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ROLAND LICHTERS  
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# Modern Derivatives Pricing and Credit Exposure Analysis

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*Theory and Practice of CSA and  
XVA Pricing, Exposure Simulation  
and Backtesting*

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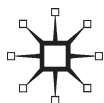
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**Theory and Practice of CSA  
and XVA Pricing, Exposure  
Simulation and Backtesting**

Roland Lichters, Roland Stamm, Donal Gallagher

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*To our families*

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# Contents

<i>List of Figures</i> . . . . .	xiv
<i>List of Tables</i> . . . . .	xxii
<i>Preface</i> . . . . .	xxv
<i>Acknowledgements</i> . . . . .	xxix
<i>List of Abbreviations and Symbols</i> . . . . .	xxx
<b>Part I Discounting</b> . . . . .	1
<b>1 Discounting Before the Crisis</b> . . . . .	3
1.1 The risk-free rate . . . . .	3
1.2 Pricing linear instruments . . . . .	4
1.2.1 Forward rate agreements . . . . .	4
1.2.2 Interest rate swaps . . . . .	6
1.2.3 FX forwards . . . . .	7
1.2.4 Tenor basis swaps . . . . .	7
1.2.5 Cross-currency basis swaps . . . . .	8
1.3 Curve building . . . . .	8
1.4 Pricing non-linear instruments . . . . .	9
1.4.1 Caps and floors . . . . .	9
1.4.2 Swaptions . . . . .	11
<b>2 What Changed with the Crisis</b> . . . . .	14
2.1 Basis products and spreads . . . . .	14
2.1.1 Tenor basis swaps . . . . .	14
2.1.2 Cross-currency basis swaps . . . . .	16
2.2 Collateralization . . . . .	17
<b>3 Clearing House Pricing</b> . . . . .	21
3.1 Introduction of central counterparties . . . . .	21
3.2 Margin requirements . . . . .	21
3.3 Building the OIS curve . . . . .	22
3.4 USD specialities . . . . .	24
3.5 Building the forward projection curves . . . . .	25
3.6 More USD specialities . . . . .	26
3.7 Example: implying the par asset swap spread . . . . .	27



3.8	Interpolation . . . . .	28
3.9	Pricing non-linear instruments . . . . .	29
3.9.1	European swaptions . . . . .	29
3.9.2	Bermudan swaptions . . . . .	30
3.10	Not all currencies are equal . . . . .	33
<b>4</b>	<b>Global Discounting . . . . .</b>	<b>35</b>
4.1	Collateralization in a foreign currency . . . . .	35
4.2	Non-rebalancing cross-currency swaps . . . . .	36
4.3	Rebalancing cross-currency swaps . . . . .	37
4.4	Examples: approximations of basis spreads . . . . .	40
4.4.1	Tenor basis spreads . . . . .	42
4.4.2	Flat cross-currency swaps . . . . .	42
4.4.3	OIS cross-currency basis spread . . . . .	42
4.4.4	LIBOR cross-currency basis spread . . . . .	43
<b>5</b>	<b>CSA Discounting . . . . .</b>	<b>44</b>
5.1	ISDA agreements and CSA complexities . . . . .	44
5.2	Currency options . . . . .	46
5.3	Negative overnight rates . . . . .	49
5.4	Other assets as collateral . . . . .	50
5.5	Thresholds and asymmetries . . . . .	51
5.6	Some thoughts on initial margin . . . . .	51
<b>6</b>	<b>Fair Value Hedge Accounting in a Multi-Curve World . . . . .</b>	<b>52</b>
6.1	Introduction . . . . .	52
6.2	Hedge effectiveness . . . . .	53
6.3	Single-curve valuation . . . . .	54
6.4	Multi-curve valuation . . . . .	59
<b>Part II Credit and Debit Value Adjustment . . . . .</b>		<b>67</b>
<b>7</b>	<b>Introduction . . . . .</b>	<b>69</b>
<b>8</b>	<b>Fundamentals . . . . .</b>	<b>71</b>
8.1	Unilateral CVA . . . . .	72
8.2	Bilateral CVA . . . . .	76
<b>9</b>	<b>Single Trade CVA . . . . .</b>	<b>79</b>
9.1	Interest rate swap . . . . .	80
9.1.1	Exercise within interest periods . . . . .	82
9.1.2	Amortizing swap . . . . .	84
9.1.3	A simple swap CVA model . . . . .	87
9.2	Cash-settled European options . . . . .	93
9.3	FX forward . . . . .	94
9.4	Cross-currency swap . . . . .	96
9.5	Rebalancing cross-currency swap . . . . .	101

<b>Part III Risk Factor Evolution</b> . . . . .	<b>103</b>
<b>10 Introduction – A Monte Carlo Framework</b> . . . . .	<b>105</b>
<b>11 Interest Rates</b> . . . . .	<b>107</b>
11.1 Linear Gauss Markov model . . . . .	107
11.1.1 Multiple curves . . . . .	108
11.1.2 Invariances . . . . .	109
11.1.3 Relation to the Hull-White model in $T$ -forward measure . . . . .	110
11.2 Products . . . . .	112
11.2.1 Zero bond option . . . . .	112
11.2.2 European swaption . . . . .	113
11.2.3 Bermudan swaption with deterministic basis . . . . .	118
11.2.4 Stochastic basis . . . . .	118
11.3 CSA discounting revisited . . . . .	125
11.4 Exposure evolution examples . . . . .	130
<b>12 Foreign Exchange</b> . . . . .	<b>134</b>
12.1 Cross-currency LGM . . . . .	134
12.2 Multi-currency LGM . . . . .	138
12.3 Calibration . . . . .	141
12.3.1 Interest rate processes . . . . .	141
12.3.2 FX processes . . . . .	141
12.3.3 Correlations . . . . .	143
12.4 Cross-currency basis . . . . .	147
12.5 Exposure evolution examples . . . . .	149
<b>13 Inflation</b> . . . . .	<b>155</b>
13.1 Products . . . . .	155
13.2 Jarrow-Yildirim model . . . . .	157
13.2.1 Calibration . . . . .	165
13.2.2 Foreign currency inflation . . . . .	166
13.3 Dodgson-Kainth model . . . . .	167
13.3.1 Calibration . . . . .	176
13.3.2 Foreign currency inflation . . . . .	177
13.4 Seasonality . . . . .	180
13.5 Exposure evolution examples . . . . .	181
<b>14 Equity and Commodity</b> . . . . .	<b>185</b>
14.1 Equity . . . . .	185
14.2 Commodity . . . . .	187
<b>15 Credit</b> . . . . .	<b>191</b>
15.1 Market . . . . .	192
15.2 Gaussian model . . . . .	196
15.2.1 Conclusion . . . . .	201

