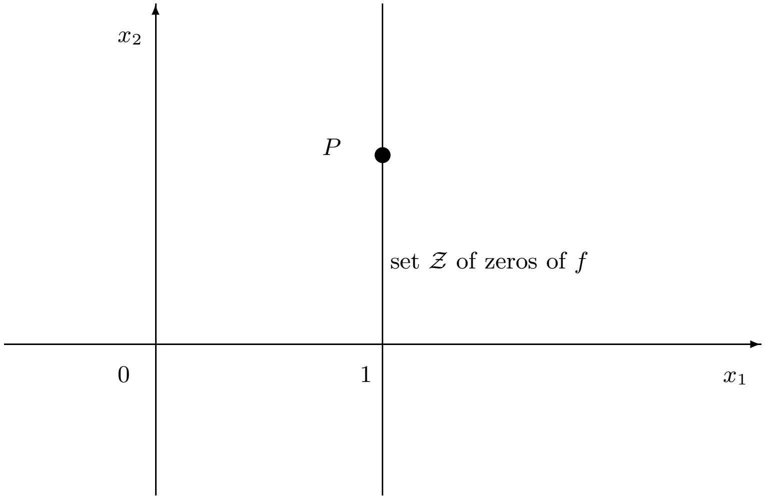

Fig. 1.3. Zeros of $f(x_1, x_2) = x_1 - 1$

By Proposition 1.6, nonzero analytic functions defined on $\mathbb{R}^n$ are different from 0 at the points of an open (since the set of zeros is obviously closed) and dense subset of $\mathbb{R}^n$, or, in other terms, they are generically different from 0. Then, it makes sense to define the *generic rank* of a matrix whose entries are analytic functions as the dimension of the maximum square submatrix having a nonzero determinant. As the determinant is an analytic function, the generic rank coincides with the rank of the matrix at the points of an open, dense subset of $\mathbb{R}^n$. Moreover, the generic rank is greater than or equal to the rank at any point of $\mathbb{R}^n$. Recalling what we said about the rank of the vector field $g(x)$ appearing in (1.1) and our interest in it being generically maximal, it should be clear, now, that we are motivated to assume that the functions f, g, and h in (1.1) are analytic.

If a function $f(x)$ is analytic, in general, the same is not true for its multiplicative inverse $1/f(x)$. However, in a suitable algebraic framework, we can give a notion of the multiplicative inverse of a nonzero analytic function. To explain this, let us first note that, with the usual notions of sum, denoted by $+$, and of product, denoted by $\cdot$, the set of analytic functions from $\mathbb{R}^r$ to $\mathbb{R}$ forms a ring, denoted by $\mathcal{A}_r$. An important ring theoretical property of $\mathcal{A}_r$ is