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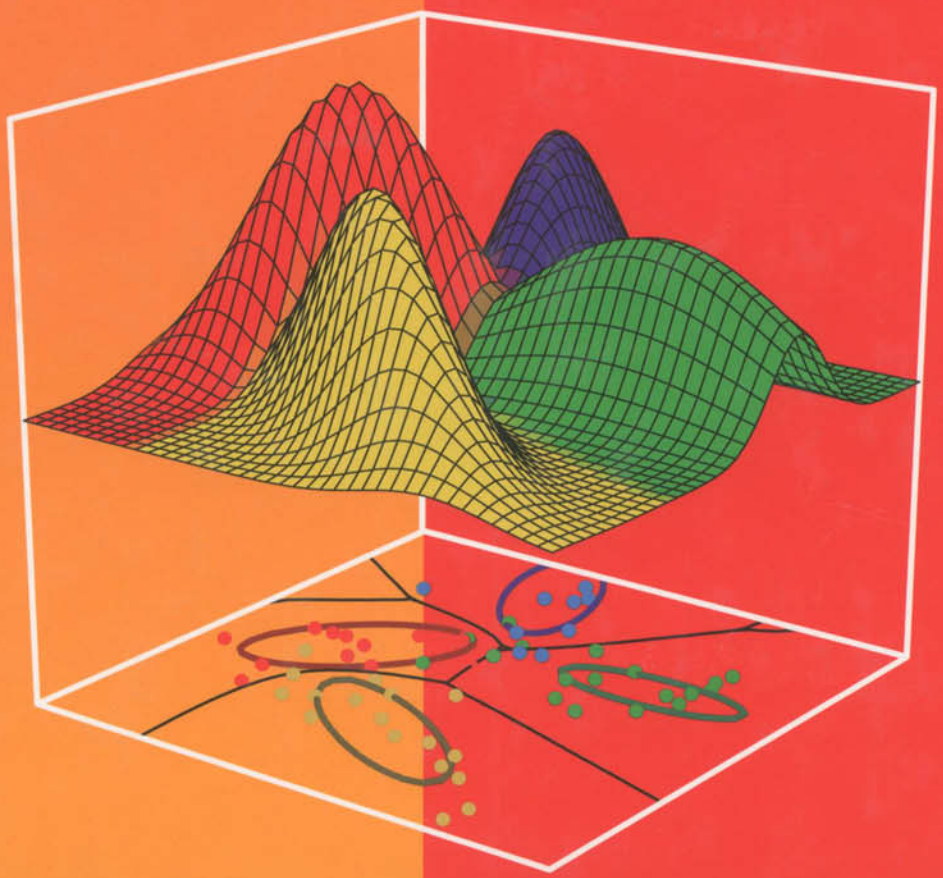
Pattern Classification



Second Edition

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To

C. A. Rosen

and

C. W. Stork

CONTENTS

PREFACE

Our purpose in writing this second edition—more than a quarter century after the original—remains the same: to give a systematic account of the major topics in pattern recognition, based whenever possible on fundamental principles. We believe that this provides the required foundation for solving problems in more specialized application areas such as speech recognition, optical character recognition, or signal classification. Readers of the first edition often asked why we combined in one book a Part I on pattern classification with a Part II on scene analysis. At the time, we could reply that classification theory was the most important domain-independent theory of pattern recognition, and that scene analysis was the only important application domain. Moreover, in 1973 it was still possible to provide an exposition of the major topics in pattern classification and scene analysis without being superficial. In the intervening years, the explosion of activity in both the theory and practice of pattern recognition has made this view untenable. Knowing that we had to make a choice, we decided to focus our attention on classification theory, leaving the treatment of applications to the books that specialize on particular application domains. Since 1973, there has been an immense wealth of effort, and in many cases progress, on the topics we addressed in the first edition. The pace of progress in algorithms for learning and pattern recognition has been exceeded only by the improvements in computer hardware. Some of the outstanding problems acknowledged in the first edition have been solved, whereas others remain as frustrating as ever. Taken with the manifest usefulness of pattern recognition, this makes the field extremely vigorous and exciting.