15.3	Cox-Ingersoll-Ross model . . . . .	201
15.3.1	CIR without jumps . . . . .	202
15.3.2	Relaxed feller constraint . . . . .	210
15.3.3	CDS spread distribution . . . . .	212
15.3.4	CIR with jumps: JCIR . . . . .	214
15.3.5	JCIR extension . . . . .	218
15.3.6	Examples: CDS CVA and wrong-way risk . . . . .	218
15.3.7	Conclusion . . . . .	220
15.4	Black-Karasinski model . . . . .	221
15.5	Peng-Kou model . . . . .	225
15.5.1	Review CDS and CDS option . . . . .	226
15.5.2	Compound Poisson process . . . . .	226
15.5.3	Compound Polya process . . . . .	227
15.5.4	Examples . . . . .	231
15.5.5	Conclusion . . . . .	233
<b>Part IV</b>	<b>XVA . . . . .</b>	<b>235</b>
<b>16</b>	<b>Cross-Asset Scenario Generation . . . . .</b>	<b>240</b>
16.1	Expectations and covariances . . . . .	241
16.2	Path generation . . . . .	249
16.3	Pseudo-random vs low discrepancy sequences . . . . .	251
16.4	Long-term interest rate simulation . . . . .	253
<b>17</b>	<b>Netting and Collateral . . . . .</b>	<b>259</b>
17.1	Netting . . . . .	260
17.1.1	Non-netted counterparty exposures . . . . .	260
17.1.2	Netting set exposures . . . . .	260
17.1.3	Generalized counterparty exposures . . . . .	260
17.2	Collateralization . . . . .	261
17.2.1	Collateralized netting set exposure . . . . .	261
17.2.2	CSA margining . . . . .	262
17.2.3	Margin settlement . . . . .	263
17.2.4	Interest accrual . . . . .	264
17.2.5	FX risk . . . . .	265
17.2.6	Collateral choice . . . . .	265
<b>18</b>	<b>Early Exercise and American Monte Carlo . . . . .</b>	<b>267</b>
18.1	American Monte Carlo . . . . .	268
18.2	Utilizing American Monte Carlo for CVA . . . . .	271
<b>19</b>	<b>CVA Risk and Algorithmic Differentiation . . . . .</b>	<b>274</b>
19.1	Algorithmic differentiation . . . . .	275
19.2	AD basics . . . . .	276

19.3	AD examples . . . . .	279
19.3.1	Vanilla swap and interest rate sensitivities . . . . .	279
19.3.2	European swaptions with deltas and vega cube . . . . .	280
19.4	Further applications of AD . . . . .	282
20	<b>FVA</b> . . . . .	283
20.1	A simple definition of FVA . . . . .	284
20.2	DVA = FBA? . . . . .	286
20.3	The role of the spreads . . . . .	288
20.4	The expectation approach . . . . .	289
20.5	The semi-replication approach . . . . .	292
20.6	CSA pricing revisited . . . . .	300
20.7	MVA . . . . .	302
20.8	Outlook . . . . .	305
21	<b>KVA</b> . . . . .	306
21.1	KVA by semi-replication . . . . .	306
21.2	Calculation of KVA . . . . .	308
21.3	Risk-warehousing and TVA . . . . .	308
	<b>Part V Credit Risk</b> . . . . .	311
22	<b>Introduction</b> . . . . .	313
22.1	Fundamentals . . . . .	314
22.2	Portfolio credit models . . . . .	317
22.2.1	Independent defaults . . . . .	317
22.2.2	Static default correlation modelling . . . . .	322
22.2.3	Dynamic default correlation modelling . . . . .	325
22.3	Industry portfolio credit models . . . . .	328
23	<b>Pricing Portfolio Credit Products</b> . . . . .	333
23.1	Introduction . . . . .	333
23.2	Synthetic portfolio credit derivatives . . . . .	334
23.2.1	Nth-to-default basket . . . . .	334
23.2.2	Synthetic collateralized debt obligation . . . . .	334
23.2.3	Synthetic CDO <sup>2</sup> . . . . .	338
23.3	Cashflow structures . . . . .	339
23.3.1	Introduction . . . . .	339
23.3.2	Cashflow CDO structures . . . . .	339
23.3.3	Overall pricing framework . . . . .	341
23.3.4	Pricing formulas . . . . .	342
23.4	Example results . . . . .	347
23.4.1	Test deal . . . . .	347

23.4.2	Test results . . . . .	348
23.4.3	Discussion of results . . . . .	348
<b>24</b>	<b>Credit Risk and Basel Capital for Derivatives . . . . .</b>	<b>351</b>
24.1	Introduction . . . . .	351
24.2	Potential future exposure . . . . .	352
24.3	Real-world measure . . . . .	353
24.3.1	Traditional approach . . . . .	353
24.3.2	Adjusted risk-neutral approach . . . . .	354
24.3.3	Joint measure model approach . . . . .	360
24.4	Standardized approach, CEM and SA-CCR . . . . .	363
24.4.1	Current standardized approach: CEM . . . . .	363
24.4.2	New standardized approach: SA-CCR . . . . .	364
24.5	Basel internal model approach . . . . .	368
24.6	Capital requirements for centrally cleared derivatives . . . . .	373
24.7	CVA capital charge . . . . .	374
24.7.1	The standard approach . . . . .	374
24.7.2	The IMM approach . . . . .	375
24.7.3	Mitigation of the CVA capital charge . . . . .	377
24.7.4	Exemptions . . . . .	377
<b>25</b>	<b>Backtesting . . . . .</b>	<b>380</b>
25.1	Introduction . . . . .	380
25.2	Backtest model framework . . . . .	380
25.2.1	Example: Anderson-Darling test . . . . .	383
25.3	RFE backtesting . . . . .	383
25.3.1	Creating the sample distance and sampling distribution . . . . .	384
25.3.2	Example I: Risk-neutral LGM . . . . .	385
25.3.3	Example II: Risk-neutral LGM with drift adjustment . . . . .	387
25.4	Portfolio backtesting . . . . .	389
25.5	Outlook . . . . .	390
<b>Part VI</b>	<b>Appendix . . . . .</b>	<b>393</b>
<b>A</b>	<b>The Change of Numeraire Toolkit . . . . .</b>	<b>395</b>
<b>B</b>	<b>The Feynman-Kac Connection . . . . .</b>	<b>398</b>
<b>C</b>	<b>The Black76 Formula . . . . .</b>	<b>400</b>
C.1	The standard Black76 formula . . . . .	400
C.2	The normal Black76 formula . . . . .	401
<b>D</b>	<b>Hull-White Model . . . . .</b>	<b>403</b>
D.1	Summary . . . . .	403
D.2	Bank account and forward measure . . . . .	407
D.3	Cross-currency Hull-White model . . . . .	410

