Mathematics for Industry 16

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Vibrations of Rotating Machinery

Volume 1. Basic Rotordynamics: Introduction to Practical Vibration Analysis

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Mathematics for Industry

Volume 16

Aims & Scope

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Volume 1. Basic Rotordynamics: Introduction to Practical Vibration Analysis

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Preface

The development of new rotating machines often encounters unexpected vibration problems. In many reported cases these vibration problems have had to be solved before the hydraulic performance of rotating machinery such as turbines, pumps, and compressors could be assessed.

The *v_BASE Databook* compiled by the Japan Society of Mechanical Engineers (JSME) contains a collection of vibration problems actually experienced in industry. Its first edition, published in 1994, includes 300 case studies. Almost two-thirds of the data case studies included in the first edition are related to rotating machinery, about half of which involve resonance. The authors learned the following lessons associated with the case studies:

1. Serious issues caused by large of rotational energy and centrifugal force:
   A seemingly slight problem in rotor dynamics may produce very dangerous vibration in a body rotating at high speed, the same as the dynamics of a car running at high speed on a highway. Furthermore, apparently vibration is reproduced unless any repair is undertaken. This vibration problem, occurring once, can be eliminated only by making specific improvements at the designing and manufacturing stages, which may entail significant effort and cost for the final correct solution. Our rotor dynamics require us to be more practical in solving the issue to avoid encountering such a dangerous situation.
2. Knowledge versus practical experience:
   Rotating machinery engineers must be accomplished with skills to achieve good machine operation, typically through balancing. Many engineers possess good theoretical knowledge of unbalance resonance phenomena through their basic engineering education. However, they may have apparent difficulty in applying that knowledge on site—for example, practical field balancing by scoping two waveform signals together with rotational pulses and rotor vibration, and reading the amplitude and phase difference without data analyzers. Such difficulty is most easily overcome by appropriate learning and experience to apply the knowledge to field practice without hesitation. Field experience in

eliminating vibration for troubleshooting based on variety of measurements and analyses offer the best educational opportunity. Experience is good teacher and the field is a good class.

3. Inertial (stationary) or rotating coordinate systems?
   Rotor vibration is characterized by the gyroscopic effect. The term "gyroscopic effect" is related to the inertial coordinate system. On the other hand, the same phenomenon observed in the rotating coordinate system is called the "Coriolis effect" as per the theory of blade vibration. This example shows the importance of a unified and seamless understanding of both dynamics described in the inertial and rotating frames of reference. The bridge to connect the knowledge gap between rotor dynamics and rotating structure dynamics is provided in this book. The introduction of the complex displacement for the analysis of rotor whirling motion may also be an effective way to facilitate such understanding.

An effective approach to analyzing various phenomena thus benefits the understandable explanation for the correction of rotor vibration problems. The purpose of the present book is to describe the general mechanisms of resonance and self-excited vibration by using models that are as simple as possible, thereby forming a common basis for addressing various vibration problems. The study of these models will also be useful for enhancing intuitive ability. The authors have placed special emphasis on the conciseness of the mathematical formulations in the process of solving vibration problems. The entire content of the book is within the realm of linear vibration theory, but the authors believe, based on their experiences on-site and in consulting, that the book provides the sufficient body of knowledge needed for practical engagement. The book will also prove useful as test-preparation material for the ISO Machinery Condition Analyst (Vibration), i.e., an examination organized by the JSME in Japan.

Although the authors have endeavored to eliminate errors and provide appropriate emphasis, critical comments from readers are welcome. We are grateful to the authors of the literature cited in this book. Special thanks are also due to Prof. Hiroyuki Fujiwara (National Defense Academy, Japan) and Dr. Naohiko Takahashi (Hitachi, Ltd.) for reviewing the manuscript and providing valuable comments.

The authors deeply thank Mr. Fumito Shinkawa, President of Shinkawa Electric Co., Ltd. for his support for our efforts.

We also wish to thank Springer for its support in publishing this work.

Yokosuka, Japan
May 2016

On behalf of the authors
Osami Matsushita

Contents

Chapter 1
Introduction of Rotordynamics

Abstract This book explains various phenomena and mechanisms that induce vibrations in rotating machinery, based on theory and field experiences (Chaps. 1–12 in volume 1). It also provides guidance in undertaking diagnosis and implementing effective countermeasures against various vibration problems in the field (in volume 2). Consequently, volume 1 is intended mainly for beginners and students, while volume 2 is mainly for design engineers and practitioners. This chapter of volume 1 emphasizes the subtlety of vibration problems in rotating machinery and the importance of reliable technologies that help to stabilize and reduce vibrations. It also outlines a wide variety of rotating machinery, vibration problems found in the field, and mathematical approaches to analyze vibration problems. In high-speed rotating machinery, the steady rotating state corresponds to a stationary equilibrium condition with a high rotational energy. Vibration brings the machine into a "dynamic" state. If the rotating system becomes unstable in the dynamic state, resulting in self-excited vibration, the machine enters a very dangerous operational condition. Since the energy source of self-excited vibration in a rotating system is provided by the spin of the rotor, the only way to avoid this dangerous situation is to stop the energy source, for example, by shutting down the power source in the case of a motor driven system. Vibrations caused by an external force, unless kept small enough, may also lead to a serious problems through contact between the rotor and the stationary part (stator).

