Quantitative Finance

Adil Reghai

Back to Basic Principles

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To my parents, my spouse Raja, and my daughters Rania, Soraya, Nesma & Amira

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Foreword I

The valuation of financial derivatives instruments, and to some extent the way they behave, rests on a numerous and complex set of mathematical models, grouped into what is called quantitative finance. Nowadays, it should be required that each and every one involved in financial markets has a good knowledge of quantitative finance. The problem is that the many books in this field are too theoretical, with an impressive degree of mathematical formalism, which needs a high degree of competence in mathematics and quantitative methods.

As the title suggests, from absolute basics to advanced trading techniques and P&L explanations, this book aims to explain both the theory and the practice of derivatives instruments valuation in clear and accessible terms. This is not a mathematical textbook, and long and difficult equations that are not understandable by the average person are avoided wherever possible.

Practitioners have lost faith in the ability of financial models to achieve their ultimate purpose, as those models are not at all precise in their application to the complex world of real financial markets. They need to question the hypotheses that are behind models and challenge them. The models themselves should be applied in practice only tentatively, with careful assessment of their limitations in each application and in their own validity domain, as these can vary significantly across time and place.

This is especially true after the global financial crisis. The financial world has changed a lot and witnesses a much faster pace of crisis. New regulations and their application in modeling have become a very important topic which is enforced through regulatory regimes, especially Basel III and fundamental review of the trading book for the banking industry.

This book nicely covers all these subjects from a pragmatic point of view. It shows that stochastic calculus alone is not enough for properly evaluating and hedging derivatives instruments. It insists on the importance of data analysis in parameters estimation and how this extra information can be helpful in the construction of the fair valuation and most importantly the right hedging strategy.

At first sight, this ambitious objective seems to be tough to achieve. As a matter of fact, Adil Reghai has done it and furthermore treated it in a very pedagogical way.

Finally, the reader should appreciate the overall aim of Adil's book, allowing for useful comparisons – some valuation methods appearing to be more robust and trustworthy than others – and often warning against the lack of reliability of some quantitative models, due to the hypotheses on which they are built.

For all these reasons, this book is a must have for all practitioners and should be a great success.

Cédric Dubois
Global Head of Structured Equity and Funds
Derivatives Trading Natixis SA London

Foreword II

Quantitative finance has been one of the most active research fields in the last 50 years. The initial push in mathematical finance was the 'portfolio selection' theory invented by H. Markowitz. This work was a first mathematical attempt towards trying to identify and understand the trade-offs between risks and returns that is the central problem in portfolio theory. The mathematical tools used to develop portfolio selection theory resulted in a rather elementary combination of the analysis of variance and multivariate linear regression. This model of assets price immediately leads to the optimization problem of choosing the portfolio with largest return subject to a given amount of risk (measured here rather simplistically as the variance of the portfolios, ignoring fat tails and non Gaussiannity). The work by H. Markowitz was considerably extended by W. Sharpe, who proposed using dimension reduction: instead of modeling the covariance of every pair of stocks, W. Sharpe proposed to identify only a few factors and to regress asset prices on these factors. For these pioneering works, H. Markowitz and W. Sharpe received 1990 Nobel prizes in economics, the first ever awarded to work in finance.

The work of Markowitz and Sharpe introduced mathematics into what was previously considered mainly as the 'art' of portfolio management. Since then, the mathematical sophistication of models for assets and markets increased quite rapidly. 'One-period' investment models were quickly replaced by continuous-time, Brownian-motion driven models of assets (among others, the emblematic geometric Brownian motion, soon followed by more complex diffusion models with stochastic volatilities); the quadratic risk measure was substituted by much more sophisticated increasing, concave utility functions. Most importantly, the early work of R. Merton and P. Samuelson (among others!) laid the foundations of thinking about assets prices, corporate finance, and exchange rate fluctuations with sophisticated mathematical (mostly stochastic) models.

