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Modern Derivatives Pricing and Credit Exposure Analysis

*Theory and Practice of CSA and
XVA Pricing, Exposure Simulation
and Backtesting*

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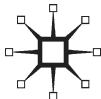
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Theory and Practice of CSA and XVA Pricing, Exposure Simulation and Backtesting

Roland Lichters, Roland Stamm, Donal Gallagher

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To our families

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Preface

The past ten years have seen an incredible change in the pricing of derivatives, a change which has not ended yet. One major driver for the change was the credit crisis which started in 2007 with the near bankruptcy of Bear Stearns, reached a first climax with the implosion of the US housing market and the banking world's downfall, and then turned into a sovereign debt crisis in Europe. While the worst seems to be over, the situation is far from normal: Central banks around the globe have injected highly material amounts of cash into a system which is still struggling to find its way back to growth and prosperity. The spectre of developed country sovereign default has become an ever present and unwelcome guest. As with many other crises, people learnt from this one that they had made serious mistakes in pricing OTC derivatives: Neglecting the credit risk and funding led to mispricing. The second major driver, which was itself triggered by the banks' heavy losses and the near-death experience of the entire financial system, is *regulation*. Banks are or soon will be forced to standardize derivatives more, clearing them through a Central Counterparty (CCP) whenever possible, thus increasing the transparency and, supposedly, robustness of the derivatives markets. Derivatives that are not cleared are penalized by increased capital requirements. Dealers are therefore caught in a bind: They either face increased funding costs due to the initial margin that has to be posted to the CCP, or higher capital costs if they trade over the counter.

A time-travelling expert for financial derivatives pricing from the year 2005 who ended up in 2015 would rub her eyes in disbelief at what has happened since then:

- The understanding of what the risk-free rate should be has changed completely. The tenor basis spread, which was a rather esoteric area of research, has turned into a new risk factor with the bankruptcy of a LIBOR bank.
- The counterparty credit risk of derivatives, which was noted but viewed to be of little relevance by the majority of banks, became a major driver of losses during the crisis and has found its way into new regulation and accounting standards in the form of value adjustments and additional capital charges.
- Features of a collateral agreement such as options regarding what collateral to post and in which currency, thresholds and minimum transfer amounts, call frequency, independent amounts, etc. have an impact on the valuation of derivatives. First of all, they turn a portfolio of individually priced trades into a bulk that has to be valued as one. Second, they make the valuation of such a portfolio unique.

- With the implosion of the repo market in the aftermath of Lehman's default came the realization that funding is not for free, and that hedging creates funding costs. The regulators enforce or strongly incentivize the usage of central clearing wherever possible. Initial margins, which are mandatory when dealing with central counterparties, hit both sides of a trade, increasing the funding requirements for hedges even further. These funding costs result in more value adjustments.
- The higher capital requirements and additional charges lead to extra costs for derivatives trading which make yet another value adjustment necessary.

As a consequence, a bank running a large book of derivatives has to be able to compute all these value adjustments – which are usually summarized under the acronym *XVA* – by simulating a large number of risk factors over a large time horizon in order to compute exposures, funding costs and capital charges for a portfolio. Capital for market risk is based on value-at-risk-like numbers, as is the initial margin; it is thus clear that on top of exposure at each time point on each simulation path, one has to compute risk numbers as well. As if that was not enough, it is also more and more important to compute the sensitivities of the adjustments to the input parameters.

The challenge in this computation is to control the following aspects:

1. Accuracy: Of course we want the numbers to be as accurate as necessary. That means that we need models that are complex enough to give good prices for time zero pricing. The accuracy is naturally limited by the uncertainties in the parameters that are fed into the models; see point 4 below.
2. Speed: Depending on the usage of the results – monthly accounting numbers, night batch for risk reporting, or near-time pricing for trading decisions – it is important that the calculations are done with the best possible performance. This need for speed obviously clashes with the requirement for accuracy.
3. Consistency: At least for internal models, the regulator has to approve the models used for exposure calculation, which means they also have to pass backtesting.
4. Uncertainty: Many of the input parameters, like future funding costs, funding strategies and capital requirements, are unknown at the time of pricing. Different assumptions can lead to largely different adjustment values. Key inputs such as probability of default and loss given defaults (or CDS spreads and recovery rates) may not be available for all derivative counterparties so that one has to resort to proxies (“similar” names) or historical estimates. This significant uncertainty puts the accuracy of pricing models for XVA into perspective and might justify relatively basic pricing approaches.
5. Model Risk: A sizeable derivatives portfolio contains a significant number of risk factors which, contrary to single trade pricing, have to be simulated in a simultaneous risk factor evolution, whose calibration is a numerical challenge.