Keywords Rotor · Bearing · Mechanism of vibration · Rotor modeling · Rotor vibration simulation · *v_BASE*

1.1 Vibration Problems in Rotating Machinery

1.1.1 Varieties of Rotating Machinery

A rotating machine consists of a rotor (comprising a shaft and disks), and bearings with associated casings/housings that support the rotor. Fluid performance based machines used in the energy and process industries, conventionally called

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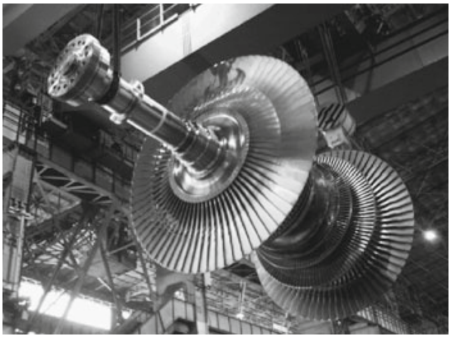
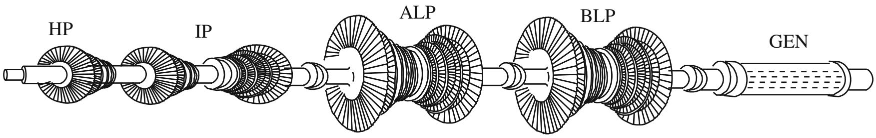
turbomachines, are well-known rotating machines. Examples of turbomachines include: steam turbine-generator sets (Fig. 1.1), gas turbines (Fig. 1.2), jet engines, waterwheels, compressors, fans, pumps, centrifuges, electric motors, and step-up/reduction gear units.



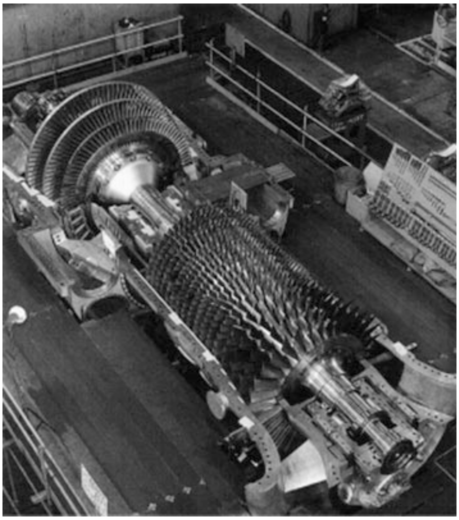
(a) LP turbine rotor (Ref. Toshiba Review)



(b) Rotor line

Fig. 1.1 Steam turbine-generator set

Fig. 1.2 Gas turbine



While these are large-sized high-power machines that rotate with high energy levels, there are also small-sized low-power machines for industrial or home applications, such as electric motors, gyroscopes, agitators, grinding machines, spinning machines, marine and automotive power trains, automotive internal combustion engines, reciprocating machines, lathes, vacuum pumps, fans, washing machines (Fig. 1.3), compressors for air conditioners (Fig. 1.4) and refrigerators.

Fig. 1.3 Washing machine

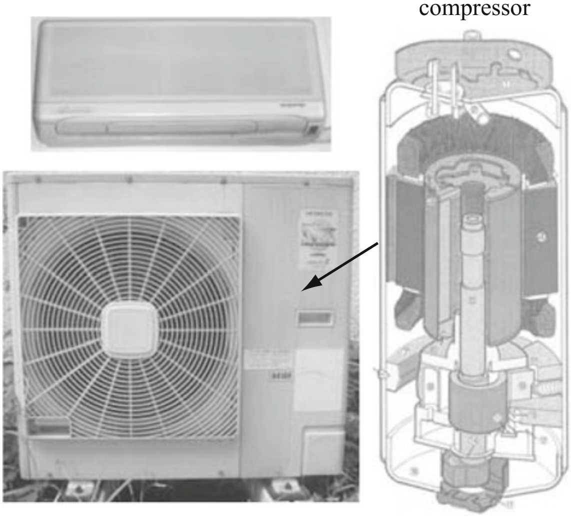
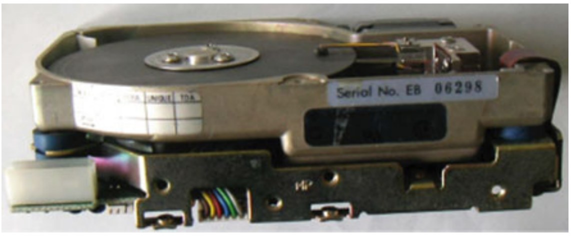


Fig. 1.4 Air conditioner

Fig. 1.5 Current type of hard disk (3.5 in.)



