Robust Control of Time-delay Systems

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With 79 Figures

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This book is dedicated to
Shuhong, Lilly and Lisa.

Preface

Systems with delays frequently appear in engineering. Typical examples of time-delay systems are communication networks, chemical processes, teleoperation systems, biosystems, underwater vehicles and so on. The presence of delays makes system analysis and control design much more complicated. During the last decade, we have witnessed significant development in the robust control of time-delay systems. The aim of this book is to present a systematic and comprehensive treatment of robust (H^∞) control of such systems in the frequency domain. The emphasis is on systems with a single input/output delay, although the delay-free part of the plant can be multi-input–multi-output (MIMO), when the delays in different channels are the same.

This book collects work carried out recently by the author in this field. It covers the whole range of robust (H^∞) control of time-delay systems: from controller parameterisation, controller design to controller implementation; from the Nehari problem, the one-block problem to the four-block problem; from theoretical developments to practical issues. The major tools used in this book are similarity transformations, chain-scattering approach and J-spectral factorisations. The main idea is to "*make everything as simple as possible, but not simpler (Albert Einstein).*" This book is self-contained and should be of interest to final-year undergraduates, graduates, engineers, researchers, and mathematicians who work in the area of control and time-delay systems.

The book is divided into two parts: Controller Design (Chapters 2–10) and Controller Implementation (Chapters 11–13). The classical control of time-delay systems is summarised in Chapter 2 and then some mathematical preliminaries are collected in Chapter 3. The J-spectral factorisation of regular para-Hermitian transfer functions is developed in Chapter 4 to prepare for the solution of the Nehari problem discussed in Chapter 5. An extended Nehari problem is solved in Chapter 6 to prepare for the solutions of the one-block problem and the standard H^∞ control problem discussed in Chapter 7, where the chain-scattering approach is applied to reduce the standard H^∞ control problem to a delay-free problem and a one-block problem. The latter is then further reduced to an extended Nehari problem. With the solution to the extended Nehari problem obtained in Chapter 6, the controllers for the one-block problem and the standard H^∞ problem are recovered. A transformed standard H^∞ problem is discussed in Chapter 8 to obtain a simpler but more conservative solution. The parameterisation of all stabilising controllers for time-delay systems are discussed in Chapter 9. All the controllers for the above problems have the same structure: incorporating a modified Smith predictor (MSP). A practical issue, a numerical problem with the MSP, is discussed in Chapter 10 and a unified Smith predictor is proposed to overcome this, followed by revisiting some well-studied problems. Another practical issue, the implementation of MSP, is tackled in Part II. The implementation of MSP, *i.e.*, a distributed delay, is not trivial because of the inherent hidden unstable poles. In Chapter 11, this is done by using discrete delays in the z-domain and in the s-domain. In Chapter 12, this is done by using rational transfer functions based on the δ-operator and then in Chapter 13 a faster converging rational implementation is discussed using bilinear transformations.

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http://come.to/zhongqc

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Notation and Abbreviations

$\mathbb{Z}$, $\mathbb{R}$ and $\mathbb{C}$	fields of integral, real and complex numbers
$j\mathbb{R}$	imaginary axis
$\operatorname{Re} s$ and $\operatorname{Im} s$	real and imaginary parts of $s \in \mathbb{C}$
$\in$	belong to
$\cap$	intersection
$\subset$	subset

I_n	$n \times n$ identity matrix (n is often omitted when not confusing)
$J_{p,q}$, J_γ and J	shorthand for $\begin{bmatrix} I_p & 0 \\ 0 & -I_q \end{bmatrix}$, $\begin{bmatrix} I & 0 \\ 0 & -\gamma^2 I \end{bmatrix}$ and $\begin{bmatrix} I & 0 \\ 0 & -I \end{bmatrix}$
A^T and A^*	transpose and complex conjugate transpose of A
A^{-1} and A^{-*}	inverse of A and shorthand for $(A^{-1})^*$
$\det(A)$ and $\rho(A)$	determinant and spectral radius of A
$\operatorname{Im} A$	image of A
$\left[\begin{array}{c:c} A & B \\ \hdashline C & D \end{array}\right]$	explicitly partitioned matrix of $\begin{bmatrix} A & B \\ C & D \end{bmatrix}$
$G(s) = \left[\begin{array}{c\|c} A & B \\ \hline C & D \end{array}\right]$	shorthand for $G(s) = D + C(sI - A)^{-1}B$
$G^\sim(s)$	shorthand for $G^T(-s) = [G(-s^*)]^* = \left[\begin{array}{c\|c} -A^* & -C^* \\ \hline B^* & D^* \end{array}\right]$
$\mathcal{F}_l(M, Q)$ and $\mathcal{F}_u(M, Q)$	lower and upper linear fractional transformations (LFT)
$\mathcal{H}_r(M, Q)$ and $\mathcal{H}_l(M, Q)$	right and left homographic transformations (HMT)
$\mathcal{C}_r(M)$ and $\mathcal{C}_l(M)$	right and left chain-scattering transformations (CST)