The time horizon of the exposure calculation for a typical portfolio is measured in decades, sometimes as far as 50 years or more. The model risk inherent in each individual risk factor's evolution model accumulates at the portfolio level. Choosing simpler models to gain performance adds to that.

The aim of this book is to address the first three points in as much detail as possible. We present at least one model for each asset class – interest rates, foreign exchange, inflation, credit, equity and commodity – which satisfies the requirement for (reasonable) accuracy and yet allows for a well-performing implementation. For credit and inflation we present alternative models and discuss the advantages of each over the others. To boost performance further, we explain different approaches to prevent simulations or complex grid calculations embedded into the risk factor simulation (American Monte Carlo) or brute force computations of sensitivities by shifting each input risk factor individually (Algorithmic Differentiation). We also explain how to bridge the gap between risk-neutral pricing and real-world backtesting.

While it is impossible to get rid of the uncertainty and model risk inherent in long-term exposure simulations and XVA computations, we want to enable the reader to fully comprehend the assumptions and choices behind the models and the calculation approaches, so he or she can make an informed decision as to model choice, implementation and calibration.

The subject of this book makes it necessary to use mathematics extensively – never trust people who say they have a simple solution for a complex problem. We have put some background material into the large Appendix, but this is not a book from which to learn financial mathematics from scratch. For an introduction into the field of stochastic calculus we recommend the text book by Steven Shreve [136]; for an overview of the vast landscape of interest rate, foreign exchange, inflation and credit models, their calibration and the pricing of various financial products, see, for example, Brigo & Mercurio's text book [33], Hunt & Kennedy [91], or the comprehensive treatise on interest rate modelling by Andersen and Piterbarg [8] – to name a few, all important training grounds and sources of inspiration for the authors. Regarding the Monte Carlo simulation techniques we present here (and which we have used extensively in our professional life), we refer the reader to the texts by Glasserman [70] and Jaeckel [95]. This book can be seen as a sequel to the book [107] by Kenyon & Stamm, which gave an overview of many of the topics we present here. Nevertheless, this text is far more detailed as to the risk factor modelling, and of course includes the significant advances that derivatives pricing has seen over the past three years.

The book is organized in five parts. The first part, Discounting, describes the basis for the pricing of all financial instruments: How to compute the value of future cash flows, that is discounting. After a brief review of pre-crisis pricing, we explain the pricing under a central clearing regime (aka OIS discounting), for

full collateralization in a currency that is different from the trade currency (which we refer to as *global discounting*), and finally for collateral agreements that contain certain options (aka CSA discounting). The final Chapter 6 in Part I describes how Fair Value Hedge Accounting under IFRS may be handled in a multi-curve world.

In Part II, Credit and Debit Value Adjustment, we lay the foundations for understanding CVA and DVA. After the basic definitions we present examples of CVA for single, uncollateralized trades.

The third part, Risk Factor Evolution, is the largest and at the same time the most technical part of the book. It contains one chapter per asset class which describes in great detail how to model the risk factors for the purpose of exposure calculation. While there are many instances where we combine the respective asset class with interest rate and FX modelling, this part is still mostly devoted to the individual asset classes.

Part IV on XVA starts with a description of a framework that comprises all the various asset classes together. It then investigates the impact of netting and collateral on the exposure simulation. Chapter 18 then introduces American Monte Carlo, and Chapter 19 Algorithmic Differentiation. The final two chapters are devoted to the funding value adjustment (FVA) and the capital value adjustment (KVA) mentioned before.

The last part, Credit Risk, looks at the “classic” credit risk rather than the pricing component linked to counterparty credit risk that is CVA. This notion of credit risk deserves special attention because of the key role it plays for the regulators. One of the great challenges in this area is the combination of market-conforming pricing and the correctness of risk factor predictions. We look at credit portfolio products in Chapter 23, since products such as Asset Backed Securities (ABS) and Collateralized Loan Obligations (CLO) are enjoying greater popularity again after an extended pause following the 2008 events. We then move on to the Basel regulations regarding capital for derivatives in Chapter 24, and close with Chapter 25 on backtesting.

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