The increase of the fluid performance of turbomachines requiring drastically high pressure and/or extremely large capacity is commonly set as a goal for engineering projects, which challenges the development of new turborotors. The turbomachines characterizing these high specifications are planed from the viewpoints of fluid performance and fluid dynamics with less consideration of mechanical vibrations. We note that the smooth rotation test with low vibration level is absolutely necessary before the fluid performance test at the rated speed operation is executed, otherwise, large vibration at rated operation might negate the fluid performance test. For example, of the 12 case studies concerning new challenging developments of rotors such as compressors, pumps, etc., introduced in the October 2009 issue of the Japanese journal "Turbomachinery", more than half were focused on how to solve vibration problems in rotating machinery, which required much effort to attenuate vibrations before the fluid performance test.

In addition to conventional machines, recent developments in mechatronics have promoted new rotational machinery, particularly for computer-related devices and devices in the field of information technology, such as drives for hard disks (Fig. 1.5), optical disks and compact disks, video tape recorders, polygonal mirrors and hybrid engines. They are progressing rapidly toward a smaller size, lower thicknesses, higher speeds or higher density information. Most of them are characterized by thin rotating disks where the gyroscopic effect is often important. Though these innovative products are often specified to have a short life cycle, dynamical design for vibration reduction will invariably be required from engineers.

1.1.2 Bearings

A bearing is located on the stationary part to support the rotor and to ensure its smooth rotation. The rotating shaft in the bearing portion is called the journal (for radial bearings) or disk (for thrust bearings). Bearings may be classified in a number of varieties, including rolling element bearings (ball bearings, roller bearings), sliding bearings (cylindrical bearings, multi-lobe bearings, tilting pad bearings, etc.), and magnetic bearings (with actively controlled electromagnet or passive/non-controlled permanent magnet), according to applications. Their characteristics [1] are summarized in Table 1.1.

Table 1.1 Comparison of various types of bearings

	Rolling element bearing	Sliding bearing	Active magnetic bearing
Load	Inferior for impact load. A deep groove / angular contact and cone roller bearing, etc., can support the load both in radial and thrust directions.	Suitable for impact load and heavy load. Acceptable specific loads are approximately: Radial dir. : less than 5 MPa Thrust dir. : less than 7 MPa	Most suitable for light load with high speed. Acceptable load pressures are approximately: Radial dir. : 0.3 to 0.5 MPa Thrust dir. : less than 0.8 MPa
Friction	Static friction coefficient is as small as 10^{-3}~10^{-2}	Static friction coefficient is as large as 10^{-2}~10^{-1}	Small
	Dynamic friction coefficient is almost the same as 10^{-3}		
Speed limit	Depending on centrifugal force and lubrication, etc. $DN < 2\times10^{5}$ mm rpm	Depending on turbulent flow transition and overheating of oil film. Generally, $V < 120$ m/s	Depending on strength of AMB rotor material. Generally, $V < 200$ m/s
Stiffness and damping	Large stiffness and no damping.	Large stiffness and high damping.	Stiffness and damping are low, but widely controllable.
Noise	Comparatively large	Comparatively small	Small
Lubricant	Grease in general.	Oil in general.	Not required.
Life and breakage	Life can be estimated using fatigue strength of material. Seizure breakage may occur at high-speed rotation.	Infinite life in hydrodynamic operation. Seizure and wear are main causes of breakage. Flaking may occur due to high load.	Nearly permanent.
Installation error	Comparatively sensitive.	Comparatively insensitive.	Insensitive because bearing clearance is large.
Influence contamination	Influential on life, wear, especially noise.	Comparatively less influential.	Less influential.
Maintenance	By using grease / oil lubrication, maintenance is easy.	Leakage from lubricant oil circulating system need to be checked and stopped.	Maintenance free in general, except some electronics parts.
Cost	Mass-produced standard bearing are inexpensive and interchangeable.	Generally in-house production. Comparatively inexpensive. Arbitrary bearing dimensions.	Expensive because custom-made manufacture is still the mainstream.

The reaction force on a bearing is often represented by spring (stiffness) and damper elements in a mechanical model using linear vibration characteristics. In these terms, the reaction force is generally characterized as follows for each type of radial bearing:

(1) Rolling bearings: The reaction force is isotropic (radially symmetric). Stiffness is relatively high, but damping is negligible or nil.
(2) Sliding bearings: The reaction force is anisotropic (radially asymmetric) because the rotor weight is supported by an oil film in the bearing gap at a slightly eccentric position with respect to the center axis of the bearing. Since the oil film also dissipates vibrational energy, resulting in vibration reduction,

a well-damped rotor system can be completed if the design is effective. Since stiffness and damping are very high in this type of bearing, they are thus required to be predicted in high-precision calculations.

(3) Magnetic bearings: The reaction force is isotropic if the journal is tuned to coincide with the bearing having an radially symmetric electromagnetic force. Both stiffness and damping are controllable over usable ranges. Errors in control may be compensated by adjusting control parameters in situ.

While the reaction force is nonlinear in general, a linear approximation (i.e., modeled by spring and damper elements) around the static equilibrium point may be used for design analysis. A small nonlinearity may, however, be evident in a normal vibration waveform, which can grow into a large amplitude waveform featuring nonlinear sources.

1.1.3 Defects in Various Elements and Induced Vibration

A rotating machine comprises a variety of elements such as shafts, disks, blades, nozzles, drums, joints, bearings, seals, coils and gears, each of which undergoes fluid forces from the process working gas/liquid or lubrication, electromagnetic forces and so on. Therefore, a rotor is practically never free from the possibility of generating vibration during its operation.