$\pi_h(G)$	completion operator
$\tau_h(G)$	truncation operator

ARE	algebraic Riccati equation
w.r.t.	with respect to
iff	if and only if
IOR	input–output representation
CSR	chain-scattering representation
LFT	linear fractional transformation
HMT	homographic transformation
SP	(classical) Smith predictor
MSP	modified Smith predictor
USP	unified Smith predictor
FSA	finite-spectrum assignment
FIR	finite impulse response
SP_{h}	standard H^∞ problem with a single delay h
SP_0	the conventional standard H^∞ problem without a delay ($h = 0$)
OP_{h}	one-block problem with a delay h
$\mathrm{ENP}_{\mathrm{h}}$	extended Nehari problem with a delay h
NP_{h}	delay type Nehari problem
SISO	single-input single-output
MIMO	multiple-input multiple-output
ZOH	zero-order hold

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Introduction

Systems with delays abound in the world. One reason is that nature is full of transparent delays. Another reason is that time-delay systems are often used to model a large class of engineering systems, where propagation and transmission of information or material are involved. The presence of delays (especially, long delays) makes system analysis and control design much more complex. In this chapter, some examples of time-delay systems are discussed and then a brief review on the control of time-delay systems, followed by an overview of this book, is given.

1.1 What Is a Delay?

Time delay is the property of a physical system by which the response to an applied force (action) is delayed in its effect [124]. Whenever material, information or energy is physically transmitted from one place to another, there is a delay associated with the transmission. The value of the delay is determined by the distance and the transmission speed. Some delays are short, some are very long. The presence of long delays makes system analysis and control design much more complex. What is worse is that some delays are too long to perceive and the system is misperceived as one without delays.

Time delays abound in the world. They appear in various systems such as biological, ecological, economic, social, engineering systems etc. For example, over-exposure to radiation increases the risk of cancer, but the onset of cancer typically follows exposure to radiation by many years. In economics, the central bank in a country often attempts to influence the economy by adjusting interest rates; the effect of a change in interest rates takes months to be translated into an impact on the economy. In politics, politicians need some time to make decisions and they will have to wait for some time before they find out if the decisions are correct or not. When reversing a car around a corner, the driver has to wait for the steering to take effect. In engineering, on which this book focuses, there are a lot of systems with delays; see the next section for examples.

A general tendency in responding to some errors in a system is to react immediately to the errors and to react more if the errors are not lessened or eliminated in time as expected. However, for a system with time delays, only after the inherent delays will the errors start to change. Hence, it is very important to properly understand the existence of delays and not to over-react. Otherwise, the system is very likely to overshoot or even become unstable. When dealing with time-delay systems, "patience is a virtue."

For a given delay element with a delay $h \geq 0$, the output $y(t)$ corresponding to the input $u(t)$ is

$$y(t) = u(t-h).$$

Hence, the transfer function of a delay element is given by e^{-sh}.

1.2 Examples of Time-delay Systems

Some typical examples of time-delay systems in engineering are discussed here.

1.2.1 Shower

A simple example of a time-delay system from everyday life is the shower,[1] as depicted in Figure 1.1. Most people have experienced the difficulty in adjusting the water temperature: it gets too cold or too warm. The actual temperature often overshoots the desired and, sometimes, it takes a while to get the temperature right. This is because it takes time for the increased (or decreased) hot/cold water to flow from the tap to the shower head (or the human body). This time is a delay, which depends on the water pressure and the length of the pipe. The change of the faucet position is almost immediate, however, the change of the water temperature has to wait until the delay has elapsed. If the faucet position is constantly adjusted according to the currently perceived temperature, then it is very likely that the temperature will fluctuate.