While we wrote then that pattern recognition might appear to be a rather specialized topic, it is now abundantly clear that pattern recognition is an immensely broad subject, with applications in fields as diverse as handwriting and gesture recognition, lipreading, geological analysis, document searching, and the recognition of bubble chamber tracks of subatomic particles; it is central to a host of human-machine interface problems, such as pen-based computing. The size of the current volume is a testament to the body of established theory. Whereas we expect that most of our readers will be interested in developing pattern recognition systems, perhaps a few will be active in understanding existing pattern recognition systems, most notably human and animal nervous systems. To address the biological roots of pattern recognition would of course be beyond the scope of this book. Nevertheless, because neurobiologists and psychologists interested in pattern recognition in the natural world continue to rely on more advanced mathematics and theory, they too may profit from the material presented here.

Despite the existence of a number of excellent books that focus on a small set of specific *techniques*, we feel that there is still a strong need for a book such as ours, which takes a somewhat different approach. Rather than focus on a specific technique such as neural networks, we address a specific *class of problems*—pattern recognition problems—and consider the wealth of different techniques that can be applied to it. Students and practitioners typically have a particular problem and need to know which technique is best suited for their needs and goals. In contrast, books that focus on neural networks may not explain decision trees, or nearest-neighbor methods, or many other classifiers to the depth required by the pattern recognition practitioner who must decide among the various alternatives. To avoid this problem, we often discuss the relative strengths and weaknesses of various classification techniques.

These developments demanded a unified presentation in an updated edition of Part I of the original book. We have tried not only to expand but also to improve the text in a number of ways:

New Material. The text has been brought up to date with chapters on pattern recognition topics that have, over the past decade or so, proven to be of value: neural networks, stochastic methods, and some topics in the theory of learning, to name a few. While the book continues to stress methods that are statistical at root, for completeness we have included material on syntactic methods as well. "Classical" material has been included, such as Hidden Markov models, model selection, combining classifiers, and so forth.

Examples. Throughout the text we have included worked examples, usually containing data and methods simple enough that no tedious calculations are required, yet complex enough to illustrate important points. These are meant to impart intuition, clarify the ideas in the text, and to help students solve the homework problems.

Algorithms. Some pattern recognition or learning techniques are best explained with the help of algorithms, and thus we have included several throughout the book. These are meant for clarification, of course; they provide only the skeleton of structure needed for a full computer program. We assume that every reader is familiar with such pseudocode, or can understand it from context here.

Starred Sections. The starred sections (*) are a bit more specialized, and they are typically expansions upon other material. Starred sections are generally not needed to understand subsequent unstarred sections, and thus they can be skipped on first reading.

Computer Exercises. These are not specific to any language or system, and thus can be done in the language or style the student finds most comfortable.

Problems. New homework problems have been added, organized by the earliest section where the material is covered. In addition, in response to popular demand, a Solutions Manual has been prepared to help instructors who adopt this book for courses.

Chapter Summaries. Chapter summaries are included to highlight the most important concepts covered in the rest of the text.

Graphics. We have gone to great lengths to produce a large number of high-quality figures and graphics to illustrate our points. Some of these required extensive

calculations, selection, and reselection of parameters to best illustrate the concepts at hand. Study the figures carefully! The book's illustrations are available in Adobe Acrobat format that can be used by faculty adopting this book for courses to create presentations for lectures. The files can be accessed through a standard web browser or an ftp client program at the Wiley STM ftp area at:

ftp://ftp.wiley.com/public/sci_tech_med/pattern/

The files can also be accessed from a link on the Wiley Electrical Engineering software supplements page at:

http://www.wiley.com/products/subject/engineering/electrical/software_supplem_elec_eng.html

Mathematical Appendixes. It comes as no surprise that students do not have the same mathematical background, and for this reason we have included mathematical appendixes on the foundations needed for the book. We have striven to use clear notation throughout—rich enough to cover the key properties, yet simple enough for easy readability. The list of symbols in the Appendix should help those readers who dip into an isolated section that uses notation developed much earlier.

This book surely contains enough material to fill a two-semester upper-division or graduate course; alternatively, with careful selection of topics, faculty can fashion a one-semester course. A one-semester course could be based on Chapters 1–6, 9 and 10 (most of the material from the first edition, augmented by neural networks and machine learning), with or without the material from the starred sections.