Rotor vibration causes various other problems in rotating machinery, which may make the situation worse. Increased cyclic stress and fatigue failure, collision of the rotor with stationary parts, seized bearings, vibrating force transmission to stationary parts, and induced vibration of peripheral units, are examples of defects caused by vibration of the rotor.

To achieve vibration reduction of a rotor, the design of rotating machinery should take analysis of such factors into account to avoid vibration under normal operating conditions. When vibration occurs in a machine, it is absolutely necessary to find its cause and take appropriate countermeasures. It is a prerequisite for engineers, therefore, to understand dynamic characteristics of machine elements, particularly with regard to possible causes for induced vibration of the rotor.

Figure 1.6 shows an example of elemental technical issues in the development of a large-scale power generator. Many factors, including large sliding bearings, seals and slot structures to hold the coil, are among important issues for avoiding rotor vibration.

1.1.4 Rotordynamics

The purpose of rotordynamics is to establish methodologies of vibration reduction/suppression. This enables design, appropriate operation and maintenance procedures of rotating machinery by elucidating causes of vibration from the

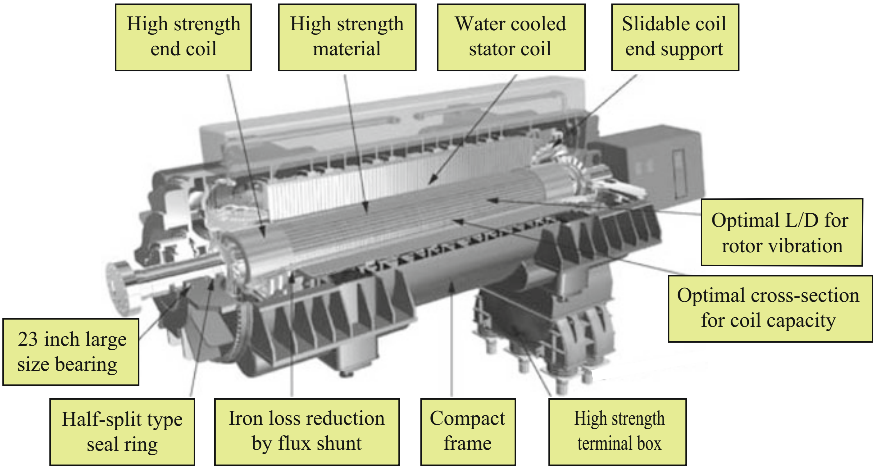


Fig. 1.6 Technical issues for 1000 MW generator development (Ref. Toshiba review)

viewpoints of excitation force and mechanisms of excitation, natural frequencies of modes excited by external influences on the rotor.

During 1950–1970, many Japanese manufacturers of rotating machinery learned know-how for solving individual vibration problems from their technological partners in Europe and the U.S. Increases in size and speed of rotating machinery since the 1960s led to development and accumulation of original technology during 1970–1980.

Since then Japanese industry has relied largely on its own technology, and now it is ranked among the highest in the field of rotordynamics.

Many cases of vibration in rotating machinery were thought to be simple "troubles" and remained obscure, even within departments of the same company. However, in April 1972 it was a surprise when the Journal of the JSME (Japan Society of Mechanical Engineers) published papers from Dr. Kazuhiro Shiraki [2] and his group, which elucidated 132 case histories on vibration problems with root cause analyses, countermeasures and their effects. Subsequently, successor engineers realized the need for openness as a means to overcome vibration problems experienced in industry. They established a Vibration Engineering Database Committee [3] (*v_BASE*) in the JSME in 1991, which has been organizing yearly meetings for collecting and discussing vibration case studies, and compiling them into database book/CD (Fig. 1.7). It is highly desirable for engineers to share solutions of vibration problems and lessons learned from them. The idea for troubleshooting came from the fundamental understanding of field data that enables them to rectify vibration phenomena, rather than from fragmentary or superficial knowledge learned in the classroom.

Some of the *v_BASE* data have been presented at conferences [4–6]. Recently, many *v_BASE* data related to rotordynamics are planned to be part of a Website of the JSME.

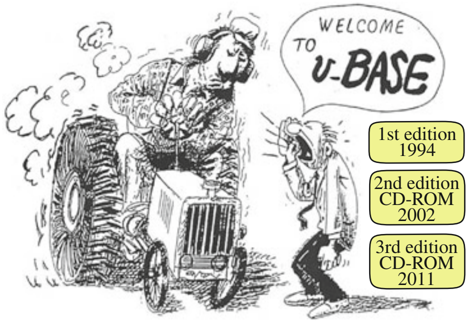


Fig. 1.7 *v_BASE* databook [3]

1.2 Types of Vibration in Rotating Machinery

From a practical point of view, frequently encountered rotor vibration modes can be categorized as follows:

(1) Bending vibration of the shaft

The shaft whirls in its rotation plane (the plane is perpendicular to the rotational axis) while maintaining its deflection mode that varies along the axis. The whirling shaft mode is measured as being rigid mode and/or flexible in nature. Most vibration problems of the shaft can be described by these bending modes.