<b>E</b>	<b>Linear Gauss Markov Model</b> . . . . .	<b>423</b>
E.1	One factor . . . . .	423
E.2	Two factors . . . . .	426
E.3	Cross-currency LGM . . . . .	430
<b>F</b>	<b>Dodgson-Kainth Model</b> . . . . .	<b>433</b>
F.1	Domestic currency inflation . . . . .	433
F.2	Foreign currency inflation . . . . .	435
<b>G</b>	<b>CIR Model with Jumps</b> . . . . .	<b>441</b>
<b>H</b>	<b>CDS and CDS Option: Filtration Switching and the PK Model</b> . . . . .	<b>446</b>
	<i>Bibliography</i> . . . . .	<b>450</b>
	<i>Index</i> . . . . .	<b>457</b>

# List of Figures

1.1	Replication of the forward rate . . . . .	5
1.2	The equivalence of discount first, convert after and convert first, discount after is proven by using the no-arbitrage principle . . . . .	7
2.1	The difference between the replicated and the quoted FRA rate for 3m×3m month EURIBOR forwards from 2004 to 2014, in percentage points . . . . .	15
4.1	Differences from the 3M EURIBOR swap rates depending on the collateral currency as of 30 September 2014 . . . . .	40
4.2	The approximate equivalence of $s^{L1,2}$ and $s^{LO,LO} + s^{O1,2}$ . . . . .	43
6.1	Fixed cash flows with annual periods vs semi-annual variable flows. The curly arrows depict the variable (stochastic) flows . . . . .	55
9.1	Vanilla swap exposure as given by Equation (9.5) as a function of time for payer swaps that are in-the-money (swap rate 0.9 times fair rate), at-the-money and out of the money (swap rate 1.1 times fair rate), notional 10,000, maturity 20 years, volatility 20%. Exposure is evaluated at the fixed leg's interest period start and end dates . . . . .	82
9.2	Cash flows on a swap with annual fixed flows vs quarterly variable flows. The curly arrows depict the variable (stochastic) flows . . . . .	83
9.3	Vanilla swap exposure as a function of time for an at-the-money payer and receiver swap with annual fixed frequency and quarterly floating frequency; other parameters as in 9.1; squared symbols follow the at-the-money curve from Figure 9.1. This shows that the CVA for the at-the-money payer and receiver swaps will differ due to the different pay and receive leg frequencies . . . . .	85
9.4	CVA approximations (9.17, 9.20) as a function of fixed rate $K$ for a receiver swap with notional 10,000, time to maturity 10; fair swap rate 3%, volatility 20%, hazard rate 1%, LGD 50% . . . . .	92
9.5	CVA approximations (9.17, 9.20) as a function of fixed rate $K$ for a payer swap with notional 10,000, time to maturity 10; fair swap rate 3%, volatility 20%, hazard rate 1%, LGD 50% . . . . .	92
9.6	CVA as a function of fixed rate $K$ for a receiver swap with notional 10,000, time to maturity 10; fair swap rate 3%, volatility 20%, hazard rate 1%, LGD 50%. This graph compares the “exact” CVA integral (9.14) to the “anticipated” and “postponed” discretizations where the exposure is evaluated at the beginning and the end of each time interval, respectively . . . . .	94

9.7	Exposure evolution (9.25) as a function of time for in-the-money (strike at 0.9 times forward), at-the-money and out-of-the-money (strike at 1.1 times forward) FX forwards, notional 10000, maturity two years, volatility 20% . . . . .	97
9.8	Exposure evolution (9.26) as a function of time for at-the-money cross currency swaps with 20-year maturity exchanging fixed payments on both legs, foreign notional 10000, domestic notional 12000, FX spot rate 1.2, flat foreign and domestic yield curve at same (zero rate) level 4%, Black volatility 20% . . . . .	98
9.9	Exposure evolution (9.26) as a function of time for cross-currency swaps as in Figure 9.8, but with flat foreign and domestic yield curve at 4% and 3% zero rate levels, respectively. The swaps are at-the-money initially and move in and out of the money, respectively, over time . . . . .	99
11.1	Mapping A: compounding the spread flows from the floating payment dates to the next fixed payment date using today's zero bond prices . . . . .	115
11.2	Mapping B: the spread flows are distributed to the two adjacent fixed payment dates using today's zero bond prices. Note that the discounted spreads use the inverse quotient from the compounded spreads (and, of course, a different index) . . . . .	116
11.3	Impact of stochastic basis spread on the valuation of vanilla European swaption and basis swaption with single-period underlyings as described in the text. The variable on the horizontal axis is basis spread volatility, i.e. square root of (11.21). The simplified model parameters are zero rate for discounting at 2%, zero rate for forward projection at 3%, $\lambda = 0.03$ and $\sigma = 0.01$ for both discount and forward curve. This means that basis spread volatility is $\propto \sqrt{1-\rho}$ according to (11.21) . . . . .	122
11.4	EONIA forward curve as of 30 September 2014 with negative rates up to two years. Under the CSA collateral is paid in EUR and based on EONIA – 10 bp . . . . .	127
11.5	Shifted EONIA forward curve compared to the forward curve from (11.27) with collateral floor; Hull-White parameters are $\lambda = 0.05$ and $\sigma = 0.004$ . . . . .	128
11.6	Comparison of the accurate Monte Carlo evaluation of the discount factor with Eonia floor (11.27) to the first order approximation (11.28) . . . . .	129
11.7	Single currency swap exposure evolution. Payment frequencies are annual on both fixed and floating legs. The swap is at-the-money, and the yield curve is flat. Note that the payer and receiver swap exposure graphs overlap . . . . .	131



11.8	Single currency payer and receiver swap exposure evolution for annual fixed and semi-annual floating payments. The symbols denote analytical exposure values (swaption prices) at fixed period start dates . . . . .	131
11.9	European swaption exposure, cash settlement, expiry in five years, swap term 5Y. Yield curve and swaption volatility structure are flat at 3% and 20%, respectively . . . . .	132
11.10	European swaption exposure with physical settlement, otherwise same parameters as in Figure 11.9 . . . . .	132
12.1	Exposure evolution for EUR/GBP FX forwards with (unusual) maturity in ten years, comparing three FX forwards which are at-the-money and 10% in and out of the money, respectively, at the start. The EUR and GBP yield curves are both kept flat at 3% so that the fair FX forward remains fixed through time, and we see the effect of widening of the FX spot rate distribution . . . . .	150
12.2	Typical FX option exposure evolution, the underlying is the EUR/GBP FX rate. The strikes are at-the-money and shifted slightly (10%) in and out of the money. Top: yield curve and FX volatility structure are flat at 3% and 10%, respectively. Bottom: flat yield curve, realistic ATM FX volatility structure ranging between 7% and 12% . . . . .	151
12.3	EUR/GBP cross-currency swaps exchanging quarterly fixed payments in EUR for quarterly 3m Libor payments in GBP. The trades start at-the-money. The upper graph shows a conventional cross-currency swap with fixed notionals on both legs, the lower graph is a resetting (mark-to-market) cross-currency swap where the EUR notional resets on each interest period start to the current value of the GBP notional in EUR . . . . .	154
13.1	Dodgson-Kainth model calibrated to CPI floors (at strikes 0%, 1% and 2%, respectively), market data as of 30 September 2014. Symbols denote market prices; the lines model prices . . . . .	177
13.2	Dodgson-Kainth model calibrated to CPI floors (at strike 1%), market data as of 30 September 2014. Symbols denote volatilities implied from market prices; lines are volatilities implied from model prices . . . . .	177
13.3	Exposure evolution for an at-the-money ZCII Swap with fixed rate 3%. Market data is as of end of March 2015; risk factor evolution model is Jarrow-Yildirim with parameters as in Table 13.6 . . . . .	182

13.4	Top: comparison of exposure evolutions of a Receiver CPI Swap to Receiver LPI Swap with varying cap strike, without floor or zero floor. Bottom: exposure evolution for a bespoke inflation-linked swap, exchanging a series of LPI-linked payments $N \times LPI(t)/LPI(t_0)$ for a series of fixed payments $N \times (1+r)^{t-t_0}$ , time to maturity about 22 years, annual payment amounts are broken into semi-annual payments on both legs. This can be decomposed into a series of LPI Swaps with increasing maturities. Risk factor evolution: Jarrow-Yildirim model with parameters as in Table 13.6 . . . . .	184
14.1	Crude Oil WTI futures prices as of 22 January 2015 . . . . .	189
14.2	Natural Gas futures prices as of 22 January 2015 . . . . .	189
15.1	History of short-dated index implied volatility since 2010 for options on CDX IG and HY, in comparison to VIX. Source: Credit Suisse, <a href="http://soberlook.com/2012/01/cdx-ig-swaption-vol-finally-followed.html">http://soberlook.com/2012/01/cdx-ig-swaption-vol-finally-followed.html</a> . . . . .	195
15.2	CIR density for model parameters in [33], p. 795 ( $a = 0.354201$ , $\theta = 0.00121853$ , $\sigma = 0.0238186$ , $\gamma_0 = 0.0181$ so that $2a\theta/\sigma^2 = 1.52154$ ) at time $t = 10$ , comparison between analytical density and histogram from Monte Carlo propagation using (15.17) . .	204
15.3	CIR density at time $t = 10$ for model parameters as in Figure 15.2 but varying $a$ such that $\epsilon = 2a\theta/\sigma^2$ takes values 1.52, 1.01, 0.51, 0.25 . .	204
15.4	CIR survival probability density at time $t = 10$ for the same parameters as in Figure 15.2 . . . . .	207
15.5	Expected exposure evolution for protection seller and buyer CDS, with notional 10m EUR and 10Y maturity, both at-the-money (premium approx. 2.4%) in a flat hazard rate curve environment (hazard rate 0.04, 0.4 recovery), compared to analytical exposure calculations (CDS option prices) at premium period start dates. The credit model used is CIR++ with shift extension, $a = 0.2$ , $\theta = \gamma_0 = 0.04$ and $\sigma$ chosen such that $\epsilon = 2a\theta/\sigma^2 = 2$ well above the Feller constraint $\epsilon > 1$ . The implied option volatilities are about 19%, highest at the shortest expiry. The simulation used 10k samples . . . . .	209
15.6	95% quantile exposure evolution for the instruments and model in Figure 15.5 . . . . .	210
15.7	Expected exposures for the same parameters as in Figure 15.5, except for $\epsilon = 2a\theta/\sigma^2 = 0.51$ . The implied option volatilities are about 33% rather than 19% with $\epsilon = 2$ . . . . .	211
15.8	Distribution of fair 5Y-CDS spreads in ten years time for the parameters in Figure 15.3, $\epsilon = 1.52$ . The position of the lower cutoff is computed as described in the text and indicated by a vertical line . . . . .	213