Because of the explosion in research developments, our historical remarks at the end of most chapters are necessarily cursory and somewhat idiosyncratic. Our goal has been to stress important references that help the reader rather than to document the complete historical record and acknowledge, praise, and cite the established researcher. The Bibliography sections contain some valuable references that are not explicitly cited in the body of the text. Readers should also scan through the titles in the Bibliography sections for references of interest.

This book could never have been written without the support and assistance of several institutions. First and foremost is of course Ricoh Innovations (DGS and PEH). Its support of such a long-range and broadly educational project as this book—amidst the rough and tumble world of industry and its never-ending need for products and innovation—is proof positive of a wonderful environment and a rare and enlightened leadership. The enthusiastic support of Morio Onoe, who was Director of Research, Ricoh Company Ltd. when we began our writing efforts, is gratefully acknowledged. Likewise, San Jose State University (ROD), Stanford University (Departments of Electrical Engineering, Statistics and Psychology), The University of California, Berkeley Extension, The International Institute of Advanced Scientific Studies, the Niels Bohr Institute, and the Santa Fe Institute (DGS) all provided a temporary home during the writing of this book. Our sincere gratitude goes to all.

Deep thanks go to Stanford graduate students Regis Van Steenkiste, Chuck Lam and Chris Overton who helped immensely on figure preparation and to Sudeshna Adak, who helped in solving homework problems. Colleagues at Ricoh aided in numerous ways; Kathrin Berkner, Michael Gormish, Maya Gupta, Jonathan Hull

and Greg Wolff deserve special thanks, as does research librarian Rowan Fairgrove, who efficiently found obscure references, including the first names of a few authors. The book has been used in manuscript form in several courses at Stanford University and San Jose State University, and the feedback from students has been invaluable. Numerous faculty and scientific colleagues have sent us many suggestions and caught many errors. The following such commentators warrant special mention: Leo Breiman, David Cooper, Lawrence Fogel, Gary Ford, Isabelle Guyon, Robert Jacobs, Dennis Kibler, Scott Kirkpatrick, Daphne Koller, Benny Lautrup, Nick Littlestone, Amir Najmi, Art Owen, Rosalind Picard, J. Ross Quinlan, Cullen Schaffer, and David Wolpert. Specialist reviewers—Alex Pentland (1), Giovanni Parmigiani (2), Peter Cheeseman (3), Godfried Toussaint (4), Padhraic Smyth (5), Yann Le Cun (6), Emile Aarts (7), Horst Bunke (8), Tom Dietterich (9), Anil Jain (10), and Rao Vemuri (Appendix)—focused on single chapters (as indicated by the numbers in parentheses); their perceptive comments were often enlightening and improved the text in numerous ways. (Nevertheless, we are responsible for any errors that remain.) George Telecki, our editor, gave the needed encouragement and support, and he refrained from complaining as one manuscript deadline after another passed. He, and indeed all the folk at Wiley, were extremely helpful and professional. Finally, deep thanks go to Nancy, Alex, and Olivia Stork for understanding and patience.

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RICHARD O. DUDA
PETER E. HART

Menlo Park, California
August, 2000

PATTERN CLASSIFICATION

CHAPTER

1

INTRODUCTION

The ease with which we recognize a face, understand spoken words, read handwritten characters, identify our car keys in our pocket by feel, and decide whether an apple is ripe by its smell belies the astoundingly complex processes that underlie these acts of pattern recognition. Pattern recognition—the act of taking in raw data and making an action based on the "category" of the pattern—has been crucial for our survival, and over the past tens of millions of years we have evolved highly sophisticated neural and cognitive systems for such tasks.

1.1 MACHINE PERCEPTION

It is natural that we should seek to design and build machines that can recognize patterns. From automated speech recognition, fingerprint identification, optical character recognition, DNA sequence identification, and much more, it is clear that reliable, accurate pattern recognition by machine would be immensely useful. Moreover, in solving the myriad problems required to build such systems, we gain deeper understanding and appreciation for pattern recognition systems in the natural world—most particularly in humans. For some problems, such as speech and visual recognition, our design efforts may in fact be influenced by knowledge of how these are solved in nature, both in the algorithms we employ and in the design of special-purpose hardware.