(2) Torsional vibration of the shaft

Torsional deformation arises when the vibrations of each part of the shaft are twisted along the rotation axis. The occurrence of the torsional vibration problem is relatively small compared with bending vibration in rotating machinery, excluding reciprocating and/or geared machines. Vibrational changes in electromagnetic forces in motors or in the torque of the load may also induce torsional vibrations.

(3) Longitudinal vibration of the shaft

This mode of vibration usually does not appear due to the high longitudinal stiffness of the shaft, but there have been cases due to a collision with a static part due to local resonance, and an excessive dynamic load in a thrust bearing.

(4) Vibration of rotating structure

Bending vibration of a rotating structure, such as turbine blades or pump and compressor impellors, is a very important issue. Three-dimensional finite element analysis is commonly used to assess vibrations in this category using detailed calculation of natural frequencies and prediction of resonance amplitudes based on the actual geometry of specific blades and impellers. The stiffness increase under centrifugal force is taken into account. The rotating structure is usually analyzed by assuming that the center of the structure is fixed, being free from the shaft movement. The coupling effect of shaft-blade vibration must be considered in some cases.

1.3 Classification of Vibration by Mechanism of Occurrence

Figure 1.8 illustrates the mechanisms of vibration occurrence from the viewpoint of the equation of motion and the associated parameters. The terms in this figure are explained below:

(1) **Equation of motion** The equation of motion describes a rotor system, where M (mass), C (damping) and K (stiffness) are linear matrices and εf is a small nonlinear term. These terms constitute a free vibration system. For forced vibration, there is a term for unbalance force U, which is generated at a frequency synchronous to the rotational speed Ω. External harmonic excitation $F(\nu t)$ with an exciting frequency ν may also be included.

(2) **Forced vibration system** The spinning rotor under an unbalance force U or a periodic external force F vibrates steadily at a frequency synchronous with the external force. When the frequency of the external force coincides with the natural frequency of the rotor, resonance occurs and the vibration amplitude becomes maximal.

The lower half of the figure shows details of the forced vibration. The resonance of the unbalance vibration synchronous with the rotational speed occurs at a rotational speed $\Omega \approx \omega_n$, which is called the "critical speed" of the rotor because it represents a potentially dangerous operating point. Resonance under harmonic excitation with an external frequency ν occurs at $\nu = \omega_n$. This type of resonance should be prevented by avoiding the resonance condition or reducing the sensitivity to resonance.

Resonance due to the existence of the nonlinear term at $N\nu = \omega_n$ (where N is a multiple or fraction) is dependent on the right hand side term εf.

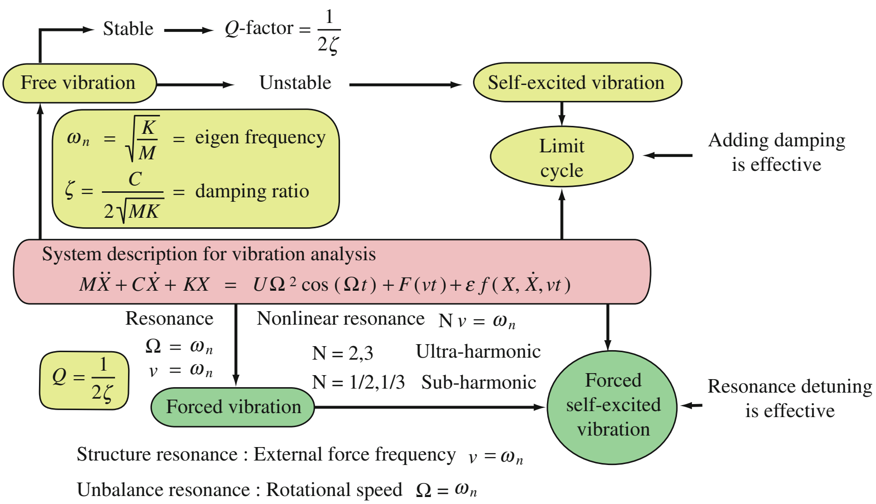


Fig. 1.8 Vibration mechanisms and parameters

(3) **Free vibration system** The upper half of Fig. 1.8 deals with free vibrations. Vibration is attenuated in a stable system, while self-excited vibration appears in an unstable system. Damping is positive in the former, and negative in the latter. The frequency of the resulting vibration is at a natural frequency of the rotor system in both cases.

An example of self-excited vibration is the gradual growth of amplitude even if no external force is applied. It is well known that a window shade/blind may vibrate frequently under certain conditions of wind velocity. Once the self-excited vibration happens, it is difficult to prevent it. Note that the only essential countermeasure is to provide the system with a positive damping factor. This solution is generally different from that to reduce resonance in forced vibrations.

The amplitude of the self-excited vibration that grows in an unstable linear system is limited by the effect of the nonlinear term, resulting in a limit cycle as noted in the figure. In terms of dynamics, this state of negative damping corresponds to an undesirable cyclic state where the restoring force (output) acts with a certain delay against the rotor motion (input). This delay between the input and output in the mechanical dynamics is sometimes qualified as "being in a negative spiral."