15.9	Distribution of fair 5Y-CDS spreads in ten years time for the parameters in Figure 15.3, $\epsilon = 1.01$ . The position of the lower cutoff is computed as described in the text and indicated by a vertical line . . . . .	214
15.10	Hazard densities at time $t = 5$ in the JCIR model for parameters $a = 0.1125$ (resp. $a = 0.2$ ), $\theta = 0.022$ , $\sigma = 0.07$ , $\gamma_0 = 0.005$ , i.e. $\epsilon \approx 1.01$ (resp. $\epsilon \approx 1.8$ ) and $\alpha = \gamma = 0.00, 0.05, 0.10, 0.15$ . The parameters result in the following fair spreads of a 5Y CDS with forward start in 6M: 58bp, 90bp, 171bp, 286bp (resp. 73bp, 101bp, 175bp, 280bp) . . . . .	216
15.11	Parameters $a = 0.1125$ , $\theta = 0.022$ , $\sigma = 0.07$ , $\gamma_0 = 0.005$ (so that $\epsilon \approx 1.01$ ) and varying $\alpha = \gamma = 0.00, 0.025, 0.05, 0.075, 0.15$ . Option expiry 6M, CDS term 5Y. The fair spread of the underlying CDS associated with the jump parameters is 58bp, 66bp, 89bp, 125bp and 285bp . . . . .	217
15.12	Unilateral CVA of a CDS as a function of the correlation between the hazard rate processes for counterparty and reference entity. The underlying CDS is at-the-money, has 10 Mio. notional and maturity in ten years. The process parameters are given in Table 15.2. The Monte Carlo evaluation used 1,000 samples per CVA calculation . . . . .	219
15.13	Linear default correlation as a function of time between three names (A, B, C) with flat hazard rate levels 0.02, 0.03 and 0.04, respectively, CIR++ model for all three processes with reversion speed 0.2 and high volatility $\epsilon = 0.51$ . The hazard rate correlations are 0.8 . . . . .	221
15.14	CDS option implied volatilities as a function of model $\sigma$ in the BK model for flat hazard rate curves at 50, 150 and 500 bp, respectively. Mean reversion speed $\alpha = 0.01$ . The option is struck at-the-money, it has 6M expiry, and the underlying CDS term is 5Y . . . . .	222
15.15	Standard deviation of the distribution of fair CDS spread ( $\ln K(t)$ ) as a function of model $\sigma$ in the BK model for flat hazard rate curves at 50, 150 and 500 bp flat, respectively. Mean reversion speed $\alpha = 0.01$ . The CDS is forward starting in six months and has a five-year term . . . . .	223
15.16	CDS Option implied volatilities as a function of standard deviation of the distribution of the fair CDS spread ( $\ln K(t)$ ) in the BK model for flat hazard rate curves at 50, 150 and 500 bp flat, respectively. Mean reversion speed $\alpha = 0.01$ . . . . .	224
15.17	CDS spread distributions at time $t = 0.5$ for hazard rate level 150 bp, $\alpha = 0.01$ and $\sigma = 0.5$ and $\sigma = 1.0$ , respectively . . . . .	225

15.18	Attainable implied volatility in the Peng-Kou model for flat hazard rate at 200bp, mean jump size $\gamma = 0.0045$ and mean jump intensity $\alpha\beta = 5$ . Parameter $\beta$ is varied while mean jump intensity is kept constant, i.e. $\alpha$ is varied accordingly. As $\beta$ increases, we hence increase the variance of the jump intensity distribution. The implied volatility refers to a CDS option with expiry in six months and a five-year term . . . . .	231
15.19	Distributions of fair 5y CDS at time $t = 5$ for the parameters in Figure 15.18 with $\beta = 1$ and $\beta = 4$ , respectively. The case $\beta = 1$ is associated with 40% implied volatility for a six-months expiry option on a five-year at-the-money CDS, $\beta = 4$ is associated with 105% implied volatility . . . . .	232
15.20	Attainable implied volatility in the Peng-Kou model as a function of $\beta$ for the cases summarized in Table 15.3 . . . . .	233
15.21	Distributions of fair 5y CDS at time $t = 5$ for cases 1/2 (top) and 7/8 (bottom) in Table 15.3 . . . . .	234
16.1	Convergence comparison for the expected exposure of 30Y maturity interest rate swap (slightly out-of-the-money) at horizon 10Y. We compare the reduction of root mean square error of the estimate vs number of paths on log-log scale (base 10) with maximum number of samples $N = 5000$ . The slope of the regressions is $-0.50$ for Monte Carlo with pseudo-random numbers (labelled MT), $-0.39$ for antithetic sampling and $-0.88$ for the quasi-Monte Carlo simulation using Sobol sequences . . . . .	251
16.2	Convergence comparison for the expected exposure of 30Y maturity interest rate swap as in Figure 16.1 but with switched pay and receive leg (slightly in-the-money). The exposure is again estimated at the 10Y horizon. Regression line slopes are in this example $-0.51$ , $-0.42$ and $-0.81$ , respectively . . . . .	252
16.3	Convergence comparison for the expected exposure of 15Y maturity interest rate swaps of varying moneyness. The exposure is again estimated at the 10Y horizon . . . . .	253
16.4	Monte Carlo estimate of (16.32) divided by the expected value $P(0, t)$ as a function of time . . . . .	255
16.5	Comparison of sampling region to the region of essential contributions to (16.32) . . . . .	256
16.6	$(H(t) - C)\sqrt{\zeta(t)}$ with different shifts applied, $C = 0$ , $C = H(30)$ , $C = H(50)$ . . . . .	257
16.7	Monte Carlo estimate of (16.32) divided by the expected value $P(0, t)$ as a function of time for $H(t)$ shifts by $-H(T)$ with $T = 50$ and $T = 30$ , respectively . . . . .	257