1.2 AN EXAMPLE

To illustrate the complexity of some of the types of problems involved, let us consider the following imaginary and somewhat fanciful example. Suppose that a fish-packing plant wants to automate the process of sorting incoming fish on a conveyor belt according to species. As a pilot project it is decided to try to separate sea bass from salmon using optical sensing. We set up a camera, take some sample images, and begin to note some physical differences between the two types of fish—length, lightness, width, number and shape of fins, position of the mouth, and so on—and these suggest *features* to explore for use in our classifier. We also notice noise or

FEATURE

variations in the images—variations in lighting, position of the fish on the conveyor, even "static" due to the electronics of the camera itself.

MODEL

Given that there truly are differences between the population of sea bass and that of salmon, we view them as having different *models*—different descriptions, which are typically mathematical in form. The overarching goal and approach in pattern classification is to hypothesize the class of these models, process the sensed data to eliminate noise (not due to the models), and for any sensed pattern choose the model that corresponds best. Any techniques that further this aim should be in the conceptual toolbox of the designer of pattern recognition systems.

PREPROCESSING
SEGMENTATION

FEATURE EXTRACTION

Our prototype system to perform this very specific task might well have the form shown in Fig. 1.1. First the camera captures an image of the fish. Next, the camera's signals are *preprocessed* to simplify subsequent operations without losing relevant information. In particular, we might use a *segmentation* operation in which the images of different fish are somehow isolated from one another and from the background. The information from a single fish is then sent to a *feature extractor*, whose purpose is to reduce the data by measuring certain "features" or "properties."

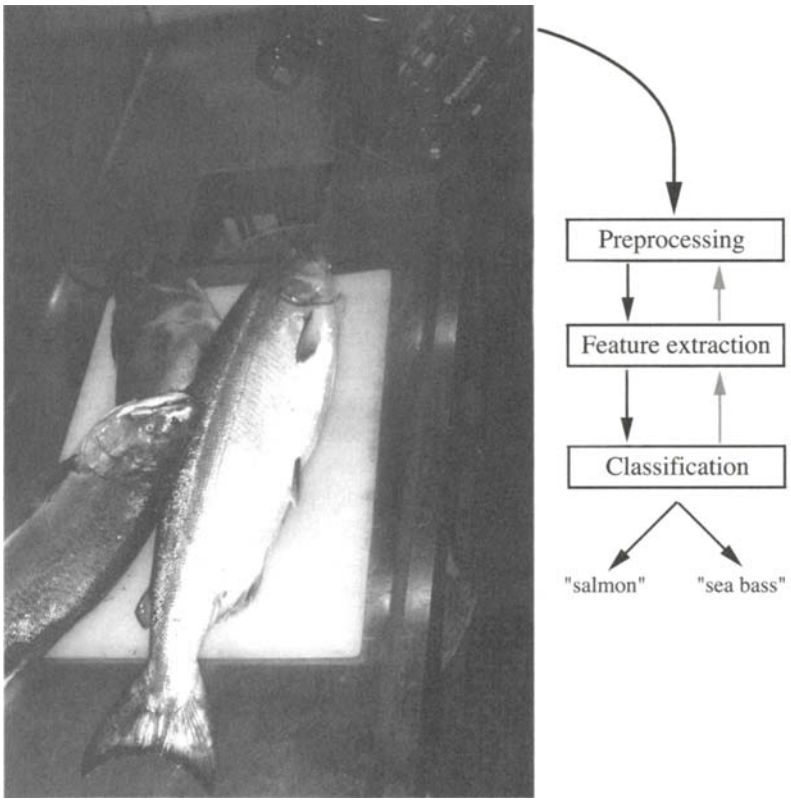

FIGURE 1.1. The objects to be classified are first sensed by a transducer (camera), whose signals are preprocessed. Next the features are extracted and finally the classification is emitted, here either "salmon" or "sea bass." Although the information flow is often chosen to be from the source to the classifier, some systems employ information flow in which earlier levels of processing can be altered based on the tentative or preliminary response in later levels (gray arrows). Yet others combine two or more stages into a unified step, such as simultaneous segmentation and feature extraction.