(4) **Parametrically excited oscillation** This is a type of self-excited vibration. The vibration amplitude may grow under certain conditions with triggered by slight changes of the parameters in time. The equation of motion includes time dependent mass, spring and damping parameters in a rotating system.
(5) **Nonlinear vibration** Vibrations caused by mechanisms involving nonlinear effects/elements (shown on the upper right and lower right of the figure) is also important. Typical examples include rubbing vibration between a rotor and a stator, and the limit cycle of oil whip appearing after the onset of self-excited vibration.

Figure 1.9 shows vibration phenomena and their causes classified according to the categories described previously. The variety of problems indicates the reality that the design optimization of actual machines is often difficult.

Wording related to mechanical vibration is summarized in ISO 2041 [7], which is definitely a prerequisite for mechanical engineering experts.

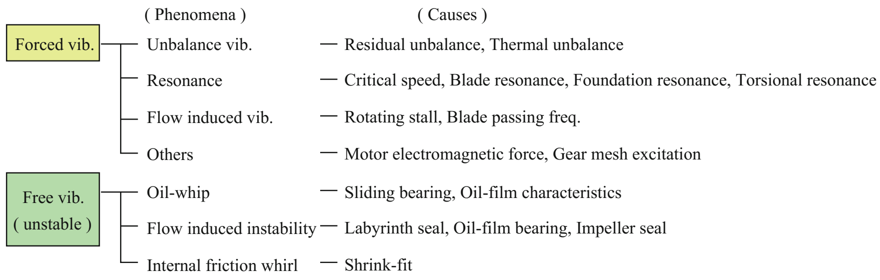


Fig. 1.9 Typical subjects to develop new compressors (*note* vib. = vibration, freq. = frequency)

1.4 Simplifying Complicated Phenomena

Since a rotating machine in a steady state has a high rotational energy, even small and localized problems tend to be amplified and lead to significant vibration problems for the entire system. It has been reported that a cupful of condensed water caused vibration of a whole turbine-generator set, and an elastic rubber coupling introduced for ease of centering rotating shafts may generate difficult vibrations in coupled machines.

However, it is true that most of these complicated vibration problems can be described by simple mechanical models, which permit identification of the causes. Appropriate countermeasures can actually eliminate vibration effectively. Engineers dealing with rotating machinery should have the ability to construct models that explain the mechanisms of how the resonance occurs or the self-excited vibration is generated, along with knowledge, experience and intuition.

Figure 1.10 illustrates an engineer contemplating various factors, such as unbalance and fluid force, causing vibration in a machine with use of a single-degree-of-freedom system. He/she tries to describe a story for solving the