Assume that the water is an incompressible fluid and stationary flow. According to the Poiseuille law, the flow rate of water is

$$F = \frac{\pi R^4}{8\mu l} \Delta p,$$

where $\mu = 0.01$ is the kinematic viscosity of water, R is the radius of the pipe, l is the length of the pipe and Δp is the pressure difference between the two ends of the pipe. The time delay h can then be found as

$$h = \frac{\pi R^2 l}{F} = \frac{8\mu}{\Delta p} \left(\frac{l}{R} \right)^2 .$$

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[1] A shower model written in SIMULINK® is available at http://www.aut.bme.hu/education/contheor/contheor/theory/model/shower/.

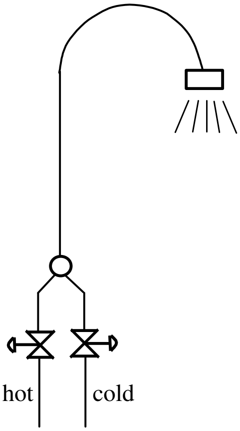


Figure 1.1. Sketch of a shower system

1.2.2 Chemical Processes

It is well known that many chemical processes contain delays. This has been well documented in the literature and there is no need to give any detailed example here. The following first-order plus dead time (FOPDT) is widely used to model chemical processes:

$$G(s) = \frac{K}{Ts+1}e^{-sh},$$

where K is the static gain of the plant, $T > 0$ is the time constant and h is the transparent delay or dead time.

1.2.3 Communication Networks

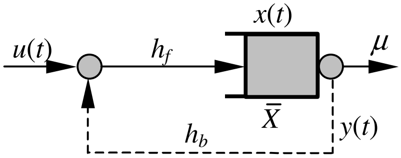
In recent years, communication networks [143] have been among the fastest-growing areas in engineering and there has been increasing interest in controlling systems over communication networks. Thanks to high-speed networks, control-over-Internet is now available [69, 122]. These systems are frequently modelled from the control point of view as time-delay systems because of the inherent propagation delays [52, 71]. These delays are crucial to the system stability and the quality-of-service (QoS).

A single connection between a source controlled by an access regulator and a distant destination node served with a constant transmission capacity μ is given as an example here. This can be described by the fluid model [52, 71] shown in Figure 1.2(a). At the source node, the access regulator controls the input rate $u(t)$, according to the congestion status of the destination node.

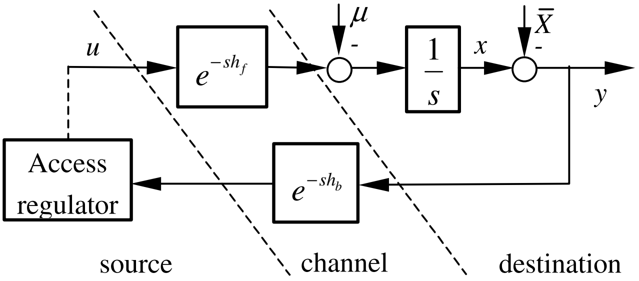
The congestion status $y(t)$ is defined as the difference between the current buffer contents $x(t)$ and the target value $\bar{X}$, *i.e.*,

$$y(t) = x(t) - \bar{X}.$$

Due to the propagation delay from the destination node to the source node, called the backward delay h_b, this status arrives at the source node (the access regulator) only after this delay period. There is also a forward delay h_f for the package to arrive at the destination node from the source node. The arrived packages are stored/accumulated in the buffer and then sent with a constant transmission capacity μ. The control objective is to adapt $u(t)$ to μ dynamically while maintaining the buffer $x(t)$ at an acceptable level. The block diagram is shown in Figure 1.2(b).



(a) fluid model



(b) block diagram

Figure 1.2. Communication networks: A single connection

The communication networks in reality, which are built from single connections, are much more complicated. The delays are often time-varying and stochastic. The information transmitted via communication networks is quantised and there exist package losses as well.