17.1	Interest rate swap NPV and collateral evolution for threshold 4 Mio. EUR, minimum transfer amount 0.5 Mio. EUR and margin period of risk two weeks . . . . .	263
17.2	Uncollateralized vs collateralized swap exposure with threshold 4 Mio. EUR and minimum transfer amount is 0.5 Mio. EUR (middle), zero threshold and minimum transfer amount (bottom). In both collateralized cases the margin period of risk is two weeks . . . .	265
18.1	Bermudan Swaption with three exercise dates on the LGM grid: conditional expectations of future values by convolution . . . . .	269
18.2	Regression at the second exercise of a Bermudan swaption with underlying swap start in 5Y, term 10Y, annual exercise dates. The AMC simulation used 1000 paths only (for presentation purposes), out of which about 50% are in-the-money at the second exercise time. The resulting quadratic regression polynomial we have fitted here is $f(x) = 0.05697 + 0.3081 \cdot x + 1.1578 \cdot x^2$ . . . . .	270
18.3	Bermudan swaption exposure evolution for the example described in the text, cash vs physical settlement . . . . .	273
20.1	Expected posted collateral compared to the relevant collateral ("Floor Nominal"), the average over negative rate scenarios. The example considered here is a 20Y Swap with 10 Mio. EUR nominal which pays 4% fixed and receives 6M Euribor. The model is calibrated to market data as of 30 June 2015 . . . . .	303
22.1	Example of a portfolio loss distribution illustrating expected loss, unexpected loss and risk capital associated with a high quantile of the loss distribution . . . . .	316
22.2	Relation between basket and tranche excess loss distributions, Equation (22.9) . . . . .	322
22.3	LHP loss probability density for $Q_i(t) = 0.3$ and correlations between 0 and 0.8 . . . . .	326
22.4	LHP excess basket loss probability for $Q_i(t) = 0.3$ and correlations between 0 and 1 . . . . .	326
23.1	Mezzanine CDO tranche payoff as difference between two equity tranches . . . . .	336
23.2	Available Interest at $t_2$ versus Pool Redemption at $t_1$ for a portfolio, and the Q-Q approximation curve. The point $(x = 0, y = 5.14M)$ corresponds to the case where none of the companies default and so there is available interest from every company but no redemptions (note there are no scheduled repayments from any of the bonds at or before $t_2$ ). The point $(x = 3.43M, y = 0)$ corresponds to the situation where every company has defaulted and so there is no interest available but there is an amount of redemption equal to the sum of all the recovery amounts: a recovery rate of $R^j = 0.01$ is assumed for all assets . . . . .	345

24.1	Exposure evolution for a vanilla fixed payer swap (top) and receiver swap (bottom) in EUR in the risk-neutral and real-world measure. The real-world measure evolution is generated from the risk-neutral evolution by means of the drift adjustment in Section 24.3.2 . . . . .	359
24.2	Exposure evolution for a vanilla fixed payer swap (top) and receiver swap (bottom) in USD in the risk-neutral and real-world measure. The real-world measure evolution is generated from the risk-neutral evolution by means of the drift adjustment in Section 24.3.2 . . . . .	360
24.3	NPV distribution and the location of expected exposure EE and potential future exposure (peak exposure) PFE . . . . .	367
24.4	Evolution of the expected exposure through time, $EE(t)$ , and corresponding $EPE(t)$ , $EEE(t)$ and $EEPE(t)$ . . . . .	367
24.5	NPV distribution at time $t = 5$ for an in-the-money forward starting single-period swap with start in $t = 9$ and maturity in $t = 10$ . The distributions are computed in three different risk-neutral measures – the bank account measure, the T-forward measure with $T = 10$ and the LGM measure. Due to calibration to the same market data, all three distributions agree on the expected NPV and expected $\max(NPV, 0)$ . However, quantile values clearly differ and are measure dependent . . . . .	371
25.1	Statistical distribution for the Anderson-Darling distance $d$ with 120 non-overlapping observations with 95% quantile at 2.49 and 99% quantile at 3.89, as computed using the asymptotic approximation from [9] . . . . .	384
25.2	Evolution of euro interest rates (continuously compounded zero rates with tenors between 6M and 20Y, derived from EUR Sswap curves) in the period from 1999 to 2015 . . . . .	385
25.3	PIT for the backtest of 10Y zero rates with monthly observations (1999–2015) of moves over a one-month horizon . . . . .	386
25.4	PIT for the backtest of 10Y zero rates as in Figure 25.3. Top: Period 1999–2006. Bottom: Period 2007–2015 . . . . .	388
25.5	PIT for the backtest of 10Y zero rates for the full period 1999–2015 using an LGM model with drift adjustment as described in Section 24.3.2 . . . . .	390
D.1	Evolution of terminal correlations in the cross-currency Hull-White model with parameters $\lambda_d = 0.03$ , $\sigma_d = 0.01$ , $\lambda_f = 0.015$ , $\sigma_f = 0.015$ , $\rho_{xd} = \rho_{xf} = \rho_{df} = 0.5$ . . . . .	418

# List of Tables

4.1	Cross-currency spreads for 3M USD LIBOR vs 3M IBOR in another currency as of 30 September 2014 . . . . .	38
4.2	3M EURIBOR swap rates as of 30 September 2014 . . . . .	39
4.3	Differences from the 3M EURIBOR swap rates depending on the collateral currency as of 30 September 2014 . . . . .	39
5.1	The various different cash flows from the example of a collateral currency option in the text. . . . .	48
11.1	“Cash flow” coefficients of the fixed and floating swap leg, with abbreviations $A_{i,j} = A_{t'_i,t'_j}$ . . . . .	114
11.2	The error resulting from the two mappings and the Jamshidian decomposition, compared with the exact results, in upfront basis points of notional . . . . .	118
11.3	CSA floor impact on vanilla swap prices in basis points of notional. The second column (“no floor”) shows prices without CSA floor, the third column (“with floor”) shows prices with CSA floor computed with full Monte Carlo evaluation of the floating leg, and the fifth column (“approx”) shows prices we get when we price the swap with the revised discount curve only. The CSA floor terminates in all cases after year five, yield curve data is as of 30 June 2015, Hull-White model parameters are $\lambda = 0.01$ , $\sigma = 0.005$ . . .	130
13.1	ZCII Cap prices as of 30 September 2015 in basis points . . . . .	161
13.2	ZCII Floor prices as of 30 September 2015 in basis points . . . . .	161
13.3	The nominal discount factors $P_n$ from the put-call parity using (13.10) for different strike pairs . . . . .	161
13.4	The real rate zero bond prices $P_n$ from the put-call parity using (13.11) for different strike pairs . . . . .	162
13.5	Pricing check for 6y maturity ZCII Cap and YOYII Caplet with unit notional and strikes 1.06 and 1.01, respectively. Model parameters: flat nominal term structure at continuously compounded zero rate of 4%, nominal LGM based on $\lambda_n = 0.03$ and $\sigma_n = 0.01$ , real rate zero rate 3%, real rate LGM based on $\lambda_r = 0.035$ and $\sigma_r = 0.007$ , CPI process volatility $\sigma_I = 0.03$ , correlations $\rho_{nr} = 0.4$ , $\rho_{nI} = 0.3$ and $\rho_{rI} = 0.5$ . . . . .	165