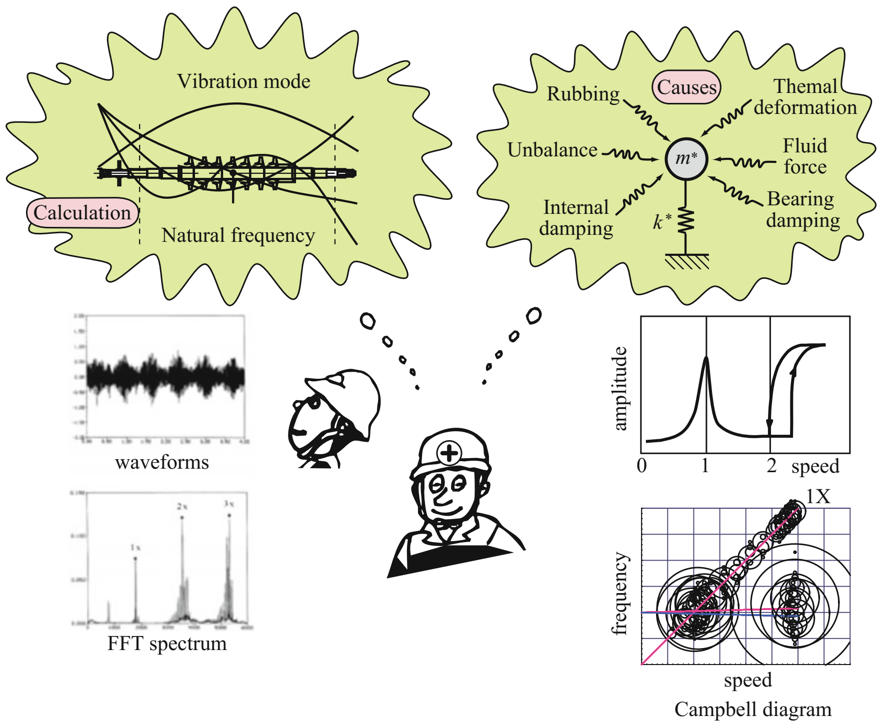


Fig. 1.10 Cause identification using simple model with measurement data

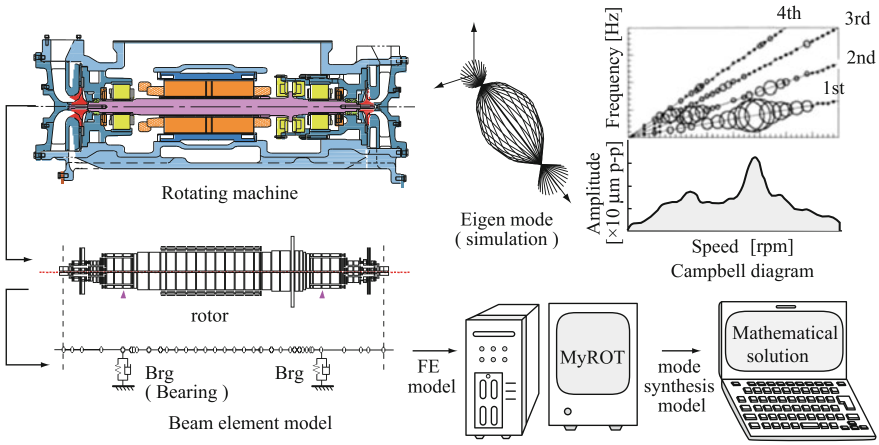


Fig. 1.11 Full model simulation using 1D-FE Method

problem by using a simple vibrating system in combination with knowledge, experience, other advice, and, of course, referring to our *v_BASE* database bank.

Simulation is another important technique in designing a rotor. Large-sized rotating machinery requires particularly detailed design calculations. Figure 1.11 shows an example of the modeling of a rotating shaft system with multiple disks by beam elements. This is a finite element method (1D-FEM) together with a vibration analysis program taking the dynamic stiffness of the bearings into account. The authors have developed MyROT (See Chap. 12), a vibration analysis program for rotating shaft systems based on 1D-FEM. Today, 1D-Finite element analysis is a standard tool in rotor design for calculations rather than 3D-FEM. 1D is enough as we do not need to “employ a steam-hammer to crack a nut?”

The 1D-FEM simulation is also a useful means to quantify the recommended countermeasures against vibration once the reason has been identified qualitatively by simple modeling. Results of simulations and countermeasures should be accumulated in a database as intellectual property for the future reference. Nevertheless it is said that vibration problems will not disappear forever, since no engineer can predetermine every cause of vibration when during the process of challenging machine design. We hope that the next generation never makes the same mistakes, at least in the field of mechanical vibration, especially rotordynamics.

Paper [8] is recommended for readers to review the state-of-art vibration technology in Japan.

Chapter 2
Basics for a Single-Degree-of-Freedom Rotor

Abstract This chapter specifies the definitions, calculation and measurement of basic vibration properties: natural frequency, modal damping, resonance and Q-value (Q-factor).

Basic properties featuring in a vibrating system, which are obtained from the free vibration waveform, are:

– Natural frequency f_n [Hz], or natural angular (or, circular) frequency $\omega_n = 2\pi f_n$[rad/s]
– Damping ratio ζ [–], or logarithmic decrement $\delta = 2\pi\zeta$ [–]

Using these parameters, the resonance caused by forced excitation can be predicted with

– Resonance frequency (critical speed in unbalanced vibration) = natural frequency f_n [Hz]
– Resonance sensitivity $Q = 1/(2\zeta)$ [–]

Since separation of resonance or reduction of the Q-value are fundamental requirements in the vibration design of rotating machinery, the placements of a natural frequency and the damping ratio are very important design indices.

Keywords Single-dof · Natural frequency · Damping ratio · Equivalent mass · Bode plot · Nyquest plot · Q-value

2.1 Free Vibrations

2.1.1 Natural Frequency

Undamped free vibration in a single-dof (degree-of-freedom) system consisting of a mass m [kg] and a spring constant k [N/m] is featured by the natural angular (circular) frequency ω_n, given by

$$\omega_n = \sqrt{k/m}\,[\mathrm{rad/s}] \tag{2.1}$$

O. Matsushita et al., *Vibrations of Rotating Machinery*,
Mathematics for Industry 16, DOI 10.1007/978-4-431-55456-1_2

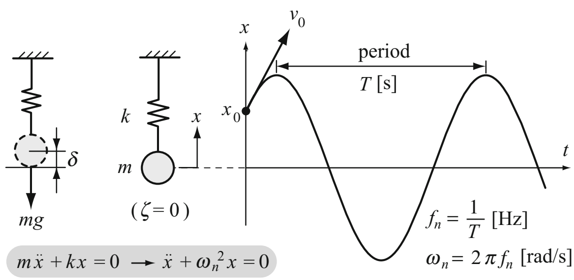


Fig. 2.1 (Undamped) Free vibration wave form

which can be converted to the natural frequency by

$$f_n = \omega_n/(2\pi)\ [\mathrm{Hz}] \tag{2.2}$$

The term "natural frequency" will be also used hereafter for the accurate term "natural circular frequency" according to the convention in industry (Fig. 2.1).

2.1.2 Calculation of Spring Constant

The spring constant k [N/m] is the reciprocal of deflection per unit load, and is determined from a static deflection calculation involving the strength of the material. Several formulae [9] to obtain spring constants are summarized in Table 2.1.