These features (or, more precisely, the values of these features) are then passed to a *classifier* that evaluates the evidence presented and makes a final decision as to the species.

The preprocessor might automatically adjust for average light level, or threshold the image to remove the background of the conveyor belt, and so forth. For the moment let us pass over how the images of the fish might be segmented and consider how the feature extractor and classifier might be designed. Suppose somebody at the fish plant tells us that a sea bass is generally longer than a salmon. These, then, give us our tentative *models* for the fish: Sea bass have some typical length, and this is greater than that for salmon. Then length becomes an obvious feature, and we might attempt to classify the fish merely by seeing whether or not the length l of a fish exceeds some critical value l^*. To choose l^* we could obtain some *design* or *training samples* of the different types of fish, make length measurements, and inspect the results.

TRAINING SAMPLES

Suppose that we do this and obtain the histograms shown in Fig. 1.2. These disappointing histograms bear out the statement that sea bass are somewhat longer than salmon, on average, but it is clear that this single criterion is quite poor; no matter how we choose l^*, we cannot reliably separate sea bass from salmon by length alone.

Discouraged, but undeterred by these unpromising results, we try another feature, namely the average lightness of the fish scales. Now we are very careful to eliminate variations in illumination, because they can only obscure the models and corrupt our new classifier. The resulting histograms and critical value x^*, shown in Fig. 1.3, are much more satisfactory: The classes are much better separated.

So far we have tacitly assumed that the consequences of our actions are equally costly: Deciding the fish was a sea bass when in fact it was a salmon was just as undesirable as the converse. Such a symmetry in the *cost* is often, but not invariably, the case. For instance, as a fish-packing company we may know that our customers easily accept occasional pieces of tasty salmon in their cans labeled "sea bass," but they object vigorously if a piece of sea bass appears in their cans labeled "salmon." If we want to stay in business, we should adjust our decisions to avoid antagonizing our customers, even if it means that more salmon makes its way into the cans of

COST

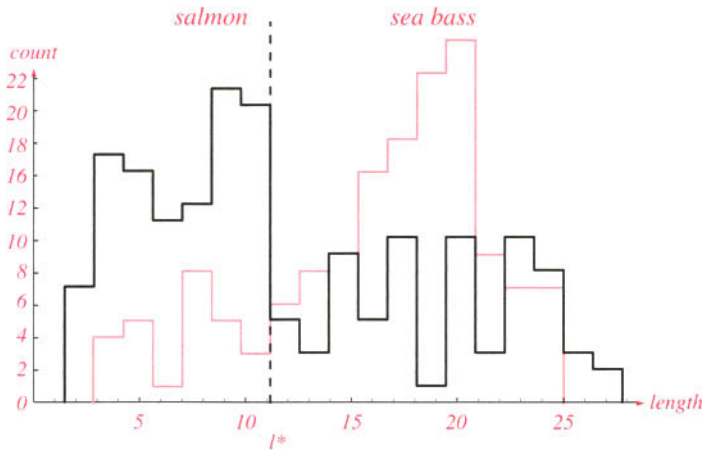

FIGURE 1.2. Histograms for the length feature for the two categories. No single threshold value of the length will serve to unambiguously discriminate between the two categories; using length alone, we will have some errors. The value marked l^* will lead to the smallest number of errors, on average.

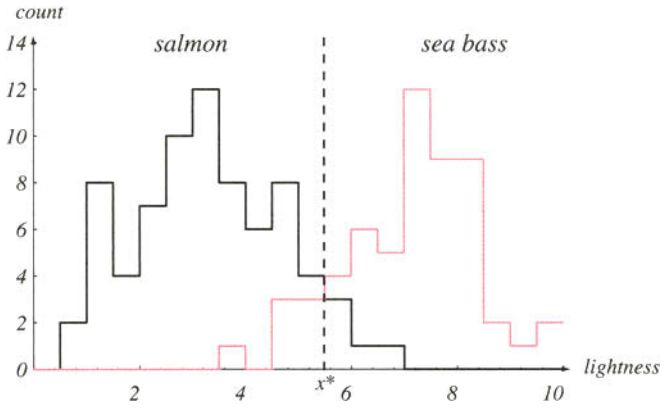

FIGURE 1.3. Histograms for the lightness feature for the two categories. No single threshold value x^* (decision boundary) will serve to unambiguously discriminate between the two categories; using lightness alone, we will have some errors. The value x^* marked will lead to the smallest number of errors, on average.