A second revolution was triggered by the appearance in the mid 1980s of the market of derivative securities. The pioneering work in this domain is due to F. Black, R. Merton and M. Scholes, who laid down the theory and the methods to price the value of an option to buy a share of a stock at a given maturity and strike price (a European call option). The basic idea to compute the price of this derivative security (etymologically, which derives its value from the underlying asset) is to construct a hedging portfolio, defined as a combination of a (fraction) of the share on which the call is written and a riskless asset, to replicate the option. The hedging portfolio is constructed in such a way that at any time the option should be worth as much as the portfolio, to eliminate arbitrage opportunities. Based on the principle of absence of arbitrage (which is highly controversial), Black and Scholes have derived an explicit expression for the price of a European call. This work, soon extended by R. Merton, was also awarded with a Nobel prize in economics.

This initial push was immediately followed by an amazing number of works providing theoretical foundations for the financial world. Most of the early developments were based on stochastic calculus making heavy use of Itô Calculus and continuous diffusion. Later, the much more demanding theory of Levy processes and jump-diffusion enters into the scene. High-frequency trading and the modeling of market microstructure naturally draws attention to discrete-event stochastic models like interacting point processes to model, at a very fine time scale, order book dynamic. Of course, all these financial models need to be calibrated to the observed prices of assets and derivatives securities using sometimes very advanced statistical estimation methods. Derivative securities pricing and arbitrage strategy also rely on accurate and fast numerical analysis. This has naturally led to an escalation in the level of mathematics (probability, statistics, partial differential equation and numerical analysis) used in finance, to a point where research problems in mathematics are completely intertwined with the theory and the practice of finance. This has also caused a proliferation of financial instruments (barrier, binary, American options, among many others), whose complexity and sophistication sometimes defies understanding! There are only a few domains in science in which there exists such an intense cross-fertilization between theory and applications.

As masterfully pointed out by the author, despite their considerable mathematical sophistications the models used today in financial mathematics are in general fragile, being based on over-simplistic models of assets and market dynamics. For example, most financial models will still presuppose the absence of arbitrage opportunities (no trading opportunities yield profit without any downside risk), which implies the existence of an equivalent martingale measure under which discounted prices have the martingale property. These are of course the very beginning of a long list of half plausible assumptions, unverifiable axioms, which in practice have proved to be sometimes devastatingly erroneous! The book proposes novel approaches to pricing and hedging financial products safely and effectively, replacing elegant but erroneous assumptions with more realistic ones, taking into account that we live in a world of more frequent crises and fatter tail risks. Based on many years of academic research and practical trading experience, Adil Reghai convincingly assembles both proven and new approaches to answer the challenges of today's highly volatile financial markets. These models not only take their inspiration from probability and statistics, but also from physics and engineering.

This book very nicely offers an original explanation covering the field. It proposes amazing new insights and techniques and opens exciting avenues of research.

The vision developed in this book, questioning the foundations of mathematical finance, is truly unique. I have learned a lot reading this amazing book and I sincerely hope that the reader will enjoy reading these pages as much as I did!

Eric Moulines
Professor at Télécom-Paris Tech

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I would like to thank Nicole El Karoui, who taught me so much, and Jean Michel Lasry, who introduced me to this fascinating field and with whom I started my career. Also I would like to thank Christophe Chazot. He was my boss back in the late 1990s in London. One day, he asked me to step in for him to give a lecture for the Masters of Finance programme in Nice. I accepted without really knowing what I was in for. This programme is unique. Most of the teachers are practitioners, while students are carefully chosen from the top engineering and business schools. The aim is to provide them with insights into the business and ensure their familiarity with the requisite tools is operative by the end of their training. I am delighted to have taken part in this unique experience, especially as this Masters is currently ranked among the top 10 in the world by the *Wall Street Journal*. I share this success with my colleagues, especially Tarek Amyuni, head of the Specialized Masters in Financial Markets, Innovations and Technologies (FMIT) at SKEMA. I would like to thank him as a friend and colleague who devotes huge energy to this programme and to working in the interest of his students.