Table 2.1 Example of spring constant [9]

shaft system	shaft system
(1) cantilever $k = \frac{3EI}{l^3}$ (a) circular cross-section $I = \pi d^4/64$ (b) rectangular cross-section $I = bh^3/12$	(3) thrust of bar $k_a = \frac{EA}{l}$
(2) torsion of bar $k_t = \frac{GJ}{l}$ (a) $J = \frac{\pi d^4}{32}$ (b) $J = \left[\frac{1}{3} - 0.2\frac{b}{a}\left(1 - \frac{b^4}{12a^4}\right)\right]ab^3$	(4) simple supported beam at both sides $k = \frac{3EIl}{l_1^2 l_2^2}$ $l_1 = l_2 = l/2$ cantilever $k = \frac{48EI}{l^3}$ (5) overhang $k = \frac{3EI}{a^2(l+a)}$

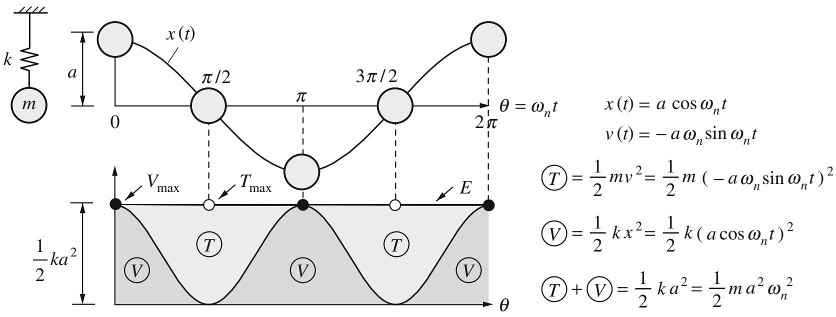


Fig. 2.2 The conservation of energy

2.1.3 Conservation of Energy

A system of moving objects is called conservative if the sum E of the kinetic energy T and potential energy (strain energy) V of the objects remains constant, i.e., $T + V$ = constant. Figure 2.2 shows that the kinetic energy and potential energy in a conservative system is complementary and their sum is thus constant. The maximum kinetic energy $T_{\max}$ is equal to the maximum potential energy $V_{\max}$:

$$T_{\max} = V_{\max} \tag{2.3}$$

This relationship determines the natural circular frequency of the system:

$$2T_{\max} = m(a\omega_n)^2,\ 2V_{\max} = ka^2 \rightarrow \omega_n = \sqrt{k/m} \tag{2.4}$$

Since free vibration is a movement with respect to a static equilibrium point and may be termed as dynamic behavior, the energy changes are represented as the deviations from the energy at that point. The equilibrium point in Fig. 2.2 is the point where gravity balances the reaction of the spring via the expansion of the spring. Therefore, the waveform of the free vibration under the gravity would remain the same even under zero gravity.

2.1.4 Mass Effects of Spring on Natural Frequency

Calculation of natural frequency is straightforward under an ideal condition of a massless spring. In actual cases, however, the natural frequency is lower than the ideal value due to the added mass of the spring. Ignoring this effect in design is dangerous because it often results in optimistic solutions having higher natural frequency than in reality.

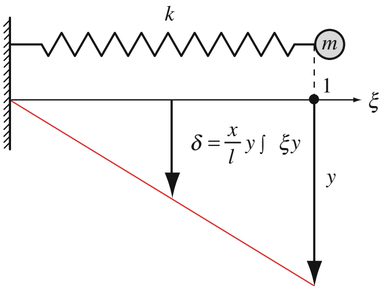


Fig. 2.3 Example of spring mass effect

In the mass m and spring k system shown in Fig. 2.3, let m_s be the mass of the coil spring and y the displacement of the tip of the spring. The displacement δ of any part of the spring can be represented by the linear equation

$$\delta = (x/l)y \equiv \xi y \tag{2.5}$$

The kinetic energy of the system is the sum of that for the mass at the tip and the distributed mass of the spring mass with a line density (mass per unit length) ρ_l:

$$T = \frac{m}{2}\dot{y}^2 + \frac{1}{2}\int_0^1 \rho_1 \dot{\delta}^2 dx = \dot{y}^2\left(\frac{m}{2} + \frac{m_s}{2}\int_0^1 \xi^2 d\xi\right) = \frac{1}{2}\left(m + \frac{m_s}{3}\right)\dot{y}^2 \tag{2.6}$$

Therefore, the formula for natural angular frequency ω_n becomes

$$\omega_n = \sqrt{\frac{k}{m + m_s/3}} \tag{2.7}$$

Thus, one third of the spring mass is added to reduce the natural frequency.

Example 2.1 Figure 2.4 shows several examples of added mass effect, when considering the spring stiffness of a uniform bar. Confirm the factor as the added mass of spring for each case.

Note: (a) $\delta(\xi) = \xi^2(3-\xi)2y$, (b) $\delta(\xi) = \xi y$, (c) $\delta(\xi) = \left[\xi\left(3-4\xi^2\right) -(1-2\xi)^3 U(\xi-12)\right]y$, (d) $\delta(\xi) = 4\left[(3-4\xi)\xi^2 - (1-2\xi)^3 U(\xi-12)\right]y$,
where the step function is $U(t) = 0$ for $t < 0$ and $U(t) = 1$ for $t \geq 0$.

Example 2.2 Figure 2.5 shows a cantilever, l in length and m_s in mass. Find the equivalent mass m_{eq} of the point located at a distance al from the supporting point for $a = \{1, 0.9, 0.8, 0.7\}$.