sea bass. In this case, then, we should move our decision boundary to smaller values of lightness, thereby reducing the number of sea bass that are classified as salmon (Fig. 1.3). The more our customers object to getting sea bass with their salmon (i.e., the more costly this type of error) the lower we should set the decision threshold x^* in Fig. 1.3.

Such considerations suggest that there is an overall single cost associated with our decision, and our true task is to make a decision rule (i.e., set a decision boundary) so as to minimize such a cost. This is the central task of *decision theory* of which pattern classification is perhaps the most important subfield.

DECISION THEORY

Even if we know the costs associated with our decisions and choose the optimal critical value x^*, we may be dissatisfied with the resulting performance. Our first impulse might be to seek yet a different feature on which to separate the fish. Let us assume, however, that no other single visual feature yields better performance than that based on lightness. To improve recognition, then, we must resort to the use of *more* than one feature at a time.

In our search for other features, we might try to capitalize on the observation that sea bass are typically wider than salmon. Now we have two features for classifying fish—the lightness x_1 and the width x_2. If we ignore how these features might be measured in practice, we realize that the feature extractor has thus reduced the image of each fish to a point or *feature vector* $\mathbf{x}$ in a two-dimensional *feature space*, where

$$\mathbf{x} = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}.$$

Our problem now is to partition the feature space into two regions, where for all points in one region we will call the fish a sea bass, and for all points in the other we call it a salmon. Suppose that we measure the feature vectors for our samples and obtain the scattering of points shown in Fig. 1.4. This plot suggests the following rule for separating the fish: Classify the fish as sea bass if its feature vector falls above the *decision boundary* shown, and as salmon otherwise.

DECISION BOUNDARY

This rule appears to do a good job of separating our samples and suggests that perhaps incorporating yet more features would be desirable. Besides the lightness

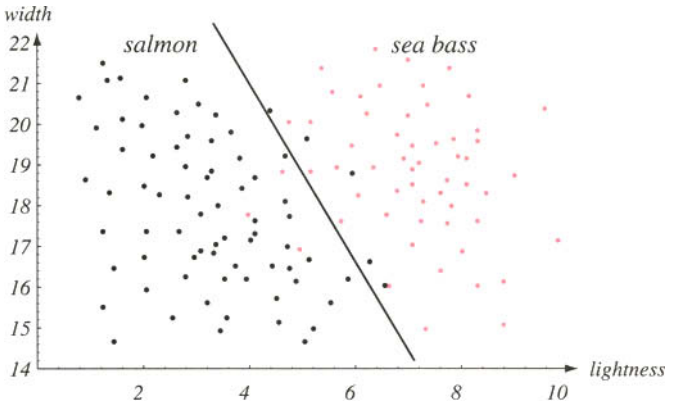

FIGURE 1.4. The two features of lightness and width for sea bass and salmon. The dark line could serve as a decision boundary of our classifier. Overall classification error on the data shown is lower than if we use only one feature as in Fig. 1.3, but there will still be some errors.

and width of the fish, we might include some shape parameter, such as the vertex angle of the dorsal fin, or the placement of the eyes (as expressed as a proportion of the mouth-to-tail distance), and so on. How do we know beforehand which of these features will work best? Some features might be redundant. For instance, if the eye color of all fish correlated perfectly with width, then classification performance need not be improved if we also include eye color as a feature. Even if the difficulty or computational cost in attaining more features is of no concern, might we ever have *too many* features—is there some "curse" for working in very high dimensions?

Suppose that other features are too expensive to measure, or provide little improvement (or possibly even degrade the performance) in the approach described above, and that we are forced to make our decision based on the two features in Fig. 1.4. If our models were extremely complicated, our classifier would have a decision boundary more complex than the simple straight line. In that case all the training patterns would be separated perfectly, as shown in Fig. 1.5. With such a "solution," though, our satisfaction would be premature because the central aim of designing a classifier is to suggest actions when presented with *novel* patterns, that is, fish not yet seen. This is the issue of *generalization*. It is unlikely that the complex decision boundary in Fig. 1.5 would provide good generalization—it seems to be "tuned" to the particular training samples, rather than some underlying characteristics or true model of all the sea bass and salmon that will have to be separated.

GENERALIZATION

Naturally, one approach would be to get more training samples for obtaining a better estimate of the true underlying characteristics, for instance the probability distributions of the categories. In some pattern recognition problems, however, the amount of such data we can obtain easily is often quite limited. Even with a vast amount of training data in a continuous feature space though, if we followed the approach in Fig. 1.5 our classifier would give a horrendously complicated decision boundary—one that would be unlikely to do well on novel patterns.

Rather, then, we might seek to "simplify" the recognizer, motivated by a belief that the underlying models will not require a decision boundary that is as complex as that in Fig. 1.5. Indeed, we might be satisfied with the slightly poorer performance on the training samples if it means that our classifier will have better performance

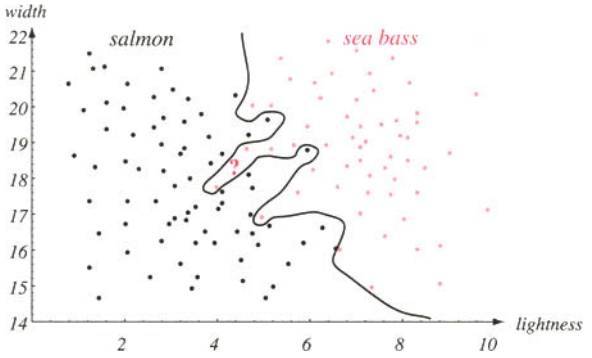

FIGURE 1.5. Overly complex models for the fish will lead to decision boundaries that are complicated. While such a decision may lead to perfect classification of our training samples, it would lead to poor performance on future patterns. The novel test point marked ? is evidently most likely a salmon, whereas the complex decision boundary shown leads it to be classified as a sea bass.

on novel patterns.* But if designing a very complex recognizer is unlikely to give good generalization, precisely how should we quantify and favor simpler classifiers? How would our system automatically determine that the simple curve in Fig. 1.6 is preferable to the manifestly simpler straight line in Fig. 1.4 or the complicated boundary in Fig. 1.5? Assuming that we somehow manage to optimize this tradeoff, can we then *predict* how well our system will generalize to new patterns? These are some of the central problems in *statistical pattern recognition*.

For the same incoming patterns, we might need to use a drastically different task or cost function, and this will lead to different actions altogether. We might, for instance, wish instead to separate the fish based on their sex—all females (of either species) from all males—if we wish to sell roe. Alternatively, we might wish to cull

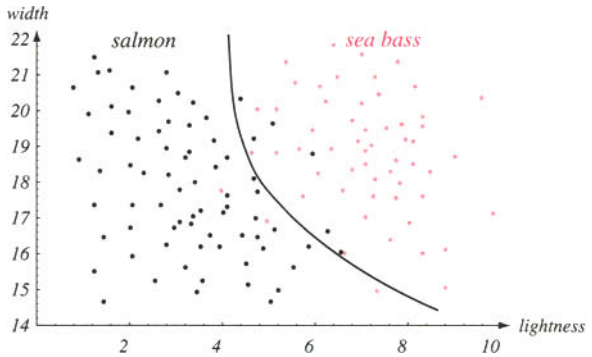

FIGURE 1.6. The decision boundary shown might represent the optimal tradeoff between performance on the training set and simplicity of classifier, thereby giving the highest accuracy on new patterns.

*The philosophical underpinnings of this approach derive from William of Occam (1284–1347?), who advocated favoring *simpler* explanations over those that are needlessly complicated: *Entia non sunt multiplicanda praeter necessitatem* ("Entities are not to be multiplied without necessity"). Decisions based on overly complex models often lead to lower accuracy of the classifier.