1.2.4 Underwater Vehicles

Recently, there have been more and more applications of underwater vehicles. They can be used for exploring ocean bottoms, installation/inspection/repair tasks and, of course, military missions. They have advantages over human divers in that they can descend to greater depths, can stay there for greater lengths of time and require less support equipment. Thus they can reach places divers cannot, and they can be less expensive to operate [64].

There are different types of underwater vehicles. One is the *Remotely Operated Vehicle* (ROV). An ROV is connected to a surface support ship via an umbilical cable, which provides power supply and a communication link, and hence the range of operation is somewhat limited. Another one is the *autonomous underwater vehicle* (AUV), which carries an on-board power unit and is equipped with advanced control capabilities to undertake tasks with the minimum of human intervention. The communication is carried out through an acoustic link. The MIT underwater vehicle Odyssey II Xanthos, shown in Figure 1.3, is taken as an example. Odyssey II Xanthos is a video survey AUV, equipped with various sensors including scanning and homing sonars, depth sensor, temperature salinity and related sensors, video, inertial sensors, acoustic modem and acoustic navigation tracking pingers. The rated operating depth is 3,000 m. More details can be found at http://auvlab.mit.edu.

Figure 1.3. The MIT underwater vehicle: Odyssey II, Xanthos

(Courtesy of C. Chryssostomidis and R. Damus, MIT Sea Grant AUV Lab, http://auvlab.mit.edu)

The control problems involved in these vehicles include navigation, task planning and the low-level autopilot. Due to the long cable or distance, there exists a long delay in the system. For AUVs, the delay is caused by the finite sound speed in water, nominally, $1,500$ m/s.

A physical system [140] is shown in Figure 1.4, where a surface ship is shown positioning an underwater vehicle through a long cable. The vehicle may be searching the ocean floor or mapping the topography of the bottom, or it may be the platform for a smaller vehicle equipped with thrusters. An

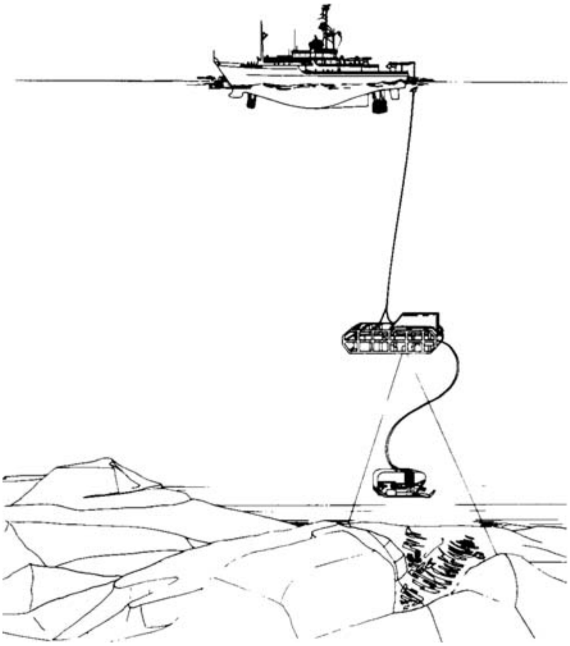


Figure 1.4. Physical underwater vehicle system

(Reprinted, with permission, from [140]. ©IEEE)

approximate model, which was validated for a cable of 2,500 m and a vehicle weighing 17,000 N in air, was given in [140] as

$$G(s) = \frac{ce^{-sh}}{as^2 + bs + c},$$

with $a = 1$, $b = 1.1 \times 10^{-4}$, $c = 2.58 \times 10^{-2}$ and $h = 40$ s. More details about this system can be found in [140] and the references therein.

1.2.5 Combustion Systems

Continuous combustion systems are widely used in power generation, heating and propulsion. Examples include domestic and industrial burners, steam and gas turbines, waste incinerators, and jet and ramjet engines. These systems are intricate and include a wide variety of dynamic behaviour. Pressure oscillations are considered the most significant in terms of the impact on system performance; much effort has been devoted to this [3, 24, 103].

There are two major dynamics in a combustion system: flame dynamics and acoustic wave dynamics. They are coupled to form a loop as shown in

Figure 1.5. Due to wave propagation, there is a delay in the wave dynamics. This often causes combustion instability [3, 24, 103]. Delays also appear in the measurement and the actuator units of the system. Detailed modelling of combustion systems can be found in [3, 24] and are omitted here.