Quant work is collaborative. Progress is achieved by talking to people, reading papers and testing. I would like to thank many of my colleagues and peers. In particular, I would like to mention here some people whom I know well and who have unconsciously shaped the way I present this book.

Nicole El Karoui: for her friendship and advice over the years. She was in favour of mixing stochastics and statistics. On my undergraduate course she insisted on avoiding the perfect fit paradigm in favour of studying errors of fit when using statistical models.

Bruno Dupire: for his friendship and timely explanations and exchanges. I always enjoy talking with Bruno. He is innovative, detail-oriented, focused on creating value, yet modest.

Jim Gatheral: for sharing ideas and concepts and his enlightening explanations on several subjects. Among other things he offers is an excellent best starting point to understand any topic.

Lorenzo Bergomi: as a friend who has brought so much to the discipline. I bear in mind one of his descriptions of a ‘quant’: “quants are like doctors, they need to prepare the medicine before the patient is sick. If they bring the medicine too late the patient has suffered for too long and it won’t work. If the medicine is provided in advance, the patient is unconscious and will not take it. A good quant is one who has the medicine and waits until the patient suffers just a little before delivering it”.

Philippe Balland: for teaching me so much when I was in his team at Merrill Lynch. He taught me how to separate technical knowledge (stochastic calculus) from transient issues (statistical nature of the knowledge). Thanks to his teaching and years of work afterwards, I have been able to reconcile the two areas of knowledge in a clear way and effectively produce models for pricing and hedging. I also learnt how to move around with my laptop and undertake fascinating work on the move (I'm not sure my wife much appreciated this!).

Alex Langnau: for a unique and timely friendship beyond description, and for sharing thoughts and exchanging ideas and concepts. His thinking, derived from his background in physics, has profoundly influenced my own. Thanks to him, I have been able to complete the bridge between derivatives quants and investment quants.

I would also like to thank Mohamed El Babsiri, Taoufik Cherif and Michel Crouhy for encouraging me to make the effort to write a book from my lecture notes.

My thanks to many who have helped me in my thinking, exchanged with me on different issues: this book is a synthesis of our common knowledge. Any errors are, however, mine.

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Finally, my thanks go to all the students over the years who have attended my classes. They asked questions and helped me progress.

1 Financial Modeling

A Introduction

At the time of writing, the term **model** has been widely overused and misused. The US subprime financial crisis in 2008 and 2009 and its subsequent impacts on the rest of the economy have prompted some journalists and magazines to launch unfair attacks on '**quants**', that is the designers of models.

This book presents a clear and systematic approach which addresses the problems of modeling. It discusses old and new models used on an everyday basis by us as practitioners in derivatives and also in quantitative investment.

The proposed approach is incremental as in the exact sciences.

In particular, we show the added value provided by the models and their use in practice.

Each model has a domain of validity and new models, with their added complexity, are only used when absolutely necessary, that is when older models fail to explain the trader's or investor's PnL.

We insist on the philosophy of modeling in the field of derivatives and asset management and offer a valuation and estimation methodology which copes with the non-Gaussian nature of returns , generally known as the Smile.

We suggest a methodology inspired from the exact sciences, allowing students, academics, practitioners and a general readership to understand developments in this field and, more importantly, **how to adapt to new situations (new products, markets, regulations) by choosing the right model for the right task at the right time**.

Derivatives pricing can be seen as extrapolation of the present (fit existing tradable products like vanilla options to price new ones like barrier options) and that quantitative investment is an extrapolation of the past (fit past patterns to predict the future). In our approach, we argue that derivatives pricing relies on extrapolation of both the present and the past. Our method allows the correct pricing of hedging instruments, as well as control of the **PnL** due to the variability of