13.6	Jarrow-Yildirim model calibration using market data as of end January 2015. The ATM Cap prices are matched with the exception of the first Cap. YoY Caps are matched only approximately. Model parameters: $\rho_{nr} = 0.95$ , $\rho_{nc} = 0.5$ , $\rho_{rc} = 0.25$ ; $\alpha_r(t) = \sigma_r e^{\lambda_r t}$ with $\lambda_r = 0.03$ and $\sigma_r = 0.005$ ; $\sigma_c = 0.01$ , $H_r(t)$ piecewise linear . . . . .	166
15.1	Maximum implied volatility $\hat{\sigma}_M$ for a 6M expiry 5Y term CDS option for three choices of $\epsilon = 1.0, 0.5, 0.25$ and five choices of $y_0 = \theta = 0.01, \dots, 0.05$ . $\hat{a}$ is the mean reversion speed for which $\sigma_M = \hat{\sigma}_M$ , $s$ is the fair spread of the underlying 5Y CDS with 6M forward start. The rightmost column contains the cumulative probability for hazard rates up to 1bp at maturity of the underlying CDS, five years plus six months . . . . .	212
15.2	CIR++ model parameters for reference entity and counterparty processes. Parameters $\theta = y_0$ are chosen to match the hazard rate level, and the shift extension is used to ensure that the curve is strictly flat . . . . .	219
15.3	Attainable implied volatilities (six months expiry, five years at-the-money CDS term) in the Peng-Kou model for several choices of market hazard rate levels, mean jump size $\gamma$ , mean jump intensity $\alpha\beta$ and scale parameter $\beta$ . . . . .	232
19.1	Comparison of interest rate sensitivity results, bump/revalue vs AAD for the stylized vanilla swap portfolio described in the text . . . . .	281
19.2	Comparison of computation times for a vanilla swap portfolio. The “unrecorded” pricing slows down by a factor of about 2.8, and the total time for 30 AD Greeks is only faster than the traditional approach of bumping and repricing by a factor 1.3 . . . . .	281
19.3	Comparison of computation times for vanilla swaption portfolios priced with analytical Black-Scholes and Monte Carlo. The overall speedup of using AAD (“Total AD Greeks”) vs the traditional bump/revalue approach (“Bumped Greeks”) is about 40 in both cases. In the latter Monte Carlo valuation, we moreover find that the “Total AD Greeks” takes only about 4.7 times the computation time for unrecorded pricing with AD<double> . . . . .	282
20.1	Quarterly FVA results as reported by <i>risk.net</i> . . . . .	285
20.2	“Exact” vs. approximate floor value and relative error for various EUR payer swaps (fixed vs. 6M Euribor, 10 Mio. EUR nominal) in single-trade netting sets. Interest rates are modelled using Hull-White with constant model parameters $\lambda = \sigma = 0.01$ and yield curve data as of 30 June 2015. Both floor values are computed via Monte Carlo simulation using 10,000 identical short rate paths . . . . .	302
23.1	Test deal characteristics . . . . .	347
23.2	Market data scenario . . . . .	348



23.3 Price results and differences in basis points without IC and OC . . . . . 348

23.4 Price results with IC and OC . . . . . 349

23.5 Credit spread sensitivity in scenario 1 . . . . . 349

24.1 Add-on factor by product and time to maturity. Single currency interest rate swaps are assigned a zero add-on, i.e. judged on replacement cost basis only, if their maturity is less than one year. Forwards, swaps, purchased options and derivative contracts not covered in the columns above shall be treated as “Other Commodities”. Credit derivatives (total return swaps and credit default swaps) are treated separately with 5% and 10% add-ons depending on whether the reference obligation is regarded as “qualifying” (public sector entities (!), rated investment grade or approved by the regulator). Nth to default basket transactions are assigned an add-on based on the credit quality of nth lowest credit quality in the basket . . . . . 364

24.2 Supervisory factors and option volatilities from [22] . . . . . 369

25.1 Anderson-Darling statistics for various horizons and zero rate tenors using a risk-neutral LGM calibration and 1999–2015 history . . . 387

25.2 Anderson-Darling statistics for the early period 1999–2006. The number of 96 observations is constant here as we can use 2007 data for evaluating the realized moves over all horizons shown here . . . . . 389

25.3 Anderson-Darling statistics for the full period 1999–2015 using an LGM model with drift adjustment as described in Section 24.3.2 . . . . . 391

# Preface

The past ten years have seen an incredible change in the pricing of derivatives, a change which has not ended yet. One major driver for the change was the credit crisis which started in 2007 with the near bankruptcy of Bear Stearns, reached a first climax with the implosion of the US housing market and the banking world's downfall, and then turned into a sovereign debt crisis in Europe. While the worst seems to be over, the situation is far from normal: Central banks around the globe have injected highly material amounts of cash into a system which is still struggling to find its way back to growth and prosperity. The spectre of developed country sovereign default has become an ever present and unwelcome guest. As with many other crises, people learnt from this one that they had made serious mistakes in pricing OTC derivatives: Neglecting the credit risk and funding led to mispricing. The second major driver, which was itself triggered by the banks' heavy losses and the near-death experience of the entire financial system, is *regulation*. Banks are or soon will be forced to standardize derivatives more, clearing them through a Central Counterparty (CCP) whenever possible, thus increasing the transparency and, supposedly, robustness of the derivatives markets. Derivatives that are not cleared are penalized by increased capital requirements. Dealers are therefore caught in a bind: They either face increased funding costs due to the initial margin that has to be posted to the CCP, or higher capital costs if they trade over the counter.

A time-travelling expert for financial derivatives pricing from the year 2005 who ended up in 2015 would rub her eyes in disbelief at what has happened since then:

- The understanding of what the risk-free rate should be has changed completely. The tenor basis spread, which was a rather esoteric area of research, has turned into a new risk factor with the bankruptcy of a LIBOR bank.
- The counterparty credit risk of derivatives, which was noted but viewed to be of little relevance by the majority of banks, became a major driver of losses during the crisis and has found its way into new regulation and accounting standards in the form of value adjustments and additional capital charges.
- Features of a collateral agreement such as options regarding what collateral to post and in which currency, thresholds and minimum transfer amounts, call frequency, independent amounts, etc. have an impact on the valuation of derivatives. First of all, they turn a portfolio of individually priced trades into a bulk that has to be valued as one. Second, they make the valuation of such a portfolio unique.

- With the implosion of the repo market in the aftermath of Lehman's default came the realization that funding is not for free, and that hedging creates funding costs. The regulators enforce or strongly incentivize the usage of central clearing wherever possible. Initial margins, which are mandatory when dealing with central counterparties, hit both sides of a trade, increasing the funding requirements for hedges even further. These funding costs result in more value adjustments.
- The higher capital requirements and additional charges lead to extra costs for derivatives trading which make yet another value adjustment necessary.

As a consequence, a bank running a large book of derivatives has to be able to compute all these value adjustments – which are usually summarized under the acronym XVA – by simulating a large number of risk factors over a large time horizon in order to compute exposures, funding costs and capital charges for a portfolio. Capital for market risk is based on value-at-risk-like numbers, as is the initial margin; it is thus clear that on top of exposure at each time point on each simulation path, one has to compute risk numbers as well. As if that was not enough, it is also more and more important to compute the sensitivities of the adjustments to the input parameters.

The challenge in this computation is to control the following aspects:

1. Accuracy: Of course we want the numbers to be as accurate as necessary. That means that we need models that are complex enough to give good prices for time zero pricing. The accuracy is naturally limited by the uncertainties in the parameters that are fed into the models; see point 4 below.
2. Speed: Depending on the usage of the results – monthly accounting numbers, night batch for risk reporting, or near-time pricing for trading decisions – it is important that the calculations are done with the best possible performance. This need for speed obviously clashes with the requirement for accuracy.
3. Consistency: At least for internal models, the regulator has to approve the models used for exposure calculation, which means they also have to pass backtesting.
4. Uncertainty: Many of the input parameters, like future funding costs, funding strategies and capital requirements, are unknown at the time of pricing. Different assumptions can lead to largely different adjustment values. Key inputs such as probability of default and loss given defaults (or CDS spreads and recovery rates) may not be available for all derivative counterparties so that one has to resort to proxies (“similar” names) or historical estimates. This significant uncertainty puts the accuracy of pricing models for XVA into perspective and might justify relatively basic pricing approaches.
5. Model Risk: A sizeable derivatives portfolio contains a significant number of risk factors which, contrary to single trade pricing, have to be simulated in a simultaneous risk factor evolution, whose calibration is a numerical challenge.

The time horizon of the exposure calculation for a typical portfolio is measured in decades, sometimes as far as 50 years or more. The model risk inherent in each individual risk factor's evolution model accumulates at the portfolio level. Choosing simpler models to gain performance adds to that.

The aim of this book is to address the first three points in as much detail as possible. We present at least one model for each asset class – interest rates, foreign exchange, inflation, credit, equity and commodity – which satisfies the requirement for (reasonable) accuracy and yet allows for a well-performing implementation. For credit and inflation we present alternative models and discuss the advantages of each over the others. To boost performance further, we explain different approaches to prevent simulations or complex grid calculations embedded into the risk factor simulation (American Monte Carlo) or brute force computations of sensitivities by shifting each input risk factor individually (Algorithmic Differentiation). We also explain how to bridge the gap between risk-neutral pricing and real-world backtesting.

While it is impossible to get rid of the uncertainty and model risk inherent in long-term exposure simulations and XVA computations, we want to enable the reader to fully comprehend the assumptions and choices behind the models and the calculation approaches, so he or she can make an informed decision as to model choice, implementation and calibration.

The subject of this book makes it necessary to use mathematics extensively – never trust people who say they have a simple solution for a complex problem. We have put some background material into the large Appendix, but this is not a book from which to learn financial mathematics from scratch. For an introduction into the field of stochastic calculus we recommend the text book by Steven Shreve [136]; for an overview of the vast landscape of interest rate, foreign exchange, inflation and credit models, their calibration and the pricing of various financial products, see, for example, Brigo & Mercurio's text book [33], Hunt & Kennedy [91], or the comprehensive treatise on interest rate modelling by Andersen and Piterbarg [8] – to name a few, all important training grounds and sources of inspiration for the authors. Regarding the Monte Carlo simulation techniques we present here (and which we have used extensively in our professional life), we refer the reader to the texts by Glassermann [70] and Jaeckel [95]. This book can be seen as a sequel to the book [107] by Kenyon & Stamm, which gave an overview of many of the topics we present here. Nevertheless, this text is far more detailed as to the risk factor modelling, and of course includes the significant advances that derivatives pricing has seen over the past three years.

The book is organized in five parts. The first part, Discounting, describes the basis for the pricing of all financial instruments: How to compute the value of future cash flows, that is discounting. After a brief review of pre-crisis pricing, we explain the pricing under a central clearing regime (aka OIS discounting), for

full collateralization in a currency that is different from the trade currency (which we refer to as *global discounting*), and finally for collateral agreements that contain certain options (aka CSA discounting). The final Chapter 6 in Part I describes how Fair Value Hedge Accounting under IFRS may be handled in a multi-curve world.

In Part II, Credit and Debit Value Adjustment, we lay the foundations for understanding CVA and DVA. After the basic definitions we present examples of CVA for single, uncollateralized trades.

The third part, Risk Factor Evolution, is the largest and at the same time the most technical part of the book. It contains one chapter per asset class which describes in great detail how to model the risk factors for the purpose of exposure calculation. While there are many instances where we combine the respective asset class with interest rate and FX modelling, this part is still mostly devoted to the individual asset classes.

Part IV on XVA starts with a description of a framework that comprises all the various asset classes together. It then investigates the impact of netting and collateral on the exposure simulation. Chapter 18 then introduces American Monte Carlo, and Chapter 19 Algorithmic Differentiation. The final two chapters are devoted to the funding value adjustment (FVA) and the capital value adjustment (KVA) mentioned before.

The last part, Credit Risk, looks at the “classic” credit risk rather than the pricing component linked to counterparty credit risk that is CVA. This notion of credit risk deserves special attention because of the key role it plays for the regulators. One of the great challenges in this area is the combination of market-conforming pricing and the correctness of risk factor predictions. We look at credit portfolio products in Chapter 23, since products such as Asset Backed Securities (ABS) and Collateralized Loan Obligations (CLO) are enjoying greater popularity again after an extended pause following the 2008 events. We then move on to the Basel regulations regarding capital for derivatives in Chapter 24, and close with Chapter 25 on backtesting.

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