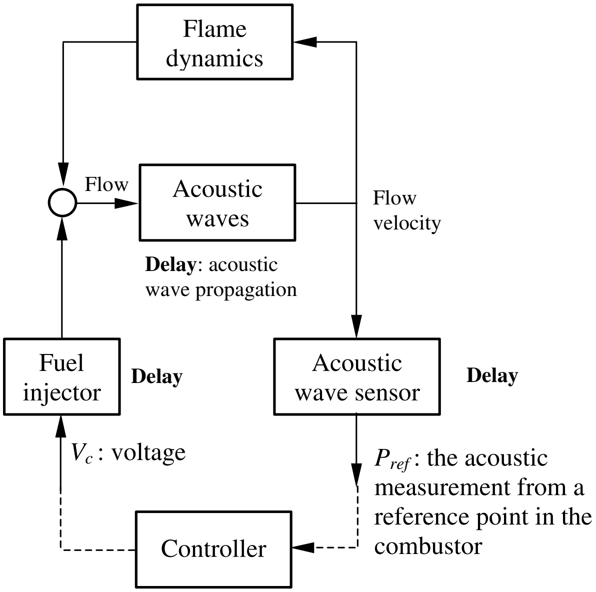


Figure 1.5. Dynamics in a combustion system

1.2.6 Exhaust Gas Recirculation (EGR) Systems

Oxides of nitrogen (NO_x) are a group of highly reactive gases that contain varying amounts of NO and NO_2. These are key elements of greenhouse gases and harmful to human health and the environment. Motor vehicles are one of the key sources of NO_x. Exhaust gas recirculation (EGR) systems were introduced in the early 1970s to reduce NO_x emissions.

The EGR valve recirculates exhaust into the intake stream. Exhaust gases have already combusted, so they do not burn again when they are recirculated. These gases displace some of the normal intake charge. This chemically slows and cools the combustion process by several hundred degrees, thus reducing NO_x formation.[2]

It is a challenge to precisely control the flow of recirculated exhaust so that the system provides good performance and economy. Too much flow will retard

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[2] http://www.asashop.org/autoinc/nov97/gas.htm.

engine performance and cause a hesitation on acceleration, and too little flow will increase NO_x and cause engine ping. Some EGR systems simply operate in open loop due to nonlinearity, engine vibrations, pressure fluctuations and high-order unmodelled dynamics [109]. The measurement, *e.g.*, of oxygen in the exhaust [1] for feedback, often introduces delay, which complicates the control design. On the other hand, it is very difficult to derive a mathematical model for the system. The model is often obtained via system identification. The following model was given in [109]:

$$G_i(z) = k(z - p)z^{-d} \qquad (i = 1, 2, 3, 4)$$

at four operating points dependent on load, speed and the desired EGR rate for an EGR system. With a sampling period of 10 ms, the vectors of the parameters are:

$$\begin{aligned} \text{gain } k &= (88,\ 110,\ 180,\ 220), \\ \text{delay } d &= (5,\ 15,\ 13,\ 6), \\ \text{pole } p &= (0.9,\ 0.9,\ 0.9,\ 0.9). \end{aligned}$$

The model changes dramatically at different operating points. The delay varies from 50 ms to 150 ms and the gain varies from 88 to 220.

1.2.7 Biosystems

This subsection is based on [10, 16], in which more examples and references can be found.

Time delay has been introduced to model biosystems to produce better consistency with nature and predictive results. The ultimate objective is to further understand the systems and then to control them. The biosystems studied in this field cover population dynamics, epidemiology, physiology, immunology, neural networks and cell kinetics.

The following delay model for population dynamics was introduced in [48], modifying the classical Verhulst model to account for hatching and maturation periods:

$$y'(t) = ry(t)\left(1 - \frac{y(t-\tau)}{K}\right).$$

Here, the nonnegative parameters r and K are, respectively, the intrinsic growth rate and the environmental carrying capacity. This simple model can explain the observed oscillatory behaviour in a single species population, without any predatory interaction of other species.

Another simple example is the model of growth in cell populations, which is given by

$$y'(t) = \alpha y(t) + \beta y(t-\tau).$$

The equilibrium solution $y(t) = 0$ becomes unstable when the value of the delay exceeds the following bound: